Demographic Change and Intergenerational Wealth Transmission

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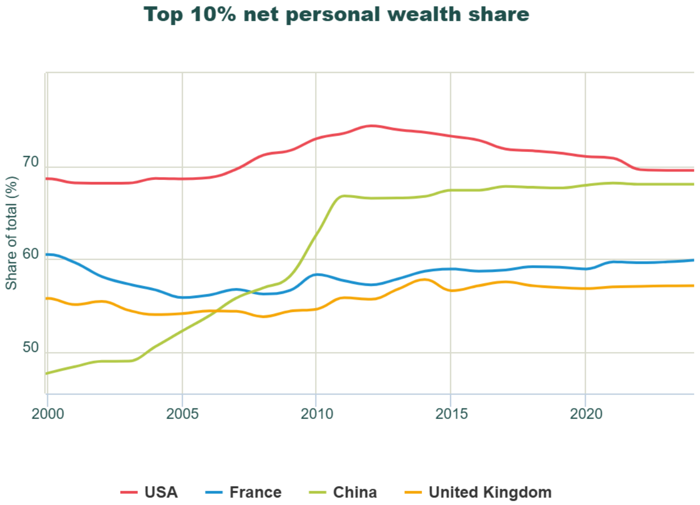
Outline

Why a concern with wealth?

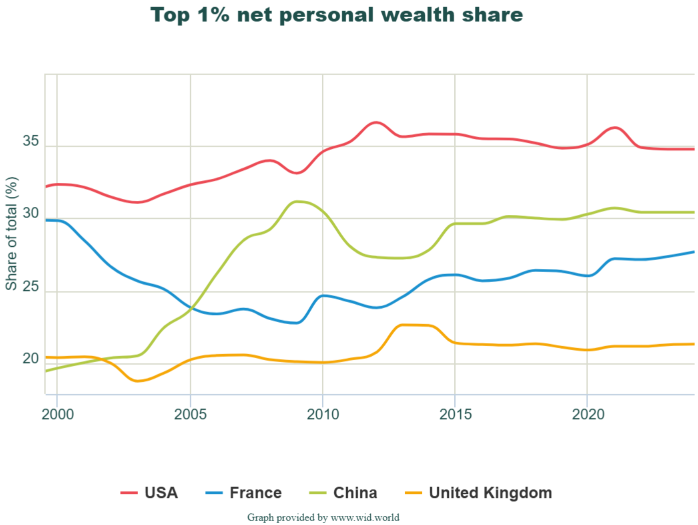
- Important component of individual wellbeing
  - housing ownership
  - security in old age
- Core of political economy questions
  - wealth and power
  - the focus on the top 1% (Alvaredo et al. 2013 , Mankiw 2013)
- Key to long-run inequality
  - asset ownership at heart of models
- Changing inequality patterns over recent years

Wealth share of the rich



Graph provided by www.wid.world

Wealth share of the very rich

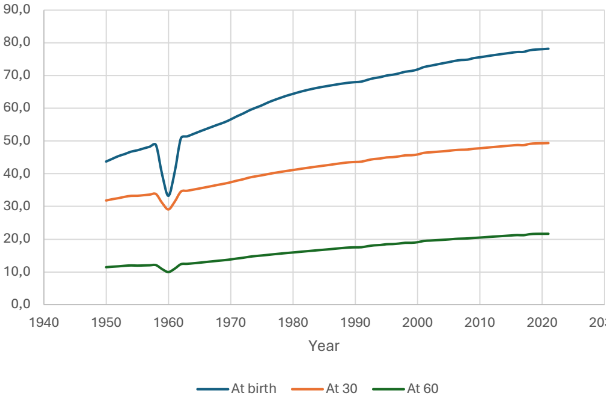


Source: World Inequality Report 2018, 4.2.1, http://wir2018.wid.world

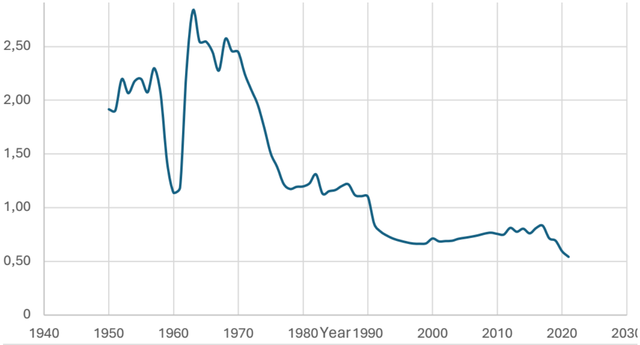
Equilibrium distribution in practice?

- Long-term evidence suggests periods of equilibrium
  - With abrupt changes from world events (Piketty and Zucman 2015)
- Effect of shocks?
  - across the board: recessions, booms
  - distributional: income, wealth inequality
- Shocks from policy?
  - equilibrium may still be relevant
  - give picture of the long run

China: life expectancy



China: net reproduction rate



See Wang et al. (2016), Zhang (2017)

Main theme

- Much of the literature focuses on the effect of market forces:
  - upper tail – role of financial asset prices
  - other key assets such as houses
  - lower tail – extent to which poor are credit constrained.
- Focus on non-market forces underlying distribution of wealth
  - Forces dividing wealth: gifts, bequests from parents to children
  - Forces uniting wealth: marriage
- Also consider the effect of outside intervention

Literature: approaches to family factors

- Literature: assumptions about family composition?
  - all families have two children (Atkinson 1980,Blinder 1973,1976)
  - reproduction is asexual and each individual has the same number of children (Stiglitz 2015)
- Literature: equilibrium analysis?
  - assume equilibrium distribution (Banerjee and Newman 1991,Galor and Zeira 1993,Laitner 1979)
  - a characteristic of the equilibrium distribution like its variance (Atkinson 1980)
  - simulate over limited number of generations (Blinder 1976)
- The approach here:
  - families are heterogeneous in size
  - establish existence and characteristics of equilibrium distribution

Time and families

- **Time**
  - Periods indexed by $t = ..., 1, 2, ...$
  - People can live for 3 ages (1 young, 2 middle age, 3 old age)
  - Children in period $t-1$ become adults in period t
  - Adults live for 1 or 2 ages; survive to old age with probability π_2
- **Adults**
  - Each family has two adults who take decisions jointly
  - Pool their wealth
  - Have at least one child, but no more than K
- **Children**
  - Proportion of families with k children: p_k, $\sum_{k=1}^{K} p_k = 1$
  - Population stationarity, $\sum_{k=1}^{K} kp_k = 2$
  - Non-degenerateness: $p_k < 1, k = 1, 2, \ldots K$

Maximisation problem: 1

- Choice variables:
  - B_t : bequests
  - C_t : consumption in period 2
  - C_t' : consumption in period 3
  - E_t : earnings ($\frac{\bar{E}-E_t}{\bar{E}}$ is leisure)
- Assume annuity A_t purchased in period 2
  - perfect insurance against uncertain length of life
  - means survival into period 3
  - fair annuity: $A_t = \pi_2 C_t'$
- Wealth acquired per adult given by $W_t = E_t + I_t$
  - $I_t \geq 0$: Inheritance received
- Budget constraint:
  - $C_t + \frac{A_t+B_t}{1+r} \leq W_t$
  - $C_t + \frac{\pi_2 C_t'}{1+r} + \frac{B_t}{1+r} \leq W_t$
  - r: per-period growth rate of wealth

Timeline

		$t-1$		t		$t+1$		$time$
Age	0	Young	30	Midle age	60	Old	90	
Period of life		1		2		3		
Surv prob	1		π_1		π_2		0	
Inheritance			I_t					
Decisons				C_t, E_t, B_t, A_t				

Maximisation problem: 2

- Utility function: $$\gamma \ln\left(B_t + \bar{B}\right) + [1-\gamma]\left[\ln\left(C_t - \overline{C}\right) + \delta\pi_2 \ln\left(C_t' - \overline{C}\right)\right] + \nu \ln\left(\frac{\bar{E}-E_t}{\bar{E}}\right)$$
- Parameters
  - γ: relative weight put on bequests rather than own consumption
  - δ: relative weight put on future consumption relative to present
  - ν: weight put on leisure.
  - $\bar{B} \geq 0$ captures the potential base aversion to altruism
  - $\bar{C} \geq 0$: precommitted consumption in each period
  - $\bar{E} > 0$: maximum possible earnings during middle age
- Problem is maximise utility subject to...
  - budget constraint: $C_t + \frac{\pi_2 C_t'}{1+r} + \frac{B_t}{1+r} \leq W_t$
  - constraints on variables: $B_t, C_t, C_t' \geq 0$; $0 \leq E_t \leq \bar{E}$

Solution

- The solution has two cases, determined by the size of inheritance
- Critical inheritance value, $\hat{I} := \frac{\xi}{\nu}\bar{E} - \frac{\overline{B}}{1+r} + \left[1 + \frac{\pi_2}{1+r}\right]\overline{C}$
  - where $\xi := 1 + [1-\gamma]\,\delta\pi_2$

Case 1: $I_t \geq \hat{I}$. For high inheritance $E_t = 0$

Case 2: $I_t < \hat{I}$. For low inheritance $E_t > 0$

- Examine detailed solution in the two cases...

Case 1 (high-inheritance) solution

- $E_t = 0$
- In general

$$\begin{aligned} C_t &= \frac{1-\gamma}{\xi}\left[I_t + \frac{\bar{B}}{1+r}\right] + \left[1 - \frac{1-\gamma}{\xi}\left[1 + \frac{\pi_2}{1+r}\right]\right]\bar{C} \\ B_t &= \max\left\{\frac{[1+r]\,\gamma}{\xi}I_t - \frac{[\xi-\gamma]}{\xi}\bar{B} - \frac{\gamma}{\xi}[1+r+\pi_2]\,\bar{C}, 0\right\} \end{aligned}$$

- If no-one survives to the third age

$$\begin{aligned} C_t &= [1-\gamma]\left[I_t + \frac{\bar{B}}{1+r}\right] + \gamma\bar{C} \\ B_t &= \max\left\{[1+r]\,\gamma I_t - [1-\gamma]\,\bar{B} - \gamma[1+r]\,\bar{C}, 0\right\}. \end{aligned}$$

Case 2 (low-inheritance) solution

- In general

$$
\begin{aligned}
E_t &= \frac{\xi \bar{E} - \nu \left[I_t + \frac{\bar{B}}{1+r} - \left[1 + \frac{\pi_2}{1+r}\right] \bar{C} \right]}{\xi + \nu}, \\
C_t &= \frac{1-\gamma}{\xi+\nu} \left[I_t + \bar{E} + \frac{\bar{B}}{1+r} \right] + \left[1 - \frac{1-\gamma}{\xi+\nu} \left[1 + \frac{\pi_2}{1+r} \right] \right] \bar{C}, \\
B_t &= \max \left\{ \frac{[1+r]\gamma}{\xi+\nu} [I_t + \bar{E}] - \frac{[1-\gamma][1+\delta\pi_2] + \nu}{\xi+\nu} \bar{B} \right. \\
&\quad \left. - \frac{\gamma}{\xi+\nu} [1 + r + \pi_2] \bar{C}, 0 \right\}
\end{aligned}
$$

Comparative statics of individual

- Demographic changes affect decisions in two ways.
  - children from larger families get a lower inheritance
  - longevity is associated with an increased π_2
- Inheritance effect
  - $\frac{\partial C_t}{\partial I_t} > 0, \frac{\partial B_t}{\partial I_t} > 0$
  - (in case 2) $\frac{\partial E_t}{\partial I_t} < 0$
- Longevity effect
  - $\frac{\partial C_t}{\partial \pi_2} < 0, \frac{\partial B_t}{\partial \pi_2} < 0, \frac{\partial \hat{I}}{\partial \pi_2} > 0$
  - (in case 2) $\frac{\partial E_t}{\partial \pi_2} > 0$

Simple inheritance mechanics

- Child will be a worker iff $I_{t+1} < \hat{I}$,
- From this get a critical value of wealth $\hat{W}$:
  - condition for a low inheritance is $W_t < \frac{k}{2\beta}\hat{W}$
  - where $\beta := \gamma[1+r]$
- Child's wealth is:
  - (Case 1) $W_{t+1} = I_{t+1} = \frac{2}{k}B_t$
  - (Case 2) $W_{t+1} = \frac{2}{k}B_t + E_t$
- Use this with the equation for I_{t+1} to get a fundamental mapping

Wealth dynamics: two groups

- For each k-family a parent-to-child wealth mapping $W_{t+1} = g_k(W_t)$
- Two convenient constants
  - $\xi = 1 + [1-\gamma]\,\delta\pi_2$
  - $\hat{W}_0 := \frac{1-\gamma}{\gamma}\frac{1+\delta\pi_2}{1+r}\bar{B} + \frac{1+r+\pi_2}{1+r}\bar{C}$
- The general form of g_k for the two inheritance cases:

  1 high inheritance $W_t \geq \frac{k}{2\beta}\hat{W}$:

  $$g_k(W_t) = \frac{2\beta}{k}\left[\frac{W_t - \hat{W}_0}{\xi}\right]$$

  2 low inheritance $W_t < \frac{k}{2\beta}\hat{W}$:

  $$g_k(W_t) = \frac{\nu\hat{I}}{\xi+\nu} + \frac{2\beta}{k}\max\left\{\frac{W_t - \hat{W}_0}{\xi+\nu}, 0\right\}$$

- Each g_k:
  - is piecewise linear in W_t
  - has two kink points

Cut-down version with $\pi_2 = \bar{B} = \bar{C} = 0$

- A restricted model: only two periods (Cowell and Van de gaer 2025)
- Let p_k be prop of families with k children
  - family structure $\mathbf{p} = \{p_1, ..., p_K\}$ defines a Markov process
  - will there be an equilibrium of the process?
  - if so, what will it look like?

Theorem: for all $\mathbf{p}$ satisfying population stationarity and non-degenerateness: (1) a globally stable equilibrium exists if and only if $0 \leq \beta \leq 1$. (2) in equilibrium, there is a non-zero lower bound on wealth

- Two scenarios in equilibrium:
  1. if $0 \leq \beta \leq 1/2$: a finite upper bound to wealth; everybody works
  2. if $1/2 < \beta \leq 1$: no finite upper bound: some rentiers are present

The process g_k and equilibrium (cut-down version)

- Focus on scenario 2, where there are people who do not work
  - simplifies the analysis
  - gives us a strikingly clear result
- What happens to top end of the wealth distribution?
  - the rentier (idle rich) part
  - mechanics are given by $W_{t+1} = g_k(W_t) = \frac{2\beta}{k} W_t$
- Equilibrium requires $F_*(W) = \sum_{k=1}^{K} \frac{kp_k}{2} F_*\left(\frac{k}{2\beta} W\right)$
  - focus on the interval $\mathbb{W}_1 := \left[\frac{K\hat{W}}{2\beta}, \infty\right)$

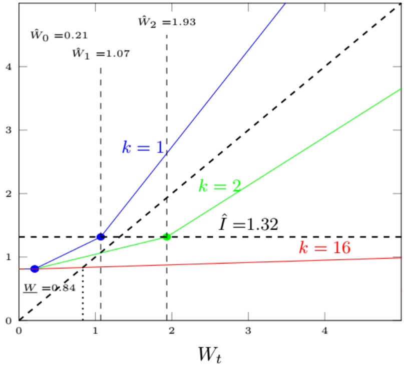
Equilibrium distribution (cut-down version)

- Focusing on $\mathbb{W}_1$ gives clear result on shape of the distribution

Theorem: for all $\mathbf{p}$ satisfying population stationarity, non-degenerateness and for $1/2 < \beta \leq 1$, over the support $\mathbb{W}_1$ the equilibrium distribution must satisfy $F_*(W) = 1 - AW^{-\alpha}$ where A is a constant and α is a root of the equation $\beta^{-\alpha} = \sum_{k=1}^{K} p_k \left[\frac{k}{2}\right]^{1-\alpha}$

- Interpretation
  - in equilibrium we have a Pareto distribution!
  - the higher is α, the lower is inequality
- What drives inequality?
  - the family structure $\mathbf{p} = \{p_1, ..., p_K\}$
  - in particular p_1, the proportion of "little emperors"

The process g_k (general case)

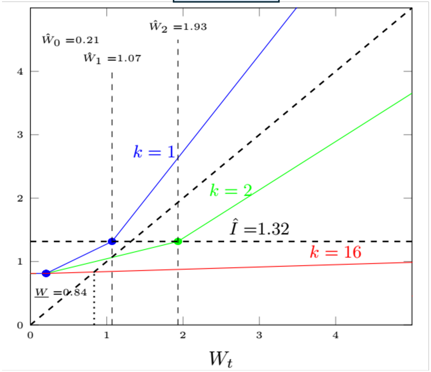


From the g_k diagram

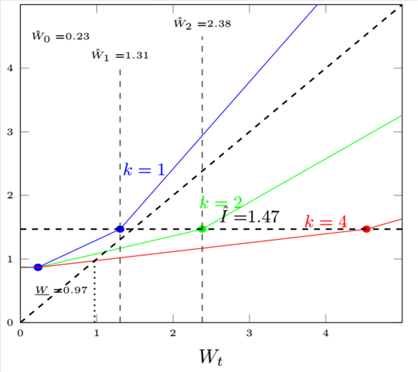
- If slope of the $g_1(W_t)$-line is above 1, no upper bound on wealth
  - all W can be reached through a succession of one-child families
- Lower bound $\underline{W}$ at intersection of $g_K(W_t)$ and the 45-deg line
  $\frac{1}{\xi+\nu-\frac{2[1+r]\gamma}{K}}\xi\bar{E}$
  $-\frac{1}{\xi+\nu\frac{2[1+r]\gamma}{K}}\left[\frac{\nu\bar{B}}{1+r}+2[\xi-\gamma]\frac{\bar{B}}{K}+2\gamma[1+r+\pi_2]\frac{\bar{C}}{K}-\nu\left[1+\frac{\pi_2}{1+r}\right]\bar{C}\right]$
  - where $\xi=1+[1-\gamma]\delta\pi_2$
- If the probability π_2 increases
  1. kink point of every $g_k(\cdot)$-function moves to the northeast
  2. slope of the rentier branch of the $g_k(\cdot)$-function decreases
  3. intercept of the worker branch of the $g_k(\cdot)$-function increases.

The process g_k after increase in π_2

Before



After



Simulation: method and parameters

- Start model from an arbitrary initial distribution of wealth for 100,000 households
    - simulate the behaviour of the following generations

(a) Basic parameters

γ	0.39	δ	1
β	0.95	ν	2
$\bar{E}$	2		

(b) Implied parameters

$\hat{I}$	1.242	$\hat{W}$	1.542

- Take as benchmark Chinese data before and during the One Child Policy

Simulation: China data

(a) Survival probability after period 2

pre-OCP		OCP	
π_{2b}	0.396	π_{2a}	0.631

(b) Distribution of the number of children per woman

pre-OCP			OCP		
# Children	p_{bi}	Cum Freq	# Children	p_{ai}	Cum Freq
0	0	0	0	0	0
1	0.04800	0.0480	1	0.4664	0.4664
2	0.10500	0.1530	2	0.4198	0.8862
3	0.17700	0.3300	3	0.0928	0.9790
4	0.21199	0.5420	4	0.0161	0.9951
5	0.18785	0.7299	5	0.0037	0.9988
6	0.13086	0.8607	6	0.0011	0.9999
7	0.07528	0.9360	7	0.0001	1
8	0.03755	0.9735			
9	0.01648	0.9900			
10	0.00649	0.9965			
11	0.00231	0.9988			
12	0.00079	0.9996			
13	0.00025	0.9999			
14	0.00008	0.9999			
15	0.00003	1			
16	0.00001	1			

Simulation: four scenarios

- a: (OCP): distribution #children $\mathbf{p}_a$, survival prob π_{2a}
- b: (pre-OCP): distribution of #children $\mathbf{p}_b$, survival prob π_{2b}
- c: counterfactual where only distribution #children changes
- d: counterfactual where only survival prob changes

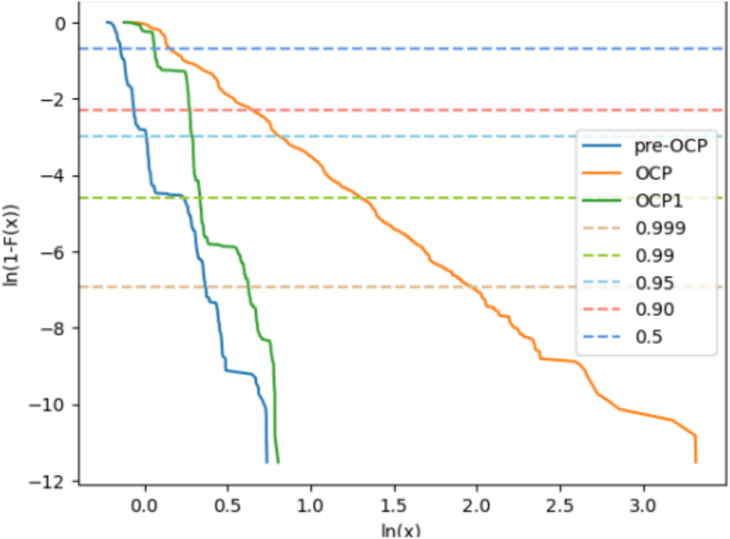
	$\mathbf{p}_a$	$\mathbf{p}_b$
π_{2a}	Scenario a	Scenario d
π_{2b}	Scenario c	Scenario b

Effects of demographic changes

	pre-OCP Scenario b	OCP1	OCP Scenario a
average W	0.833	1.115	1.380
average E	0.579	0.392	0.248
average I	0.304	0.723	1.131
lower bound on wealth	0.796	0.882	0.890
fraction of rentiers	0.011	0.015	0.311
correlation between E and I	-0.996	-0.996	-0.747
Gini W	0.033	0.064	0.162

- pre-OCP: long-run equilibrium before the policy
- OCP1: situation after one generation of the policy
- OCP: long-run equilibrium after the policy

Equilibrium wealth distributions



Effects of demographic change in China: summary

- Important effects even after one generation
- Smaller size of families: children receive larger inheritances
- Average inheritances increase by 138% from pre-OCP value
    - lowers labour supply by 32%
- Gini coefficient almost doubles. Pareto line flattens
- Reinforced in long run

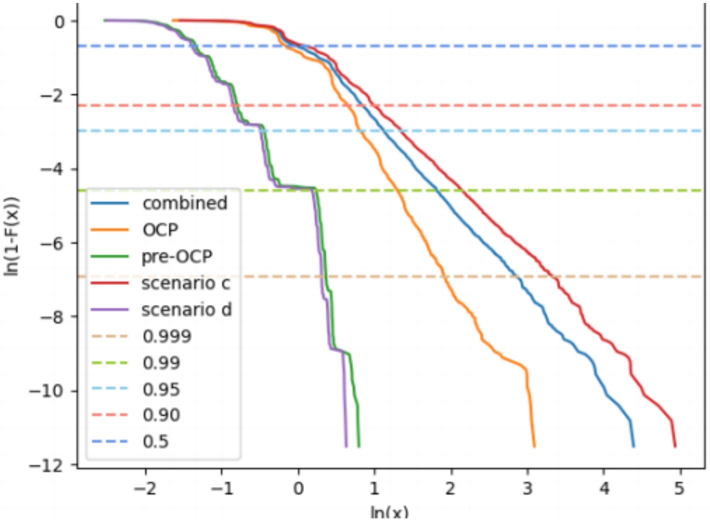
Results decompositions

-

	Total	OCP	ILE
average W	0.547	0.678	-0.131
average E	-0.331	-0.413	0.082
average I	0.827	1.065	-0.238
lower bound on wealth	0.094	0.041	0.053
fraction of rentiers	0.300	0.382	-0.082
correlation between E and I	0.249	0.436	-0.187
Gini W	0.159	0.222	-0.063

- OCP, ILE each push up the the lower bound on wealth
- Other variables. OCP, ILE are opposed
- The OCP effect outweighs that of ILE by a factor from 3 to 5

Equilibrium wealth decompositions



Conclusions

- A three-age model gives enough flexibility:
  - to model major life decisions
  - to represent major demographic effects
  - to construct a full OLG family model
- The OLG model leads to an equilibrium distribution
  - takes the Pareto form in the upper tail
  - little emperors increase equilibrium inequality
- The China simulation:
  - both OCP and ILE have effects in the expected direction
  - OCP effect is much stronger than ILE

Bibliography I

Alvaredo, F., A. B. Atkinson, T. Piketty, and E. Saez (2013). The Top 1 Percent in international and historical perspective. *Journal of Economic Perspectives 27*, 3–20.

Atkinson, A. B. (1980). Inheritance and the distribution of wealth. In G. A. Hughes and G. M. Heal (Eds.), *Public Policy and the Tax System*, Chapter 2, pp. 36–66. London: George Allen and Unwin.

Banerjee, A. V. and A. F. Newman (1991). Risk-bearing and the theory of income distribution. *The Review of Economic Studies 58*(2), 211–235.

Blinder, A. S. (1973). A model of inherited wealth. *Quarterly Journal of Economics 87*, 608–626.

Blinder, A. S. (1976). Inequality and mobility in the distribution of wealth. *Kyklos 28*, 607–638.

Cowell, F. A. and D. Van de gaer (2025). Condorcet was wrong, Pareto was right: Families, inheritance and inequality. *Journal of Public Economic Theory 27*, https://tinyurl.com/tx7xm2t5.

Galor, O. and J. Zeira (1993). Income distribution and macroeconomics. *Review of Economic Studies 60*, 35–52.

Laitner, J. (1979). Household bequests, perfect expectations and the national distribution of wealth. *Econometrica 47*, 1175–1193.

Mankiw, N. G. (2013). Defending the One Percent. *Journal of Economic Perspectives 27*, 21–34.

Piketty, T. and G. Zucman (2015). Wealth and inheritance in the long run. In A. B. Atkinson and F. Bourguignon (Eds.), *Handbook of Income Distribution*. Elsevier B.V.

Stiglitz, J. E. (2015). The origins of inequality, and policies to contain it. *National Tax Journal 68*, 425–448.

Wang, Z., M. Yang, J. Zhang, and J. Chang (2016). Ending an era of population control in China: was the one-child policy ever needed? *75*, 929–979.

Zhang, J. (2017). The evolution of China's one-child policy and its effects on family outcomes. *7*, 141–160.