Shadow Economy

- **What?** - “all market-based legal production of goods and services that are deliberately concealed from public authorities” – Schneider et al. (2010)
- **How much?** - India: 22.2% US: 8.6% (1997-2007)

**Cash lends anonymity facilitating shadow economy**

- **So, phase out cash?** How?
“...any plan to drastically scale back the use of cash needs to provide heavily subsidized, basic debit card accounts for low-income individuals... A simple idea to jump-start the process is to create debit accounts through which all government transfer payments are made.”

Demonetization

- **Intervention:**
  - RBI withdrew old Rs 500 and Rs 1000 notes
  - comprised 86% of amount in circulation
  - the two largest denominations
  - remonetization slow and inefficient

- **Objectives:**
  - black money (tax compliance)
  - forced digitization
    - counterfeit
    - terrorism
Qs How effective are subsidized digitization and demonetization to reduce shadow *transactions*?

- Can some shadow be optimal for revenue maximization?
Lagos and Wright (2005) + cash and digital + sales tax

- Useful feature:
  explicit role for money as medium of exchange

- Limitation:
  no wealth effect/persistence
  (but makes the model tractable)
Key Insights

- Size of shadow (which follows from extent of digitization)
  - depends on fundamentals, but also beliefs
  - can be stuck in a ‘bad’ equilibrium

- No-shadow eqm maximizes private welfare, not tax revenue

- Policies:
  - if multiple eq, demonetization can have LR effect
  - if unique eqm, subsidize digitization vs. reduce tax
Monetary Framework: Lagos and Wright (2005)
Cash vs. No Cash: Rogoff (2016)
Shadow Economy Theory: Gomis-Porqueraz et al. (2014)
Shadow Economy Empirics: Schneider et al. (2010)
FRAMEWORK
(based on Lagos and Wright, 2005)
Market Structure

Discrete time, infinite horizon
- Discount factor, $\beta$

Markets
- Decentralized goods market: DM
- Centralized market: CM

Households
- Sellers (in DM)
- Buyers (measure 1):
  - two types: $l$ (w.p. $\pi_l$), $h$
  - $l$’s DM utility $\epsilon u(q)$, $h$’s $u(q)$
Goods and Preferences

- Perishable *consumption* good, $q$:
  - produced and consumed in DM

- Good, $x$ as *numeraire*
  - produced and consumed in CM
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- Perishable \textit{consumption} good, \( q \):
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- Good, \( x \) as \textit{numeraire}
  - produced and consumed in CM

\[
U^b(q, x, l) = u(q) + U(x) - l
\]
\[
U^s(q, x, l) = -q + U(x) - l
\]
Medium of Exchange

- Buyers cannot produce, sellers do not consume in DM

  ➔ Role for trade

- Decentralized market with anonymity: no unsecured credit

  ➔ Role for means of payment
Medium of Exchange

- Buyers cannot produce, sellers do not consume in DM

⇒ Role for trade

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⇒ Role for means of payment

- Money supply, $M$ in CM
- real value of money, $\phi$
Portfolio Options

- Carry money to DM as digital or cash
  - conversion (fixed) cost to digital, $\kappa$
  - digital transactions \emph{fully} taxed (tax rate, $\tau$)
  - cash cannot be taxed but is lost with probability, $\eta$
Timing

- Buyers carry $m$
- Sellers dispense $m$

Portfolio choice: $m: c, d$

- Buyers + sellers $q_{t+1}$
EQUILIBRIUM
Steps

- **FOC:**
  1. Household’s Problem in CM
     - Choice of $m'$: $\max_{m'}\{-\phi m' + \beta V(m')\}$
  2. Bargaining in DM: cash vs digital
  3. Plug in for $V$

- Market Clearing: $\phi(\pi_l m_l + \pi_h m_h) = \phi M$
Bargaining

- If \( h \) type carries cash,
  \[
  \max_{q^c, g^c \in [0, \phi m]} [u(q^c) - g^c] \quad \text{s.t.} \quad -q^c + (1 - \eta)g^c \geq 0
  \]

- digital,
  \[
  \max_{q^d, g^d \in [0, \phi m - \kappa]} \{ u(q^d) - g^d \} \quad \text{s.t.} \quad -q^d + (1 - \tau)g^d \geq 0
  \]

  - Similarly, \( l \)-type: replace \( u(q) \) with \( \epsilon u(q_l) \)
Equilibrium Types

1. All-cash/Full-shadow
2. All-digital/No-shadow
3. $l$-cash, $h$-digital/Partial-shadow
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- obtain $\phi, m_i$ from FOCs, MC assuming equilibrium type
- check if this equilibrium maximizes surplus among others
All-cash Equilibrium

- FOC, \( h \):
  \[
  \iota = \alpha \left[ u' \left( \phi^c m_h^c \right) - \frac{1}{1 - \eta} \right]
  \]

- FOC, \( l \):
  \[
  \iota = \alpha \left[ \epsilon u' \left( \phi^c \left( \frac{M - \pi_h m_h^c}{\pi_l} \right) \right) - \frac{1}{1 - \eta} \right]
  \]

- Is an equilibrium if and only if \( S_i^c > S_i^d \)

- Similarly, all-digital and \( l \)-cash, \( h \)-digital equilibria
RESULTS
Shadow Economy

- Size of shadow follows from extent of digitization
  - as enforcement rate $\in \{0, 1\}$
- For digital payments to be viable, $\tau < \eta$
  - or, model heterogeneous tax rates across sectors
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Equilibrium Regions
Output and Bargaining Surplus

- Unique: no-shadow eqm higher output, $q$
  (low $\tau$ less distortionary)

- Under multiplicity: no-shadow maximizes output, $q$
  ($\tau < \eta$)
  - and also match surplus net of cost, $u(q) - q - \kappa$
    (output under no-shadow much higher to cover $\kappa$)
Tax Revenue

- Under multiplicity: no-shadow maximizes revenue, $\tau q^t$
  (same $\tau$, higher output)

- Unique: partial may be better than no-shadow for revenue
  (for low $\epsilon$, $m_l << m_h$, $\tau << \bar{\tau}$)

- Optimal tax rate for revenue maximization, $\tau^o$

$$\tau^o = \begin{cases} 
\tau & \text{if } \epsilon \geq \tilde{\epsilon}, \\
\bar{\tau} & \text{if } \epsilon < \tilde{\epsilon}.
\end{cases}$$
To move to partial from full shadow:

1. Demonetization: cash holdings in CM worthless
   - no behavior change, no effect except one-time loss

2. Subsidized digitization: reduce $\kappa$
   - works if tax rate not too high

3. Tax reduction: $\tau = \bar{\tau}$
   - which maximizes revenue?

If in multiplicity region then to ... need a shock
   - demonetization might help coordinate on no-shadow
   - tax red. or digi. - no effect if in same region
QUANTITATIVE
(preliminary)
### Calibration

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<th>Parameter</th>
<th>Source</th>
<th>India</th>
<th>US</th>
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<td>illiquid bonds rate, $\iota$</td>
<td>govt bonds rate data average assume (max tax)</td>
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<td>sales tax, $\tau$</td>
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<td>loss of cash, $\eta$</td>
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<td>utility elasticity, $\sigma$</td>
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<table>
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<th>Parameter</th>
<th>Target</th>
<th>India</th>
<th>US</th>
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<tbody>
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<td>utility low $\epsilon$</td>
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<td>prop of low value, $\pi_l$</td>
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</table>
Results

- Partial shadow is the unique equilibrium when
  - \( \kappa / \text{GDP} \): US [3.4% - 8.7%] India [8.5% - 21.4%]
  - Size of shadow: US 1.2% India 2.8%

- Policy
  - demonetization: no effect unless changes behavior
  - digitization vs. tax reduction: involve similar cost
Conclusion

- A shadow economy model to analyze policy interventions
  - cash lends anonymity facilitating shadow

- Extensions:
  - denominations, income tax: analysis
  - probability of enforcement: shadow and digitize rel.
  - distribution of types and sectors: data
  - seller’s mark-up: data

- Richer quantitative analysis
All Cash Equilibrium Surplus

\[ u(\phi^c m^c_h) - \frac{\phi^c m^c_h}{1 - \eta} > u(\phi^c m^c_h) - \frac{\phi^c m^c_h}{1 - \tau} - \kappa \]

- If \( h \) carries cash then \( l \) too: \( \epsilon < 1, \ m^c_l < m^c_h \)
All Digital Equilibrium

- FOC, \( l \):
  \[
  \nu = \alpha \left[ \epsilon u' (\phi^d m^d_l) - \frac{1}{1 - \tau} \right]
  \]

- FOC, \( h \)
  \[
  \nu = \alpha \left[ \phi^d (M - m^d_h) - \frac{1}{1 - \tau} \right]
  \]

- Is an equilibrium if and only if:
  \[
  \epsilon u(\phi^d m^d_l) - \frac{\phi^d m^d_l}{1 - \eta} < \epsilon u(\phi^d m^d_l) - \frac{\phi^d m^d_l}{1 - \tau} - \kappa
  \]

  - if \( l \) carries digital then \( h \) too: \( \epsilon < 1, m^d_l < m^d_h \)
$l$-cash, $h$-digital Equilibrium

- **FOC, $l$:**
  \[ \nu = \alpha \left[ \epsilon u' (\phi m_l) - \frac{1}{1 - \eta} \right] \]

- **FOC, $h$:**
  \[ \nu = \alpha \left[ u' (\phi (M - m_h)) - \frac{1}{1 - \tau} \right] \]

- Is an equilibrium if and only if:
  \[ \epsilon u(\phi m_l) - \frac{\phi m_l}{1 - \eta} > \epsilon u(\phi m_l) - \frac{\phi m_l}{1 - \tau} - \kappa \]

  and,
  \[ u(\phi m_h) - \frac{\phi m_h}{1 - \eta} < u(\phi m_h) - \frac{\phi m_h}{1 - \tau} - \kappa \]