Left and right. A tale of two tails of the wealth distribution.

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Introduction

- The aim of the paper is to show that heavy (right) tail in the distribution of wealth is a feature which is consistent with the presence of credit market imperfections;

- We study the wealth distribution arising in a standard model of educational investment with occupational choice and financial bequests. Returns to labor in the skilled and unskilled sector as well as the returns to financial wealth are exogenous random variables at the lineage level;

- The main result is that the limit distribution is a heavy tail distribution both in the case in which the parameter of the model induce ergodicity and in the case in which the limit distribution depends on initial distribution.
Model Details

- Preferences:

\[ U_{i,t} = \min\{(1 - \alpha)c_{i,t}; \alpha b_{i,t}\} \quad \text{with} \quad \alpha \in (0, 1) \tag{1} \]

where \(c_{i,t}\) denotes consumption and \(b_{i,t}\) denotes the bequest, \(\alpha\) is a measure of altruism.

- Budget constraint

\[ c_{i,t} + b_{i,t} = (1 + r_t)(b_{i,t-1} - e \cdot x) + y_{e,t} \tag{2} \]

where \(e \in \{0, 1\}\) denotes education investment to be chosen along with bequest.

- Wealth transitions turn out to be described by

\[ \Phi(b_{t-1}) = \begin{cases} 
\Phi_u(b_{t-1}) := (1 - \alpha)v + \theta_t b_{t-1} & b_{t-1} < x \\
\Phi_s(b_{t-1}) := (1 - \alpha)w_t + \theta_t(b_{t-1} - x) & b_{t-1} \geq x 
\end{cases} \tag{3} \]

where \(\theta := (1 - \alpha)(1 + r)\)
Sources of shocks in the economy both in the labor income and in the returns to capital.

**Wage in the skilled sector** is a i.i.d. process $W$, where $W$ takes value over the support $[w, \bar{w}]$.

**Wage in the unskilled sector** The wage in the unskilled sector is a constant $v > 0$, moreover $v < w$.

**Returns on financial wealth** i.i.d. random process $R$, on the support $[r, \bar{r}]$.

**Human capital** Simplifying Assumption: human capital is a dominant asset vis a vis financial wealth $x < \frac{w-v}{1+E[R]}$. 
Heavy tail and Ergodicity: Assumptions

In order to study the evolution of wealth distribution the following technical assumptions are made:

i) $E \log |\theta| < 0$, it means that $(1 - \alpha)(1 + E[r]) = E[\theta] < 1$;
ii) The conditional law of $\log |\theta|$ is not arithmetic;
iii) (Cramér Condition) There is $\lambda > 1$ such that $E|\theta|^\lambda = 1$;
iv) For each $c \in \mathbb{R}$ we have that $P(W + \Theta c = c) < 1$;
v) The minimum skilled wage $w$ is less than $\frac{x}{1 - \alpha}$.

See Benhabib et al. (2011) for analogous assumptions in a model where transition functions define a Markov chain.
Results

Proposition

Under the above assumptions the stochastic process defined in (3) is ergodic.

Proposition

The stationary distribution defined in (3) features a heavy upper tail.
Results about the Social Mobility

We define downward and upward social mobility as simple as possible

\[ DM_t(\Theta) = \text{Prob}(b_t < x | b_{t-1} \geq x) \quad UM_t(\Theta) = \text{Prob}(b_t \geq x | b_{t-1} < x) \]

On the stationary distribution \( DM_\infty(\Theta) = UM_\infty(\Theta) \).

**Proposition**

Under the same assumptions of Proposition 1 and if \( \Theta' \geq_{FSD} \Theta \), then the new stationary distribution \( \Phi'(x) \leq \Phi(x) \).

**Proposition**

Under the same assumptions of Proposition 1 and if \( \Theta' \geq_{SSD} \Theta \), then \( UM_\infty(\Theta') \geq UM_\infty(\Theta) \).
Heavy tail and Poverty Traps: Assumptions

We study the limit distribution when financial shock $\Theta^e$ depends on the occupational choice $e$. In particular we study the case in which the support of $\Theta^0$ is a subset of the support of $\Theta^1$, moreover the following assumptions hold:

i) as above;
ii) as above;
iii) as above;
iv) same as above;
v) $(1 - \alpha)w > \max\left\{x, \frac{(1-\alpha)v}{1-\max\{\Theta^0\}}\right\}$ (no downward occupational mobility).
vi) Each $\theta^0 \in \Theta^0$ is less than 1, (stability for non i.i.d shocks);
Proposition

Under assumptions i)-vi) bis we have the following cases:

i) [Poverty Trap] If \( x > \frac{(1-\alpha)v}{1-\max\{\Theta^0\}} \), then the limit distribution of (3) presents a poverty trap.

ii) [Ergodicity] If \( x \leq \frac{(1-\alpha)v}{1-\max\{\Theta^0\}} \), then the process define in (3) is ergodic.

In both the case, ergodicity or poverty trap, the limit distribution present a right heavy tail in the limit distribution.

Notice: the left tail is determined by initial conditions.
Conclusions

- A limit distribution with a heavy *right tail* emerges in a model with investment indivisibilities and credit market imperfections;
- A right heavy tail emerges also under conditions in which the model produces a poverty trap (or, more generally, when the limit distribution depend on the initial distribution);
- conclusion 1: if credit market imperfections are binding in the long run the left tail (wealth dynamics in the segment of the poors) is determined by initial conditions;
- conclusion 2: The specific market failure cannot be recovered by investigating the properties of the right tail (Benhabib et al., 2011 or Piketty (2013)) without knowing the properties at the bottom of the wealth distribution (the left tail).
Bibliography


