Objectives – To understand

• What single-name credit derivatives are

• How single name credit default swaps work

• How to compute CDS spreads from risk-neutral survival probabilities (and vice versa)

• Recent changes to CDS contract in “CDS Big Bang”
Single Name vs. Basket Credit Derivatives

- A *single-name credit derivative* is a contract whose payoff depends on the default of a *single underlying credit*.
  - example: single name credit default swap

- A *basket credit derivative* is a contract whose payoff depends on the default of *several underlying credits*.
  - example: CDO tranche
Main Credit Derivative Products: Single Name

• **Single name credit default swap (CDS)** is a contract that provides *protection against a default event on the part of a single issuer (“name”)*
  ✓ protection buyer pays premium and, in event of ‘credit event’, receives par in exchange for eligible obligation of “name”

Single Name Products, contd.

• **Rate-of-return swap** – is a contract in which one side pays the rate-of-return on one asset (A) and the other side pays the rate-of-return on a different asset (B)

• **Example**: asset A is a *corporate bond* and asset B is a *Treasury bond*
  ✓ Similar to (but not the same as) a CDS
Single Name Products, contd.

• Many other types of single-name credit derivative contracts but **CDS** has emerged as by far the **most heavily traded**
Single Name Credit Default Swaps

- The **buyer** of protection pays
  - **EITHER** a constant premium per year \((d)\) until the maturity of the contract **OR** the occurrence of the default event (whichever comes first)
  - **OR** (since 2009) a combination of a constant premium per year **AND** an upfront fee

- The **seller** pays
  - if the *default event does occur*: the difference between the promised (face) value of the underlying issue \((100)\) and the market value of the defaulted bond \((Y)\)
  - if the *default event does not occur*: zero

- **Initially** we assume (pre-2009) situation of *no upfront fee*

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Credit Default Swap: Mechanics

- if **no default**: only cash flow is premium of \(d\) bps p.a
- if **default**: transaction stops and transaction settled using a recovery rate that is determined in an auction process
CDS: Critical Items in Contract

- **Reference entity**: company / country on which contract is written
- **Reference obligation**: identifies relevant seniority of claims (i.e., point in the capital structure)
- **Credit events**: describes what events can trigger default (see next page)
- **Obligation category**: describes what types of obligation can trigger default
- **Deliverable obligations**: describes what obligations can be delivered to the seller in settlement

CDS Big Bang – 1. Fixed premium and Upfront Fee

- **Originally** CDS premium was determined so that net value of contract was zero, i.e., so that – as in most swaps – no initial fee passed between buyer and seller at inception.
- Since **Big Bang**, annual premium is fixed at either 100 bps or 500 bps (depending on credit quality) and an upfront fee is paid by the buyer to the seller (or vice versa).
- Initially we use old convention. Later we derive the relation between the upfront fee and the old “zero fee” premium.
CDS Big Bang – 2. CDS Auction

• The process of determining the recovery value of bonds in order to determine CDS payments following a default has sometimes proves troublesome (lack of liquidity in bond markets etc.)

• CDS Big Bang has introduced a mandatory auction mechanism that consists of two stages:

  1. In the first stage: Market makers supply two-way quotes for defaulted assets (with pre-defined maximum spread) and CDS holders submit physical settlement requests (limited to physical bond positions)
     – the bids are used to determine the “inside market midpoint” – an initial reference point

  2. In the second stage: dealers and investors submit limit bids and the final price is the price that just satisfies the demand for physical settlement.

The Default Event

• ISDA documentation (2003) defines SIX trigger events:
  1. bankruptcy
  2. obligation acceleration
  3. obligation default
  4. failure to pay
  5. repudiation / moratorium
  6. Restructuring

• In practice THREE principal credit events:
  1. bankruptcy
  2. failure to pay
  3. Restructuring

• The tough one is restructuring
Credit Default Swap Cash Flows

- Buyer of protection pays $d$ per period until default when he receives face value ($100$) minus market value of underlying note $100*R$

Synthetic Credit Default Swap

**Default-free floating rate note (long)**

- Default-free floating rate: $L$
- Payment in Default $100R$

**Credit Default Swap**

- Default-free payment $100$
- Loss in default $100(1 – R)$
- Payment per period: $d$

**Defaultable floating rate note (short)**

- Defaultable floating rate: $L + S$
- Even though time of default is unknown, value of default-free floater will equal 100 at each coupon date
Pricing Default Swaps I: **Supply** (Dealer Perspective)

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cash Flow</th>
<th>Default Event Payment</th>
<th>Cash Flow at Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Now</td>
<td>Period</td>
<td></td>
</tr>
<tr>
<td>Write default protection</td>
<td>0</td>
<td>d</td>
<td>-100(1-R)</td>
</tr>
<tr>
<td>Borrow bond and sell</td>
<td>100</td>
<td>-(L+S)</td>
<td>-R*100</td>
</tr>
<tr>
<td>Invest Proceeds</td>
<td>-100</td>
<td>r</td>
<td>100</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0</td>
<td>d - [S + (L-r)]</td>
<td>0</td>
</tr>
</tbody>
</table>

R: recovery rate; L: Libor rate; S: floating rate spread; r: repo rate; d: CDS rate

CDS rate (ask) = Spread + (Libor – repo rate) => d = S + (L – r)

Pricing Default Swaps II: **Demand** (Dealer Perspective)

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cash Flow</th>
<th>Default Event Payment</th>
<th>Cash Flow at Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Now</td>
<td>Period</td>
<td></td>
</tr>
<tr>
<td>Buy default protection</td>
<td>0</td>
<td>-d</td>
<td>+100(1-R)</td>
</tr>
<tr>
<td>Buy bond</td>
<td>-100</td>
<td>+ (L+S)</td>
<td>+R*100</td>
</tr>
<tr>
<td>Finance bonds</td>
<td>+100</td>
<td>-r_B</td>
<td>-100</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0</td>
<td>-d + [ S + (L – r_B)]</td>
<td>0</td>
</tr>
</tbody>
</table>

R: recovery rate; L: Libor rate; S: floating rate spread; r_B: financing rate; d: CDS rate

CDS rate (bid) = Spread - (financing – Libor) => d = S - (r_B – L)
CDS Pricing – Arbitrage Limits vs. Supply/Demand

- These values $S + L - \text{Repo}$ and $S - (\text{Financing} - L)$ set the upper and lower bounds for the CDS spread but, most of the time, the market bid and offer will lie within these.

In the crisis the basis became strongly negative:

- **CDS-bond basis** is CDS premium minus bond spread

- In principle, a **strongly negative basis** (bond yield higher than CDS spread) means that it is **profitable** to:
  - buy the bond; and
  - buy credit protection using CDS

- If you can afford to **hold the position** up to maturity it would be profitable but, in the crisis, many people could **not maintain financing** of the bond position.
“The trade that ate Wall Street”

Valuing a CDS Contract
Risk Neutral Survival Probability

• If the risk-neutral probability of state $s$ at time $T$ (really a forward price!!) is $p_s$ then the value of a contract that pays $1$ in that state is:

$$V = e^{-rT} \hat{E}(CF) = e^{-rT} (p_s \times 1) = e^{-rT} p_s$$

• Therefore, if the risk-neutral survival probability for time $t$ is $Q_t$, then the value of $1$ paid only if the credit has survived to time $t$ is:

$$D_t Q_t$$

where $D_t$ is the riskless discount factor: $e^{-rT}$

Next …

1. We first assume that we know the risk-neutral survival probabilities (the $Q$’s) and work out the no-arbitrage value of the CDS spreads.

2. In practice the $Q$’s are obtained from CDS spreads via “bootstrapping” and so we derive the formula to do this

3. We will also see later that, under certain assumptions, the market is complete in this case and so we can expect to derive unique $Q$’s from prices

4. There are many (slightly) different versions of these formulae depending on small changes in the assumptions. We will assume that, if default occurs, it occurs only on a payment date (and not between dates)
Determining the CDS premium – the idea

- Cash flows on "premium leg"
  - payment of premium up to maturity or default event.

- Cash flows on "protection leg"
  - payment of loss amount, i.e., 100 \times (1-R), in event of default

- CDS premium:
  - level of premium that makes PV of premium leg equal to PV of protection leg.

- Initially we assume "old" form of contract with no upfront payment

Cash flows on a CDS Contract – Premium Leg

- Suppose the CDS contract has maturity of N periods and that the premium, \( S_N \), is paid once per period.

- In this case, if the credit has survived to period \( t-1 \) then the payment at period \( t \) is \( S_N \) and the PV of the payment is:

  \[ S_N D_t Q_{t-1} \]

  where \( D_t \) is the riskless discount factor for time \( t \)

- The total PV of the premium leg is the value of the payments up to the final period \( N \) and is therefore:

  \[ S_N \sum_{i=1}^{N} D_i Q_{i-1} \]

Note: \( \sum_{i=1}^{N} D_i Q_{i-1} \times 0.0001 \) is referred to as the PV01 of the contract, i.e., the value of 1 bp paid at each of the premium payment dates
Cash flows on a CDS Contract - – Protection Leg

- In the event of default the CDS pays \((1-R)\) where \(R\) is the recovery rate; otherwise it pays zero.

- The RN probability that the reference entity survives to time \(t\) and defaults at time \(t\) is \((Q_{t-1} - Q_t)\) and so the value of the protection payment for default at time \(t\) is:

\[
(1 - R) D_t (Q_{t-1} - Q_t)
\]

- And so the PV of the protection leg is:

\[
(1 - R) \sum_{t=1}^{N} D_t (Q_{t-1} - Q_t)
\]

No-Arbitrage Value of CDS Premium

- The CDS spread is the value of the premium, \(S_N\), that makes the net value of the contract zero, i.e., makes the value of the premium leg equal to the value of the protection leg:

\[
S_N \sum_{t=1}^{N} D_t Q_{t-1} = (1 - R) \sum_{t=1}^{N} D_t (Q_{t-1} - Q_t)
\]

and so the CDS spread is given by:

\[
S_N = \frac{(1 - R) \sum_{t=1}^{N} D_t (Q_{t-1} - Q_t)}{\sum_{t=1}^{N} D_t Q_{t-1}}
\]
Example: Calculating the CDS Spread from $Q$ and $D$

\[
S_N = \frac{(1 - R) \sum_{t=1}^{N} D_t (Q_{t-1} - Q_t)}{\sum_{t=1}^{N} D_t Q_{t-1}}
\]

<table>
<thead>
<tr>
<th>$D(t)$</th>
<th>Spread (bps)</th>
<th>$Q(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9710</td>
<td>0.00102</td>
</tr>
<tr>
<td>2</td>
<td>0.9380</td>
<td>0.00117</td>
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<tr>
<td>3</td>
<td>0.9050</td>
<td>0.00134</td>
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<tr>
<td>4</td>
<td>0.8720</td>
<td>0.00154</td>
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<tr>
<td>5</td>
<td>0.8400</td>
<td>0.00177</td>
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<tr>
<td>6</td>
<td>0.8080</td>
<td>0.00204</td>
</tr>
<tr>
<td>7</td>
<td>0.7770</td>
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</tr>
<tr>
<td>8</td>
<td>0.7460</td>
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<tr>
<td>9</td>
<td>0.7160</td>
<td>0.00309</td>
</tr>
<tr>
<td>10</td>
<td>0.6870</td>
<td>0.00355</td>
</tr>
</tbody>
</table>

“Bootstrapping” the CDS Curve to Derive RN Survival Probabilities

- In most cases we are interested in calculating the RN-probabilities from CDS spreads (rather than the other way round).

- This can be done by “bootstrapping”:
  - starting with a 1-period CDS contract it is simple to work out the implied value of $Q_1$.
  - for a 2-period CDS contract, knowing $Q_1$ it is simple to work out $Q_2$
  - … and so forth
How is it that we can work out RN Survival Probabilities?

- We know that we can only work out unique RN probabilities in the case of a complete market.

- Why is the market complete in this case?
  - if we ignore interest rate uncertainty and also assume that the recovery rate is known, then, most unusually, uncertainty in future cash flows is actually binomial (“default” or “no default”);
  - and so,
  - with two assets (the CDS and a riskless bond we can work out all the RN default probabilities)

Default / No-Default is a Binomial Event

- Because default is a binomial event, with a full term structure of:
  - CDS contracts
  - LIBOR borrowing and lending
  - the market is complete and we can calculate unique risk-neutral survival and default probabilities

\[ t = 1 \quad t = 2 \quad t = 3 \]
“Bootstrapping” the CDS Curve to Derive Survival Probabilities

• For those of you who like formulae, here are the formulae for working out the RN-survival probabilities from spreads:

Definitions: $M_N = \sum_{t=1}^{N} D_t Q_{t-1}$, $\hat{M}_N = \sum_{t=1}^{N} D_t \hat{Q}_t$, $\alpha_N = \frac{S_N}{(1-R)}$

$$Q_N = \left(\frac{1}{D_N}\right)\left[ M_N (1-\alpha_N) - \hat{M}_{N-1} \right]$$

Note: $M_N$ does not depend on $Q_N$

• Using the formulae you can work out $Q_1$, then $Q_2$ etc.

Technical note

• The formulae given above are simplified in the sense that they assume that default can occur only on one of the payment dates of the CDS premium.

• They also assume (this is realistic) that the premium is paid up to the time of default.

• In fact, default can occur at any time (including between payment dates) and a more refined version of the formulae would take this into account.
Standardised Coupons

• After the ‘big bang’ in 2009, CDS contracts started to trade with a fixed coupon set at either 100bp or 500bp.
  ✓ standardised coupon would be chosen nearest to where the CDS was trading at the time.
  ✓ this does not change regardless of the actual level of spread the CDS is traded at.
• The PV of the difference between the CDS quoted in running terms and the fixed coupon standardised contract is settled up-front.
• Example: an investor buying protection for 150bp over five years will pay:
  ✓ 100bp running over five years; and
  ✓ an upfront amount equivalent to the present value of 50bp per year over five years.
  ✓ if the value of the (risky) annuity is, say, 4.6 then the up-front fee would be 50 x 4.6 = 230 bps

Relation between the upfront fee and the premium under the old contract

• The protection payment is the same under both contracts and so the value of:
  ✓ the premium payments under the “old” contracts; AND
  ✓ the sum of the upfront payment and the standardised premium under the new payments must be the same.

\[ S_N \sum_{t=1}^{N} D_t Q_{t-1} = U_N + S^C_N \sum_{t=1}^{N} D_t Q_{t-1} \]

so, \[ U_N = \left( S_N - S^C_N \right) \sum_{t=1}^{N} D_t Q_{t-1} \]

where \( S^C_N \) is the standardised premium and \( U_N \) is the upfront fee.
Valuing a CDS Contract (After Inception)

- Suppose an investor has sold protection for 5-years at 250 bps per year.

- What is the value of this contract one year later (assuming no credit event has occurred) and current 4-year CDS rate is 200 bps points?

- Idea: if investor buys protection for 4 years:
  ✓ if credit event: payment of notional x (1 – R) from long and short position cancel
  ✓ net payment is annuity of 250 – 200 = 50 bps per year
  ✓ annuity is risky since stops if credit event occurs.
Value of “Risky Annuity”

- We value the *risky annuity* using the PV01 calculated before:

\[ \sum_{t=1}^{N} \text{Notional} \times \frac{\text{Annuity}}{\text{Level}} \times Q_{t-1} \times D_t \equiv \text{Notional} \times (C - S) \times PV01 \]

- Where (for a position that has sold protection):
  - \(C\): contract level of spread
  - \(S\): current level of spread

- *Note*: this is precisely how the upfront payment is determined.

---

Example: Valuing a CDS on GM

- 7 December 2005 CDS Spreads on GM
- Implied risk-neutral survival probabilities calculated as described earlier

<table>
<thead>
<tr>
<th>(D(t))</th>
<th>Spread (bps)</th>
<th>(Q(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9569</td>
<td>0.7173</td>
</tr>
<tr>
<td>2</td>
<td>0.9115</td>
<td>0.5458</td>
</tr>
<tr>
<td>3</td>
<td>0.8680</td>
<td>0.4415</td>
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<td>4</td>
<td>0.8260</td>
<td>0.3774</td>
</tr>
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<td>5</td>
<td>0.7855</td>
<td>0.3172</td>
</tr>
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<td>6</td>
<td>0.7474</td>
<td>0.2702</td>
</tr>
<tr>
<td>7</td>
<td>0.7093</td>
<td>0.2385</td>
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<tr>
<td>8</td>
<td>0.6751</td>
<td>0.2000</td>
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<tr>
<td>9</td>
<td>0.6408</td>
<td>0.1714</td>
</tr>
<tr>
<td>10</td>
<td>0.6066</td>
<td>0.1504</td>
</tr>
</tbody>
</table>

- CDS Contract: Protection *bought* at 800 bps on $10 million notional.
- Time remaining: 5 years

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Face value</td>
<td>10,000,000</td>
</tr>
<tr>
<td>Initial spread (b.p.)</td>
<td>800</td>
</tr>
<tr>
<td>Current spread (b.p.)</td>
<td>1350</td>
</tr>
<tr>
<td>Annuity (b.p.)</td>
<td>550</td>
</tr>
<tr>
<td>Annuity ($)</td>
<td>550,000</td>
</tr>
</tbody>
</table>
### Value of Contract

- Time remaining: 5 years
- Assume annual payments (for simplicity)

<table>
<thead>
<tr>
<th>Annuity ($)</th>
<th>D(t)</th>
<th>Q(t)</th>
<th>D(t) x Q(t-1)</th>
<th>Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 550,000</td>
<td>0.9569</td>
<td>0.71728</td>
<td>0.9569</td>
<td>526,322</td>
</tr>
<tr>
<td>2 550,000</td>
<td>0.9115</td>
<td>0.54581</td>
<td>0.6538</td>
<td>359,578</td>
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<tr>
<td>3 550,000</td>
<td>0.8680</td>
<td>0.44154</td>
<td>0.4738</td>
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<td>4 550,000</td>
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<td>0.37739</td>
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<tr>
<td>5 550,000</td>
<td>0.7855</td>
<td>0.31721</td>
<td>0.2964</td>
<td>163,033</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td>2.7457</td>
<td>1,510,109</td>
</tr>
</tbody>
</table>

**PV01**

### Risk Exposure of CDS Position

- The value of a CDS position has **exposure** to:
  - the **spread**
  - the **interest rate**
  - a **default** event

- Changes in the **spread** typically have a much larger impact than changes in the interest rate
Risk Exposure of CDS Position

• Because value of CDS position is:

\[ V_{CDS} = \text{Notional} \times (C - S) \times PV01 \]

\[ DV01 = \frac{\partial V_{CDS}}{\partial S} = -\text{Notional} \times PV01 + \text{Notional} \times (C - S) \times \frac{\partial PV01}{\partial S} \]

impact of change in spread on size of annuity
impact of spread on PV01 via RN-Q

with PV01 unchanged.

• An increase in the spread has two effects (protection sold):
  ✓ decrease on the size of the annuity (impact of PV01 per unit change in S)
  ✓ decrease in PV01 through a decrease in the RN survival probability, \( Q \).

Approximate impact on PV01 – Credit Duration

• The RN survival probability is approximately:

\[ Q_t \approx \exp \left[ -\frac{S}{1-R} \right] \]

where \( \frac{S}{1-R} \) is approximately equal to the RN "default intensity" (to be discussed)

• Using this approximation we can calculate the impact of a small change in the spread on the PV01 as:

\[ PV01 = \sum_{t=1}^{N} D_t \times Q_{t-1} \times (t-1) = -\frac{1}{(1-R)} \sum_{t=1}^{N} D_t \times Q_{t-1} \times (t-1) \times PV01 \]

\[ \Rightarrow \frac{\partial PV01}{\partial S} \approx -\frac{1}{(1-R)} \sum_{t=1}^{N} D_t \times Q_{t-1} \times (t-1) \times PV01 \]

credit "duration"
Accuracy of Approximation: Impact on PV01 of Parallel Shift in CDS Spread Curve

- When spreads are relatively low the approximation on the previous slide is quite accurate.

![Graph showing change in PV01 vs. change in spread (bp)]

DV01 vs. PV01

Coupon = 500 bps, current spread = 220 bps

![Graph showing DV01 vs. PV01, with current spread indicated]
CDS Exposure to Interest Rates

• Because value of CDS position is:

\[ V_{CDS} = \text{Notional} \times (C - S) \times PV01 \]

so:

\[ \frac{\partial V_{CDS}}{\partial r} = +\text{Notional} \times (C - S) \times \frac{\partial PV01}{\partial r} \]

\[ PV01 = \sum_{t=1}^{N} D_t Q_{t-1} = \sum_{t=1}^{N} Q_{t-1} e^{-rt} \]

\[ \Rightarrow \frac{\partial PV01}{\partial r} \approx -\sum_{t=1}^{N} Q_{t-1} \times D_t \times t \]

\[ = -\left( \frac{1}{PV01} \sum_{t=1}^{N} Q_{t-1} \times D_t \times t \right) \times PV01 \]

= "credit duration" \( \times PV01 \)

---

**Note:** definition of *credit duration* here is very slightly different from one given above (“t” rather than “t-1”) because here we use continuous compounding.
Three Ways to Unwind a CDS Position

• An investor with a long or short position in a CDS can unwind this position in one of three ways:
  – in cases (1) and (2) the investor pays or receives the mark-to-market value of the contract

1. Agree to unwind position with original counterparty
  – all future cash flows from contract cancelled
  – “legal certainty” but little bargaining power

2. Assign the transaction to another counterparty ("novation")
  – transaction is now between new counterparty and other old counterparty

3. Enter an offsetting transaction
  – removes exposure to default event but counterparty risk remains

Central Clearing

• Clearinghouse is counterparty to CDS trades put through clearing house:
  ✓ concern during crisis over extent of counterparty risk
  ✓ central clearing would “mutualise” counterparty risk
  ✓ mid-Sep 2009, 15 banks (which include Barclays Capital, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, JPMorgan Chase and Morgan Stanley) have made a pledge to the Fed to clear the majority of credit and interest rate derivatives through central counterparties by the end of the year
  ✓ Clearing houses are (so far) mainly exchanges, e.g., CME, ICE (Intercontinental Exchange) etc.
Takeaways

• **CDS highly liquid** pre-crisis
  ✓ post crisis .. ??

• Calculation of **risk-neutral survival** (and default) **probabilities**
  ✓ unusual case where outcomes are *actually binomial* (default/no default)
  ✓ means (approximately) we can compute risk-neutral probabilities directly since *market* is (approximately) *complete*
  ✓ provides *no-arbitrage method* to value contracts with different coupons

• Next .. The **intensity approach** and how it relates to these calculations