Outline

• Why do firms default?
• Modelling early default
  – concept of default boundary
• Valuing credit risky debt with early default
• Credit risk models and corporate financial policy
  – Leland model: optimal capital structure
Why do firms default? – The Default “Mechanism”

• In *Merton* model *default* can occur only *at maturity* of the (single) zero coupon bond that the firm has outstanding.
• In particular: the *financial condition* of firm prior to maturity – no matter how poor – *never leads* to *default*

• Important: *default* is a *decision* taken by the *equity holders*
  – always free to put in more cash to continue to service debt

Why do firms default? – The Default “Mechanism”

• In practice *default* *prior to maturity* can occur for many reasons:
  – *Lenders force default* (through covenants etc.) – if firm violates covenants, some control of firm passes from borrowers to lenders. If violation is sufficiently serious, lenders may be able (and wish) to force the firm to liquidate, declare bankruptcy etc.
  – *Equity holders choose to default (coupon debt)*: if equity value is sufficiently low then it will not be worthwhile for equity holders to continue paying the coupons to keep their option alive (unless they can sell assets to do so). Decision to default maximises equity value.
The Default “Mechanism”, contd.

• **Strategic Default**: 
  - In some (relatively unusual) cases *equity holders* have considerable *bargaining power* in default.
  - **Example**: firm, *previously low* credit rating *now high* credit rating: *bonds* sell at *premium* to par (high coupon) but in *bankruptcy* lenders would get *par*.
  - Possibility of *strategic default* by equity holders and therefore room to bargain for lower coupon with lenders now. (Anderson-Sundaresan)

• Despite *differences in motivation*, most of these approaches imply existence of a *boundary value* for the firm’s assets at which default is triggered.

---

Cross-Default Clauses and Early Default

• Most bond agreements include a “*cross-default clause*” which says that, if a firm has several bonds outstanding, then default on any one obligation means that the firm is in default on all its other obligations.
  - necessary for lenders to protect interests and prevent other lenders “cherry-picking” assets.

• This means that, *prior to maturity*, default on one bond could be caused by, for example, default *at maturity* on another.
  - i.e., another mechanism for early default
Early Default

- With early default we need
  - to model the default event, i.e., what is the condition that triggers default?
  - to value recovery payments made at these times
- In most cases default is modelled as the first time, $\tau$, that the firm’s assets fall to a lower boundary value, $V_B$:

$$\tau = \min\{ t : V_t \leq V_B \}$$

The default boundary

[Graph showing the value of the firm over time with a default boundary at $V_B = 60$. The graph illustrates the no default scenario and the default time $\tau$.]

Does introduction of early default increase the credit spread?

• Suppose a firm has a single zero-coupon bond outstanding (as in Merton)

• What would be the value of equity if the firm value were to hit the boundary
  – without early default
  – with early default

• What does this imply for:
  – value of debt (now)?
  – value of equity (now)?

Default probabilities with early default.

• The natural probability that $V$ hits $B$ before time $T$ is given by:

$$
N\left(\frac{\ln(B/V) - mT}{\sigma_V\sqrt{T}}\right) + \left(\frac{B}{V}\right)^{2m/\sigma_V^2} N\left(\frac{\ln(B/V) + mT}{\sigma_V\sqrt{T}}\right),
$$

where $m = \mu - \frac{1}{2} \sigma_V^2$


• Intuitively, this must be higher than the probability that $V < B$ at $T$ since there $V$ may hit $B$ before $T$ and still end above $B$ at time $T$.

• In fact the first term is prob($V_T < B$), i.e., $N(-D2D_N)$, where $D2D_N$ is the natural distance-to-default, and since the second term is positive, the probability given by the expression above is clearly higher.
Valuing a Payment at Default

- Suppose the firm pays no dividends and has current value $V_t > V_B$ that:
  - behaves like the stock price in the Black-Scholes model with volatility $\sigma$ (Geometric Brownian Motion); and therefore has a
  - distribution of future value that is lognormal
- Suppose we wish to value a security (somewhat Arrow-Debreu (A-D) like) that pays:
  - $\$1$ the first time that the firm value, $V_t$, hits the boundary value $V_B$ from above. (Hitting “from above” implies that $V$ is initially above $V_B$)
  - otherwise pays zero.
- The value of this security is given by:
  $$G(V) = \left( \frac{V_t}{V_B} \right)^{-\alpha} \text{ where } \alpha = \frac{2r}{\sigma^2}$$
- This is a very important and extremely useful result. basis for many
  “second generation” structural models
What does $G(V)$ mean?

- $G(V)$ is like a *discount factor* (or Arrow-Debreu price) for a cash flow that occurs when the value of the firm hits the default boundary *whenever that occurs*.
- Useful because *many important* corporate *cash flows* are *triggered* (started or stopped) *by default* and, if we model default as the firm’s value hitting a threshold, then we can value these components of the cash flow:
  - *Examples*:
    - bond cash flows (coupon and principal)
    - recovery cash flows to bond holders
    - tax deductions from coupon payments
    - dissipation of value or transfer to third parties in default

Value of First Hitting-Time Security

- An *increase* in $V_t$ decreases the RN probability of hitting $V_B$ and so decreases the value.
- An *increase* in the *interest rate* increases the (risk-neutral) upward drift of the asset value, decreases the RN hitting probability and so decreases the value.
- An *increase* in the *volatility* increases the RN probability of hitting the boundary and so increases the value.
Valuing Risky Perpetual Debt: I – No recovery

- Suppose a firm has risky perpetual debt outstanding that pays a constant coupon of \( c \) per period and defaults when the firm’s value hits a boundary value \( V_B \). The riskless rate is \( r \).
- If the debt were riskless it would sell for \( c/r \), the value of a riskless perpetuity.
- Suppose first that, in the event of default, the recovery amount is zero, then cash flows are same as:
  - holding a default-free perpetuity (value = \( c/r \))
  - losing the right to the future cash flows (i.e., to the default-free perpetuity) when \( V \) hits \( V_B \) (value = \( G(V) c/r \)).
- Total value:

\[
D(V) = \frac{c}{r} - G(V) \frac{c}{r} = \frac{c}{r} - \left( \frac{V}{V_B} \right)^{-\alpha} \frac{c}{r} = \frac{c}{r} \left( 1 - \left( \frac{V}{V_B} \right)^{-\alpha} \right)
\]

Valuing Risky Perpetual Debt: II – with Recovery

- Now suppose that, in default, there is a known recovery amount, \( R \).
- Value becomes:

\[
D(V) = \frac{c}{r} - G(V) \frac{c}{r} + G(V)R = \frac{c}{r} - \left( \frac{V}{V_B} \right)^{-\alpha} \left( \frac{c}{r} - R \right)
\]

Example: \( V_B = 40, R = 40, \sigma_v = 20\%, \ c = 3, r = 5\% \)
What happens when shareholders decide when to default?

**Endogenous** default boundaries

- As we have just seen, if the recovery amount is $R$, the bond value is:
  
  $\frac{d}{c - \left(\frac{V}{V_B}\right)^\alpha \left(\frac{c}{r} - R\right)}$ where $\alpha = \left(\frac{2r}{\sigma^2}\right)$

- If, at default, the bondholders take over the firm and receive the firm value $V = V_B$, (i.e., $R = V_B$) then the value is:
  
  $D(V) = \frac{c}{r} - \left(\frac{V}{V_B}\right)^\alpha \left(\frac{c}{r} - V_B\right)$

- Now suppose shareholders can choose $V_B$ to maximise the value of equity, i.e., minimise the value of debt, in this case the optimal default boundary is given by:
  
  $V_B = \frac{c}{r} \left(\frac{\alpha}{1 + \alpha}\right)$
Endogenous Default Boundary with Perpetual Debt, contd.

- **Example:** \( r = 5\%, \ c = 3, \ \sigma_V = 20\% \) **Riskless value** = \( c/r = 60 \)
- The corresponding value of \( \alpha \) is 2.5 \((= 2r/(\sigma^2))\) and the optimal value of \( V_B \) is therefore:

\[
V_B = \frac{c}{r} \left( \frac{\alpha}{1 + \alpha} \right) = \frac{3}{0.05} \left( \frac{2.5}{3.5} \right) = 42.9
\]

- The value of \( V_B \) that minimises the debt value is (naturally) independent of the **current value** of the firm’s assets

![Graph showing bond value vs. default boundary]

An aside: the perpetual put

- The situation where the **stockholders choose the default boundary** is equivalent to the stockholders:
  - having **issued riskless perpetual debt** (value \( c/r \))
  - holding a perpetual put to sell the assets of the firm to the bondholders for an amount equal to \( c/r \) (the exercise price) in exchange for the firm’s assets (value \( V_B \) when the firm defaults).
- The stockholders will exercise the put when the firm value hits the optimal exercise value \( V_B \) (previous slide), at which point they give up the firm in exchange for \( c/r \) and use this amount \((c/r)\) to repay the “riskless” debt
  - i.e., the stockholders get zero – and the bondholders get the firm \((V_B)\).

Choosing \( V_B = \frac{c}{r} \left( \frac{\alpha}{1 + \alpha} \right) \), we have:

\[
D(V) = \frac{c}{r} \left( \frac{V(1 + \alpha)}{(c/r)\alpha} \right)^{-\alpha} \left( \frac{1}{1 + \alpha} \right)
\]

```markdown
Structural Models III 19
```

```markdown
Structural Models III 20
```
Perpetual Debt Example: Final Points

- The *trade-off* for the stockholders is between
  - *benefit* of *maintaining their interest in the firm*; and the
  - *cost* of continuing to *service the debt*.

- The value \( \alpha = \frac{2r}{\sigma^2} \) holds only when the firm makes *no cash payments (zero payout ratio)*. If the firm has debt outstanding, this means that all the coupon and principal payments must be met by the shareholders (via rights issues) and do not detract from existing value firm value, \( V \).

- If the firm has a *positive payout ratio*, the form of the relation still holds but the value of \( \alpha \) is different.

Structural Models and Corporate Finance

- Models of this type are not only useful for trying to understand *pricing* of credit risk but also firms’ capital structure decisions
  - ✓ much of this work has been done by Leland
Structural Models and Corporate Debt Policy

- Corporate debt is typically quite long term (average maturity of outstanding debt ~ 10 years) and over this time many aspects of the firm may change including, particularly, **LEVERAGE**

- Value of corporate debt *today* depends strongly on *future leverage* which is a decision of the firm

- Second generation structural models (particularly, Leland, Leland & Toft) have been used to try to understand *corporate leverage* and *bankruptcy* decisions ($V_B$).

Leland Model: Key innovations

- **Bankruptcy costs**: at default boundary a fraction $\delta$ of the value of firm ($V_B$) is dissipated and bondholders receive only $(1-\delta)V_B$.
  - **note**: the shareholders bear the cost of the bankruptcy costs (because they receive less when the debt is originally issued).

- **Tax benefits of debt**: The debt in Leland’s model is perpetual and he assumes that for each $1$ of coupon payment, the shareholders benefit from a tax deduction of $\tau (\tau$ is the corporate tax rate)
  - the shareholders benefit from the tax deduction because it reduces their cost of servicing the debt
Leland Model – Components of the Firm Value

- The value of the tax-benefits (another defaultable stream, just like the coupons themselves) is:
  \[ TB(V) = \frac{\tau c}{r} (1 - G(V)) = \frac{\tau c}{r} \left( 1 - \left( \frac{V}{V_B} \right)^{-\alpha} \right) \]

- The value of the bankruptcy costs:
  \[ BC(V) = \delta V_B G(V) = \delta V_B \left( \frac{V}{V_B} \right)^{-\alpha} \]

- The total value of the firm – i.e., the value of debt plus equity – is therefore the asset value, \( V \), plus the value of the tax benefits, minus the bankruptcy costs
  \[ v(V) = V + TB(V) - BC(V) = V + \frac{\tau c}{r} \left( 1 - \left( \frac{V}{V_B} \right)^{-\alpha} \right) - \delta V_B \left( \frac{V}{V_B} \right)^{-\alpha} \]

Leland Model – Optimal Financial Policy

- There are two main financial policy variables that affect the value of the firm:
  - the coupon \((c)\): this determines the amount of debt – since the debt is perpetual this is a measure of leverage
  - the default boundary, \( V_B \), where the shareholders give up the firm

- Increasing debt
  - increases the value of the tax benefits but also ….
  - increases the bankruptcy costs

- So Leland’s model is traditional trade-off model of tax benefits versus bankruptcy costs
Optimal Bankruptcy Level, $V_B$

- If shareholders can choose $V_B$, in other words if there are no covenants that protect bondholders, then the value that maximises the value of equity is:

$$V_B = \frac{(1 - \tau)c}{(r + \sigma^2 / 2)} = \frac{(1 - \tau)c}{r} \left( \frac{\alpha}{1 + \alpha} \right)$$

i.e., as before but with after-tax coupon

Optimal Bankruptcy does not Depend on Bankruptcy Costs

- Optimal bankruptcy level ($V_B$) is independent of the bankruptcy cost parameter $\delta$ because the value of equity is:

$$E(V) = V - D(V) = V - \frac{(1 - \tau)c}{r} \left( 1 - \left( \frac{V}{V_B} \right)^{-\alpha} \right)$$

i.e., it is the asset value minus the debt value (i.e., the value of the (defaultable) stream of after-tax coupon payments minus the recovery value ($V_B$)).

- This last amount – the recovery value – is divided between the debtholders (($1-\delta)V_B$) and the amount that is lost, e.g., to the lawyers, ($\delta V_B$) but it doesn’t matter to the equity holders what proportion goes to each and so the optimal value of $V_B$, the one that maximises the equity value, is also independent of the bankruptcy cost, $\delta$. 
Takeaways

• Different *mechanisms* that lead to default

• Early default can *protect bondholders* and so will *not* necessarily lead to *higher spreads*

• Modelling early default – *formula* for $G(V)$

• *Valuing* a defaultable perpetual bond

• *Leland* model of optimal leverage with default