The Black-Scholes-Merton Option Pricing Model

“… options are specialized and relatively unimportant financial securities …”.

Robert Merton – Nobel prize winner for work on option pricing – in 1974 seminal paper on option pricing

- Great hope for the new theory was the valuation of corporate liabilities, in particular:
  - equity
  - corporate debt
Equity is a call option on the firm

- Suppose a firm has borrowed \( \€5 \text{ million} \) (zero coupon) for 5 years (say) and that at the maturity of the loan there are two possible scenarios:

  ✓ **Scenario I:** the assets of the firm are worth \( \€9 \text{ million} \):
    - lenders get \( \€5 \text{ million} \) (paid in full)
    - equity holders get residual: \( \€9 - \€5 = \€4 \text{ million} \)

  ✓ **Scenario II:** the assets are worth, say, \( \€3 \text{ million} \)
    - firm defaults, lenders take over assets and get \( \€3 \text{ million} \)
    - equity holders receive zero

- Payments to equity holders are those of a call option written on the assets of the firm with a strike price of \( \€5 \text{ million} \), the face value of the debt

**Payoffs to Debt and Equity at Maturity**

- Firm has single 5-year zero-coupon bond outstanding with face value \( B = \€5 \text{ (million)} \)

- **Equity** is a call option on the assets of the firm

- Payoff on risky debt looks like this
Prior to maturity … Equity as a Call Option

- Face value $B = €5$ (million); riskless PV of debt $= €3.5$ (million)

![Graph showing the value of equity as a function of the value of assets of the firm.](image)

- **value of equity when assets are risky**
- **value of equity if assets are riskless**

Prior to maturity …

- value of the debt is value of firm’s assets less the value of the equity (a call)

![Graph showing the value of debt and equity.](image)

- **value of debt**
- **value of equity**
- **asset value**
• The sensitivity of a credit risky bond to the value of the collateralising assets (the “firm value”) is a useful way of thinking about credit exposure.

\[
\text{put-call parity: } \text{underlying asset} = \text{riskless bond} - \text{put option} + \text{call option}
\]

\[
\text{Modigliani-Miller: } \text{value of firm assets} = \text{bond value} + \text{equity value}
\]

• Since equity is a call option

\[
\text{value of risky debt} = \text{riskless bond} - \text{option on assets}
\]

• Merton model uses Black-Scholes to value the (default) put.
Limited Liability and the “Default Put”

- *Limited liability* of equity means that no matter how bad things get, equity holders can walk away from firm’s debt in exchange for payoff of zero

- *Limited liability* equivalent to equity holders:
  - issuing *riskless debt*
  - **BUT**
  - lenders giving equity holders a *put* on the firm’s *assets* with a *strike* price equal to the *face amount* of the debt (*“default put”*)

Relation between Bond Spread and Value of Equity

- Bond value increases and the spread declines with the value of firm assets (*left hand panel below*)
- Equity value increases with the value of firm assets (*right hand panel below*)
- Combining these two, the spread also declines with the equity value (*next slide*)
Understanding Credit

- The idea that corporate debt and equity can be viewed as derivatives written on the assets of the firm is of fundamental importance.

- This idea is the basis of the structural approach – one of the two most useful ways of thinking about and analysing credit risky instruments.
Valuation Theory: Merton Model

- **Merton model**: value *credit risky bond* as value of equivalent *riskless bond minus Black-Scholes value of put* on assets

- Assumptions as those of *Black-Scholes* model
  - ✓ lognormal distribution for value of assets of firm
  - ✓ no uncertainty in interest rates
- Merton model:
  - ✓ *basis for all structural models*
  - ✓ has been generalised and extended
The Merton Model: Assumptions

- **Parameters**
  - constant interest rate:
  - constant volatility of firm value

- **Structure of debt**
  - zero coupon bond is only liability – dealing with realistically complex capital structure is problematic

- **Nature of bankruptcy**
  - costless bankruptcy:
    - “undramatic” – simply allocation of property rights between equity and debt holders
    - no loss of value in default (e.g., *nothing for the lawyers*)
  - strict priority of claims preserved: *defines recovery rate* $(1-L)$
  - bankruptcy triggered only at maturity when value of assets falls below face value of debt: *defines default event*

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Bond Prices in the Merton Model
Black-Scholes Formula for Call Option Value

- The Black-Scholes formula for value of a call option on a stock with current price \( S \) and exercise price \( X \), is:

\[
C = SN(d_1) - PV(X)N(d_2)
\]

\[
d_1 = \left( \frac{\ln(S/X) + (r + \frac{1}{2}\sigma_S^2)T}{\sigma_S \sqrt{T}} \right)
\]

and

\[
d_2 = d_1 - \sigma_S \sqrt{T}
\]

Bond Prices in the Merton Model

- The Black-Scholes Value for a call on the firm assets \( V \) with exercise price \( B \), i.e., the value of equity, \( E \), is:

\[
E = VN(d_1) - PV(B)N(d_2);
\]

\[
d_1 = \left( \frac{\ln(V/B) + (r + \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}} \right)
\]

and \( d_2 = d_1 - \sigma_V \sqrt{T} \)

- Since the bond value, \( D \), is the firm value minus the equity value:

\[
D = V - E
\]

\[
= V - \left[ VN(d_1) - PV(B)N(d_2) \right]
\]

\[
= V(1 - N(d_1)) + PV(B)N(d_2)
\]

\[
= VN(-d_1) + PV(B)N(d_2)
\]

since \( 1 - N(x) = N(-x) \)
Credit Spreads in the Merton Model

• The promised yield on the bond in the Merton model is

\[ y = r + s \]

where \( s \) is the "credit spread" and \( y \) is defined by:

\[
D \equiv e^{-yT} B = e^{-(r+s)T} B = e^{-sT} PV(B)
\]

and

\[ D = VN(-d_1) + PV(B)N(d_2) \]

• If we now define the “quasi leverage ratio”, \( L \), as \( PV(B)/V \), i.e., the debt-to-firm value ratio, but valuing the debt using the riskless rate, then we can express the spread simply as a function of \( L \) and \( \sigma \sqrt{T} \)

\[ s = -\frac{1}{T} \ln \left( \frac{1}{L} N(-d_1) + N(d_2) \right), \]

where \( d_1 = \frac{\ln(1/L)}{\sigma \sqrt{T}} + \frac{1}{2\sigma \sqrt{T}} \), and \( d_2 = d_1 - \sigma \sqrt{T} \)

What are Reasonable Values of Leverage and Asset Volatility?

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<th>All</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
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<tr>
<td>Mean</td>
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<tr>
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<td>0.08</td>
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• Quasi-market leverage ratio

\[
\frac{\text{Book Value of Debt (Compustat items 9 and 34)}}{\text{Book Value of Debt + Market Value of Equity}}
\]

• Estimated asset volatility

\[
\sigma_{A_{jt}}^2 = (1 - L_{jt})^2 \sigma_{E_{jt}}^2 + L_{jt}^2 \sigma_{D_{jt}}^2 + 2L_{jt} (1 - L_{jt}) \sigma_{ED_{jt}}
\]

Source: Schaefer & Strebulaev (2009)
The Merton Model, contd.

- For a given maturity, the credit spread in the Merton model depends on only two variables: **asset volatility** and the **quasi-leverage ratio**: \( L = \frac{PV(B)}{V} \)
- When the likelihood of default is high (right-hand panel) the term structure of spreads is likely to be downward sloping;
- When the risk of default is lower (LH panel) it will be upward sloping or hump shaped.

Merton model: spreads vs. leverage and volatility

- In the Merton model the spread over riskless rate increases with volatility and leverage \((L)\)
Hedging Corporate Debt with Equity

• An important implication of the Black-Scholes framework is that it is possible to hedge (in principle perfectly) an option with the underlying stock.

• In the same way, the same framework implies that it should be possible to hedge the credit risk on a corporate bond with a position in the equity.

• This is fundamental to the model because the idea is that both the debt and equity values are driven by changes in the firm value.

• This implies that it should be possible to hedge debt with equity (or vice versa)
Hedging Corporate Debt with Equity

- For a call option that has been sold (bought) the amount of the underlying stock that must be bought (sold) to hedge the position is given by the option delta $\Delta = N(d_1)$

- For a long position in a bond in the Merton model the amount of the underlying stock that must be sold per unit investment in the bond is given by:

$$\left(\frac{1}{\Delta} - 1\right)\frac{E}{D},$$

where $E$ is the value of equity, $D$ is the value of debt and $\Delta$ is the delta of the firm's equity (call) - $N(d_1)$ - against the value of the firm's assets ($V$)

Estimating Hedge Ratios Via Regression

- Running a regression of the rate of return on a bond (say, monthly) against the return on equity will give a good idea of how well (or badly) this hedge will work.

$$R_{bond} = \alpha + \beta R_{Equity} + \epsilon$$

- If the Merton model worked perfectly the $R$-squared in this regression would be very high (although not 100% because the theoretical value of $\beta$ in the Merton model changes as the equity value changes)

- In practice the $R$-squared is much less than 100%
The Lucent Exercise (A)

- The Lucent Exercise (A) will give you the opportunity to see how well or badly the idea of hedging debt with equity works in practice.
- The objective is to get a sense of the relation between debt and equity returns and whether this relation is consistent with the predictions of the Merton model.

Merton - Takeaways

- **Important first step** in modelling default
- Idea of credit risky debt as *riskless debt minus a put* AND using Black-Scholes to value put.
  - ✓ basis for all structural models of credit risk
- Predictions of *credit spreads* appear *too low* (more later)
- **Regression** as a way of investigating the effectiveness of hedging corporate debt with equity
- Occurrence of default only at maturity is major limitation
  - ✓ **BUT**, inclusion of early default does not necessarily increase spreads.