Structural Models of Credit Risk are Useful: Evidence from Hedge Ratios on Corporate Bonds*

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Abstract

It is well known that structural models of credit risk provide poor predictions of bond prices. We show that, despite this, they provide quite accurate predictions of the sensitivity of corporate bond returns to changes in the value of the equity of the issuing firm. This is important since it allows us to identify much better the reasons for model failure. The main result of this paper is that even the simplest of the structural models (Merton (1974)) produces hedge ratios that are not rejected in either time series or cross-sectional test. As well as providing insight into the determinants of corporate bond prices our results are also useful to practitioners who wish to hedge their positions in corporate debt. The paper also shows that corporate bond prices are strongly related to VIX, an index of implied volatility on equity index futures, and the Fama-French SMB factor in way that is not predicted by structural models.

Keywords: Credit risk, structural models, hedge ratios, credit spreads.

I Introduction

It is commonly agreed that structural models of credit risk over-value corporate bonds.[1] Structural models employ the contingent claims approach to value the put option inherent in the contract between lenders and equityholders. Widely used in practice, contingent claims models are seen as one of the major successes of financial theory. The failure of such models to explain satisfactorily actual corporate debt prices and spreads is therefore surprising. While the poor performance of these models in this area has been recognized for many years their failure continues to surprise.

This paper makes two simple but important points. First, while structural models provide a poor prediction of prices and returns, they may perform much better as a predictor of the *sensitivity* – or hedge ratio – of debt to equity. This is important because hedge ratios determine the composition of the replicating portfolio which, according to the theory, determines the price. Thus, if we find that a model provides a good prediction of hedge ratios but a poor prediction of the price, we are better able to identify the reasons for model failure. In fact, we find that even the simplest structural model (Merton (1974)) predicts hedge ratios that are *in line* with those observed empirically. This leads us to reconsider the possible explanations for the failure of structural models to explain better the level of prices and yields.

At present, there are two main explanations for the poor performance of structural models. First, that they fail to predict accurately the probability of default and/or the recovery rate. This explanation has weight because all current models violate at least some known facts about capital structure and/or the circumstances of corporate default. Consequently, the development of much of the theory has been in pursuit of improved ways to model credit events. As a result, we possess an arsenal of models that include stochastic default boundaries, dynamic capital structure and opportunistic behavior on the part of claimholders (e.g., Anderson and Sundaresan (1996), Collin-Dufresne and Goldstein (2001)). Empirical tests have showed, however, that these modifications do not substan-

[1] For early empirical investigation of the Merton model see Jones, Mason, and Rosenfeld (1984). For a more recent analysis see Eom, Helwege and Huang (2003) who study the empirical performance of a number of structural models and find a significant estimation error with some models over-valuing and other models under-valuing corporate bonds.

tially improve the ability of the models to explain the level of corporate bond prices (Eom, Helwege and Huang (2002), Huang and Huang (20002)).

A second explanation is that the pricing of corporate bonds is influenced by factors that are not related to credit risk, and are therefore not included in structural models. It could even be the case that structural models account *very well* for the credit risk component of bond prices and returns while, at the same time, credit risk is actually responsible for only a part, perhaps not even a very large part, of spreads and returns. Some recent results tend to support this view. Elton, Gruber, Agrawal, and Mann (2001) find that differences in taxation account for about a third of credit spreads. Huang and Huang (2002) estimate that credit risk accounts only for a small fraction of the observed credit spread. Collin-Dufresne, Goldstein, and Martin (2001) find that the variables present in structural models explain only a small fraction of the variation in spreads.

On the positive side, a recent paper by Leland (2002) shows that the default probability prediction of structural models are indeed roughly consistent with observed default frequency.[2,3]

Our paper contributes to this on-going debate but, unlike many previous authors (e.g., Huang and Huang (2002)), we do not focus on the *level* of prices or the size of the spread. Instead we investigate the ability of structural models to predict hedge ratios and ask whether these models can be used to hedge corporate bond returns. Using data on monthly returns for a large sample of U.S. corporate bonds over a five-year period, we find that the variables present in structural models explain a large fraction of the returns on investment grade bonds and a smaller but significant fraction for high yield bonds. This result is in itself not surprising, since it has been known for some time that investment grade and government bonds have to a large extent similar driving factors (Campbell and Ammer, 1993). We also find that debt returns are significantly related to returns on the underlying asset and that the pattern of sensitivities is broadly consistent with the level of credit exposure.

[2]This finding means that models that make predicted spreads closer to the actual ones, may substantially overpredict default frequency.

[3]This result is also in line with the eponymous KMV method that uses a Merton-based approach to predict "distance to default".

Thus, our paper focusses attention on the *second-moment predictions* of structural models. In these models, any change in the value of a credit risky bond is the result of a change in the value of the assets that collateralize the debt or in riskless rates. In our empirical analysis we ask whether *the sensitivities* of corporate bond returns to the issuing firm's equity and riskless bond returns are consistent with the model. Our main result is that even the Merton (1974) model produces equity sensitivities that are roughly in line with those observed empirically. Our test is supportive of the view that structural models account well for the credit risk component of corporate bond returns. This positive result is also consistent with Leland's (2002) recent and favorable findings on the default probability predictions of structural models.

A number of authors have found that the returns on corporate bonds are also related to a number of factors that are not present in structural models. These include the Fama-French SMB and HML factors (Elton et. al. (2001)), returns on a broad index of equity prices and "VIX", an index of implied volatility from options on equity indices (Collin-Dufresne et. al. (2001)). In the second half of the paper we include these variables in our analysis of hedged corporate bond returns.

These results represent the second main finding of this paper. We find that while returns on corporate bonds are indeed significantly related to changes in the VIX implied volatility index the sensitivities of bond returns to VIX are not related to any standard measure of credit exposure such as rating, leverage or asset volatility. We also find that returns on corporate bonds are significantly related to the Fama-French SMB factor but this sensitivity does not arise as a result of the bond's exposure to equity (or interest rates). Thus VIX and SMB have significant effects on the prices of corporate bonds but not in any way that is consistent with the predictions of structural models.

Our results also have potential interest for practitioners who wish to hedge positions in corporate debt and suggest that structural models are in fact more useful for this purpose than might be supposed from their performance in explaining the size of credit spreads.

Other authors have previously studied these issues. Blume, Keim, and Patel (1991) also study the behavior of corporate bond returns but not within the framework of structural models. Huang and Huang (2002) use a variety of models to determine whether structural models explain the average level of yield spreads but do not study hedge ratios. Collin-Dufresne, Goldstein, and Martin (2001) analyze changes in yield spreads in a regression framework where the choice of regressors is motivated by structural models. They do not, however, examine whether the size (as distinct from the sign) of the estimated coefficients is consistent with the theory.

Our paper proceeds as follows. Section II provides a description of the data set and the sample selection procedure and also gives descriptive statistics. Section III contains a preliminary regression analysis of the ability of structural models to explain returns on corporate bonds. In section IV the procedure is refined and the regressions take into account the effect on the hedge ratios predicted by the structural model of changes in asset values, volatility and leverage. Section V examines the sensitivity of bond returns to other variables such as VIX and the Fama-French SMB and HML factors. Section VI concludes. Appendix A contains a description of the procedure we develop to estimate the standard error for the cross-sectional mean of hedge ratios in time-series regressions.

II Data, Sample Selection and Descriptive Statistics

II.1 Data

We use monthly prices on corporate bonds that are included either in the Merrill Lynch Corporate Master index or the Merrill Lynch Corporate High Yield index. These indices include most rated U.S. publicly issued corporate bonds. The data covers the period from December 1996 to September 2002. Table I provides descriptive statistics on the bonds in the entire data set. The data set contains more than 308,000 bond-month observations, with about 2800 issuers and 8700 issues. Our analysis requires the bond return data to be matched with CRSP and COMPUSTAT and this allows us to use about 37% of the total number of observations. All rating categories (from AAA to CCC) are represented in the matched sample. As we would expect, as we move down the ratings the average time-to-maturity decreases and the average coupon rate increases. The median size at issuance is $200 million dollars. Detailed information on each bond is obtained from the Fixed Income Securities Database (FISD) as provided by LJS Global Services and equity and treasury bond returns are from CRSP. For riskless rates of return we use constant-maturity US Treasury monthly returns as reported by CRSP.

II.2 Sample selection

For the purposes of our analysis we construct two subsamples. The first, which we call the "cross-sectional sample", is used in the cross-sectional regression analysis, and the specific bonds included in the analysis satisfy the following criteria: (1) the bond is issued by a U.S. company and denominated in $U.S.;[4] (2) it is possible to match unambiguously the bond issuer with a company in CRSP using the CUSIP; (3) the bond is issued by a non-financial corporation. The second, which we call the "time-series sample", is used for the time-series analysis, and, in addition to the criteria above, the following two are added: (4) the bond has an initial maturity of at least four years and (5) the bond has at least 25 consecutive monthly price observations. Table II gives summary statistics for two subsamples of 3631 and 1595 bonds, respectively.[5]

II.3 Descriptive statistics on returns

For each bond j we calculate the rate of return between months t and $t-1$ as follows:

$$r_{j,t} = \frac{P_{j,t} + AI_{j,t} + I_{j,t}C_j/N_j}{P_{j,t-1}},$$

where $P_{j,t}$ is the price of bond j at the end of month t and $AI_{j,t}$ is the change in accrued interest between $t-1$ and t. Since the calculation of the accrued interest restarts with each coupon payment, if the coupon date falls between $t-1$ and t, C_j/N_j is added to the price, where C_j is the annual coupon rate and N_j is the coupon frequency per annum. $I_{j,t}$ is an indicator function taking the value of 1 if the coupon is due between $t-1$ and t. The excess return is then calculated as

$$\overline{r}_{j,t} = r_{j,t} - rf_{1m,t},$$

[4] More specifically, the company is of the U.S. origin according to the FISD definition. In particular, its headquarters should be located in the U.S. and is subject to the U.S. legal practice.

[5] For a number of further descriptive tables we use only the time-series sample. The results for the cross-sectional sample are similar.

where $rf_{1m,t}$ is the return on one-month Treasury bills up to month t.

Table III provides summary statistics on raw and excess returns on individual bonds for the entire time-series sample data set and by rating (where rating, as provided by the S&P, is defined as the rating on the first date that a bond is present in the dataset). The realized average monthly return on corporate bonds is 0.52% and the average excess return is 0.15%.[6] Over this period low-grade bonds had a *lower* average return than investment-grade bonds and a substantially higher standard deviation of the rate of return.

III Structural models and returns

In this section we study the determinants of corporate bond returns implied by structural credit risk models. We take initially a conservative view and include in the regressions only those factors which are explicitly *stochastic* in the Merton model. Later other variables are included – specifically VIX and the Fama-French factors – and the sensitivity of debt returns to equity re-examined.

The value of the firm, V, is the driving state variable in most structural models; indeed, the presence of V is the distinguishing feature of the structural approach. In this section we estimate the sensitivity of debt values to changes in the value of the firm by regressing excess rates of return on corporate bonds against the excess return on the *equity* of the issuing firm.[7] In a one-factor model the elasticity of the value of debt to equity is related to the sensitivity ("delta") of the debt against V by:[8]

$$\frac{\partial D}{\partial E}\frac{E}{D} = \left(\frac{\frac{\partial D}{\partial V}}{\frac{\partial E}{\partial V}}\right)\frac{E}{D} = \left(\frac{1}{\frac{\partial E}{\partial V}} - 1\right)\frac{E}{D}, \tag{1}$$

Clearly, this elasticity is a function of both V and interest rates and therefore varies

[6]We include these statistics on mean returns only for comparison. The period covered by our data is too short to be able to identify means with any precision. The focus of our study, however, is on *second moments* (e.g. hedge ratios) and for this purpose the data is adequate.

[7]Holding volatility constant, an increase in V increases both E and D. Holding V constant, an increase in asset volatility results in a wealth transfer from debtholders to equityholders. The latter issue has been investigated in the structural framework by Leland (1998). An alternative rationale for such a wealth transfer has been investigated in strategic debt service models such as Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997).

[8]We also refer to these sensitivities as hedge ratios since they determine the proportions in the hedging portfolio.

over time. These regressions are, therefore, simply a first step. Later in the paper we use the Merton model to account for time variation in the elasticity. In our regressions we also control for changes in the riskless term structure by including returns on a ten-year constant maturity Treasury bond. While in the first generation of structural models the risk-free rate was held constant, more recent versions include a stochastic interest rate (e.g., Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995)). An increase in the risk-free rate has two offsetting effects on the price of debt. First, it decreases the value of debt by decreasing the present value of all future cash flows. Second, it increases the risk-neutral drift of the value of the firm, V, and so increases the value of debt by decreasing the likelihood of default. The second effect will be relatively more important for bonds with a higher likelihood of default.

Thus we regress the excess return on each bond, $\overline{r}_{j,t}$, on the excess return on the issuing firm's equity, $\overline{r}_{E,t}$, and return on riskless bonds, $\overline{rf}_{10y,t}$:

$$\overline{r}_{j,t} = \alpha_{j,0} + \alpha_{j,E}\overline{r}_{E,t} + \alpha_{j,rf}\overline{rf}_{10y,t} + \epsilon_{j,t}, \tag{2}$$

Table IV reports the average value of the coefficients and their t-statistics computed. However, calculating reliable estimate of these averages is not straightforward since the time series of observations for different bonds often overlap significantly and this may lead to correlation between estimates of hedge ratios. Ignoring what we find to be a significant level of correlation between residuals in our time-series regressions may introduce a substantial downward bias in estimated standard errors. Therefore, we compute the standard error of the average in a way that takes these features into account. This is described in Appendix A.

A number of points are worth noting. Both factors are significant for the whole sample ("All") and for most of the rating categories. For the whole sample, a one percent return on the riskless bond leads to 0.45% increase in the corporate debt price. The standard deviation of treasury bond returns is about 1.5% per month and so the average impact on one-month returns on corporate bonds of a one standard deviation return on government bonds is about 0.75%. The impact of the riskless rate becomes smaller for lower credit rating categories: the loading is significantly positive for investment-grade bonds and negative but insignificant for B and CCC grade bonds. This suggests, quite interestingly, that for very low quality bonds the effect of an increase in the riskless rate – and thus the risk-neutral drift of V – in reducing credit exposure may outweigh the resulting decline in the present value of the promised cash flows.

Ignoring the effects of credit risk and assuming parallel shifts in the riskless term structure, the coefficient on the return on Treasuries would equal the ratio of the duration of the corporate bond to the duration of the Treasury. As the average maturity of our time-series sample of corporate bonds is just over 13 years (see Table II) and the Treasuries have a constant maturity of 10 years, the actual coefficient of 0.45 is clearly lower than the ratio of the durations. Since some of the bonds in our sample are callable, a simple comparison between our estimated coefficient and the ratio of durations (that clearly exceeds one) is not legitimate. However, in a corresponding analysis performed on the sample of non-callable bonds (not reported here) the estimated coefficient was still substantially below unity. This is consistent with a negative correlation between changes in the yield spreads on corporate debt and changes in Treasury yields (see, for example, Longstaff and Schwartz (1995)).

For the full sample, the return impact of a 1% return on equity is 0.041%. If we assume that the standard deviation of monthly equity returns is 12%, then a one standard deviation return on equity increases a bond's return by 0.4% on average. This is smaller than, but of the same order of magnitude as the effect of the risk-free rate. The sensitivities of bond returns to equity are convex in the credit rating: a one percent increase in the stock price increases returns by 1-2 basis points (bp) for AA-A bonds, 4 bp for BBB bonds, and 7-9 bp for BB-B bonds. The average coefficient on equity is significant for each of the ratings (apart from AAA).[9]

In univariate regressions (not reported here), returns on the riskless bond explain a larger fraction of the variability of returns on high-grade bonds than equity, while for low-grade bonds the reverse is true. The two factors combined explain about half the

[9]This is a different result from Collin-Dufresne, Goldstein, and Martin (2001) who find that changes in equity and quasi-market leverage do not have a significant impact on changes in credit spreads.

variation in returns on AAA-A-grade bonds and 20% for low-grade bonds. Even when other regressors are included and the results are computed for subperiods, the sensitivities of corporate bond returns to both equity and riskless bonds remain significant and the coefficients exhibit the same relationship with credit ratings.

As a check on our time series results we have also estimated the equity hedge ratios using cross sectional regression. Our approach has two steps: first, for a given month t, we estimate the average sensitivity of corporate bond returns to equity returns in a cross section:

$$\overline{r}_{j,t} = \alpha_0 + \alpha_E \overline{r}_{E,j,t} + u_{j,t}. \tag{3}$$

We then repeat this regression for each month and compute the average sensitivity over time.

For the monthly regressions the conventional standard errors are both large and, as a result of cross sectional correlation in the residuals, unreliable. We therefore employ the Fama-MacBeth (1973) approach and calculate the standard error of the mean coefficient as the time-series standard error of the monthly coefficients divided by the square root of the number of months. Independence of the rates of return over time means that these standard errors should be reasonably reliable.[10]

The results are contained in Table V which shows that the estimated hedge ratios are significant for the credit rating categories of A and below. The estimated hedge ratios for BBB, BB and B are very similar to the time series estimates in Table IV; for CCC they are somewhat lower. For example the cross-sectional estimate for BBB is 3.80 compared with 3.78 from time series; for B the cross-sectional estimate is 10.52 versus 9.39 from time series. For CCC we obtain 18.35 from the cross-sectional regression and 12.28 from time series. Overall, however, these results are broadly consistent with our time series results and this suggests that the latter are relatively reliable.

[10] One potential problem with this regression is possible bias in the estimated coefficients as a result of omitted variables. For example, we know that differences in duration across bonds in the cross-section will lead the errors to be correlated with duration. Similarly, later in the paper we discuss the impact of the Fama-French SMB and HML factors and changes in VIX. So far we have investigated the effect of omitting the effect of yield curve changes and found that the impact on the estimated ratios is minimal.

IV Debt sensitivity to equity

IV.1 Sensitivity in structural models

Having established that the sensitivities of corporate debt returns to the underlying equity and riskless debt are significant, we now ask whether the magnitude of these sensitivities is consistent with the predictions of structural models. In this paper we employ the Merton (1974) model. This may be a surprising choice because, not only are its assumptions regarding capital structure clearly oversimplified, but it is also well known to underestimate credit spreads by a wide margin (Jones, Mason, and Rosenfeld, 1984). However, it remains an open question whether this simple model is able to explain the *sensitivity* of debt returns to equity and this is the question we now address.

In the Merton model the value of equity is simply the value of a European call option on the assets of the firm with exercise price equal to the face value of the debt. Using equation (1) we may therefore write the sensitivity of the return on a credit risky bond to the return on equity as:

$$h_E = \left(\frac{1}{\Delta_E} - 1\right)\left(\frac{1}{L} - 1\right), \tag{4}$$

where L is market leverage (defined as the market value of debt as a percentage of the market value of the firm) and Δ_E is the delta of a European call option on the value of the firm (equity). In the Merton model the parameters of interest are *book* leverage, B, the volatility of the firm's assets, σ_A, time to maturity, $T - t$, and the risk-free rate, r. The table below shows the comparative statics of h_E (assuming that the option to default is out-of-the-money).

Parameter	h_E
B	$+$
σ_A	$+$
$T - t$	$+/-$
r	$-$

For short maturity bonds the effect of an increase in time to maturity on the dispersion of asset values at maturity is greater than the effect on the risk-neutral expected value of the firm's assets and the hedge ratio increases. For long maturity bonds the second effect dominates and an increase in time to maturity decreases the hedge ratio. Our calculations suggest that, for reasonable parameter values, the actual times-to-maturity in our sample are too small for the second effect to dominate.

Table VI shows the values of h_E for the Merton model for asset volatilities between 10% and 50% and values of L between 10% and 70%. The risk-free rate is 5% per annum and the time-to-maturity is 10 years. For each pair of values of volatility and leverage we simulate a time-series of monthly asset values over a five-year period and compute the corresponding bond and equity values under the Merton model. Using these values we estimate the hedge ratio in a regression of bond returns on corresponding equity returns. The numbers reported in the table are the averages of the estimated hedge ratios for each pair over 1000 simulated paths. The values of volatility and leverage have been chosen to reflect those encountered in our sample. The table shows that the sensitivity varies significantly from zero (less than 0.01), when both leverage and volatility are low, to about 0.3 when both leverage and volatility are high.

IV.2 Preliminary comparison of sensitivity in the Merton model and actual data

As a first step in investigating the relationship between the theoretical Merton model sensitivities and those estimated empirically we perform a similar simulation. The object of this simulation is to calculate the mean value of the sensitivity of bond returns to equity in our data when (a) the Merton model holds and (b) the sensitivities are estimated using linear regression on monthly data that has the same characteristics as our sample. For a given rating class, the difference between the mean sensitivity we obtain in this way and the sensitivity calculated from Merton model using the mean values by rating class, leverage, volatility, and time to maturity can be attributed to (a) non-linearity and (b) the discrete time interval between observations.

In Table VII we aggregate the seven principal rating classes into three: "AAA-A" (including AAA, AA, and A bonds), "BBB", and "Junk" class (BB, B, and CCC bonds). For each of the three classes we find the 5% and 95% quantiles of leverage,[11] volatility, and time to maturity and we take these to be the minimum and maximum points of unform distributions from which we draw values in the simulation.[12] The table below gives the upper and lower limits for the three variables for each rating class.

Parameter		AAA-As	BBB	Junk
Leverage				
	min	0.15	0.2	0.3
	max	0.35	0.45	0.7
Asset volatility				
	min	0.1	0.15	0.2
	max	0.3	0.35	0.5
Time to maturity				
	min	5	5	5
	max	20	20	20
Frequency		0.48	0.35	0.17

We now generate 2000 time series of "bond returns" as follows. First, we assign a rating class to each time series according to the proportions found in the actual data. Second, again for each time series, we randomly draw values for leverage, volatility, and time to maturity from the distributions for the relevant credit class and generate a time series of asset values. Using the Merton model we then calculate monthly equity and bond prices and, from these, monthly returns. Finally, we estimate the hedge ratio, h_E, by running a regression of the simulated bond returns on simulated equity returns (i.e., a regression similar to (2) but excluding the return on the riskless bond.)[13]

Table VII reports the mean values of h_E and $\bar{R}^2$ for these regressions. Comparing these results and those in Table IV we see that the sensitivities are surprisingly similar. For high quality (AA-A) bonds, the average sensitivity is found to be about 0.01 for simulations and 0.01–0.02 for the actual data;[14] for BBB bonds both for the model and the actual data produce a sensitivity of 0.04 and for junk bonds we find 0.15 for the simulations and 0.07–

[11]The ratio of the face value of the firm's debt to the market value of assets.

[12]We also tried to account better for the distribution patterns of leverage and volatility within the rating class; it did not produce any significant changes.

[13]We also run regressions on the actual data using only equity returns to be consistent. The results are broadly similar with the equity sensitivities slightly larger.

[14]We exclude the estimate of AAA as it is insignificant.

0.12 bp for the actual data. The mean values of the simulated and empirical hedge ratios are not significantly different at a 5% confidence for the entire sample and the "AAA-A" and "BBB" subsamples and at the 10% level for the "Junk" subsample. These results are the more surprising since, for the same simulated data, the model underestimates the observed level of credit spreads by more than 80%, or from 50bp for AAA bonds to more than 250 basis points for low-grade bonds.

IV.3 Preliminary Analysis of Hedge Ratios

The results in the previous section raise the possibility that, although the Merton model leads to poor predictions of credit spreads, it may perform better as a predictor of hedge ratios. This result, if substantiated, would be important because, in contingent claims pricing theory, the hedge ratios define the composition of the replicating portfolio which, in turn, defines the contingent claim price. Thus, if we find that the model provides good predictions of hedge ratios but poor predictions of the bond price, we are better able to identify the reasons for model failure.

To address this issue, we next test the second-moment predictions of the model in a more rigorous manner. Note that if the model is correct then in equation (2) $\alpha_{j,E}$ is equal to h_E and we can therefore rewrite the regression as

$$\overline{r}_{j,t} = \alpha_{j,0} + \beta_{j,E} h_{E,j,t} \overline{r}_{E,t} + \alpha_{j,rf} \overline{rf}_{10y,t}, \tag{5}$$

where $h_{E,j,t}$ is the model hedge ratio for firm j at time t and under the null hypothesis that the Merton model holds, $\beta_{j,E}$ is equal to one. To implement (5) the following parameters need to be estimated for each firm: the ratio of the book value of debt to the market value of assets, B/V, the volatility of assets, σ_A, time to maturity, $T - t$, and the riskless rate.

To estimate $\frac{B}{V}$, we take the ratio of the book value of debt (sum of COMPUSTAT items 9 (long-term debt) and 34 (debt due within a year)) to the quasi-market value of assets (sum of COMPUSTAT items 9 and 34 plus the number of shares outstanding times the stock price (both CRSP)). The COMPUSTAT data are taken at the date of the last annual accounting report and the CRSP data are taken on the date of observation.

The estimation of asset volatility is a challenging task and here we consider a number of alternatives. First, we compute upper and lower bounds on asset volatility as follows. The maximum equates asset and equity volatility, σ_E, i.e., assuming zero leverage. The minimum is calculated as $\sigma_E(1-L)$, where L is the market leverage, i.e., assuming that the debt bears no asset risk. In this case, the theoretical hedge ratio is zero.

A more realistic estimate of asset volatility recognizes that debt bears some asset risk and that equity and debt covary. For firm j at time t we have:

$$\widehat{\sigma^2_{Ajt}} = (1-L_{jt})^2\sigma^2_{Ejt} + L_{jt}{}^2\sigma^2_{Djt} + 2L_{jt}(1-L_{jt})\sigma_{ED,jt}, \tag{6}$$

where σ_{Djt} is the time t volatility of firm j's debt and $\sigma_{ED,jt}$ is the time t covariance between returns on firm j's debt and equity.[15] In principle, we could estimate firm j's debt volatility using the returns on each of firm j's bonds but this approach has two drawbacks. First, it assumes that all of a firm's outstanding debt has the same volatility as its publicly traded debt. Second, much corporate debt is relatively illiquid and some of the observed volatility may be spurious.

We therefore estimate equation (6) as follows. Firm j's equity volatility at time t is estimated as the time series volatility of returns on firm j's equity using three years of monthly data up to month t. For the volatility of returns on firm j's debt we first calculate the average volatility of debt returns by credit rating. Thus, for rating category BBB and firm j, for example, we take the returns on firm j's debt for each month that the debt was rated BBB at the start of the month. If the bond in question is rated BBB in at least 15 months we then compute the time series volatility. Averaging these volatilities over all firms we obtain the average volatility for BBB debt. The volatility of firm j's debt in month t is then set equal to the average volatility for the rating category of firm j at month t.[16]. The covariance between equity and debt returns, $\sigma_{ED,jt}$, is estimated as $\rho_{ED,jt}\sigma_{D,jt}\sigma_{E,jt}$ where $\rho_{ED,jt}$ is estimated in a similar manner to $\sigma_{D,jt}$.

Finally, we take time to maturity as equal to the median time to maturity for each

[15] This calculation assumes that leverage is measured instantaneously.

[16] One can think of this as a form of "switching regime" where the volatility for firm j switches between the volatilities of the different rating categories

rating class and the riskless rate as 5%.

Table VIII reports summary statistics for these estimates. As expected, both leverage and equity volatility are higher for lower rating categories. The average values of equity volatility and quasi-market leverage are broadly consistent with similar results reported in other studies.[17] Deleveraging equity volatility using L, but taking no account of the asset risk borne by debtholders (the third panel of Table VIII), results in estimates of asset volatility ("$(1 - L)\sigma_E$") that are relatively constant across the rating categories.

The fourth panel of Table VIII gives estimates of $\widehat{\sigma_A}$ using equation (6) and the method described above. Here the mean values of asset volatility are very similar for AA, A and BBB (investment grade) bonds (all equal to 22%) but noticeably higher for junk bonds: 26% for BB, 31% for B and 38% for CCC. The range of values, as measured by the 5% and 95% quantiles, is also much wider for the lower rated bonds, e.g.,11-32% for AA vs. 21-57% for B.

Table IX shows estimates of the hedge ratio $h_E(\sigma_E)$. The second and third panels set σ_A equal to σ_E and $(1 - L)\sigma_E$ respectively. The first – more realistic – panel shows estimates of the hedge ratio using the estimates of σ_A derived using equation (6). As expected, these rise monotonically as the rating category declines from around 0.5-1.5 basis points for AA-A, 2.3 basis points for BBB and 10.8 basis points for B.

IV.4 Testing Merton Model Predictions of Hedge Ratios

We now use our estimates of asset volatility to test more formally whether hedge ratios calculated using the Merton model are consistent with the empirical relation between equity and corporate bond returns.

For firm j we take the time t estimate of asset volatility described above and use this to calculate the time t hedge ratio, $h_{E,jt}(\widehat{\sigma_A})$.[18] We then estimate the following regression for each firm, j:

$$\overline{r}_{j,t} = \alpha_{j,0} + \beta_{j,E} h_{E,jt}(\widehat{\sigma_A})\overline{r}_{E,t} + \alpha_{j,rf}\overline{rf}_{10y,t}. \tag{7}$$

[17] For example, Davydenko and Strebulaev (2003), using a different sample of corporate bonds, arrive at similar numbers.

[18] The other inputs – book leverage, time to maturity and the riskless rate – are as described earlier.

Under the null hypothesis that the Merton model correctly estimates the sensitivity of returns on firm j's debt to firm j's equity, the coefficient $\beta_{j,E}$ should be unity.

The results are given in Table X. For the entire sample the mean estimate of $\beta_{j,E}$ is 1.18 and the t-statistic against unity, the value of $\beta_{j,E}$ under the null, is 0.30. For the seven rating categories the mean value of $\beta_{j,E}$ is significantly different from unity only for the CCC category where the mean estimate of 0.60 is five standard errors below unity. For the other six categories the mean value ranges from -2.91 (AAA) to 1.61 (BBB) and none is significantly different from unity.

As in the case of the raw time series regressions on equity returns and riskless bond returns (Tables IV and V) we supplement our time series results in Table X with a cross sectional analysis. The regression equation:

$$\bar{r}_{j,t} = \alpha_0 + \alpha_E h_E \bar{r}_{E,j,t} + u_{j,t}, \tag{8}$$

is the cross-sectional analogue to equation (7) where, once again, the expected value of α_E is unity.

The results are given in Table XI where, using the entire sample, we find that the mean estimated coefficient is 0.98 with a t-statistic (against unity) of -0.19. For BBB the estimated value is rather high at 2.03 but all the other six estimates lie in the range 0.63 (AA) to 1.46 (BB) and none of the seven is significantly different from unity.

Both the time-series and the cross-sectional results are supportive of the structural approach, and the Merton model in particular, in a way that previous analyses focussing on the level of prices or credit spreads have not been. They are also complementary to the results recently obtained by Leland (2002) who shows that the default frequency predictions of structural models are broadly consistent with the data.

Apart from the size of the yield spread, there is a further prediction of structural models that is inconsistent with the results in Table X. This is the $\bar{R}^2$ in the regression which, if the Black-Scholes assumptions supporting the structural approach applied, would be much higher. In our simulations reported in Table VII the $\bar{R}^2$ varied from 0.65% for AAA-A to

0.936% for BB-CCC.[19] In Table X the $\bar{R}^2$ are much lower (24% for CCC and 53% for AA).

The small fraction of rate of return volatility explained by equity returns and changes in interest rates has a number of possible explanations. One is simply that it reflects noise in the bond return data (or, possibly in the equity or riskless bond data). The first of these almost certainly accounts for some of the unexplained variability of corporate debt returns since it is well known that liquidity in many corporate bond issues is limited. Another is that the model is mis-specified and that either the functional form of the hedge ratio is incorrect or that other variables are necessary to account for the credit exposure of corporate debt. For example, the conditions for the Merton model could hold except that volatility is stochastic and this would mean that other variables are necessary to predict volatility. Finally, returns on corporate debt could be related to other variables in a way that is not directly related to credit risk. For example, returns on corporate debt might be related to variables that proxy for changes in liquidity.

V Other Determinants of Returns on Corporate Bonds

In this section we consider the impact of variables that other authors have found to be significant explanators of returns on corporate bond. These include (i) changes in the 10-year minus 2-year yield spread on US Treasuries, (ii) the return on the S&P 500 index, (iii) changes in the VIX index of implied volatility of options on the S&P 100 index and (iv & v) the Fama-French SMB and HML factors. All five factors are included in the recent study by Collin-Dufresne et. al. (2001) and the Fama-French factors are included by Elton et. al. (2001).

The results are shown in Table XII. The mean coefficients on 10-year Treasury returns are very similar to, but around 5-10 basis points higher than, those in Table IV and those on equity are also similar but a little lower.

The mean coefficients on the S&P are not significant for any credit category. The mean coefficients on HML are significant for the whole sample and for three out of the seven

[19]Note that in these simulations the interest rate is held constant. Since the great majority of the risk of high grade bonds is interest rate related, including interest rate risk in the simulations would increase the $\bar{R}^2$ substantially.

rating subsamples while those on SMB are significant for the whole sample and all the subsamples except for CCC.

The most interesting results, however, are those for VIX and SMB. Collin-Dufresne et al. (2001) had previously found this variable to be significant in regressions of changes in yield spreads on a very similar list of regressors. The mean coefficients for $\Delta(VIX)$ are significant for each of the investment grade credit categories; in fact, in these cases the t-statistics are higher than those on the firm's own equity. A natural interpretation of the role of $\Delta(VIX)$ in the regression is as a proxy for changes in the volatility of equity (or assets). In this interpretation a bond's sensitivity to VIX is related to its credit exposure via the effect of volatility on the value of the default put. However, when we examine the magnitude of the coefficients for the different rating subcategories we see that they are essentially the same: for the rating categories between AAA and BB all the estimated coefficients are in the range -0.047 to -0.060. For example, the mean coefficients for AAA and AA are -0.058 and -0.060 respectively and for BBB and BB are -0.047 and -0.056 respectively. If $\Delta(VIX)$ were acting as a proxy for changes in equity volatility the (absolute) sensitivity for the lower credit categories, as in the case of the coefficients on equity returns, would be much larger.[20] The $\bar{R}^2$ in Table XII are significantly higher than in Table IV but particularly so for the non-investment grade bonds, for example, the average $\bar{R}^2$ for BB and B increases to 33% from 24%.

For SMB the t-statistics are even larger than those on $\Delta(VIX)$ and here the coefficients generally increase with maturity. The average coefficient is significant for the entire sample and for each of the rating categories apart from CCC. Just as one might (incorrectly) imagine that the sensitivity of corporate bonds to VIX is related to their exposure to asset volatility, so it might be supposed that they are sensitive to the Fama-French SMB factor because the corporate bond value is related to the value of the issuing firm's equity and the equity is related to SMB. However, the regressions in Table XII already control for changes in the value of issuing firm's equity and so this explanation is unlikely to be valid. To investigate this point further we have taken the residuals from the regressions

[20]We have also carried out cross-sectional regressions of the individual coefficients on $\Delta(VIX)$ on simple descriptors of credit risk, such as equity volatility and leverage, and found no relation.

in Table IV and regressed these on all the variables in Table XII except for equity and riskless bond returns. The table below reports the coefficients of SMB in this regression and, for a comparison, those from Table XII:

All	AAA	AA	A	BBB	BB	B	CCC
Panel (a): SMB coefficients from regressions on residuals from Table IV							
9.743	6.044	6.104	8.524	9.439	15.895	11.756	11.570
(6.945)	(3.475)	(5.679)	(10.418)	(6.343)	(5.754)	(2.424)	(0.921)
Panel (b): SMB coefficients from regressions from Table XII							
10.73	6.46	6.69	9.22	10.42	18.28	12.66	14.45
(7.11)	(3.34)	(5.89)	(10.68)	(6.65)	(6.07)	(2.37)	(0.98)

The residuals in this regression are, by construction, orthogonal to the returns on the issuing firm's equity and yet we find the coefficients on SMB (and the other variables) are essentially unchanged.

To summarize, it appears that corporate bond returns are (a) highly sensitive to the implied volatility on equity index options but in a way that is unrelated to their credit exposure and (b) highly sensitive to the Fama-French SMB factor but independently of their exposure to the equity of the issuing firm. We do not have explanations for either of these surprising results but both suggest that there is an important component in the returns on corporate bonds that is not explained by the "fundamentals" that appear in structural models such as the Merton model.

VI Conlusion

This paper studies the ability of structural models to predict the hedge ratios of corporate bonds against the equity of the underlying firm. Using data on monthly returns for a large sample of U.S. corporate bonds over a five-year period, we find that those variables included in structural models – returns on the issuing firm's equity and on riskless bonds – explain a large fraction of the returns on investment grade bonds and a smaller but significant fraction for high yield bonds. Further, and this is a principal result of the paper, we find that, for most rating categories, the equity ratios predicted by the Merton model are not rejected in either time series or cross-sectional tests.

The next step is to account for other factors. We include in our regression variables that in previous studies have been shown to influence corporate bond prices. The variables we use are: (i) changes in the 10-year minus 2-year yield spread on US Treasuries, (ii) the return on the S&P 500 index, (iii) changes in the VIX index of implied volatility of options on the S&P 100 index and (iv & v) the Fama-French SMB and HML factors. We find that when these variables are included the sensitivities of corporate bonds to the risk-free rate and equity are largely unchanged. Our main result here, and the second main result of the paper, is that changes in both VIX and SMB have an impact on corporate bond returns that is both significant and not predicted by structural models. It seems clear, therefore, that returns on credit risky bonds are systematically related to at least two factors that lie outside standard measures of "credit risk". Whether there are still important other factors, and the precise role of $\Delta(VIX)$ and SMB in the determination of risky bond prices, is a question for further research.

Appendix A Computation of Standard Errors for Cross-Sectional Mean of Hedge Ratios

For the time series regressions described in this paper (e.g. Tables X, XII and IV) we report average values of the estimated coefficients. However, calculating reliable estimate of these averages is not straightforward since the time series of observations for different bonds often overlap significantly and this may lead to correlation between estimates of hedge ratios. Here, we compute the standard error of the average in a way that takes these features into account and discuss its properties.

Consider the regression equation (2)

$$\overline{r}_{j,t} = \alpha_{j,0} + \alpha_{j,E}\overline{r}^{E}_{j,t} + \alpha_{j,rf}\overline{rf}_{10y,t} + \varepsilon_{j,t} \tag{A1}$$

For each bond j returns are observed for only T_j periods. Define X_j as a $T \times 1$ vector, where $x_{j,t}$ is equal to the regressor value if the bond return is known at date t and 0 otherwise. Also, define ε_j as a $T \times 1$ vector where $\varepsilon_{j,t}$ is equal to the residual term if the bond return is observed at date t and NaN otherwise. NaN has two properties: the multiplication or summation of NaN and any number (including NaN) is NaN; when summation is done, all values equal to NaN are ignored.[21]

Then, under the usual regression assumptions – and using conventional notation – the estimation error in the parameter vector α_j can be written as

$$\left(X_j' X_j\right)^{-1} X_j' \varepsilon_j \equiv A_j' \varepsilon_j \tag{A2}$$

and, assuming $E(\varepsilon^2_{j,t}) = \sigma^2_{\varepsilon,j}\ \forall t$,

$$Var(\alpha_j) = \left(X_j' X_j\right)^{-1} \sigma^2_j \tag{A3}$$

Similarly, assuming $E(\varepsilon_{j,t}\varepsilon_{k,t}) = \sigma_{\varepsilon,j,k}\ \forall t$, the variance-covariance matrix of the estimates

[21] We have to introduce this notation for non-overlapping observations since, if NaN were replaced by 0, the mean error would be calculated over all T observations (rather than T_j) and this would result in a downward bias.

of α for two firms j and k is:

$$Cov(\widehat{\alpha_j}, \widehat{\alpha_k}') = \sigma_{\varepsilon,j,k} A_j' I^{j,k} A_k \tag{A4}$$

where $I^{j,k}$ $(T \times T)$ is a diagonal matrix with element $I_{t,t}^{j,k}$ equal to one if observations of the return on both bonds j and k are available in month t and zero otherwise. Thus, $Trace(I^{j,k})$ is the number of months for which observations on both bonds are available.

The mean value of the average of $\widehat{\alpha_j}$ over j is:

$$\overline{\widehat{\alpha}} \equiv \frac{1}{N} \sum_{j=1}^{N} \widehat{\alpha_j} \tag{A5}$$

and therefore the variance-covariance matrix of $\overline{\widehat{\alpha}}$ is:

$$\begin{aligned} Cov(\overline{\widehat{\alpha}}, \overline{\widehat{\alpha}}') &= \frac{1}{N^2} \sum_{j=1}^{N} \sum_{k=1}^{N} Cov(\alpha_j, \alpha_k) \\ &= \frac{1}{N^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \sigma_{\varepsilon,j,k} A_j' I^{j,k} A_k. \end{aligned}$$

To calculate t-statistics for $\overline{\widehat{\alpha}}$ we need to compute the diagonal elements in the variance-covariance for $\overline{\widehat{\alpha}}$ above and this requires an estimate of the correlation, $\rho_{\varepsilon,j,k}$, between pairs of residuals in the N regressions. Under the assumption that $\rho_{\varepsilon,j,k} = \overline{\rho_\varepsilon}\ \forall j, k, j \neq k$, the estimation of the standard error is derived from this formula:

$$\widehat{Cov(\overline{\widehat{\alpha}}, \overline{\widehat{\alpha}}')} = \frac{1}{N^2} \sum_{j=1}^{N} \left(X_j' X_j\right)^{-1} \widehat{\sigma_{\varepsilon,j}^2} + \overline{\rho_\varepsilon} \frac{1}{N^2} \sum_{j=1}^{N} \sum_{k \neq j} \widehat{\sigma_{\varepsilon,j}} \widehat{\sigma_{\varepsilon,k}} A_j' I^{j,k} A_k. \tag{A6}$$

This is a different approach from that adopted by Collin-Dufresne et al. (2001) who estimate the cross-sectional standard deviation in estimated values of each α_j and then scale by $\sqrt{N}$. This would be the appropriate procedure if (a) the cross-sectional variation in true parameters were zero (implying that all cross-sectional variation in estimated sensitivities is the result of estimation error) and (b) each estimate in the cross-section were independent. In fact, neither of these conditions hold.

To gauge the importance of using the revised standard error, consider the behavior of the standard error in the limit. Under mild technical assumptions on the finiteness of variance and regressors, the first term vanishes as $N \to \infty$ while the second term converges to a positive constant We see that ignoring the dependence in the cross-sectional estimates understate standard errors and, even for very large N, the understatement can be substantial even for small values of correlation. Conversely, ignoring only partially overlapping nature of the data overstates standard errors. In our procedure we take account of both.

Finally, we report below estimated values of $\overline{\rho_\varepsilon}$ for the whole dataset and for various ratings.

	All	AAA	AA	A	BBB	BB	B	CCC
$\overline{\rho_\varepsilon}$	0.112	0.459	0.192	0.079	0.157	0.211	0.228	0.230
N	(1595)	(11)	(125)	(553)	(540)	(145)	(208)	(13)

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Table I
Summary statistics for the entire data sample

This table reports summary statistics for the entire data set on corporate debt returns over the period 12.1996–12.2001. The number of observations is given in thousands. $T-t$ is time to maturity remaining on the date of each observation. The coupon rate is in percent. Nominal Value is the nominal amount at issue in million $US dollars. "% in CRSP" reports the fraction of observations that have been matched with CRSP and "% in COMPUSTAT" – the fraction of observations that have been matched with COMPUSTAT.

	All	AAA	AA	A	BBB	BB	B	CCC
No. Observations	308.78	6.89	31.21	113.51	90.40	25.72	31.48	9.58
No. Issuers	2807	74	237	812	1019	653	1039	514
No. Issues	8717	287	1125	3738	3243	1461	1608	768
Mean $T-t$	10.59	11.64	11.02	10.91	12.04	9.29	6.83	6.64
Median $T-t$	6.96	7.30	6.47	6.77	7.87	6.63	6.84	6.05
Mean Coupon	7.83	7.39	7.07	7.31	7.58	8.42	10	10.38
Median Coupon	7.50	7.25	6.88	7.13	7.38	8.35	9.88	10.25
Mean Nominal Value	279	339	338	292	269	246	230	225
Median Nominal Value	200	250	200	200	200	200	175	179.30
% in CRSP	37.21	8.10	29.13	37.58	42.52	35.65	38.86	28.92
% in COMPUSTAT	34.16	7.89	28.18	33.60	39.85	32.11	35.71	25.97

Table II
Summary statistics for the final sample

This table reports summary statistics for the final samples over the period 12.1996–12.2001. Observations are given in thousands. $T - t$ is time to maturity remaining on the date of each observation. Coupon rate is in %. Volume is in million $US dollars. Panel 1 reports for the sample used in cross-sectional analysis. Cross-sectional sample selection criteria are: (1) the bond is issued by a U.S. company and denominated in $U.S.; (2) it is possible to match unambiguously the bond issuer with a company in CRSP using the CUSIP; (3) the bond is issued by a non-financial corporation. Panel 2 reports for the sample used in time-series analysis. The time-series sample selection criteria are (1)-(3) above and also: (4) the bond has an initial maturity of at least four years and (5) the bond has at least 25 consecutive monthly price observations.

	All	AAA	AA	A	BBB	BB	B	CCC
Panel 1: Cross-Sectional sample								
No. Observations	104.65	0.54	8.77	37.90	35.74	8.20	11.07	2.44
No. Issuers	1047	9	52	276	413	217	358	141
No. Issues	3631	17	325	1446	1484	541	597	231
Mean $T - t$	11.44	20.92	11.16	12.83	12.32	8.02	7.07	7.37
Median $T - t$	7.67	25.27	6.88	8.26	8.12	6.67	7.18	6.84
Mean Coupon	7.79	6.96	6.98	7.39	7.51	8.41	9.82	10.04
Median Coupon	7.48	6.80	6.75	7.13	7.25	8.35	9.75	9.88
Mean Nominal Value	267.21	331.24	338.25	281.98	251.18	260.05	221.62	234.06
Median Nominal Value	200	300	250	200	200	200	175	200
Panel 2: Time-Series sample								
No. Observations	67.57	0.49	4.88	21.71	25.75	6	7.32	1.42
No. Issuers	536	4	30	155	240	136	170	68
No. Issues	1593	11	133	642	778	308	277	105
Mean $T - t$	13.23	22.56	13.35	15.92	13.86	8.60	7.15	8.18
Median $T - t$	8.34	25.64	8.21	12.10	8.76	6.84	7.15	6.96
Mean Coupon	7.78	6.96	7	7.44	7.49	8.31	9.62	9.83
Median Coupon	7.48	6.80	6.75	7.15	7.25	8.13	9.50	9.75
Mean Nominal Value	260.26	324.90	315.54	271.46	254.22	252.19	219.74	229.14
Median Nominal Value	200	300	250	200	200	200	175	200

Table III
Summary statistics on returns

This table reports summary statistics on excess and raw returns for the time-series sample (using the selection procedure described in the paper) of corporate debt returns over period 12.1996–12.2001. Returns are first calculated for each bond separately and then averaged across bonds. Excess returns are given in parentheses. N is the number of bonds.

	All	AAA	AA	A	BBB	BB	B	CCC
Mean	0.52	0.81	0.66	0.63	0.55	0.42	0.19	-0.67
	(0.15)	(0.46)	(0.30)	(0.26)	(0.19)	(0.06)	(-0.18)	(-1.06)
5%	-0.76	0.57	0.46	0.35	-0.22	-1.10	-2.87	-5.16
	(-1.14)	(0.17)	(0.08)	(-0.02)	(-0.58)	(-1.46)	(-3.24)	(-5.59)
95%	1.01	1.16	0.88	0.90	1.00	1.01	1.27	2.90
	(0.66)	(0.83)	(0.54)	(0.55)	(0.65)	(0.64)	(0.93)	(2.49)
std	3.18	2.14	1.83	2.15	3.01	4.25	5.97	10.84
	(3.18)	(2.15)	(1.83)	(2.15)	(3.01)	(4.24)	(5.97)	(10.84)
N	1595	11	125	553	540	145	208	13

Table IV
Regressions of Excess Returns: Time-Series Sample

For the time-series sample, this table reports the results of regressing excess returns on corporate bonds over the period 12.1996–12.2001 on equity returns and risk-free bond returns. For each bond j the following regression was estimated:

$$\overline{r}_{j,t} = \alpha_{j,0} + \alpha_{j,E}\overline{r}_{E,t} + \alpha_{j,rf}\overline{rf}_{10y,t} + \epsilon_j, t$$

The table reports the means of the estimated coefficients across bonds. The t-statistics are given in parenthesis and calculated using a formula given in Appendix A. rf_{10y}^{ret} is the excess return on the 10-year constant maturity U.S. Treasury bond and E^{ret} is the excess return on the issuer's equity. All coefficients are given in basis points. N is the "average" number of observations per bond (in parenthesis the total number of bonds in the sample).

	All	AAA	AA	A	BBB	BB	B	CCC
Intercept	-0.06	-0.14	-0.11	-0.12	-0.10	0.20	0.15	-0.73
	(-0.93)	(-1.77)	(-2.32)	(-3.45)	(-1.63)	(1.72)	(0.79)	(-1.53)
rf_{10y}^{ret}	44.54	87.51	69.01	60.41	51.97	19.71	-11.59	-35.79
	(14.23)	(21.13)	(27.14)	(32.72)	(15.36)	(3.14)	(-1.11)	(-1.34)
E^{ret}	4.08	-0.20	1.44	2.06	3.78	7.09	9.39	12.29
	(10.09)	(-0.23)	(2.76)	(6.44)	(7.89)	(9.60)	(11.04)	(8.65)
$\bar{R}^2$	0.39	0.71	0.57	0.48	0.36	0.24	0.14	0.28
N	42.43	47.72	48.74	42.75	42.09	41.59	39.39	36.54
	(1595)	(11)	(125)	(553)	(540)	(145)	(208)	(13)

Table V
Regression of Excess Returns: Cross-Sectional Method

This table reports results of a cross-sectional regression analysis of excess returns on corporate bonds over the period 12.1996-12.2001 for the cross-sectional sample (using the selection procedure described in the paper) of corporate bonds. Standard error are calculate using the Fama-MacBeth (1973) estimation procedure. For each of 60 months the following cross-sectional regression is run

$$\overline{r}_{j,t} = \alpha_0 + \alpha_E \overline{r}_{E,j,t} + u_{j,t}.$$

Then the resulting coefficients are averaged across time. E^{ret} is the excess return on the issuer's equity. N is the average number of observations in each month. t-statistics are given in parentheses.

	All	AAA	AA	A	BBB	BB	B	CCC
Intercept	0.55	0.71	0.63	0.60	0.49	0.61	0.48	1.57
	(3.29)	(3.05)	(3.72)	(3.40)	(2.47)	(3.73)	(2.44)	(2.85)
E^{ret}	7.80	0.33	0.25	0.85	3.80	5.98	10.52	18.35
	(8.48)	(0.17)	(0.71)	(2.46)	(3.49)	(5.69)	(10.01)	(7.81)
$\bar{R}^2$	0.08	0.12	0.01	0.02	0.05	0.08	0.14	0.17
N	1430.45	8.68	95.55	443.82	537.12	131.43	175.98	37.87
	(60)	(60)	(60)	(60)	(60)	(60)	(60)	(60)

Table VI
Sensitivities of bond returns to equity in the Merton (1974) model

This table reports sensitivities of bond returns to equity in the Merton (1974) model for a range of values of leverage and asset volatility that covers most of the sample. To estimate sensitivities, for each value of volatility and leverage, 1000 25-month bond returns were simulated and time-series regressions were run for each bond. The means of hedge ratio and t-stats (in parenthesis) are reported. Leverage is the ratio of the quasi-market value of debt over the market value of assets. The quasi-market value of debt is the face value of debt discounted at the riskless rate of interest. σ_A is asset volatility. Leverage and asset volatility are given in percent. The riskless rate is assumed to be equal to 5% and the time to maturity, 10 years. Sensitivities are given in basis points (0.01%).

	Volatility, σ_A						
	10	15	20	25	30	40	50
Leverage, L							
10	0.00	0.00	0.04	0.58	2.53	9.25	17.19
	(0.23)	(8.17)	(10.65)	(14)	(18.26)	(26.86)	(33.20)
20	0.00	0.03	0.70	2.63	4.80	13.52	21.39
	(5.94)	(8.04)	(13.19)	(19.22)	(19.95)	(33.39)	(39.42)
30	0.00	0.86	3.42	4.60	13.60	15.31	29.03
	(5.48)	(15.70)	(22.11)	(20.52)	(33.71)	(33.53)	(48.97)
40	0.10	1.42	6.88	10.41	16.44	23.73	31.58
	(10.30)	(13.97)	(30.34)	(32.63)	(40.95)	(50.45)	(55.86)
50	0.49	5.19	8.05	11.88	17.50	24.17	29.68
	(10.04)	(25.37)	(28.51)	(32.29)	(44.62)	(51.17)	(54.74)
60	2.06	4.47	13.25	16.60	19.53	24.79	34.08
	(20.66)	(18.37)	(45.26)	(46.44)	(43.40)	(52.19)	(62.05)
70	3.20	7.98	15.25	14.73	21.79	27.83	29.87
	(17.23)	(32.19)	(51.41)	(40.06)	(55.52)	(60.84)	(49.06)

Table VII
Sensitivities of debt returns to equity implied by the Merton (1974) model for simulated monthly returns

This table reports sensitivities of bond returns to equity in the Merton (1974) model. The sensitivities were obtained by running a simulation as described in the paper. Sensitivities are given in basis points (0.01%). t-statistics are in parentheses.

	All	AAA-A	BBB	BB-CCC
h_E	4.62	1.10	4.09	15.35
	(29.50)	(17.95)	(22.87)	(32.54)
$\bar{R}^2$	0.74	0.65	0.78	0.93

Table VIII
Leverage and Volatilities

This table reports summary statistics on estimates of leverage and volatility for the final sample used in time-series analysis. L is quasi-market leverage defined as the ratio of book value of debt to the sum of book value of debt and market value of equity. σ_E is historical equity volatility over the last three years before an observation. $\widehat{\sigma_A}$ is estimated asset volatility according to formula (6) in the paper. Volatilities and leverage are given in %/100. N is the number of observations. The further details of the estimation procedure are given in the paper.

	All	AAA	AA	A	BBB	BB	B	CCC
L								
Mean	0.36	0.04	0.15	0.24	0.38	0.47	0.60	0.75
Median	0.31	0.02	0.11	0.21	0.35	0.47	0.62	0.81
Std. Dev.	0.23	0.11	0.13	0.15	0.20	0.20	0.22	0.20
5% quantile	0.07	0.01	0.03	0.06	0.11	0.14	0.21	0.35
95% quantile	0.84	0.04	0.34	0.54	0.77	0.84	0.94	0.98
σ_E								
Mean	0.37	0.26	0.25	0.29	0.34	0.44	0.57	0.84
Median	0.33	0.27	0.25	0.27	0.32	0.44	0.53	0.78
Std. Dev.	0.18	0.05	0.07	0.09	0.11	0.13	0.22	0.35
5% quantile	0.18	0.19	0.15	0.17	0.19	0.26	0.29	0.49
95% quantile	0.70	0.35	0.37	0.47	0.56	0.67	1.00	1.37
$(1-L)\sigma_E$								
Mean	0.22	0.25	0.22	0.22	0.21	0.23	0.23	0.20
Median	0.20	0.26	0.22	0.21	0.20	0.22	0.19	0.15
Std. Dev.	0.11	0.06	0.07	0.08	0.10	0.12	0.16	0.19
5% quantile	0.07	0.18	0.11	0.12	0.08	0.06	0.04	0.02
95% quantile	0.40	0.34	0.32	0.36	0.39	0.45	0.55	0.62
$\widehat{\sigma_A}$								
Mean	0.24	0.25	0.22	0.22	0.22	0.26	0.31	0.38
Median	0.22	0.26	0.22	0.21	0.20	0.24	0.27	0.33
Std. Dev.	0.10	0.05	0.07	0.08	0.09	0.10	0.13	0.13
5% quantile	0.12	0.18	0.11	0.12	0.10	0.13	0.21	0.31
95% quantile	0.42	0.34	0.32	0.37	0.39	0.46	0.57	0.68
N	63071	462	4400	20226	23599	5179	7547	1658

Table IX
Hedge Ratios

This table reports descriptive statistics on estimated hedge ratios using the Merton (1974) model for the final sample used in time-series analysis. Hedge ratios are estimated for three different assumptions on asset volatility ($sigma_A$). In panel (a) $\widehat{\sigma_A}$ is estimated using formula (6) in the paper. In panel (b) σ_A is set equal the $(1-L)\sigma_E$, where σ_E is historical equity volatility over the last three years before an observation. L is quasi-market; in other words, estimation in panel (b) assumes that the firm debt bears none of the riskiness. In panel (c) σ_A is set equal to σ_E. Hedge ratios are given in basis points (0.01%) and N is the number of observations. The further details of the estimation procedure are given in the paper.

	All	AAA	AA	A	BBB	BB	B	CCC
				Panel (a): $h_E = h_E(\widehat{\sigma_A})$				
Mean	3.60	0.31	0.47	1.54	2.33	4.91	10.82	19.04
Median	1.11	0.01	0.01	0.19	1.03	3.76	10.57	18.72
Std. Dev.	5.32	0.65	1.33	2.82	3.36	4.50	5.64	4.46
5% quantile	0	0	0	0	0	0.26	2.35	13.25
95% quantile	15.60	2.02	2.81	7.36	9.11	13.99	21.46	27.76
				Panel (b): $h_E = h_E((1-L)\sigma_E)$				
Mean	2.34	0.31	0.46	1.48	2.03	3.69	5.08	6.24
Median	0.56	0.01	0.01	0.167	0.71	2.16	2.51	3.38
Std. Dev.	4.09	0.65	1.32	2.77	3.26	4.41	6.39	7.99
5% quantile	0	0	0	0	0	0	0	0
95% quantile	10.66	2.01	2.79	7.19	8.65	12.50	18.62	24.47
				Panel (c): $h_E = h_E(\sigma_E)$				
Mean	10.70	0.60	1.51	5.12	10.18	16.61	23.78	35.21
Median	6.77	0.02	0.10	1.74	7.97	16.85	24.21	35.85
Std. Dev.	11.16	1.81	3.87	7.07	9.06	9.50	10.10	6.87
5% quantile	0	0	0	0	0.09	1.85	6.13	23.47
95% quantile	33.20	2.82	6.89	21.01	27.63	31.77	39.74	46.05
N	63071	462	4400	20226	23599	5179	7547	1658

Table X
Hedge Ratio Regressions: Time-Series Sample

This table reports results of a regression analysis of hedge ratios. The regression is

$$\overline{r}_{j,t} = \alpha_{j,0} + \beta_{j,E} h_{E,j,t} \overline{r}_{E,t} + \alpha_{j,rf} \overline{rf}_{10y,t},$$

where $h_{E,j,t}$ is the hedge ratio for bond j at time t as implied by the model. Under the null hypothesis that the Merton model holds, $\beta_{j,E}$ is equal to unity. rf^{ret}_{10y} is the excess return on the 10-year constant maturity U.S. Treasury bond. The regression is estimated for each bond and the table reports the cross-sectional means of the coefficients. The method used the estimate the t-statistics is described in Appendix A. The t-statistics for $h_E \times E^{ret}$ is with respect to the difference from unity. The further details of the estimation procedure are given in the paper. N is the number of observations.

	All	AAA	AA	A	BBB	BB	B	CCC
Intercept	-0.07	-0.17	-0.15	-0.14	-0.11	0.10	0.12	-0.89
	(-1.16)	(-1.89)	(-2.31)	(-3.60)	(-1.61)	(0.82)	(0.63)	(-1.67)
$h_E \times E^{ret}$	1.18	-2.91	0.53	0.92	1.61	1.57	0.88	0.60
	(0.30)	(-0.22)	(-0.30)	(-0.10)	(1.01)	(1.04)	(-1.24)	(-5.05)
rf^{ret}_{10y}	46.36	94.93	84.05	64.27	53.82	25.03	-8.74	-31.80
	(13.60)	(20.61)	(24.96)	(31.34)	(15.18)	(3.72)	(-0.86)	(-1.10)
$\bar{R}^2$	0.34	0.70	0.53	0.43	0.34	0.22	0.16	0.24
N	42.94	49.56	51.40	43.69	42.44	42.39	39.95	36.50
	(1328)	(9)	(70)	(426)	(487)	(132)	(192)	(12)

Table XI
Hedge Ratio Regressions: Cross-Sectional Sample

This table reports results of a cross-sectional regression analysis of excess returns on corporate bonds over the period 12.1996-12.2001for the cross-sectional sample (using the selection procedure described in the paper). The Fama-MacBeth (1973) estimation procedure is implemented. For each of 60 months the following cross-sectional regression is run:

$$\overline{r}_{j,t} = \alpha_0 + \alpha_E h_E \overline{r}_{E,j,t} + u_{j,t},$$

where h_E is the average Merton-implied ratio for a "fine" rating (e.g., AA1,AA2 and AA3). Further details are given in the paper. Then the resulting coefficients are averaged across time. E^{ret} is the excess return on the issuer's equity. $\bar{h_E} \times E^{ret}$ is the excess equity return multiplied by the average Merton(1974)-implied hedge ratio for the rating class. N is the average number of observations in each month. t-statistics are given in parentheses. t-statistics on $\bar{h_E} \times E^{ret}$ are on difference with unity.

	All	AAA	AA	A	BBB	BB	B	CCC
Intercept	0.60	0.71	0.63	0.60	0.49	0.62	0.49	1.54
	(3.73)	(3.05)	(3.73)	(3.39)	(2.53)	(3.78)	(2.50)	(2.80)
$\bar{h_E} \times E^{ret}$	0.98	1.13	0.63	0.79	2.03	1.46	0.98	1.01
	(-0.19)	(0.02)	(-0.39)	(-0.66)	(1.80)	(1.78)	(-0.22)	(0.10)
$\bar{R}^2$	0.12	0.12	0.01	0.02	0.05	0.08	0.15	0.17
N	1430.45	8.68	95.55	443.82	537.12	131.43	175.98	37.87
	(60)	(60)	(60)	(60)	(60)	(60)	(60)	(60)

Table XII
Regression of Excess Returns

This table reports results of regression analysis of excess returns on corporate bonds over the period 12.1996-12.2001 for the time-series sample (using the selection procedure described in the paper) rf_{10y}^{ret} is the excess return on the 10-year constant maturity U.S. Treasury bond. E^{ret} is the excess return on the issuer's equity. Slope ($\Delta(rf_{10y} - rf_{2y})$) is the slope of the term structure (the difference between the yield on ten-year and two-year constant-maturity U.S. Treasury bonds. $S\&P^{ret}$ is the return on the $S\&P$ index, VIX is the volatility implied by the S&P100 index options. SMB and HML are the Fama-French Small minus Big and High minus Low factors. N is the number of observations. t-statistics are given in parentheses and the methodology of their estimation is given in Appendix A.

	All	AAA	AA	A	BBB	BB	B	CCC
Intercept	-0.13	-0.16	-0.15	-0.18	-0.18	0.05	0.06	-0.70
	(-2.14)	(-2.19)	(-3.25)	(-5.26)	(-2.76)	(0.41)	(0.31)	(-1.41)
rf_{10y}^{ret}	50.80	93.73	74.03	65.55	58.64	29.86	-4.67	-41.01
	(16.15)	(24.59)	(30.70)	(36.29)	(17.68)	(4.68)	(-0.43)	(-1.47)
Slope	-1.45	-1.12	-0.81	-1.14	-0.80	-2.50	-3.03	-11.72
	(-3.98)	(-2.66)	(-2.85)	(-5.24)	(-2.06)	(-3.59)	(-2.43)	(-3.34)
E^{ret}	2.91	-0.32	0.56	0.86	2.76	5.00	8.29	11.91
	(6.33)	(-0.35)	(1.01)	(2.38)	(5.17)	(5.60)	(8.42)	(6.23)
$S\&P^{ret}$	0.88	1.68	0.47	1.69	-0.26	4.17	-1.15	13.30
	(0.49)	(0.79)	(0.35)	(1.60)	(-0.14)	(1.14)	(-0.19)	(0.89)
VIX change	-0.06	-0.06	-0.06	-0.05	-0.06	-0.05	-0.07	-0.12
	(-2.96)	(-2.30)	(-4.30)	(-4.41)	(-2.89)	(-1.35)	(-1.08)	(-0.76)
SMB	10.73	6.46	6.69	9.22	10.42	18.28	12.66	14.45
	(7.11)	(3.34)	(5.89)	(10.68)	(6.65)	(6.07)	(2.37)	(0.98)
HML	2.74	2.95	1.48	1.14	0.99	8.01	7.00	28.05
	(2.08)	(2.01)	(1.45)	(1.43)	(0.69)	(3.19)	(1.59)	(2.75)
$\bar{R}^2$	0.47	0.77	0.65	0.56	0.46	0.33	0.22	0.30
N	42.44	47.73	48.74	42.75	42.09	41.60	39.39	36.54
	(1595)	(11)	(125)	(553)	(540)	(145)	(208)	(13)