Extended Rational Addiction: Endogenous Discounting and Heterogenous Habits

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Abstract

This study extends the Rational Addiction theory with endogenous discounting and heterogeneity in the habit formation process. The model describes how heterogenous habits affect consumption paths via a subjective rate of time preference varying the rate of habit adjustment and the patience-dependence trade off. The intertemporal structure of preferences explains how an increase in the rate of habits depreciation implies a decrease in the rate of time preference. This makes a forward-looking agent more patient than a myopic one. We suggest a possibly estimable Euler equation and further extensions for a further generalization to dynamic demand systems.

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The usual assumption in economics is that discount rates on future utilities are constant and fixed to each person, although they may differ between persons. This assumption is a good initial simplification, but it cannot explain why discount rates differ by age, income, education and other personal characteristics or why they change over time for the same individual, as when a person matures from being a child to being an adult.


1 Introduction

The habit formation process is affected by economic variables and other exogenous factors such as demographic characteristics and the psychological state of consumers. A habit is formed when past and current consumption are linked by a positive relation. The higher is previous consumption, the larger the habit, and the higher should the current consumption level be to deliver the same utility. It follows that utility depends on the difference between current consumption and a weighted sum of the quantities consumed in the past. Utility reaches a peak after consumption rise to a permanently higher level. Then it declines over time as the person becomes accustomed to that level. Similarly, utility reaches a minimum just after consumption fall to a permanently lower level. Comparisons with past consumption may be so effective that past consumption can be weighted more heavily than present consumption. When the habit formation process is sustained, then a consumer may turn a habit into a state of addiction. A habit may evolve into addiction by being exposed to habit itself. Becker and Murphy (1988) define a person addicted to some goods when an increase in current consumption increases future consumption.

Consumption of an addictive good is not equally harmful to all individuals. For instance, many people can drink regularly without becoming alcoholic. Addiction involves an interaction between people and goods. Each individual possesses a subjective belief structure concerning his potential to become addicted. People of comparable wealth and education but with different past experiences do not share the same risk to become addicted. In essence, people have different rates of time preference, and part of this heterogeneity may be explained by personal differences in past experiences, demographic characteristics, genetic patrimony, and other exogenous factors. The rate of time preference is a subjective indicator of impatience representing the desire of an agent to anticipate and enjoy the benefits stemming from higher current consumption. A high rate of time preference lowers the propensity towards future utility in determining current consumption choices. The main objective of the present paper is to treat the rate of time preference as endogenous and dependent upon demographic characteristics in order to capture heterogeneity in the habit formation process.

We believe that by introducing heterogeneity, we can develop a useful tool to evidence which policies can effectively reduce alcoholism avoiding to penalize much people who enjoy a moderate consumption of alcoholic beverages. To most people, current consumption of alcohol in moderation provides enjoyment without serious side effects. To others the same pattern of consumption may lead to a state of dependence and eventually addiction. This development path is critically affected by personal characteristics. Analyzing the causes for the low marriage rate, Akerlof (1998) observes that there are noticeable differences in the lifestyle of married and unmarried people.

\[2\text{In general, addiction creates physical abstinence or withdrawal symptoms, when the use of the substance is discontinued, and generates tolerance, which is a physiological phenomenon requiring the individual to use more and more of the substance (Kennedy (1987) and Stein et al. (1988)). Tolerance for a substance may be independent of the drug ability to produce physical dependence which manifests itself by the symptoms of abstinence when the drug is withdrawn.}\]
men. Married men stay longer in the labour force, are less inclined to substance abuse, commit less crime and are less likely to be victims of crime. Moreover, they have better health and are less accident prone. A simple explanation is advanced: low marriage rates, or, in general, solitude, will lead to increases in some social pathologies such as crime, drugs and alcohol addiction.

Given that data seem to support our working hypothesis (Aristei et al. (2006a) and Aristei et al. (2006b)), we extend the Rational Addiction model of Becker and Murphy (1988) by assuming an endogenous discount rate depending on and index of past consumption. In doing so, we follow the specification proposed by Epstein and Shi (1993).

This specification of the model nests as special cases many models of habit formation and addiction, such as:

- the Ramsey model, characterized by a constant rate of time preference;
- the Rational Addiction model of Becker and Murphy (1988), which has a constant discount rate and habits nested in the utility function in an additive way;
- the habit forming model of Boyer (1978), in which utility depends on current \( t \) and past \( t-1 \) consumption, and the discount rate is constant;
- the hyperbolic discounting model of Harris and Laibson (2001) and, specifically for addiction, Gruber and Köszegi (2001), in which utility depends also on the stock of habits and the discount rate is hyperbolic;
- the multiplicative habits model proposed by Carroll (2000), characterized by a constant discount rate and a stock of habits entering in the utility function in a multiplicative way;
- the Uzawa (1968) or Obstfeld (1990) models, that assume an endogenous discount rate depending on current consumption;
- the Epstein and Shi (1993) model, characterized by an endogenous discount rate depending on the stock of habits.

With respect to the analysis of Becker and Murphy (1988) and Becker and Mulligan (1997), we extend the model in order to study the impact of habits on intertemporal consumption paths by the endogenous rate of time preference. One implication given by the endogenous discounting framework is that agents are no more assumed to be time-consistent. In fact the discount rate depends upon an index of past consumption levels, and hence the consumption decision is influenced by the time at which the decision is taken, since different time periods imply different indexes of past consumption (which evolve along time) and hence different discount rate levels. This result is in line with the critic of Gruber and Köszegi (2001) to the Rational Addiction model, although implemented in a different way.

We further extend in order to include heterogeneity of consumers who may have different rates of time preference for different sets of personal characteristics (as suggested, among others, by Fuchs (1982) and Lawrance (1991)). Heterogeneity is included in the habit formation process, which influences both the discount rate and the rate of time preference.

The study is structured as follows. Section 2 proposes the theoretical exposition of the Extended Rational Addiction model, where the consumer maximizes an instantaneous utility function discounted by an endogenous discount rate defined with respect to the stock of habits. Section 3 propose the inclusion of heterogeneity in the habit formation process and consequently in the rate of time preference. Section 4 presents an estimable Euler equation. The last section concludes the work.

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3 For a sketch of the proofs see Appendix A1.
2 The Extended Rational Addiction (ERA) Model

2.1 Theory and Notation

The seminal works of Blanchard and Fisher (1989), Deaton (1992) and Romer (1986) have criticized the assumption of a constant rate of time preference as suggested more by convenience than economic rationales. Critics come also from empirical works as Bishai (2004) or Laibson et al. (2005), which rejects the hypothesis of a constant discounting. Part of the economic literature represents preference structure in a dynamic context using the Ramsey model through functionals in which an additive utility function is discounted by a constant rate. Additivity implies that the marginal rate of substitution between the consumption at time $t$ and $t + 1$ is independent of consumption for each $t$ different from $t$ and $t + 1$ (time consistency). Situations such as addiction in alcohol consumption (but also in drug use or cigarette smoking), or the existence of goods as holidays and works of art whose benefits continue over the consumption act, cannot be described properly by an additive preference structure.

The reference model in the field of addiction is the Rational Addiction (RA) by Becker and Murphy (1988). This model endows some characteristics that make it an easy tool for applied works, since it brings to a linear estimable Euler equation. To obtain such a result, one critical assumption is that of a constant discount rate, which is set equal to the rate of return to savings. Such an assumption have never been supported by theoretical reasoning or empirical evidence. It follows that a model like the Rational Addiction may be inadequate to properly describe behavior of possibly addicted agents. For this reason we relax the assumption of a constant discount rate in favour of an endogenous specification. In fact, addiction is likely to influence the discount rate and the rate of time preference in the sense of an increased impatience.

In doing so, we follow the works of Obstfeld (1990) and Epstein and Shi (1993), which propose an endogenous discount rate. As a consequence, the rate of time preference turns out to be endogenous, making the analysis more interesting in terms of optimizing behavior. Also the intertemporal elasticity of substitution is enriched, incorporating other variables like, for example the shadow price of habits or the present value of the future stream of utility, which, even if not generally observable in the data, can help in characterizing consumer’s behavior.

Epstein and Shi (1993) introduces an endogenous discount rate as a function of an index of past consumption, called stock of habits and denoted as $z(t)$\(^4\). Our proposal is to extend the Rational Addiction model in this direction.

Felicity of consumers depends on the consumption of two categories of goods. The first good, $g(t)$, is a generic composite good, and does not cause addiction. The second good, $c(t)$, is a potentially addictive good\(^5\), which generates habits in consumption. Felicity also depends on the strength of habits, synthesized by the stock of habits. In particular we assume that a high level of habits affects negatively felicity, so that it is necessary to consume more to obtain the same felicity that would be reached in absence of habits. Hence, felicity of consumers is synthesized by an utility function $u(g(t), c(t), z(t))$ which depends con $g(t)$, $c(t)$ and $z(t)$.

Being a potentially addictive good, $c(t)$ generates habits, and the more the good is consumed, the less importance it has in generating felicity, since it increases $z(t)$ which enters negatively into the utility function. The degree of habits of an individual is represented by the stock variable\(^6\).

\(^4\)We index with $t$, $τ$ or $s$ any variable depending on time or taken in some standpoint in time.

\(^5\)When we say “potentially addictive” good, we mean that if that good is consumed under certain conditions (for example in high quantity), it generates addiction. Alcoholic beverages, for example, are potentially addictive goods. A moderate consumption of alcohol does not lead to addiction, but excessive consumption does.

\(^6\)Becker and Murphy (1988) call $z(t)$ the “consumption capital” in a framework in which “past consumption of $c$ affects current utility through a process of learning by doing... ”. We prefer to use the definition proposed by Ryder and Heal (1973), which call $z(t)$ the “stock of habits.”
$z(t)$, which accumulates according to a dynamic process. Generally, this process depends on the personal characteristics of the individual and on the consumption level of the addictive good $c(t)$.

In the literature there are mainly two approaches to the habit formation process: the partial adjustment approach and the adaptive approach\(^7\).

The partial adjustment approach consists in considering the habit formation as an investment process. The stock of habits $z(t)$ accumulates as if it was capital, investment is represented by current consumption and the stock of habits depreciates at rate $\sigma$, which may depend upon personal characteristics of individuals\(^8\). The dynamic equation which describes this process is

$$\dot{z}(t) = c(t) - \sigma z(t). \quad (2.1.1)$$

In the adaptive approach, habits accumulation process is due to the difference between current consumption of the potentially addictive good $c(t)$ and the current stock of habits $z(t)$, through the rate of habits adjustment $\lambda$. If current consumption $c(t)$ exceeds the current stock of habits $z(t)$, there will be formation of further habits, at the rate $\lambda$, otherwise, the stock of habits decreases. The dynamic equation which describes this approach is

$$\dot{z}(t) = \lambda (c(t) - z(t)), \quad (2.1.1)$$

where the variables are the same as above, except for $\lambda$, which is the rate of habits adjustment.

In this work we follow the partial adjustment approach, for consistency with the Rational Addiction model. However, using the adaptive approach (like Epstein and Shi (1993) do) causes no additional issues.

Given our behavioral hypothesis, the consumer problem is

$$\max \int_0^\infty u(g(t), c(t), z(t)) e^{-\int_0^t \theta(z(\tau)) d\tau} \, dt \quad (2.1.2)$$

s.t.

$$\dot{a}(t) = ra(t) - g(t) - pc(t), \quad a(0) \geq 0 \quad \text{given}$$

$$\dot{z}(t) = c(t) - \sigma z(t), \quad z(0) \geq 0 \quad \text{given},$$

where

$$u(g(t), c(t), z(t)) : \quad \text{instantaneous utility function}$$

$$g(t) : \quad \text{current consumption of a non-addictive good}$$

$$c(t) : \quad \text{current consumption of an addictive good}$$

$$z(t) : \quad \text{stock of habits}$$

$$\theta(z(t)) : \quad \text{endogenous discount rate}$$

$$a(t) : \quad \text{wealth}$$

$$p : \quad \text{relative price of the addictive good}^9$$

$$r : \quad \text{rate of return to savings}$$

$$\sigma : \quad \text{rate of habits depreciation},$$

and the rate of habits depreciation $\sigma$ is assumed to be positive ($\sigma \geq 0$).

\(^7\) We propose these names, which are not found in the literature, since each of these approach reminds respectively the Partial Adjustment model and the Adaptive Expectation model. Refer to any time series econometrics textbook for details about these models.

\(^8\) As in Becker and Murphy (1988), “the rate of habits depreciation $\sigma$ measures the exogenous rate of disappearance of the physical and mental effects of past consumption of $c$.”

\(^9\) Price of $g(t)$ is normalized to 1.
The utility function is assumed to be increasing with respect to \( g(t) \) and \( c(t) \), decreasing with respect to \( z(t) \) and concave in \( g(t), c(t) \) and \( z(t) \). The discount rate \( \theta(z(t)) \) is a twice continuously differentiable function assumed to be strictly positive \( (\theta(z(t)) > 0) \), strictly increasing \((\theta'(z(t)) > 0)\) and linear\(^{12}\) \((\theta_{zz}(z(t)) = 0)\).

The above program (2.1.2) can be reformulated as

\[
\max \int_0^\infty u(g(t), c(t), z(t)) e^{\Theta(t)} e^{-rt} dt
\]

s.t. \( \dot{a}(t) = ra(t) - g(t) - pc(t), \quad a(0) \geq 0 \text{ given} \)

\( \dot{z}(t) = c(t) - \sigma z(t), \quad z(0) \geq 0 \text{ given} \)

\( \Theta(t) = \theta(z(t)) - r \quad \Theta(0) = 0, \)

where \( \Theta(t) \) is the cumulative subjective discount rate\(^{13}\) and is obtained integrating back from 0 to \( t \) its motion equation

\[
\Theta(t) = \int_0^t (\theta(z(\tau)) - r) d\tau.
\]

We can obtain a formulation for the stock of habits \( z(t) \) solving the differential equation

\[
\dot{z}(t) = c(t) - \sigma z(t), \text{ i.e.}
\]

\[
z(t) = z(0) e^{-\sigma t} + \int_0^t c(\tau) e^{\sigma(\tau-t)} d\tau \quad \text{with} \quad \sigma \geq 0.
\]

This formulation, that involves non separability of preferences, is suggested by Ryder and Heal (1973) who introduce the notion of adjacent complementarity\(^{14}\). An increase or decrease in consumption at \( t - 1 \) can induce a variation of the marginal rate of substitution of current and future consumption at \( t + 1 \). Complementarity is represented by a utility function that depends on both current consumption, \( c(t) \), and an index of past consumption which is a weighted average of past consumption levels. Weights decline exponentially in the past at the exogenous depreciation rate \( \sigma \), which can be interpreted as a measure of permanence of physical and mental effects of past consumption on present consumption \( c(t) \). As \( \sigma \) gets larger, less weight is given to past consumption in determining \( z(t) \).

In the economic literature there is an open discussion if the endogenous discount rate must be considered increasing or decreasing with respect to consumption. Koopmans (1960) suggests a decreasing rate of impatience, while Lucas and Stokey (1984) observe that an increasing rate of impatience is necessary to obtain a single, stable, non degenerate equilibrium point into wealth.

\(^{10}\) Becker and Murphy (1988) assume a quadratic utility function, which clearly is not globally increasing (decreasing) with respect to \( g(t) \) and \( c(t) \) and have a global maximum point. For the sake of generality we will stick to an undefined utility function until we decide to specify an estimable Euler equation, where we will use a quadratic function as in the RA model.

\(^{11}\) We denote the derivative of a generic function \( f \) with respect to some variable \( x \) \( (\partial f/\partial x) \) as \( f_x \). The second derivative \( (\partial^2 f/\partial x^2) \) is denoted as \( f_{xx} \).

\(^{12}\) Linearity is not essential assumption for the solution to the problem, but we set it for the sake of simplicity.

\(^{13}\) We use this formulation, in which the discount rate is “divided” in two terms because it allows for some mathematical simplifications. This specification also introduce a direct treatment of the concept of impatience, with no further effects on the founded solutions. The two formulation are in any case equivalent, since

\[
e^{-\Theta(t)} e^{-rt} = e^{-\int_0^t (\theta(z(s)) - r) ds} e^{-rt} = e^{-\int_0^t \theta(z(s)) ds + rt} e^{-rt} = e^{-\int_0^t \theta(z(s)) ds}
\]

\(^{14}\) Adjacent complementarity occurs when past consumption of a good raises the marginal utility of present consumption.
distribution in a deterministic horizon with a finite number of agents. According to Blanchard and Fisher (1989), the assumption of an increasing rate of impatience is difficult to defend \textit{ex ante}. On the other side Epstein (1987b) and Epstein (1987a) argues that the more a person consumes, the more discounts the future. In line with Epstein, we assume that the endogenous discount rate, $\theta(z)$, is strictly increasing with respect to the stock of habits, and hence consumption$^{15}$. This condition is necessary to ensure the stability of the long-run optimal consumption plan, because it guarantees that consumptions in different dates are substitutes. In this case as wealth and consumption rise, the marginal private return to further savings, which depends on the marginal utility of future consumption, falls. If $\theta_{c} < 0$, consumption in different dates are complements, and a rise in present consumption rises the marginal utility of future consumption. Such an assumption is plausible in a model with habit formation, but it does not seem much coherent when we consider consumption in general.

The implication of a discount rate strictly increasing with respect to consumption, is that a higher consumption level at time $t$ increases the discount rate applied to utility after $t$. An increase in current consumption in $t$ induces an increase in the rate of time preference: the consumer’s desire to anticipate effects of future consumption is picked up by more current consumption at $t + 1$. An increase in current consumption at $t + 1$ rises the stock of habits $z(t)$, increasing a further increase in the discount rate: the higher is previous consumption, the larger the habit, and the higher must be the current level of consumption to deliver the same effect. An increase in the discount rate rises the degree of adjacent complementarity and hence strengthens the commitment to all habits.

Control variables of the optimization problem are per-capita consumption $g(t)$ and $c(t)$, while the real assets per person $a(t)$ and the stock of habits $z(t)$, are the state variables. We assume a constant, non-negative rate of habits depreciation $\sigma$.

The control problem (2.1.3) is solved according to the Maximum Principle of Pontriagin (Optimal Control theory). The present value Hamiltonian function$^{16}$ is

$$H_{d} = e^{-\Theta(t)}u(g(t), c(t), z(t)) + \tilde{q}(t) [ra(t) - g(t) - pc(t)] - \tilde{\varphi}(z(t)) - r + \tilde{\Psi}(t) [c(t) - \sigma z(t)],$$

where $\tilde{q}(t) = e^{rt}\tilde{q}(t)$, $\tilde{\varphi}(t) = e^{rt}\tilde{\varphi}(t)$ and $\tilde{\Psi}(t) = e^{rt}\tilde{\Psi}(t)$ are the discounted costate variables.

The first order necessary conditions for an interior solution are

$$\frac{\partial H_{d}}{\partial c(t)} = 0 \rightarrow \tilde{q}(t)p = u_{c}(g(t), c(t), z(t))e^{-\Theta(t)} + \tilde{\Psi}(t)$$

$$\frac{\partial H_{d}}{\partial g(t)} = 0 \rightarrow \tilde{q}(t) = u_{g}(g(t), c(t), z(t)) e^{-\Theta(t)}$$

and

$$\frac{\partial H_{d}}{\partial a(t)} = r\tilde{q}(t) - \tilde{\varphi}(t) \rightarrow \tilde{q}(t) = r\tilde{q}(t) - r\tilde{q}(t) = 0$$

$$\frac{\partial H_{d}}{\partial \Theta(t)} = r\tilde{\varphi}(t) - \tilde{\varphi}(t) \rightarrow \tilde{\varphi}(t) = r\tilde{\varphi}(t) - [u(g(t), c(t), z(t))]e^{-\Theta(t)}$$

$$\frac{\partial H_{d}}{\partial z(t)} = r\tilde{\Psi}(t) - \tilde{\Psi}(t) \rightarrow \tilde{\Psi}(t) = (r + \sigma) \tilde{\Psi}(t) + \tilde{\varphi}(t)\theta_{z}(z(t)) - u_{z}(g(t), c(t), z(t)) e^{-\Theta(t)}.$$

$^{15}$Note that this assumption does not imply an always increasing discount rate. The discount rate may also decrease if, for instance an agent quits consuming the addictive good. In this case, the stock of habits $z(t)$ smoothly depreciates at rate $\sigma$, and consequently also the discount rate lowers.

$^{16}$Note that the relation between current value and present value Hamiltonian is

$$H_{d} = e^{rt} H,$$

where $H_{d}$ is the current value Hamiltonian, $H$ is the present value Hamiltonian and $r$ is a constant discount rate.
It is convenient to re-scale the costate variables in order to eliminate $\Theta(t)$. Let $q(t) = \tilde{q}(t)e^{\Theta(t)}$, 
$\varphi(t) = \tilde{\varphi}(t)e^{\Theta(t)}$ and $\Psi(t) = \tilde{\Psi}(t)e^{\Theta(t)}$. Then, the first-order necessary conditions take the form

\begin{align}
q(t) &= \frac{1}{p}[u_c(g(t), c(t), z(t)) + \Psi(t)] \\
q(t) &= u_g(g(t), c(t), z(t)) \\
\end{align}

(2.1.5a)

(2.1.5b)

and, given that $q(t) = \tilde{q}(t)e^{\Theta(t)}$, and

\begin{align}
\dot{q}(t) &= \tilde{\dot{q}}(t)e^{\Theta(t)} + \tilde{q}(t)e^{\Theta(t)}\dot{\Theta}(t) = 0 + q(t)\dot{\Theta}(t),
\end{align}

(2.1.7a)

(2.1.7b)

the other first order conditions are

\begin{align}
\dot{\varphi}(t) &= r \varphi(t) - u(g(t), c(t), z(t)) \\
\dot{\Psi}(t) &= (r + \sigma) \Psi(t) + \varphi(t)\theta_z(z(t)) - u_z(g(t), c(t), z(t)).
\end{align}

(2.1.6a)

(2.1.6b)

(2.1.6c)

To simplify notation, from now on we refer to $u(g(t), c(t), z(t))$ as simply $u$. Differentiating equations (2.1.5a) and (2.1.5b) with respect to time we obtain

\begin{align}
\dot{q}_t &= \frac{1}{p}[u_c\dot{c}(t) + u_{cg}\dot{g}(t) + u_{cz}\dot{z}(t) + \dot{\Psi}(t)] \\
\dot{q}_t &= u_{gg}\dot{g}(t) + u_{gc}\dot{c}(t) + u_{gz}\dot{z}(t),
\end{align}

(2.1.7a)

(2.1.7b)

and using equation (2.1.7b) with (2.1.5b), (2.1.6a), (2.1.6c) and (2.1.1), we obtain the following expressions for $\dot{g}(t)$ and $\dot{c}(t)$

\begin{align}
\dot{g}(t) &= \frac{1}{u_{gg}}[[\theta(z(t)) - r] u_g - u_{gc}\dot{c}(t) - u_{gz}(c(t) - \sigma z(t))]
\end{align}

(2.1.8)

Note that equating equations (2.1.5a) and (2.1.5b) we obtain a simple expression for $\Psi(t)$, which is

\begin{align}
\Psi(t) &= pu_g - u_c.
\end{align}

(2.1.9)

The differential equation (2.1.6b) gives a continuous-time specification of the recursive structure of consumer preferences for every feasible consumption path. If we solve the differential equation (2.1.6b) we obtain

\begin{align}
\varphi(t) &= \int_t^\infty [u(g(\tau), c(\tau), z(\tau))]e^{-\int_\tau^\tau \theta(z(s))ds}d\tau,
\end{align}

(2.1.10)

which is the present value of future utilities\(^{18}\) at time $t$ and corresponds to the shadow price of the accumulated impatience rate $\Theta(t)$.

\(^{17}\text{Recall that the solution for a differential equation with no constant coefficients as } \dot{y}(t) + P(t)y(t) = Q(t) \text{ is } y(t) = e^{-\int P(t)dt} \int Q(t)e^{\int P(t)dt}dt + Ce^{-\int P(t)dt}. \text{ The value that the solution approaches is referred to as the steady state so the limit for } t \to \infty \text{ of the solution is } y(t) = \int Q(t)e^{\int P(t)dt}dt.\)

\(^{18}\text{In the light of equation 2.1.10, it is clear that the shadow price of the cumulative discount rate } \Theta(t) \text{ may well be seen as an index of impatience. In fact, being } \varphi(t) \text{ the sum of all future utilities, the higher future utilities are, the higher is patience, since the agent is willing to wait for the realization of his desires of consumption. For this reason, from now on we will refer to } \varphi(t) \text{ as the rate of impatience, which the higher it is, the more patient is the individual.}\)
By equating the two equations for \( \dot{q}(t) \) ((2.1.6a) and (2.1.7b)), we can solve for \( c(t) \) and find the Euler Equation

\[
\dot{u}_g[\theta(z(t)) - r] = u_{gg}\dot{g}(t) + u_{gc}\dot{c}(t) + u_{gz}\dot{z}(t) \quad \implies \\
\frac{\dot{c}(t)}{c(t)} = \eta^c(g(t), c(t), z(t)) \left[ \rho^c(g(t), c(t), z(t), \varphi(t)) - r \right],
\]

or, in terms of interest rate

\[ r = \rho^c(g(t), c(t), z(t), \varphi(t)) - \frac{1}{\eta^c(g(t), c(t), z(t))} \frac{\dot{c}(t)}{c(t)}. \]

where, using (2.1.1), (2.1.8), (2.1.6c), (2.1.5b) and (2.1.9), the rate of time preference for good \( c \) is

\[ \rho^c(t) = \theta(z(t)) + u_{gg}\theta(z(t))\varphi(t) + u_{gg}(r + \sigma)(pu_g - u_c) + (u_{gg}u_{cx} - u_{cg}u_{gz})(c(t) - \sigma z(t)) - u_{gg}u_{z}, \]

and the elasticity of intertemporal substitution is

\[ \eta^c(t) = \frac{u_g(pu_{gg} - u_{cg})}{(u_{gg}u_{cc} - u_{cg}u_{gc})c(t)}. \]

As regards \( g(t) \), we can derive an Euler equation in a similar fashion, namely

\[ \frac{\dot{g}(t)}{g(t)} = \eta^g(g(t), c(t), z(t)) \left[ \rho^g(g(t), c(t), z(t), \varphi(t)) - r \right], \]

where the rate of time preference for \( g \) is

\[ \rho^g(t) = \theta(z(t)) + u_{gg}\theta(z(t))\varphi(t) + u_{gc}(r + \sigma)(pu_g - u_c) + (u_{gg}u_{gc} - u_{cg}u_{cc})(c(t) - \sigma z(t)) - u_{gc}u_{z}, \]

and the elasticity of intertemporal substitution is

\[ \eta^g(t) = \frac{u_g(u_{cc} - pu_{gc})}{(u_{gg}u_{cc} - u_{cg}u_{gc})g(t)}. \]

The two rates of time preference in (2.1.11) and (2.1.12) embed

1. memory of past events through the stock of habits \( z(t) \) and the rate of habits depreciation \( \sigma \);
2. perception of present events by the current consumption levels \( c(t) \) and \( g(t) \);
3. the anticipation of future events by the present-value of future utilities, \( \varphi(t) \).

Consumer behavior is non separable along time, revealing complementarity and time inconsistency\(^{19}\). Present consumption of the potentially addictive good \( c(t) \) and future consumption...
(represented by $\varphi(t)$), depend on past consumption of the addictive good through the rate of habits depreciation $\sigma$, and need not be valued equally along a locally constant consumption path. The rate of time preference expresses the propensity that a person reveals towards future utility in determining current choices. This depends on the ability to anticipate benefits of future consumption and the related physical and mental consequences of present and past consumption effects.

The Euler equation is different from the canonical expression, because it comprehends the complementarity between past consumption $z(t)$, current consumption $c(t)$, and future consumption by the rate of impatience $\varphi(t)$, through the endogenous rate of time preference and the elasticity of intertemporal substitution. The complementarity allows us to explain why an increased rate of return to savings, $r$, tends to induce more patience in consumers. First consider the simple case where an increased rate of return is compensated holding the marginal utility $q(t)$ constant. All future consumption rises since the rate of return is higher and current consumption is unchanged by marginal utility assumption holding the growth rate of consumption constant. The effect is picked up by an increase in the elasticity of intertemporal substitution: the agent is more in favour to the intertemporal substitution between future and current consumption, given the increased rate of return to savings. The problem can be more complicated. We do not consider a constant marginal utility and so an increased rate of return can lower the rate of time preference inducing more patience on the agent. The growth rate of consumption declines thus increasing savings. Therefore future consumption and the simultaneous effect on the growth rate of consumption is picked up by an increase in elasticity to allow the model to approach another consumption equilibrium level. The impact of an increased rate of return to savings on the rate of time preference and the elasticity of intertemporal substitution and indeed the growth rate of consumption changes from person to person because people are not equally patient and have an heterogenous structure of preferences. The analysis of the effects induced by habits on consumption paths reveals how habits can influence the reaction of an agent with respect to an increased rate of return to savings.

2.2 Dynamic and Local Stability Properties\(^{20}\)

The dynamic behavior of the model is described by the following set of equations

\[
\begin{align*}
\dot{z}(t) &= c(t) - \sigma z(t) \tag{2.2.1a} \\
\dot{\varphi}(t) &= r\varphi(t) - u(\cdot) \tag{2.2.1b} \\
\dot{c}(t) &= c(t)\eta_c(\cdot)\left[\rho_c(\cdot) - r\right] \tag{2.2.1c} \\
\dot{g}(t) &= g(t)\eta_g(\cdot)\left[\rho_g(\cdot) - r\right] \tag{2.2.1d} \\
\dot{a}(t) &= ra(t) - g(t) - pc(t) \tag{2.2.1e}
\end{align*}
\]

\(^{**}\)

The steady state solution to the system consists in assuming that none of the relevant variables change over time, i.e.

\[
\begin{align*}
\dot{z}(t) &= 0; & \dot{\varphi}(t) &= 0; & \dot{c}(t) &= 0; & \dot{g}(t) &= 0; & \dot{a}(t) &= 0,
\end{align*}
\]

\(^{20}\)Under construction. Any comment is very wellcome.
finding the locus where the condition is satisfied

\[ \begin{align*}
0 &= c(t) - \sigma z(t) \\
0 &= \tau \varphi(t) - u(g(t), c(t), z(t)) \\
0 &= \rho^c(\cdot) - r \\
0 &= \rho^\theta(\cdot) - r \\
0 &= \tau a(t) - g(t) - pc(t).
\end{align*} \]

\(^{22}\) Every function indexed with an asterisc is intended as that function evaluated at the steady state. For example \(\rho^c(\cdot)\) corresponds to \(\rho^c(g^*, c^*, z^*, \varphi^*)\), as well as \(\rho^\theta(\cdot)\) correspond to \(\frac{\partial}{\partial \tau} \rho^\theta(g^*, c^*, z^*, \varphi^*)\).

\(^{*}\) To analyze the local stability first we should impose each equation in (2.2.1) to be equal to 0, and find the equilibrium condition for the system, i.e. the steady state. Then we should linearize the system by a first order Taylor expansion\(^{21}\), obtaining\(^{22}\)

\[ \begin{align*}
\dot{a}(t) &\equiv ra^* - g^* - pc^* \\
\dot{g}(t) &\equiv \rho^c_0(\cdot)(g - g^*) + \rho^\theta_0(\cdot)c - z^* + \rho^\theta_0(\cdot)(\varphi - \varphi^*) \\
\dot{c}(t) &\equiv \rho^c_0(\cdot)(g - g^*) + \rho^\theta_0(\cdot)c - z^* + \rho^\theta_0(\cdot)(\varphi - \varphi^*) \\
\dot{z}(t) &\equiv c^* - \sigma z^* \\
\dot{\varphi}(t) &\equiv r(\varphi - \varphi^*) - u^g_0(\cdot)(g - g^*) - u_c(\cdot)(c - c^*) - u_z(\cdot)(z - z^*),
\end{align*} \]

\(^{21}\) Note that equations (2.2.1a) and (2.2.1e) are already linear, so there is no need to apply a Taylor expansion to them.

\(^{22}\) We suppress asterisks to lighten notation.
We know that for polynomials of degree four there is an analytical solution for its roots. However these solutions are by far too complicated to carry out analytically whether the system is stable or not, even if we decide to use the well known Routh-Hurvitz method. To examine stability of the system we need to know the number of stable and unstable roots for matrix $J$ (see Gantmacher 1964 for details). The cyclical behavior of the system can be deduced from the number of stable complex roots of the system (see Epstein and Shi (1993) for details on how to calculate it).

To overcome the problem, we provide a numerical simulation of the model and find the eigenvalues of the $J$ matrix for different set of parameters.

The setup of the simulation is the following: for consistency with the RA model we chose a quadratic utility function, defined as

$$u(g(t), c(t), z(t)) = \alpha_g g(t) + \frac{\alpha_{gg}}{2} g(t)^2 + \alpha_c c(t) + \frac{\alpha_{cc}}{2} c(t)^2 + \alpha_z z(t) + \frac{\alpha_{zz}}{2} z(t)^2$$  \hspace{1cm} (2.2.3)

where $\alpha_g$, $\alpha_c$, $\alpha_{zz}$ are assumed to be non-negative, while $\alpha_{gg}$, $\alpha_{cc}$, $\alpha_z$, $\alpha_{gz}$, $\alpha_{cz}$ are assumed to be non-positive. The sign of the left parameter, $\alpha_{gc}$, cannot be predetermined by theory. To simplify, we further assume that all the parameters have an absolute value less than 1. Finally, the discount rate is linear in $z(t)$, namely

$$\theta(z(t)) = \mu_0 + \mu_1 z(t),$$  \hspace{1cm} (2.2.4)

where

$$\mu_0 \geq 0; \quad 0 \leq \mu_1 \leq 1.$$  

\*\*\* under construction \*\*\*

Intuition suggests that the critical parameters in this framework are the rate of habits depreciation $\sigma$ and $\mu_1$. Hence, the numerical strategy is as follows: we assign 3 values to each parameter of the utility function (in absolute value 0.1, 0.5 and 0.9) except for the interaction term $\alpha_{gc}$ which having an undetermined sign we set to -0.1, 0 and 0.1 (which respectively means that good $g$ and good $c$ are substitutes, indifferent or complements). For each combination of these parameters we provide a more complete range of values for $\sigma$ and $\mu_1$. If we want to observe habit formation we should keep $\sigma$ below the unity, so the range will be from 0.05 to 1 with step 0.05. On the other hand, $\mu_1$ in principle should be quite a small parameter (especially if $\mu_0$ were different from 0), hence we test it from 0.01 to 0.2 with step 0.01. All the variable will be set to their steady state values, determined by the parameters' values.

\*\*\* verify with cited works by Epstein and Shi in the appendix \*\*\*

Cyclical convergence (as suggested by the case of a $\sigma \approx 0.5$) toward the steady state may have a behavioral correspondence at economic level. According to Epstein and Shi (1993) "... the cycles are local and they dampen towards the steady state. Cycles are more likely if habits adjust slowly or the steady state rate of time preference is more sensitive to the level of consumption or the desire to smooth consumption is weaker" as it is the case of a myopic agent in response of a rise in interest rates.

\*\*\* up to here \*\*\*

2.3 Further Extensions\textsuperscript{24}

In section 2.1 we have shown that we can derive an Euler equation for both $g(t)$ and for $c(t)$. This implies that the model can be straightforward generalized to any number goods, say $n$ non-addictive goods $g^i$, $i = 1, 2, ..., n$, and $m$ possibly addictive goods $c^j$, $j = 1, 2, ..., m$.

\textsuperscript{24} Under construction. Any comment is very welcome.
In this sense, there are two possible extensions to a dynamic demand system. On one side, we still maintain the assumption of non-separability among goods but assume that all addictive goods concur to form an unique stock of habits. On the other side, we can assume that each good forms its own stock of habits.

In the first case there will be a unique rate of habits adjustment $\gamma$, but it would be necessary to include an additional parameter for each good in the habits motion equation. The consumer problem would configure as follows

$$\max \int_0^\infty u(c^1(t), c^2(t), \ldots, c^m(t); z(t))e^{-\int_0^t \theta(z(\tau))d\tau} dt$$

s.t. $\dot{a}(t) = ra(t) - \sum_{j=1}^m p^j(t) c^j(t)$, \quad $a_0 \geq 0$ \quad given

$\dot{z}(t) = \left(\sum_{j=1}^m \gamma_j c^j(t)\right) - \sigma z(t)$, \quad $z_0 \geq 0$ \quad given.

Note that we can simplify notation eliminating the non-addictive goods $g$, since a possibly addictive good $c^j$ is non-addictive if its parameter $\gamma_j = 0$. In this case, clearly good $j$ does not concur in forming the stock of habits, and so is non-addictive. The model proposed in section 2.1, is a special case of this model, in which there are only 2 goods, and good one (which is called $g$ instead of $c^1$) have $\gamma_1 = 0$.

On the other hand we could assume that each good $c^j$ form its own stock of habits $z^j$. In this case the discount rate would depend on a number of stock of habits, rather than on an unique one, and so the utility function would do. This way, each addictive good would have its own rate of habits depreciation $\sigma^j$, and this allows us to avoid the distinction between addictive and non-addictive goods, since a good $c^j$ would be non-addictive if its rate of habits depreciation $\sigma_j$ would equal 1 (i.e. all habits disappears from one period to another). The problem would be specified as

$$\max \int_0^\infty u(c^1(t), c^2(t), \ldots, c^m(t); z^1(t), z^2(t), \ldots, z^m(t))e^{-\int_0^t \theta(z^1(\tau), z^2(\tau), \ldots, z^m(\tau))d\tau} dt$$

s.t. $\dot{a}(t) = ra(t) - \sum_{j=1}^m p^j(t) c^j(t)$, \quad $a(0) \geq 0$ \quad given

$\dot{z}^j(t) = c^j(t) - \sigma^j z^j(t)$, \quad $j = 1, 2, \ldots, m; \quad z^j(0) \geq 0$ \quad given.

These specification could be useful in panel-data analysis. In fact the framework would be that of a dynamic demand system in which habit formation is taken into account and the discount rate is endogenous. There should be no particular issues in estimating such a dynamic demand system, provided that an estimable specification of the Euler equation could be derived for each good.

The choice of one of these specification should rely firstly on theory, and only secondly on empirical issues. In particular, the first model should be preferred since a common stock of habits can help in explaining what is called the “gateway” effect, i.e. the fact that consumption of an addictive substance may increase the probability to consume also other addictive substances (see, among others, Pierani and Tiezzi (2006)). This is possible introducing interaction terms in the habit formation process and verifying whether their parameters are positive or not (if they are positive there is evidence for the gateway effect). The choice of this model should be preferred also from a strictly empirical point of view. In fact the second specification introduces a large number of new parameters to be estimate in the utility function (linear terms parameters, quadratic term parameters plus all the interaction terms parameters) and in the discount rate function (one parameter for each stock of habits) and $\sigma_j$. On the other hand, in the first specification the only additional parameters are the $\gamma_j$.

Going further in the analysis of these extended models will be the subject of a future work.
3 Heterogeneous Habits and Comparative Dynamics

3.1 Heterogeneity in the Habits Formation Process

In this section we characterize the main features of the reference model "extended" to endow heterogeneous characteristics of agents and analyze the behavior induced by an endogenous rate of time preference.

Heterogeneity is widely recognized as a necessary feature for any empirical cross-sectional work on consumer demand, in fact it is a common opinion that each individual has its own preference structure and that the "representative consumer" models do not describe properly how society behaves. Then, the natural extension to dynamic models, beyond heterogeneity in the utility function, should regard the rate of time preference, as suggested by Fuchs (1982) and Lawrance (1991). We concentrate our analysis on the rate of time preference, leaving heterogeneity in preferences aside in order to keep things simple.

The definition of an endogenous rate of time preference permits separating the effects of heterogeneity from a generic habit effect. This is achieved through the definition of a rate of habits depreciation $\sigma$ as a function of a set of personal characteristics, say $d$. This imply that each individual develops habits and eventually addiction with a different consumption-habits path and achieve a different steady state. Hence heterogeneity is taken into account by means of an individual process of habits formation, which directly reflects on the rate of time preference and on the discount rate (via the stock of habits $z(t)$ which is formed differently for each individual).

Our assumption is that $\sigma(d)$ is a function that depends on a set of characteristics of individuals and of society describing consumers' heterogeneity, which may not be constant over time. An example can help understanding our hypothesis.

Define two categories of agents which compose the society, say myopic ($m$) and forward-looking ($f$).

**Definition (Myopic agent).** An agent is myopic if it is characterized by strong a preference towards actual consumption, with a large degree of impatience (small values of $\varphi(t)$). The impatience is caused by a large discount rate $\theta(z(t))$, which in turn is caused by a large stock of habits $z(t)$. A large stock of habits may be caused by a rapid growth of $z(t)$ induced by a small rate of habits depreciation $\sigma(d)$.

**Definition (Forward-looking agent).** An agent is forward looking if it is characterized by a preference towards future consumption, with a small degree of impatience (large values of $\varphi(t)$). Consequently the discount rate $\theta(z(t))$ is small, due to a small stock of habits $z(t)$, which remain small thanks to a large rate of habits depreciation $\sigma(d)$.

From these definitions, it comes out that the model implies that myopic agents have a smaller rate of habits depreciation ($\sigma^m(d)$) respect to the forward-looking ($\sigma^f(d)$). Each rate of habits depreciation depends on the individual’s characteristics and some of them (for example age, as suggested by Bishai (2004)) may vary over time, and so the rate of habits depreciation does, providing a further behavioral improvement: not only the model allows for heterogeneity in the rate of time preference and in the discount rate, it also allows for an evolution over time of the key parameter of heterogeneity.

The comparative dynamic analysis investigates how the variables of interest, such as $c(t)$, $\varphi(t)$, $\sigma(d)$ and $d$, affect the endogenous rate of time preference $\rho^c(\cdot)$ (and $\rho^g(\cdot)$). Since $\sigma(d)$ is assumed to be a function depending only upon exogenous variables, from now on we refer to the rate of time preference as $\rho^c(g(t), c(t), z(t), \varphi(t), \sigma(d))$.

---

25Under construction. Any comment is very welcome.
As regards the shape of the rate of habits depreciation $\sigma(d)$, it is necessary to bound it between 0 and 1 ($\sigma(d) \in (0, 1]$), ensuring the needed mathematical and behavioral properties. In fact, allowing for a negative rate of habits depreciation would rule out the existence of a steady state since habits accumulates over time even without consumption. On the other hand, for values of $\sigma(d)$ greater than 1, the stock of habits $z_t$ responds more than proportionally to a change in consumption $c_t$ and would allow for a negative stock of habits $z(t)$ which would have no sense from a behavioral point of view, and is ruled out by assumption. The limiting case of $\sigma(d) = 1$ imply that the stock of habits fully depreciates each period, thus excluding the possibility for habits to accumulate and for addiction to arise. **Also a small value of $\sigma(d)$ could cause instability since the stock of habits grows much rapidly and may lead to a cyclical convergence path rather than a saddle point behavior. The limiting case of a $\sigma(d) = 0$ imply $\dot{z}(t) = c(t)$, and hence again exclude the possibility of a steady state. **

Other characteristic of $\sigma(d)$, which are not relevant from the purely mathematical point of view, like slope and concavity, have important implications for the behavioral analysis. These characteristics are determinant for the hypothesis testing process. For instance, one appealing feature could be inverse-U-shaped with respect to age, which would imply that an individual is more likely to develop addiction when young or older, rather than when he is middle-aged. This could be easily achieved introducing a quadratic form for age. Many studies suggest that men and women have different predisposition to addiction caused by a different genetic structure. This may lead to a physiological difference in the process that generates addiction, in which women may be more exposed to the risk of become addicted. According to these studies, women need less alcohol than man to become addicted. Also we can reasonably expect that a higher education decreases the probability of becoming addicted. In fact, a higher education should imply a better knowledge about risks connected to the addictive substances abuse, and hence should reduce the probability of becoming addicted.

In general, the idea for an empirical work would be to separate variables $d$ into two groups of characteristics which can be expected to have a positive ($d^+$) and negative ($d^-$) impact on the memory process and the rate of time preference affecting the degree of myopia. We may also want to consider an additional group of relevant variables for which we are not able to infer about their impact. This is interesting for testing a set of conjectures which are usually made by psychologists, medical doctors and social scientists about the causes of addiction to alcohol, tobacco or drugs. Just as a purely illustrative example, we may consider

<table>
<thead>
<tr>
<th>variable</th>
<th>$d^-$</th>
<th>$d^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>education</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>age</td>
<td>old</td>
<td>young</td>
</tr>
<tr>
<td>well-being</td>
<td>rich</td>
<td>poor</td>
</tr>
<tr>
<td>gender</td>
<td>male</td>
<td>female</td>
</tr>
<tr>
<td>peer effects</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>household</td>
<td>stable</td>
<td>divorce</td>
</tr>
<tr>
<td>living area</td>
<td>residential</td>
<td>degraded</td>
</tr>
<tr>
<td>working condition</td>
<td>employed</td>
<td>unemployed</td>
</tr>
<tr>
<td>satisfaction with work</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>ethnicity</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>family dependency</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

26 An example is a research conducted by Istituto Superiore della Sanità (a branch of Italian Health Department), which links the stronger effects produced by alcohol abuse, with a scarce presence of Alcohol Dehydrogenase (ADH), which is an enzyme involved in the metabolism of Alcohol. It is also observed that, in general, women have less ADH than men.
Let us consider the case of a myopic agent. We may think, for example, at a middle-aged woman who becomes jobless in a certain moment of her life and has dependent children. This situation is one of the most frequent causes of the “feminization” of alcohol consumption in Italy. As time passes, the myopic agent builds habits, evolving into addiction when the accumulation of a stock of past consumption grows beyond some critical level. The forward-looking, instead, tends to accumulate less habits that a myopic, reducing the risk of becoming addicted.

We intend to compare the behavior of a myopic agent with that of a forward-looking. For a forward-looking agent, we may think about a forty-year-old single, happily employed, who does not disdain a couple of glasses of wine per meal but he is a health-friend and sport-loving. The likelihood that the myopic agent reveals addiction with respect to alcohol because jobless is picked up by a rate of habits depreciation lower than the rate of the forward-looking agent. The larger is σ(d), the less weight is given to past habits in determining current habits z_t. Therefore, the risk of addiction is more intense for a lower σ(d).

Considering the assumption of a rate of habits depreciation bounded between 0 and 1 (σ(d) ∈ (0, 1)), then a forward looking agent should have a rate of time preference greater than 0.5, while a myopic agent should have σ_m(d) < 0.5.

This is relevant and important from a policy point of view, since it may be desirable to affect the number of forward looking agents in the society and this may be achieved by rising the rate of habits depreciation through one or more of the variables which compose it.

### 3.2 Comparative Dynamics

The following propositions analyze the dynamic behavior of the rate of time preference with respect to the variables of interest (c(t), ϕ(t), σ(d) and d) in the neighborhood of the steady state. This kind of analysis is in part dependent on the shape of the preferences, in the sense that it may be eased considering a quadratic utility function as in (2.2.3), which imply that all third derivatives of the utility function are 0. We further assume independency between g and c, in the sense that they are not substitutes or complements (u_{gc} = u_{cg} = 0), and that there is no interaction between the non-addictive good g and the stock of habits z (u_{gz} = 0). Proof of these propositions are given in Appendix A1.

**Proposition.** The rate of time preference ρ'(g(t), c(t), z(t), ϕ(t), σ(d)) is strictly increasing with respect to current consumption c(t).

\[
\frac{\partial}{\partial c(t)} \rho'(g(t), c(t), z(t), \varphi(t), \sigma(d)) > 0.
\]

The rate of time preference is increasing with respect to an increase in current consumption of the possibly addictive good c_t, which indicates that the more an agent consumes, the more he is willing to consume. This imply that he is less concerned with future consumption rather than immediate consumption. In such cases there may be no need to “save against a rainy day.” In other words a rise in current consumption causes an increase in the impatience level.

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27 Italian Secretary of State for Health, on Sirchia, Relazione sullo stato sanitario del paese 2001 - 2002.
28 The choice of \( \sigma(d) = 0.5 \), as the discriminant value is purely arbitrary and have been chosen only for illustrative purposes.
29 Under construction. Any comment is very wellcome.
Proposition. The rate of time preference $\rho^c(g(t), c(t), z(t), \varphi(t), \sigma(d))$ is strictly decreasing with respect to the present value of future utility $\varphi_t$.

$$\frac{\partial}{\partial \varphi(t)} \rho^c(g(t), c(t), z(t), \varphi(t), \sigma(d)) < 0.$$  

An increase in the rate of impatience $\varphi_t$ indicates that the consumer is less impatient, giving more weight to future consumption. Hence the rate of time preference is reduced and the consumption growth of the addictive good becomes smaller or even negative, if the rate of time preference become smaller than the interest rate $r$.

Proposition. The rate of time preference $\rho^c(g(t), c(t), z(t), \varphi(t), \sigma(d))$ is strictly decreasing with respect to the rate of habits depreciation $\sigma(d)$ if $u_c < pu_g - z(\cdot)u_{cz}^\ast$.

$$\frac{\partial}{\partial \sigma(d)} \rho^c(g(t), c(t), z(t), \varphi(t), \sigma(d)) < 0 \quad \text{iff} \quad u_c < pu_g - z(\cdot)u_{cz}^\ast.$$  

An increase in the rate of habits depreciation implies that the rate of time preference declines, i.e. a decrease in the habit effects of the possibly addictive good tends to reduce the growth of consumption of $c$. This is true provided that the marginal utility of the possibly addictive good $u_c$ is smaller than the sum of the marginal utility of the non addictive good $g$ multiplied by the relative price $pu_g$ and the cross effects of habits on consumption multiplied by the stock of habits $z(\cdot)u_{cz}^\ast$. Hence, unless preferences are strongly distorted towards the possibly addictive good, an increase in the rate of habits depreciation lowers the rate of time preference, reducing the growth path of the possibly addictive good.

It is worth noting that being the rate of habits depreciation $\sigma(d)$ a function itself, one may be interested in the effect that demographic variables $d$ have on the rate of time preference. This is straightforward since

$$\frac{\partial}{\partial d} \rho^c(g(t), c(t), z(t), \varphi(t), \sigma(d)) = \frac{\partial \rho(\cdot)}{\partial \sigma(d)} \frac{\partial \sigma(d)}{\partial d} \begin{cases} > 0 & \text{if } \frac{\partial \sigma(d)}{\partial d} < 0 \\ < 0 & \text{if } \frac{\partial \sigma(d)}{\partial d} > 0. \end{cases}$$

The analytical and behavioral properties of the rate of time preference (Propositions 2 and 3) allow us to describe the dynamic evolution of an agent from a condition of potential habit to a state of addiction.

Reconsider the case of the myopic and forward-looking agent introduced above. The degree of impatience of the myopic agent is generally higher (lower $\varphi$) than the degree of the forward-looking and so will be the degree of habits ($\sigma^m < \sigma^f$). The propensity to exchange current for future consumption becomes less and less considerable. The myopic agent reveals an increasing impatience since his stock of habits is higher and higher. The subjective rate of time preference of the myopic agent encloses reinforcement and tolerance, two behavioral factors that are closely related to the concept of adjacent complementarity. Reinforcement means that greater current consumption of a good rises its future consumption in accordance, while tolerance means that given levels of consumption are less satisfying when past consumption has been greater. On the other hand, the forward-looking agent is patient, since has greater capability to anticipate the future consequences of present and past consumption.

The analysis clearly reveals a patience-dependence trade-off. A patient person tends to have a lower stock of habits than an impatient person, since the desire to anticipate future consumption

30Note that $u_{cz}$ is assumed to be negative, hence the term $-z(\cdot)u_{cz}$ is positive.
is lower. It is not surprising that addiction is more likely for people who discount the future heavily since they pay less attention to the adverse consequences. Becker et al. (1994) suggested that poorer and younger persons discount the future more heavily while Chaloupka (1991) found that less educated persons may have higher rates of time preference. Capability of anticipating the consequences of present and past consumption depends on income, education, rank and degree of awareness of dangers.

According to Becker and Mulligan (1997) "... the analysis of endogenous discount rates implies that even fully rational utility-maximizing individuals who become addicted to drugs and other harmful substances or behavior are induced to place less weight on the future, even if the addiction itself does not affect the discount rate." In their analysis, addiction affects the discount rate through the rate of habits depreciation. The degree of impatience is higher for lower values of the discount rate and so the likelihood that the consumer reveals addiction to a good is greater.

In doing so, we perform a simulation analysis imposing a quadratic utility function, a linear discount rate and arbitrary but plausible values to structural parameters as specified in the previous Section except for \( \sigma \), which will be \( \sigma^f = 0.95 \) for the forward-looking and \( \sigma^m = 0.5 \) for the myopic.

Figure 4 shows time paths of consumption for the forward looking and myopic agents. This graph represent the system’s speed of convergence to the steady state. The comparison of the two paths put in evidence a significantly different behavior: the myopic agent have a higher steady state level of consumption and reach it much faster than a forward looking. Moreover, for the myopic individual future utility is discounted more heavily since the habits stock adjust more rapidly and toward a higher level, and hence also the discount rate. This leads to a higher degree of impatience which explain the different behavior of the two curves. *** insert curves ***

Does an increase in the rate of return lower the rate of time preference of the myopic and forward-looking agents making them more patient? An increase in the rate of return should have a more considerable impact on the rate of time preference of the forward-looking than the myopic agent. This depends on the stock of habits held by the two agents. The myopic agent is addicted to alcohol and his rate of habit depreciation, \( \sigma^m(d) \), approaches to \(* * * zero* * * \). This implies a flat elasticity of intertemporal substitution for the myopic agent: at all the values of future utility the elasticity assumes the same values given a current consumption level. The myopic's capability to abstain from current consumption in favour of savings is reduced and the agent does not react to an increase of the interest return.

*** up to here ***

4 Specification of an Estimable Euler Equation: ERA in Discrete Time

4.1 Discrete-Time Extended Rational Addiction

In this section we propose a discrete time model, analytically equivalent to the Extended Rational Addiction model presented in Section 2, which may serves as a base for an empirical work. Then we propose a specification of the utility function and the discount rate that allow to obtain an estimable Euler equation and discuss identification of structural parameters.

In developing the discrete-time version of the ERA model we adopt a continuous-time formulation for the discount rate. The reason relies on the tractability of the motion equation for \( \Theta_t \), which results simpler, and allows in any moment to switch to the discrete-time discounting framework, without loss of generality.
Let the discrete version of the program proposed in section 2 (equation (2.1.3)) be
\[
\max_{c_t} \sum_{t=0}^{\infty} e^{-\Theta_t} u(g_t, c_t, z_t) \tag{4.1.1}
\]
\(s.t.
\begin{align*}
a_{t+1} &= e^r a_t - p c_t - g_t \\
z_{t+1} &= c_t + (1 - \sigma) z_t \\
\Theta_{t+1} &= \Theta_t + \theta(z_t) \\
\Theta_0 &= 0.
\end{align*}
\]

The discount rate \( \Theta_t \) is formed by the cumulative sum of each instantaneous discount rate \( \theta(z_t) \), which, in turn, depends on the stock of habits \( z_t \). To ensure the concavity of the program, it is sufficient for \( \theta(z_t) \) to be positive and increasing. As in Epstein and Shi (1993), we assume \( \theta(z_t) \) to be linear, even if this condition is not necessary to ensure a unique solution to the problem.

To solve the program in (4.1.1) we follow a procedure proposed by Manuelli (2004)\(^{33}\). The basic idea is that the analysis of dynamic problems can be successfully approached using a simple extension of the standard Kuhn Tucker theorem and its conditions. Note that, under standard economic conditions, all variables involved in the optimization \( (g_t, c_t, a_t, z_t \) and \( \Theta_t \)) are interior, so we can omit from the analysis the non-negativity constraints for all these variables, since they do not bind by assumption. We also omit the transversality conditions, which, of course, must hold.

The current value Lagrangian function for this problem is
\[
L = \sum_{t=0}^{\infty} e^{-\Theta_t} \left\{ u(g_t, c_t, z_t) + q_t \left[ e^r a_t - p c_t - g_t - a_{t+1} \right] + \varphi_t \left[ \Theta_t + \theta(z_t) - \Theta_{t+1} \right] \right\} ,
\]
with the following first order conditions\(^{34}\)
\[
\begin{align*}
g_t : \frac{\partial L}{\partial g_t} &= u_{g_t} - q_t = 0 \\
c_t : \frac{\partial L}{\partial c_t} &= u_{c_t} + \Psi_t - p q_t = 0 \\
a_{t+1} : \frac{\partial L}{\partial a_{t+1}} &= -q_t + q_{t+1} e^{r-\theta(z_t)} = 0 \\
z_{t+1} : \frac{\partial L}{\partial z_{t+1}} &= -\Psi_t + (1 - \sigma) \psi_{t+1} e^{-\theta(z_t)} + \theta(z_t) \varphi_{t+1} e^{-\theta(z_t)} + u_{z_{t+1}} e^{-\theta(z_t)} = 0 \\
\Theta_{t+1} : \frac{\partial L}{\partial \Theta_{t+1}} &= -\varphi_t - u_{t+1} e^{-\theta(z_t)} + \varphi_{t+1} e^{-\theta(z_t)} = 0.
\end{align*}
\]

\(^{31}\) Here we change the time notation, indicating the time index as a subscript. For instance, \( g(t) \) will be denoted as \( g_t \). This change is due to the fact that here we not only need to indicate that a variable is function of time, but also to which time period it refers to (as \( g_{t+1} \) for example).

\(^{32}\) If we solve the difference equation in (4.1.1), we find that
\[
\Theta_t = \sum_{s=0}^{t-1} \theta(z_s).
\]

\(^{33}\) For an exhaustive treatment of this procedure, please refer to the author’s notes.

\(^{34}\) To save on notation, we write the utility function \( u(g_t, c_t, z_t) \) as \( u_t \) and its partial derivatives as \( u_{g_t} \), where the time index of the subscripted variable is intended to determine also the reference period for the utility function, i.e. \( u_{g_t} \equiv \partial u(g_t, c_t, z_t) / \partial g_t \).
where we have already substituted $e^{-\Theta_{t+1}} / e^{-\Theta_t} = e^{\Theta_t - \Theta_{t+1}} = e^{-\theta(z_t)}$, from the last constraint in 4.1.1. Rewriting optimality condition, we obtain

$$q_t = u_{gt}$$  \hspace{1cm} (4.1.2a)

$$\Psi_t = pu_t - u_{ct}$$  \hspace{1cm} (4.1.2b)

$$q_{t+1} = e^{-\theta(z_t)}$$  \hspace{1cm} (4.1.2c)

$$\frac{\Psi_t}{\Psi_{t+1}} = (1 - \sigma)e^{-\theta(z_t)} + \theta'(z_{t+1}) \frac{\varphi_{t+1} - \theta(z_t)}{\Psi_{t+1}} + \frac{\psi'_{t+1}(z_{t+1}, z_{t+1})}{\Psi_{t+1}} e^{-\theta(z_t)}$$  \hspace{1cm} (4.1.2d)

$$\varphi_t = \varphi_{t+1} e^{-\theta(z_t)} - u_{t+1} e^{-\theta(z_t)}$$  \hspace{1cm} (4.1.2e)

and dividing (4.1.2b) by its counterpart in $t + 1$, substituting (4.1.2a) and replacing $\Psi_t / \Psi_{t+1}$ with (4.1.2d), we obtain one discrete time version of the Euler equation

$$\frac{mu_{gt} - u_{ct}}{mu_{g_{t+1}} - u_{c_{t+1}}} = (1 - \sigma)e^{-\theta(z_t)} + \theta'(z_{t+1}) \frac{\varphi_{t+1} - \theta(z_t)}{\Psi_{t+1}} + \frac{\psi'_{t+1}(z_{t+1}, z_{t+1})}{\Psi_{t+1}} e^{-\theta(z_t)}.$$ \hspace{1cm} (4.1.3)

From equations (4.1.2a) and (4.1.2b) we can derive an analytical expression for $\Psi_t$, which is

$$\Psi_t = mu_{gt} - u_{ct}.$$

Being the discount rate linear\textsuperscript{35}, and considering that $e^{\theta(z_t)} \approx 1 + \theta(z_t)$ for small values of $\theta(z_t)$, we may rewrite equation (4.1.3) as

$$\frac{mu_{gt} - u_{ct}}{mu_{g_{t+1}} - u_{c_{t+1}}} = \frac{1}{1 + \theta(z_t)} \left( 1 + \frac{\theta(z_t) \varphi_{t+1} + u_{z_{t+1}}}{mu_{g_{t+1}} - u_{c_{t+1}}} - \sigma \right).$$

Note that this is not the unique possible Euler equation, and nor is the one used to obtain the econometric specification. In fact we might have used (4.1.2c) or have specified it in term of a difference instead of a ratio, and that is exactly what we do in the next section specifying the proper utilities and discount rate functions.

We can easily find an Euler equation for $g_t$, which is obtained using (4.1.2a) and its counterpart in $t + 1$, with equation (4.1.2c), and shifting to discrete-time discounting,

$$\frac{u'_{g_{t+1}}}{u_{gt}} = \frac{1 + \theta(z_t)}{1 + r}.$$  \hspace{1cm} (4.1.4)

### 4.2 Econometric Specification\textsuperscript{36}

The task of proposing an econometric specification of the ERA model is a departure from the classical scheme of a theoretical article, at least for two reasons: first people may not be interested in applying such models, and second the econometric problem may be by far too complicated to be treated in a single section of a paper. Nevertheless, without the ambition of providing a complete econometric specification and an application of the proposed model, we propose some simple guidelines for the econometric treatment of this particular model.

In applied works, the main decision is about which question we are going to answer to, with a given information set. Usually microeconomics panel-data have information on consumption path

\textsuperscript{35}Linearity of the discount rate imply that its derivative with respect to the stock of habits is a constant.

\textsuperscript{36}Under construction. Any comment is very welcome.
for several goods (macroeconomic datasets may have only one composite good), and, sometimes, their prices or unit values (macro datasets usually have a general price index). In these cases, what we know is $c_t$, $g_t$ and, possibly, $p_t$, together with the rate of return to savings $r$ and a set of demographic variables $d$.

The other variables which enter the proposed Euler equation, and which we will never observe in any dataset, are the discount rate (in our case a discount function) $\theta(z_t)$, the stock of habits $z_t$ and the rate of impatience $\varphi_t$.

Given this information set, to state the issue of which question we are willing to answer to is equivalent to establish which of the structural parameters are important to be recovered from the estimates. In our case the answer is simple: the rate of habits depreciation $\sigma$ and the rate of impatience $\varphi_t$, since we think that these to objects are important to better understand the dynamic behavior of an individual, we should focus our attention on obtaining a specification which allows to estimate these parameters.

The first thing to notice is that $\varphi_t$ is not a parameter nor a parametric function of other exogenous variables, but an endogenously determined costate variable, which we could very unlikely ever recover. However, our simulation studies suggest that this variable is nearly constant for a wide range around the steady state, and for this reason we may consider it as a constant parameter $\varphi$ (once the Euler equation is specified), with a small error.$^{37}$

Regarding the rate of habit depreciation $\sigma$, by now we consider it as a parameter. In Section 3 we have shown that it can be considered as a parametric function of exogenous variables, and that this specification have important policy and economic implications. However to maintain things simple, and without loss of generality, we stick to the simple parameter $\sigma$, and leave for a future development a specification of $\sigma(d)$, also because this specification may have important implications on the econometric technique to be used.

*** under construction ***

For two reasons, we are not much interested about $z_t$. The stock of habits $z_t$, when the system is near to the steady state, is equal to a constant fraction of consumption of the addictive good $c_t$ (namely, $z_t \approx c_t/\sigma$) implying a non-relevant behavior. Moreover we use this relation to overcome estimation issues in the Euler equation. In fact, $z_t$ appears in the Euler equation for both $c_t$ and $g_t$, and being not observable we need to get rid of it to obtain estimable form. On the other hand we could also use a simulation based econometric technique, but eventually we will discuss in more details in a future work.

***

Finally, it remain the discount rate $\theta(z_t)$. In this model, this is an endogenously determined variable and its values can be recovered later. Since we assume it to be linear, it could be interesting to recover the parameter relative to $z_t$, which in (2.2.4) we called $\mu_1$. This parameter represents the marginal impact of an increase to the stock of habits $z_t$ on the discount rate $\theta(z_t)$, which could be of some interest.

Proceeding through the specification of the Euler equation, we assume that the utility functions are quadratic as in (2.2.3) and that the discount rate is linear

$$u(g(t), c(t), z(t)) = \alpha_g g(t) + \frac{\alpha_{gg}}{2} g(t)^2 + \alpha_c c(t) + \frac{\alpha_{cc}}{2} c(t)^2 + \alpha_{cz} z(t) + \frac{\alpha_{zz}}{2} z(t)^2 + \alpha_{gc} g(t) c(t) + \alpha_{gz} g(t) z(t) + \alpha_{cz} c(t) z(t)$$

and

$$\theta(z_t) = \mu_0 + \mu_1 z_t.$$  

*** work in progress ***

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$^{37}$ However we strongly suggest, that once the structural parameters have been estimate, one should perform a further simulation to verify the assumption of a constant rate of impatience. In a further work we may suggest a simulation based test for the stability of $\varphi$. 

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To obtain an estimable Euler equation for $g_t$, we use directly equation (4.1.4), which, substituting $z_t$ with $c_t/\sigma$ leads to\textsuperscript{38}

$$g_{t+1} = \gamma_0 + \gamma_1 g_t + \gamma_2 c_t + \gamma_3 g_t c_t + \gamma_4 c_t^2 + \gamma_5 c_{t+1},$$

with

$$\gamma_0 = \frac{\alpha_g}{\alpha_{gg}} \left( \frac{\mu_0 - r}{1 + r} \right),$$
$$\gamma_1 = \frac{1 + \mu_0}{1 + r},$$
$$\gamma_2 = \frac{(1 + \mu_0)(\sigma \alpha_{gc} + \alpha_{g_2}) + \alpha_g \mu_1}{\sigma \alpha_{gg}(1 + r)},$$
$$\gamma_3 = \frac{\mu_1}{\sigma(1 + r)},$$
$$\gamma_4 = \frac{\mu_1(\alpha_{g_2} + \sigma \alpha_{gc})}{\sigma^2 \alpha_{gg}(1 + r)},$$
$$\gamma_5 = -\frac{\alpha_{g_2} + \sigma \alpha_{gc}}{\sigma \alpha_{gg}}.$$

About the identification of the structural parameters, a couple of things should be noted. Firstly, the only structural parameter that we identify is $\mu_0$, which is the fixed part of the discount rate. Secondly we cannot recover any parameter of the utility function, but at least we have the proportion among them ($\alpha_g/\alpha_{gg}$). Finally we cannot say anything about $\sigma$ and $\mu_1$, which as stated above are two parameters of interest, but we have to keep in mind these relations, which could be helpful if coupled with informations coming from the Euler equation for $c_t$.

Following the same strategy to get rid of $z_t$ in the Euler equation, we can obtain an estimable form also for $c_t$. The advantage of this strategy is clear: we can obtain a couple of Euler equations which can be jointly or independently estimated in a dynamic demand system framework with panel-data. Having only two kind of good is rather restrictive for this kind of analysis, but the model can be extended to any number of goods.

Before going through the Euler equation for $c_t$, note that we can rewrite equation (4.1.2d)\textsuperscript{2}\textsuperscript{2}\textsuperscript{2} by means of a discrete-time discounting framework, i.e.

$$\Psi_t = \frac{1 - \sigma}{1 + \theta(z_t)} \Psi_{t+1} + \frac{\theta(z_t)}{1 + \theta(z_t)} \varphi_{t+1} + \frac{u_{z_{t+1}}}{1 + \theta(z_t)}.$$  \hspace{1cm} (4.2.1)

If we write equation (4.1.2b) for period $t + 1$ and subtract (4.1.2b) multiplied by $\frac{1 + \theta(z_t)}{1 - \sigma}$, we obtain

$$p q_{t+1} - p q_t \frac{1 + \theta(z_t)}{1 - \sigma} = \Psi_{t+1} - u_{c_{t+1}} - \frac{1 + \theta(z_t)}{1 - \sigma} (\Psi_t - u_{c_t}),$$

which, substituting $q_t$ and $g_{t+1}$ with (4.1.2a), and $\Psi_t$ with (4.2.1), becomes

$$p u_{g_{t+1}} - p u_{g_t} \frac{1 + \theta(z_t)}{1 - \sigma} = u_{c_t} - u_{c_{t+1}} - \frac{\theta(z_t)}{1 - \sigma} \varphi_t - \frac{u_{z_{t+1}}}{1 - \sigma}.$$  \hspace{1cm} (4.2.2)
\textsuperscript{38}{$c_t/\sigma$ is the steady state value of $z_t$.}
We can do the same, for the time periods $t$ and $t-1$, obtaining

$$\frac{pm_{g_{t+1}}}{pm_{g_{t}} - \frac{1 + \theta(z_{t-1})}{1 - \sigma} = \frac{u_{c_{t}}}{1 - \sigma} - \frac{\theta_{z}(z_{t-1})}{1 - \sigma} - \frac{u_{z}}{1 - \sigma},}$$  \tag{4.2.3}

Finally, if we subtract equation (4.2.3) multiplied by $\sigma$ to equation (4.2.2), substitute the utility functions, the discount rate function, $z_{t}$ and consider $\varphi_{1}$ as a constant, we obtain

$$c_{t+1} = \beta_0 + \beta_1 c_t + \beta_2 c_{t-1} + \beta_3 g_t + \beta_4 g_{t-1} + \beta_5 c_t - \beta_6 c_{t-1} + \beta_7 c_t + \beta_8 c_{t-1} + \beta_9 g_t - \beta_10 c_t^2 + \beta_11 c_{t-1}^2,$$

where

$$\kappa = (\sigma - 2)\alpha_{c} - \alpha_{z} + (\sigma - 1)(\alpha_{cc} + \rho \alpha_{g} + \rho \alpha_{g})$$

$$\beta_0 = \frac{1}{\kappa} (1 + \sigma) \alpha_{z} - (1 - \sigma)(\mu_0 + \sigma)(\alpha_{c} + \rho \alpha_{g}) + (1 - \sigma)\mu_1 \varphi$$

$$\beta_1 = \frac{1}{\kappa} [(\sigma^2 - \sigma - 1 - \mu_0)(\rho \alpha_{g} + \rho \alpha_{g}) + \sigma \alpha_{cc} + \alpha_{cz} - \mu_1 ((1 - \sigma) \alpha_{c} + \rho \alpha_{g}) \sigma \alpha_{zz}]$$

$$\beta_2 = \frac{1}{\kappa} [(1 + \mu_0) \alpha_{cc} + \alpha_{cz} + \rho \alpha_{g} + \rho \alpha_{g}) + \mu_1 \alpha_{g}]$$

$$\beta_3 = \frac{1}{\kappa} [(1 - \sigma) \alpha_{gc} + \rho \alpha_{gg}) + \alpha_{gz}]$$

$$\beta_4 = \frac{1}{\kappa} (1 + \mu_0) \alpha_{gc} + \rho \alpha_{gg}) - \sigma \alpha_{gz}]$$

$$\beta_5 = \frac{1}{\kappa} (1 + \mu_0) \alpha_{gc} + \rho \alpha_{gg})$$

$$\beta_6 = \frac{1}{\kappa} \mu_1 \alpha_{cc} + \alpha_{cz}$$

$$\beta_7 = \frac{1}{\kappa} \mu_1 \alpha_{gc} + \rho \alpha_{gg})$$

$$\beta_8 = \frac{1}{\kappa} \mu_1 \alpha_{gc} + \rho \alpha_{gg})$$

$$\beta_9 = \frac{1}{\kappa} \mu_1 \alpha_{gc} + \rho \alpha_{gg})$$

$$\beta_{10} = \frac{1}{\kappa} \mu_1 \alpha_{gc} + \rho \alpha_{gg})$$

$$\beta_{11} = \frac{1}{\kappa} \mu_1 \sigma(\alpha_{cc} + \rho \alpha_{g}) + \alpha_{cz} + \rho \alpha_{g}).$$

The identification of the structural parameters is complicated by the introduction of the constant $\kappa$, however we can still recover some parameters of interest. For example $\sigma$, which is obtained by the ration between $\beta_3$ and $\beta_2$, recalling that we already know $\mu_0$ from $\gamma_1$. In the similar way, we can obtain $\mu_1$ from the ration between $\beta_7$ and $\beta_2$, recalling that we already know $\mu_0$ and $\sigma$.

The only parameter of interest (beyond the structural parameters of the utility functions) which we cannot recover is the rate of impatience $\varphi$.

The two Euler equations we present can be estimated provided that we have reliable information on prices. If we think that our information on prices is problematic (for example because we had to estimate the unit values), we may consider the relative price constant, assuming that

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39This further step is not necessary from a strictly mathematical point of view, since we could obtain an Euler equation in two periods only. However, since we need to identify at least parameters $\sigma$ and $\mu_1$, we use this mathematical “trick,” which extends the Euler equation to a third period.

40Note that these Euler equations are linear in the parameters and can be estimated using standard dynamic techniques.

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the relative price should not vary much in a wide time period unless a chance in the tax policy. This relative price, which we should know and may include in the parameters, would simplify the specification of the Euler equation, at the cost of having less and more complicated parameters. This can lead to further problem in the identification of the structural parameters.

Finally note that if we have information about only one good (say $c_t$) further assumptions are needed, as assuming $q_t$ or $\Psi_t$ to be constant, while our simulations and the mathematical analysis of the model suggest that it is not the case. We would stress, however, that even if the Rational Addiction model brings to fully independent Euler equations for the two goods (see Chaloupka (1991)), the authors rely on some assumption that an economist may not consider acceptable (see Gruber and Köszegi (2001) for a general critic to the RA model, and add the unjustified assumption of a discount rate equal to the interest rate$^{41}$). For this reason we believe that our model does not introduce much complexity with respect to the RA model, although it relaxes many restrictive assumption as is general with respect to all the previous models of addiction and habit formation.

\*\*\* up to here \*\*\*

5 Conclusions$^{42}$

Traditionally, the economic literature represents the structure of preferences in a dynamic context through functionals where a utility function is discounted by a constant rate. This choice, often adopted for the sake of mathematical tractability, does not allow to explain why the discount rate changes over time for the same individual (see Blanchard and Fisher (1989), Deaton (1992), Romer (1986), Bishai (2004) and Laibson et al. (2005) among others).

Assuming an endogenous discount rate depending on past consumption as adopted in Epstein and Shi (1993), the study develops analytically a new formulation of the rate of time preference that generalizes many rational models of habit formation and addiction. The rate of time preference supports a subjective structure of preferences that comprehends the memory of past events, the perception of present events and the anticipation of future events revealing adjacent complementarity. The behavioral contents delivered by the dynamic comparative analysis are in line with the results of the theory of Rational Addiction proposed by Becker and Murphy (1988).

Extending further the model to incorporate heterogeneity among individuals in the habit formation process, allows us to suggest the relevant policies in the field of addiction to substances, as differentiated policies for males and females, for example.

The dynamic analysis of addiction proposed by the model is characterized by the following behavioral properties:

1. an increase in current consumption of the possibly addictive good increases the growth rate of consumption itself, suggesting that the model support for reinforcement and tolerance effects. The higher is previous consumption, the larger the habit, and the higher must be the current level of consumption to deliver the same effect. The behavioral dynamics of an agent who evolves from habits to a state of addiction are delivered, deriving a patience-dependence trade-off. A patient agent reveals himself forward-looking valuing the future more than a myopic agent, whose level of habits is higher than the first one, because he is less worried about the consequences of an excessive current consumption.

2. an increase in the rate of habits depreciation decrease the growth rate of consumption of the addictive good. This means that if it is possible to intervene to modify the characteristics

$^{41}$ The assumption of $\theta = r$ is necessary in order to obtain a constant $q_t$ (see equation (4.1.2c)), which simplify by much the analysis.

$^{42}$ Partial, under construction.
which determine the predisposition to habit formation ($\sigma$), there is space for a policy of diminishing consumption even avoiding an intervention on prices.

3. if there is a high predisposition to habits (small $\sigma$) the optimal path that converge towards a steady state may be cyclical (or oscillatory).

This means that this model is potentially useful in applications with dynamic micro-data, specially if one can use panel-data or create a pseudo-panel. Observing individual behavior over time, together with demographic informations, may help to identify the parameters. The proposal of an estimable Euler equation, suggests that the model is highly demanding, however we believe that the habit formation process could play an important role in explaining part of unobserved heterogeneity in individual data. The task of an empirical work based on this model will be objective of a further work.
References


A Appendices

A.1 Proofs

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A.2 Figures

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