Theories of distributive justice - by John E.
Roemer

A summarizing exposition

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1 The measurement of utility and Arrow’s
Theorem main results (Cf. Chapter 1)

The main result of the Arrow’s impossibility theorem states that if there is a finite set of $X$ formed of at least 3 social alternatives, and individuals have preferences over $X$ that satisfy certain properties (completeness and transitivity) under some axiomatic assumptions ($WP, IIA, ND$), there is no social choice function satisfying all the three axioms simultaneously (or in other words, a social functional satisfying $WP$ and $IIA$ is a dictatorship). The application of this theorem in the voting rules systems show us that no voting rule can possibly define a social choice function satisfying Weak-Pareto, Independence of irrelevant Alternatives and Non-Dictatorship simultaneously (where we deal with the aggregation of several individuals’
preferences over a set of social alternatives). The theorem nevertheless may have several interpretations, one of which may be the individual multi criteria decision-making (it is the case of one individual that forms preferences over some set of alternatives summarized by a list of criteria) in order to form a global ranking of these alternatives. The result shows that when we use more information than just the event-specific rankings of the alternatives, one possible solution is to assign a global score to each alternative (summarizing the ‘performances’ of each alternative by event). Therefore in this case it is meaningless to say which rule satisfies or violates Arrow’s axioms.

Although, reformulating the Arrow’s theorem in a different formal setup (starting from individual’s utility functions, rather than from their references) will allow us to reach conclusions about the global ranking under measurability and comparability of alternatives. The goal still remains that of aggregating utility functions into a social ordering of alternatives (social evaluation functions).

1.1 Measurability and comparability of utility

The concept of welfarism is introduced by defining a new axiom (Pareto – Indifference) conjointly with the existing one of IIA that will have a strong implication in simplifying the analysis (social evaluation function will consist in an ordering of utility vectors). PI and IIA operate now under the name of welfarism (term used initially by Amartya Sen) and is equivalent to strong – neutrality that states that the social evaluation function \( \mu \) ignores all non-utility features of the alternatives (for example, their nominative is anonymous if for all \( \mu \) and \( \mu' \in U^N \) and all \( x, y, z, w \in X \), if \( \mu(x) = \mu'(w) \) and \( \mu(y) = \mu'(z) \) then \( R_\mu \) makes the same relative ranking of \( x \) and \( y \) as \( R_{\mu'} \) makes of \( w \) and \( z \)). In this context if a social evaluation function \( F \)
satisfies $PI - IIA$ iff it satisfies strong neutrality and we managed to build a social evaluation function that consists of an ordering of utility vectors (it is generated by an utility ordering $R^*$ on $\mathbb{R}^N$).

The axioms reflecting different assumptions about different levels of measurability and comparability of individuals’ utility limit the information conveyed by utility profiles (used at their turn for building the social ordering). The five axioms respectively $OF$ (ordinal measurability - full comparability), $CN$ (cardinal measurability - non-comparability), $CU$ (cardinal-measurability - unit-comparability) and $CF$ (cardinal measurability - full-comparability) make assumptions on the usable information in building the social ordering (independent increasing transformations of utility functions, ordinal and interpersonal statements). The most information-demanding rule is $CF$ (therefore it is the weakest because it requires multiple restrictive assumptions to be made over the general set in study, before it happens) whereas at the opposite side we find $ON$ (that unfortunately does not coexist with the three axioms in Arrow’s impossibility theorem). Therefore, two issues are to be answered at this point: how much is to be weaken the $ON$ in order to break the impossibility result and which will be the resulting social evaluation function in this case?

1.2 Main results

The results show that there is the need to assume utility comparability in order to break impossibility and this will lead to social evaluation functions including utilitarism and leximin. With a new definition for dictatorship (an ordering $R^*$ on $\mathbb{R}^N$ in which $\exists n$ such that $\forall u, u' \in \mathbb{R}^N$ if $u_n > u'_n$ then $u' P * u$) we obtained now that a social evaluation function (SEF) $F$ satisfies $PI$, $WP$, $IIA$ and $CN$ if it is generated by a dictatorship (the
impossibility still exists). This new reformulation of the Arrow theorem is easily proved diagrammatically for \( N = 2 \) individuals with the help of rankings and translation of choice vectors grouped by regions of a quadrant. Discussion is made about the boundaries’ ranking.

Furthermore, interpersonal comparability of utility is addressed with the objective of removing the dictatorship situation, while maintaining the Arrow’s axioms valid. In this occasion, new social evaluation functions arise (implicitly restricting attention to SEF that satisfy \( PI \) and \( IIA \)). A new axiom is added, the \textit{Anonimity} (considering that if we operate a permutation \( u' \) of \( u, \forall u', u \in \mathbb{R}^N \) then \( uI \ast u' \)), that is incompatible with \textit{Dictatorship} stating that the social ordering should treat all individuals anonymously, by not considering their names’ importance. This new axiom together with the Arrow’s axioms form the \textit{positional dictatorship}, in which the dictator is a certain rank \( r \) in the individual utility \( r(u) \) (provided that now utility is assumed to be fully comparable). The most utilized orderings in the class of positional dictatorship are:

- The \textit{maximin} ordering \((uR^*u' \iff \min_{n=1..N}u_n \geq \min_{n=1..N}u'_n)\);
- The \textit{maximax} ordering \((uR^*u' \iff \max_{n=1..N}u_n \geq \max_{n=1..N}u'_n)\);
- The \textit{leximin} ordering \((uI^*u' \text{ and } uP^*u' \iff \exists r^* \text{ such that } u_{r(u)} = u'_{r(u')} \forall r > r^* \text{ and } u_{r^*(u)} \geq u'_{r^*(u')}; \text{ therefore if } r = N \text{ the lowest ranked individual is the dictator.} \)
- The \textit{leximax} ordering .

We obtain now that a SEF satisfies \( PI, WP, IIA, A \) and \( OF \) iff it is generated by a positional dictatorship. In the case of \textit{leximin} ordering two
new axioms are derived: $SP$ (strong Pareto-optimality) that imposes some
strict preference over alternatives, and $ME$ (minimal equity) that establishes
who is the dictator (if we construct a utility vector $u'$ starting from an existing
one $u$ where individual $i$ is better off than individual $j$, if we add even more
‘welfare’ to the individual better ranked and we ‘worsen’ the situation of
the already poor individual, the society should rank the final situation $u'$
below the initial one $u$). The ordering that satisfies $PI$, $SP$, $IIA$ $A$ and $ME$
axioms is generated by the unique SEF $leximin$ ordering (remark that $OF$
is satisfied by default in the $leximin$ ordering case). In the next section we
assess the further weakening of the $ON$ axiom.

2 Utilitarianism (Cf. Chapter 4)

The weakening of axiom $ON$ to $CU$ together with $A + Arrow$ axioms allow us
to characterize utilitarianism. The utilitarian ordering $R^*$ on $\mathbb{R}^N$ is defined
if $\forall u, u' \in \mathbb{R}^N, u R^* u'$ if and only if $\sum_{n=1}^{N} u_n \geq \sum_{n=1}^{N} u'_n$. Therefore, a SEF now
that satisfies $PI$, $WP$, $IIA$, $A$ and $CU$ is generated by the unique utilitarian
ordering. A new (but weaker) axiom is introduced to replace $A$, in order to
provide generality to the utilitarian ordering, respectively $C$ (continuity) that
states the boundedness of sets \{ $u' \in \mathbb{R}^N : u R^* u'$ \} and \{ $u' \in \mathbb{R}^N : u' R^* u$ \}.

There are some other characterizations of utilitarianism some of which
relax axiom $CU$ (to $CP$ for instance) in order to explain the context of life-
expectancy utility functions (functional that ranks states according to the
value of $\sum (u^i)^2$ that satisfies axioms $WP$, $IIA$, $U – domain$, $AN$ and ratio-
scale full comparability). To explain the fact that maximizing average life
expectancy in a population is the goal-behavior of the society, we will need
a characterization theorem (similar to one that defines the utilitarian social
ordering rule) but with the 'cardinal-unit comparability’ replaced by 'ratio-scale full comparability’ (that is weaker than the former one) and without any other strengthening of axioms, there will be no unique determination of the social welfare functional.

It was needed therefore that another axiom be defined (EL elimination of influence of indifferent individuals) in order to achieve the results in Maskin’s theorem. The axiom EL states that two social orderings $F(u^1)$ and $F(u^2)$ are identical on the set of social alternatives $X$, if $\forall x, y \in X$ if $u^{1j}(x) = u^{1j}(y)$ and $u^{2j}(x) = u^{2j}(y)$, with $u^1, u^2 \in U$ and $i \in M, j \in H - M$. Consequently we can find a $F$ that satisfy EL together with SP, C, AN, U, SP and I that were previously defined and respect cardinal full comparability, then $F$ is utilitarianism (that is $xF(u)iff \sum u^i(x) \geq \sum u^i(y)$) (Maskin 1978). This new way of defining utilitarianism through its new requirement of continuity eliminates some mechanisms such as the lexicomin (lexicographic minimum) that is not continuous, which corresponds to egalitarianism. This has proved to be also an argument against it from the point of distributive justice (as the author specifies). Nevertheless, when utility is assumed to be ratio-scale comparable RFC, the utilitarianism should be provided with an axiomatic characterization (problem debated by Donaldson and Blackorby 1982) who show that if RFC is substituted by FC in Maskin’s theorem then a class of social choice functionals is characterized. If utility is restricted to be non-negative, this class includes Bergson-Samuelson-type functionals of the form $F(u^1, u^2, ..., u^H) = (\sum ra^h)^{1/r}$ with $r \geq 0$. In order to obtain the generalized utilitarian welfare functionals of the form $\sum g(u^i)$ from Maskin’s theorem, we have to replace 'fully measurable and comparable’ with 'absolutely measurable and comparable'.
2.1 Lotteries and individual maximization under uncertainty

If we imagine that heterogeneous individuals make choices with a certain probability over some alternatives (lotteries), that is they have von Neumann-Morgenstern preferences over a set of lotteries, the utilitarianism should deduce a social preference ordering over $L$ (the set of lotteries). This is the problem first addressed by Harsanyi that argued that if it is reasonably enough to assume utility functions that obey to $vNM$ axioms than it is reasonably to assume that society’s preference order also obeys to these axioms. The result is that, if we introduce a new Pareto-axiom over lotteries, together with $vNM$, the social ordering must be of the form $\sum r^h u^h$ for $r^h \in \mathbb{R}$, that is society must order lotteries by weighted utilitarian rule. Another interesting result (derived by Epstein and Segal 1992, as a solution to the Diamond (1967) striking arguing against the fact that the social welfare functional must necessarily satisfy $vNM$ if individual preferences over $L$ obey $vNM$ axioms) states that the preference rule for the society is represented by a quadratic, strictly increasing, quasi-concave Bergson-Samuelson SWF. This is true - authors state - under some minimalistic assumptions, namely:

- individual preferences $R^h$ over $L$ satisfy $vNM$ axioms;
- the social preference $R$ on $L$ satisfies $SP$, $C$ (upper and lower contour sets under the preference order of a given lottery to be closed), mixture symmetry (if $pIq$ than $\forall \alpha \in [0, 1], (\alpha p + (1-\alpha)q)I((1-\alpha)p + \alpha q)$) and randomization preference. This last requirement states that a mixture of probability-weighted risky lotteries is preferred to the certain non-weighted risk by the society.
The Harsanyi aggregation theorem is a different result about the representation of social preferences in terms of vNM utilities. Because the lottery which maximizes $\sum r^h u^h$ is not considered as a maximizer of meaningful sum of individuals utilities (because of its restricted domain, $CNC^* \neq CNS$ in the vNM case.), it is not representative for utilitarianism. Therefore, unfortunately it cannot be seen as a social welfare measure, but only as a 'marginal' one. If instead we suppose that each individual a-priori, when it is still an unborn 'soul', has some probability $\pi^h$ to born as a type $h$ person (characterized by a certain type of income), the individual faces some decision problem behind a veil of ignorance, where it knows the probability distribution of types in the actual world, but not which person it shall become in the birth lottery. Harsanyi argues that the soul has preferences over these lotteries that obey the vNM properties. Nevertheless the soul’s maximization problem $\max_{\phi}(\pi(x)) = \sum \pi^h r^h u^h(x) + K$ with $X$ set of states (say possible distributions of income), rests some 'marginal' feature in measuring welfare (due to its contested role of accounting for some well-being across the persons). This result is due to Harsanyi’s Impartial Observer Theorem, that unfortunately has been contested as having nothing to do with the utilitarianism, but it rests a representation theorem. In a few words, the demonstration shows that it is never socially optimal to give the hole wealth to individual 1 if $F(x) = \sum_h \pi^h r^h u^h(x)$ is the social objective, that leads to the result that a society will give the whole wealth to individual 1 if it were to maximize an average utility.

Nevertheless some optimistic conclusion is reached when a difference is made between expected utility and average utility, that is if we consider the a society formed by identical individuals with vNM preferences, that chooses the insurance vector $M^1, M^2, ..., M^s$ such as to:
with $s$ states of the world, $x^s$ income of individual for a state of the world. In this case, the program implies the smoothening of consumption over states of the world, that leads to the conclusion that it is incorrect to view optimization under uncertainty by an agent with $vNM$ preferences over lotteries as resulting in a kind of intra-personal utilitarianism.

### 2.2 Optimal population size

The issue studied here is related to compare two possible *worlds* in terms of both its population size and its distribution of resources among their population, issue that does not found solution under the utilitarianism approach (as far as for the utilitarian approach, the more people, the better even if the number $n$ of people gets infinitely large!). New principles are introduced in order to be able to compare the two worlds, respectively:

- **$PP$** (Pareto Population principle), according to which, for any positive population-size $n$ and any vector of average utility $\pi \in \mathbb{R}^n$, $(\pi, 0)Iu$ an indifference relation on $\mathbb{R}$. It says that if a person whose life is neutral is added to a $n$-individuals world, the new world is ranked as indifferent w.r.t. the old one;

- **$CLP$** (Critical-level Population principle) according to which there exists a positive integer $\alpha$ that takes the place of the 0 in the previously mentioned $PP$, such that indifference of the new world ranking is maintained. Therefore, we can add up to $\alpha$ life-values to the old world ans still maintain the same ranking of it;

- **axiom $IUD$** (Independence of Utilities of the Dead): the dead in state $M$ of the world at period $t$ consists of all persons whose life has ended
before period $t$. If each member of a society who are dead by period $t$ are considered to have the same lifetime utility in two states for instance $M_1$ and $M_2$, then the utility profiles of other members in $t$, matrices $M^1$ and $M^2$ will be ranked by $R$ in the same manner, independently of the dead-persons’ choice.

The main result is that if the preference ordering $R$ on $M$ satisfies $SP$, Anonymity, Continuity, IUD if and only if it is characterized by a social evaluation function $W(n, v) \xrightarrow{ord\text{-}equiv} n(g(v) - g(\alpha)) = \sum_{h \in N}(g(\pi^h) - g(\alpha))$ where $g$ is continuous, increasing with a fixed point $g(0) = 0$. Here $\pi^h$ is the lifetime utility of individual $h$ and $v^h$ is the equally distributed equivalent utility if the state $M$ with $n$ persons having the lifetime utilities just defined (Blackorby, Bossert and Donaldson result, 1993).

Still the most striking ethical criticism to utilitarianism remains the fact that it is insensitive to inequality of utility among individuals, and this becomes even worse if utilities are absolutely measurable and comparable.

3 Axiomatic Bargaining (Cf. Chapter 2)

The move from viewing a particular society as the main object to the image of a one point in the domain of a function may have resulting in giving the theorems more importance in the political philosophy context. The study of Nash’s bargaining problem as one way of articulating justice as mutual advantage applies the same mathematical tools as the articulation of justice as impartiality. This approach was necessary in order to provide responses to why people cannot reasonably expect to get more and what they cannot reasonably claim more under the two senses: bargaining and ethical.
3.1 The Nash solution

Nash describes the general bargaining problem as an abstract pair \((S,d)\) where \(S\) is a compact and convex set in \(\mathbb{R}^2\) and \(d \in S\). Furthermore we concentrate on a mapping function \(F\) (analog to the Arrow’s social choice functional) that projects the pairs \((S,d)\) into points in \(S\). This mapping function is called the solution that mainly differs from the SWF by the fact that it just chooses which allocation is the best, and there are established the axioms to which \(F\) must obey (Nash):

- Pareto Efficiency \((P)\) according to which \(F(S,d)\) is Pareto optimal in the utility possibility set \(S\), for whatever pair \((S,d)\);

- Scale Invariance \((S.INV)\) that is motivated by the implicit assumption that \(S\) and \(d\) are associated with a particular choice of \(vNM\) utility functions, then the bargaining solution should not change its underlying allocations if the problem in the utility space changes because of a change in the the \(vNM\) choice, preferences unchanged;

- Symmetry \((S)\) asserting that the players have equal bargaining skill as argued by Nash, that is, if the two players have identical preferences over lotteries, than the problem can be represented by choosing the same \(vNM\) utility function for them, giving rise to a symmetric set \(S\) wit a threat point \((0,0)\).

- Contraction Consistency \((CC)\) similar to \(IIA\) in which if we consider two set of lottery bargaining problems (one larger than the other), the solution chooses a lottery that is feasible in the smaller lottery;

- Unrestricted domain \((U)\) according to which the domain of \(F\) consists of all pairs \((S,d)\) with \(S\) some convex, compact set.
With the aid of these new axioms and the definition of the *Nash bargaining solution* and $F^N$ that maps a problem $(S,d)$ into a bi-dimensional point that maximizes a scalar product on the set of individually rational points in $S$: $(\pi^1, \pi^2) = \text{Argmax} \{(u^1 - d^1)(u^2 - d^2)\}$. Therefore, if $F$ obeys to $U, P, S.INV, S$ and $CC$, then it is the Nash bargaining solution. One of the criticisms to the Nash solution lies on its chosen axioms, in particular $CC$, that faced some other axioms to be replaced with. Another questioning problem was if it worth to model a bargaining solution rather then the bargaining process itself, seen that the economic reality lies on non-cooperative games rather then cooperative agreements (Rubinstein’s model of alternating offers, 1982). There are various models of bargaining with alternate offers that display non-cooperative bargaining games and show that in the limit the equilibrium in each game is the Nash bargaining solution.

### 3.2 Egalitarian solutions and welfarism

A new theorem was needed to provide the possibility of interpreting the Nash solution as a prescription for justice and impartiality as far as we interpret justice as mutual advantage. Kalai and Smorondinsky found a mapping-solution, $F^{KS}$ that associates to each bargaining problem in the plane $(S,d)$ the point on the Pareto-frontier of $S$ that lies at its intersection with the line joining $d$ and $a(S,d)$ vector whose $h^{th}$ coordinate is $a^h(S,d) = \max\{s^h | s \in S, s \geq d\}$ (the most that individual $h$ can possibly get at points in $S$ that dominate the threat point). The main result is that $F = F^{SK}$ is the only social functional satisfying $U, P, S, S.INV$ and individual monotonicity (in two bargaining problems, having different utility possibility sets, at equal maximum strategy-values, the greater social functional is the one corresponding to the larger set). The introduction of individu-
ual monotonicity had to do with the relative strength of players bargaining positions, even though there were provided alternative characterizations in which $I.\text{MON}$ was replaced with a normative-type axiom. Further solutions involve finding a solution that must in any problem equalize the utilities of individuals, provided that the problem is normalized such that $u^h(0) = 0, \forall h$ and some ethical properties are fulfilled (Pareto, symmetry and monotonicity). Unfortunately, without any measurability properties of the underlying utility functions (such as scale invariance), an impossibility result would have been obtained. If scale invariance is substituted by symmetry, a dictatorship-possibility result may be obtained (an undesired result!).

A different statement arguing that the solution should depend only on the utility possibilities set and the threat point is the welfarism. In this new approach the Nash's world provides micro-foundations that is, it can be deduced from more primitive assumptions about the behavior of rational bargainers. One important view about the consequences of bargaining in a large economy with many players each being provided with endowments of goods at the beginning of the game, is that a solution that was proposed in this case is the core of the economy. An allocation is in the core of the economy iff no coalition of traders can arrange trades (by withdrawing from the game together with the endowments of its members) among themselves and obtain better-off situations than the proposed allocation (Pareto). Thus the core focuses on possibility of coalitions and a basic result in the General Equilibrium theory is that the core consists of exactly the Walrasian equilibria of the economy, and as economies get large the core shrinks to the set of Walrasian equilibria (characterized by the fact that $Z(x, p) = 0$, excess demand at given prices is null, or more, all markets - factors and final goods - are cleared). Nevertheless, the mechanism that assigns to an economy its
Walrasian equilibria is not welfarist (one cannot find the Walrasian equilibria of the economy, or the inferred utility points associated to it, simply from information on the utility possibilities set and the initial endowment allocation of the economy).

The welfarism axiom $W$ states that if $F$ is a mechanism on a specific class of economic environments, under some conditions two economic environments $e, e'$ that give rise to the same utility possibility set $A(e) = A(e')$, then the mechanism must assign solutions for the two problems that are indistinguishable in terms of utility $u^1(\phi(e)) = v^1(\phi(e'))$ and $u^2(\phi(e)) = v^2(\phi(e'))$. The class of economic environments that we are considering contains utilities possibilities set generalized by the environment $e$ such that $A(e) = \{(a, b) \mid \exists p \text{ such that } u^1(p) = a, u^2(p) = b\}$. An alternative solution to the bargaining problem (which also accounts for coalition) is the bargaining set proposed by von Neumann and Morgenstern. Even if Mas-Colell (1989) showed that for large economies the bargaining set also reduces onto a Walrasian equilibria set, is still remains unclear that welfarism is a property is a property if the solution to the many-person bargaining problem.

A further elegant result developed by Binmore 1987, is the determination of the solution $F$ as a Walrasian equilibrium allocation as a functional that satisfies some assumptions:

- $S.INV^*$, $S^*$, $P^*$ and $CC^*$, that are the corresponding reformulated Nash-axioms on economic environments, such as to allow for welfarism;

- $U^B$ that establishes the domain where axioms are defined as $G$, that is formed by feasible elements $(u, E)$ where the Walrasian equilibrium of that economy is unique and lies in $E$, the set of possible trades between players and lotteries of those lotteries.
CC\textsuperscript{B} that is the Binmore consistency which states that if preferences differ in a certain way on two elements in the domain with the same set of possible lotteries, \( s = F(u, E) \) and \( s' = F(u', E) \) such as \( u(s) = u'(s) \), then the solution functional chooses the same trade in those two problems. In other words, the utilities of both players in one problem must be less or equal than utilities of both players in the other, for all utilities.

4 Axiomatic Mechanism of Economic Environments (Cf. Chapter 3)

The theorem that defines the Nash mechanism satisfying the restated Nash-traditional axioms, tells us that Nash axioms appropriately restated on a domain of economic environments, hardly restrict the mechanisms at all, in that one can choose any regular collection of problems in the domain and choose arbitrary \( P - \text{optimal} \) lotteries in these problems, and find a mechanism obeying to the axioms and picking the specified lotteries as solutions to the given problems. This is to prove that Nash axioms when appropriately stated (capturing economic intuitions at the base of their motivation), are weak in characterizing a unique allocation mechanism.

Therefore, a new requirement is necessary in order to 'activate' the characterization theorems presented in the previous section, on economic environments. Precisely, the axiom called CONRAD (see Roemer,ch.3,1996) represents a weaker version of welfarism and together with other axioms will imply welfarism. Thus, some axioms on economic environments are defined, which are weaker than Nash-type axioms, in order to assure us over the following commitments:
• Welfarism arguing that an allocation mechanism must treat identically (in terms of utility distribution) any two economic environments with the same UPS (utility possibility set);

• Monotonicity stating that as long as one UPS is contained in another, neither utility should decrease under the functional’s action, in the richer environment;

• Resource Monotonicity that commits the mechanism to increasing both utilities only when the two considered environments are the same except for a certain situation of increase in resources;

• Individual’s utility should not decrease in the richer environment, Individual resource monotonicity;

• When two individuals have the same utility function, the mechanism divides the resources equally between them;

• Cardinal non-comparability (or ‘miopy’ of functionals) over differing-by-a-constant environments;

• Continuity of the utility mapping $\mu$ in its arguments $\pi, u$ and $v$;

• Consistency of resource allocations across dimension CONRAD, that commits the mechanism to choosing resource allocations in the two environments $E$ and $E^*$ which generate the same point in utility space when these environments are related to each other by a strong relationship. It is a weaker version of welfarism.

By combining these axioms and alternative assumptions on the domain of $F$ regarding ‘strictly comprehensive’ environments ($\Sigma$ and $\Gamma$), some already known mechanisms (such as $F^N$ - Nash mechanism; $F^{KS}$ - Kalai-Smorodinsky...
mechanism that selects the resource allocations generating utility point at which the line connecting the optimal point and origin intersects Pareto-frontier; $E$-the egalitarian mechanism choosing resource allocations at which utility is equalized at the highest possible level and dictatorship) are obtained. The demonstration of the theorems recovering characterization results in the previous section, lie on representativeness theorems for the utility functions in a $UPS = \mathbb{R}^H$.

If we consider further that goods may be distinguished by their names (because specific goods are associated with specific functioning and capabilities, following Amartya Sen’s theory), a refinement of these axioms is needed in order to distinguish sets of different named goods. According to Amartya Sen’s notion of functioning, certain kinds human functionings are necessary preconditions for any possible conception of a life-plan (like the essential needs in Maslow’s pyramid). On the other side, there are certain resources that are required to deliver these functionings. It is therefore wrong to assume unrestricted domain for human preference orderings over certain sets of goods (including primary and luxury goods), if for example utility measures the success of achieving one’s life-plan of overall wellbeing.

5 Primary goods and fundamental preferences
(Cf. chapter 5)

We can account for a clear distinction between primary social goods and economic resources and commodities. The only primary good that is conventionally private is the wealth (comprising income). Goods like powers and positions of responsibility, social bases of self-respect are viewed as local public goods. These goods are of course, not distributed to people like
private goods, but are parts of institutions. Individuals that take part of these institutions are their consumers. Each of these charges is characterized by location and attributes (income and non-income linked). like the degree of autonomy that the position provides to its holder, etc. Rawls states that there is a free choice of occupation and the amount of time worked. This does not imply that society cannot design tax schedules that induce people to work a large quantity of time, and these schedules being motivated by redistribution towards the poorer individuals. An index of primary goods should then account for the non-income attributes that location $l$ embodies, namely $q^l(l)$ where $I$ are locations, the income $m$ and the labor $L$. Furthermore, the Rawls’ extremely interesting views of happiness seen as the relation between primary goods and the achievement of life plans, together with the existence of an objective index for primary goods, may be inconsistent in that the index does not necessarily account for any social welfare. Following Rawls (1971), we are happy when our rational plans are going well and our more important aims are fulfilled, and we are sure that our fortune will continue to support us. In this context we may suppose that the utility function of each of us has the following form $u^h(q^l(l), m, L)$, that represents the extent to which each life-plans will be fulfilled. It is a function of the primary social goods, and of labor and leisure chosen. It has been shown that each individual’s index of primary goods is ordinally equivalent and indices are different across persons. It is no way to claim then that a single index of primary goods can be used (required to be fulfill OC condition in order to compare two world states).

In order to be able to calculate Rawl’s maximin allocation, we must redefine the mean of $u^h$, supposing that there are several degrees of fulfillment of one’s life-plans ($\text{degrees } \in [0, 1]$). Given a set of institutions $I$ and a dis-
tribution of relevant resources, we aim to measure for the degree of life-plan fulfillment in an uncertain environment (space of probabilities, e.g. $\Pi^h(D; x)$ that is the probability that with a primary social good allocation $x$, $h$’s life-plan will be fulfilled to some degree $\delta$). A measure for the degree of life-plan fulfillment may be computed under the formula (as a function of his primary good consumption, including labor):

$$E^h = f_0^1 \delta \pi^h(\delta; x) d\delta$$

that is the area under the density function of probabilities. Individual $h$ has preferences over probabilities (=lotteries that must not satisfy vNM properties), and utility is represented by a function $v^h(\Pi)$. It must be clear though that persons will chose more expensive or ambitious life-plans than others. Rawlsian theory must then contain a rule for ranking profiles of expected degrees of fulfillment in terms of life-plans. While Rawls claims that society has the responsibility to equalize primary social goods (and not degree of fulfillment ion lifeplans), individuals must be held responsible for their choice in terms of life-plans. A complement of a social primary good enable a person to choose his L-P, but Rawls does not provide for a determination of the profile of expected fulfillment degrees $\phi$ of monotone functions that are necessary in order to define a generalized index of primary goods (indices which are justifiable despite life-plans heterogeneity).

In the system of natural liberty that is a regime of private property rights where everybody may benefit through the trade of his property with others, only the formal equality of opportunity is guaranteed and discrimination is not allowed - there are not inherited income or wealths (originated in Barry, 1989). In this mainframe, persons with similar native talents should face similar prospects of life (the concept of fair equality of opportunity). By comparison, in a democratic regime individuals may not benefit neither from inheritance nor from native talents (these being morally arbitrary, including...
effort at the limit). In the Pareto sense, it may exist an allocation of resources that outcomes a distribution of primary goods that gives everyone a higher index of those goods than the equal distribution (passing from equality to the maximin set of allocations, having leximin as favorite). If the emphasis is put on equality rather than on primary goods, the equality of welfare would be more plausible and if we recall axioms regarding economic environments where distributions of resources are at issue, $u$ must be interpreted as a utility-profile, and an argument for maximin requires a justification of COAD (see previous sections). Recall that COAD does not name products and so we have no way of distinguishing resources which represent genetic and familiar links from conventional ones (a vector of resources). measure-theoretic approach is put in place in order to allow us to ignore groups of people that are small (measure close to zero) that in the Rawlsian tax system they may be individuals whose welfare is lower that a certain level $\bar{v}$ such that \( \{ w | v(\tau; w) < \bar{v}, G = 0 \} \) (where \((u, G)\) is a profile of utility-probability functions and $\tau$ is the tax schedule). Thus any $\tau \in T$ induces a probability measure on the real space, specifying probabilities for welfare of souls behind the veil of ignorance. The aim is that of specifying the number of members of society that will end up being embodied in a person with a certain welfare level $S$, if $\tau$ is a chose tax schedule. the decision problem of a soul consists then in choosing one of the lotteries represented by a probability measure (depending on $\tau$), therefore the soul is provided by preferences. For example $vNM$ preferences will induce a choice of the maximin tax schedules. The idea is that of defining a function on lotteries representing preferences over lotteries (that obey $vNM$), which will introduce a choice of Rawlsian tax schedules.

To summarize, either a life-plan of a person and his propensity to effort...
are morally arbitrary or they are not so. In the affirmative situation, souls will stay away from them in the original position (but they will be aware of the position they’ll be distributed within the society). If, as in this case, all souls are identically distributed into the society and the social contract becomes an individual optimization program of the representative soul. It follows that the choice of a Rawlsian tax schedule is not justified by rational choice, therefore souls’ may not have attributed preferences. In the second case, if one is morally responsible of its effort choices and life-plan, then souls behind the veil of ignorance should know the life-plans and effort propensities of persons they will be embodied into, without knowing other resources distribution is (Dworkin’s set-up of distributive justice). The way of generating the Rawlsian tax schedule (RTS) is that in which the representative soul faces a decision problem under ignorance (unknowing the probability distribution of traits in the actual society). Preserving the major element in RTS (stating that the index of primary goods must be such that its increase gets better prospects for a person’s life-plan to be realized) requires that the primary-good index be personalized and labor-leisure to be primary goods (PG). If this is the case, then in order to maintain the fact that primary goods a person enjoys are due to social choices - tax schedules and to morally arbitrary features of individual’s features, we will need to argue that occupation and labor-leisure choice are not responsibility of individuals (arbitrarily determined). But then, we cannot state that the choice of a person’s life-plan is his entire responsibility, inconsistency that becomes one of the Rawls’ reasoning critics. Also Kolm (1972) took a methodological more formal approach, saying that if two persons have preferences which appear different, there is a reason for it, because there is something that renders them different (in the commodity space). In this way, we can eventually
represent two individual’s preferences as identical on an extended space of goods (prooved in a certain sense by Howe(1987), that claimed that if a set of $H$ persons had concave, monotone, cont non-negative utility functions on a commodity set formed by $n$ goods, each utility function can be represented as a projection of a single concave-mon-cont utility function in a space with $n + H$ goods). For Kolm, an allocation of transferable goods, which puts all individuals on the same indifference surface of fundamental preferences (equalizes utility) is unique and just (the fundamental preferences argument was criticized by Broome (1993) by making the distinction among objects and causes of preferences).

Rawls pioneered a method of analytical political philosophy, as Arrow did in social choice, Nash in bargaining theory whereas Sen improved Rawls’ theory by locating a type of human advantage in between the access to goods and welfare (measured even nowadays through the index of functionings per capita that is computed for every country in the world in the form of human development index).

6 Neo-Lockeanism and Self-Ownership (Cf. chapter 6)

The Nozick’s view of the rights of persons over themselves, in particular the self ownership postulate states that a person has a moral right to use his powers to its own benefit, without harming another person. A definition of no harming condition has been bring in front by the same Nozick as what the welfare of others would appear if the resource have had no owner, or be in common use. By contrary, Gibbard (1976) proposed the hard libertarian position in conformity to which everybody has an equal right to use all things.
there is no social contract establishing ownership rights, the situation degen-
erates in *miserable life*, as its author claims (due to the fact that no-one
would have an incentive to cure land, grow grains, etc). In the generalization
of the Lockean preamble of situation in which external resources are scarce
is the joint ownership of the external world where the distribution problem
becomes a bargaining one. In the absence of an agreement on how to dis-
tribute the proceeds form production, no good is produced and individuals
consume nothing. If we consider a two-consumer economy that have both
$vNM$ utility functions, formed by consumed good and leisure, the Nash so-
lution of the bargaining game will be the same whether or not individuals are
seen as owners of their skills. It is obtained that the less skilled worker works
less than the more skilled one. Nevertheless the output is shared equally,
therefore, in a society with many individuals, the bargaining theory may be
insufficient. The solution may be studied with the help of classes of eco-
nomic environments $e = \langle u, f, s_1, s_2 \rangle$ where $f$ is a production function that
efficiently transforms units of work in single consumption good ($x$). In this
context an axiomatic approach as the one presented in the previous sections
may be needed.

New axioms are defined in order to define an egalitarian mechanism that
equalized utilities of individuals at the highest feasible level, namely:

- Domain of F : the space of economic environments where $u$ and $x$ have
  well-behaved properties;

- Pareto optimality of F;

- Self-ownership: more skilled individuals do not envy the less skilled
  individual at the solution point;

- technical monotonicity, according to which from two economic envi-
environments differentiated by production technologies one that is greater than the other, individuals are better off in the environment with the greater production; otherwise said, if one world is more abundant than another, individuals being the same, then no one should end-up worse-off in the more abundant world

• Protection of infirms.

The generalization of Locke on economic environments is possible if we consider a class of EE of type: \( \langle u^1, ..., u^H, s^1, ..., s^H, f \rangle \) where \( s \) are skills of individuals and there is one good produced from labor, measured on efficiency units, using technology \( f \) and the external world. In order for Locke’s proviso to be met, we need a set of assumptions:

• increasing or constant-returns of scale of \( f \)

• if \( u \) is strictly concave - it leads to a unique Pareto-efficient solution

• \( u^h \) quasi-concave, increasing in \( x \) and decreasing in \( L \) then an unrestricted common ownership equilibrium always exists (where by this we understand that since a resource is held in unrestricted common ownership, as opposed to jointly every participant may use as much as he wants to.)

There are several solutions to the problem generalizing the Lockean solution to the domain \( D \) that must fulfill several requirements: they must coincide with the free access solution on linear economies (accordingly to FALE axiom when \( f \) is linear) and they must be Pareto-efficient. The equal benefits solution is the Walrasian equilibrium associated with equal distribution of firms’ profits. An interesting characterization of this solution is proved by Moulin(1990) that by defining two additional axioms, ULB (unanimity lower
bound, or \textit{full individual rationality} as referred by Gevers) and TCC (technological contraction consistency, some equivalent to IIA version of Nash contraction consistency axiom) he finds that this particular solution \(F\) is full and satisfies ULB and TCC.

Furthermore Roemer finds several characteristics of Pareto-efficient allocations in the environment \(D\) and proves their uniqueness (by using Brower’s fixed point theorem). He also claims that the Walrasian equilibrium allocation associated with initial shares in the firm, has a proportionality property (because of individuals’ budget constraints).

A CRE (constant-returns to scale equivalent) mechanism discovered by \textit{Mas—Collel} (1980) defines a set of allocations in \(e\) that generates a point in the linear economy, where free-access allocation produces a utility-allocation lying on the Pareto-efficient frontier that lies on the \(UPS\) of this environment. The free access solution (efficiency-preserving generalizations of free-access Lockean solution on linear economies) on linear economies enjoys some important properties:

- it weakly Pareto-dominates the Nash solution;
- all individuals enjoy equal utilities;
- consumed quantities are proportional with labor-quantities invested in it;
- it is itself a linear economy;
- these properties characterize a unique solution on the domain \(D\).

The author also demonstrates that there is no efficient mechanism on \(D\) that Pareto-dominates the common ownership equilibrium and that can be implemented as a Nash equilibrium. This means that the allocation rule
determined by the privatization must be describable as a mechanism \( F \) that Pareto-dominates equilibrium under unrestricted common ownership. If that mechanism is also Pareto-optimal, that it cannot be decentralized in a Nash implementation. Another reported result is the implementation on large environments of Makowski and Ostroy (1992) in which allocation mechanisms on finite type economies which are Pareto-efficient and which can be implemented in dominant strategies, are characterized. Nevertheless, as Rawls states it, the thesis of self-ownership may be challenged, even if neither of the four mechanisms described does (whereas the \( FALE \) axiom contains self-ownership by definition).

### 7 Equality of welfare vs. equality of resources

(Cf. chapter 7)

The following two chapters provide a descriptive and criticized framework of Ronald Dworkin’s major contributions to distributive justice, in particular regarding the equality in welfare and resources. He argues that there is ethically unattractiveness of equality of welfare and proposes a way of conceiving equality of resources as a proxy to distributive justice. Dworkin’s argues that justice requires compensating individuals only for aspects of their situation for which they are not responsible (and which can influence negatively their value achievement during life). Individuals are responsible for their preferences as long as there’s a perfect identification between the two concepts (the idea of personal responsibility that originates in Rawls and Sen is furthermore developed).
7.1 On equality of welfare

Dworkin’s motivation against welfarism comes from at least some fundamental considerations:

- the extent to which welfare reflects needs or desires;
- the extent to which expensive tastes which are voluntarily cultivated deserve consideration;
- the inherent value in different conceptions of the good;

Dworkin concludes that equality of welfare, at least in its naive interpretation (and assuming interpersonal comparability), is not attractive when it comes to confront different types of tastes (involuntarily cultivated cheap tastes and voluntarily cultivated expensive tastes). For Dworkin there are at least two concepts of welfare that provide comparative and measurable welfare issues: *relative success* which reflects the degree to which one fulfills his life-plans, and *overall success* which measures the degree to which one’s life is a success. The link that all these have with resources is that *fulfillment degrees* are subject to a cause of *reasonable regret* measure on behalf of individuals, and this measure is related to the comparison of one’s situation with situations of others (that directly depend on resources available). In other words, no one is motivated to reasonably regret his life if resources are *fairly* distributed. It still remains to define what exactly we mean by *a fair* distribution. It was also demonstrated (a contradictory result of *overall success* definition) that if a society adopts a distribution of resources that equalizes degrees of overall success, then no individual would have an incentive to adopt expensive tastes (because if he does so, he would end with a lower degree of overall success as before, as it would be irrational in a society to cultivate expensive tastes).
An interesting conclusion is derived: a person has no cause reasonably to regret not being treated like another in his own class (assuming that individuals in a society are divided in quasi-homogeneous classes), because all members of his class receive the same bundle of external resources, other differences between him and the other members of the class being irrelevant. In this framework persons care only about expected relative success (they have risk-neutral $vNM$ utility-functions over relative success) and problem arise when it deals with involuntarily acquired cheap tastes (some persons may unfairly suffer for appearing to have expensive tastes, when in reality they don’t, by the means of adopting a falsely truncated conception of success from the beginning). Therefore, no coherent concept of equality of equality in overall success of a life can be adopted, because of implicit definition of reasonable regret that distorts results.

At this point, an alternative approach that comes as a solution to this problem is the issue of equality in resources.

### 7.2 On equality of resources

If all individuals in a society were identical, a solution will be to give to each individual an equal share of society’s external resources. In real life, not only individuals are not identical, but neither differential circumstances (part of his resource endowment) that characterize them do not completely explain differences among individuals. Additionally, many of circumstantial resources cannot be easily transferred from person to person (e.g. family, handicaps, talents). The question is then which distribution of external resources appropriately compensates persons for their different bundles of circumstantial resource endowments?

The Dworkin’s equality-of-resources proposal consists in making distinc-
tion between preferences and circumstances. All souls (under the veils of ignorance) have the same opportunity to acquire the same insurance policies and each know the distribution of handicaps in the real world is transparent for all and difference in insurances are due only to preferences which are the person’s responsibility. This is an issue than cannot be always considered reasonable, in that a person should be held responsible for the consequences of his tastes as long as he is glad he had them and identifies with them (just because at a certain point in time he has be imposed to accept them).

To explain the way in which differences in talents (seen as resources) can be compensated, Dworkin states that wages associated to talented labor are state—contingent, therefore they depend on what state of the world is realized and persons can insure against having a low income.

By adopting an axiomatic approach, Roemer describes a mechanism $F$ (a full correspondence) which assigns each environment $E$ a resource allocation or a set that implements comprehensive equality of resources. The proof is made under economic environments of the form $e = n, \pi, u, v, U, V, \Pi$ where $u, v$ measure the degree of overall success of a life (on the interval $\in (0, 1)$) and $U, V$ are representations of individuals’ preferences over lotteries of resources $\pi$. $\Pi$ is a set of states of the world and a probability distribution over such states consisting of distributions of resources over individuals. The central idea states that if we were to compare two worlds in which the two sets of persons have the same profiles of overall success functions, and in world I the resource vector weakly dominates the resource vector in world II, then resource egalitarianism would require that no individual have a lower degree of success in W-I compared to his counterpart in W-II (axiom presented in a previous section as resource monotonicity $RMON$). Also, $F$ should satisfy $CONRAD$ and in these framework, it must be the allocation
mechanism that equalizes utilities (intended as degrees of overall success). However, a resource egalitarianism may coexist with welfare while allowing interpersonal welfares to differ (due to some non morally arbitrary characteristics and actions of persons). In that case, the measure of size of the resource bundle should entail for example, the vector of primary goods it delivers (egalitarianism requires equalizing the degree of functioning of all persons).

8 Equality of opportunity for welfare (Cf. chapter 8)

The boundary placed by Dwarkin between what a person is responsible for and what not (to distinguish between preferences and resources), requires a particularly conception of responsibility denoting a person’s situation in which he has the control. Arneson and Cohen proposed revisions of this theory with the intention of separating responsibility situations in which a person may be placed, resulting in restricted kinds of egalitarianism. Arneson states that equality of opportunity for welfare has been equalized if transferable resources have been redistributed so that the decision trees of any two individuals are effectively equivalent (each of them is aware of the definition of his tree and each path is accessible in the same manner for both individuals). The author also distinguishes between first best and second best ideal preferences. Individuals then choose some path $p$ in order to maximize $U^i(p)$, persons having control over formation of preferences $U^i$. nevertheless, Cohen argues that persons who have similar decision trees may not choose identical paths (because they may have different criteria in deciding a path, e.g. altruism, morality, etc.). Thus persons may have different $U$’s, even if
they have same decision trees. One Ministry of Justice allocation of resources would be said to equalize the opportunity for welfare of persons if under that distribution, persons form decision trees that are effectively equivalent. Nevertheless a strong criticism to Arneson's approach is linked to the attempt to construct preferences under condition of ample opportunity (which would lead individuals to deliberate in an institution-free environment, that is a contradictory result). What Cohen proposes instead is a cut between responsibility (intended as having control over one's actions) and bad luck, rather than between preferences and resources, which he calls equality of access to advantage, preferring access to opportunity and advantage to welfare.

Furthermore, Roemer proposes a second-best approach having as main objective that of calibrating the degree of responsibility that a person has by virtue of his voluntary choice via a two-step procedure, by partitioning individuals in equivalence classes and deciding upon accessibility of different options to individuals (through the empirical examination of the choice distribution for each type). An environment describing the equality of opportunity is formalized over a continuum of individuals with characteristics $s \in S$, a sample space in $\mathbb{R}^n$ and endowed with a probability measure $H$. The utility function of individuals $u(w, e, s)$ (monotone increasing in $x$ and decreasing in $e$) outcomes the advantages detained by some individual with characteristics $s$, who receives resources $x$ and spends effort $e$. The social planner has a fixed supply of resources to distribute to individuals according to a linear rule $\phi(e) = \gamma e$, $\gamma > 0$. the economic environment therefore is characterized by the bundle $(u, S, H, e^*(\cdot))$. Societies are derived for each environment and an advantage function is defined for each type of individuals (divided in classes). The social planner that does not observe the environment, calculate an advantage function for individuals of each type $i$ as
the average of functions of individuals in that category. He also observes a
distribution of effort responses of individuals in each category to a certain
transfer rule $\phi(e) = \gamma e$ through which the social planner distribute resources
to individuals, in an undifferentiated manner within groups, under its bud-
get constraint. A difference is made in treating individuals that belong to
different types. An allocation mechanism is defined as a mapping functional
$F$ that associates to each society a transfer rule. Furthermore three types of
allocation mechanisms with particular characteristics are detailed, namely:

- **Equality of opportunity**, situation which is formalized by the following
  social planner maximization program:

  $$\left\{ \phi_j = \max_{\phi} \min_i v^i(j, \phi) \mid \text{s.t. } \sum_i \sum_j p^i \phi^i(e^j) \right\}$$

  where $j$ represents the effort quantile observed by the $SP$ for type
  $i$ individual facing a transfer rule $\phi^i$. The overall equality of oppor-
tunity rule is defined by averaging the $j$ different objectives in
  the previous maximization program. Therefore the rule now will be
  $\phi_E = \arg\sum_{\phi} \sum_j j^i \min_i v^i(j, \phi^i)$. The data used to calculate this
  mechanism are available in this discrete quantile formulation and the
  calibration may be easily computed. Interesting numerical examples
  are found in Roemer that help us understand how variance in data
  powerfully influence the interpretation of an egalitarian distribution
  rule.

- Some related approaches to the equality of opportunity are the ones
taken by Flauerbaey (1994) and Bossert (1995) which consider that indi-
nviduals are characterized by traits (of responsibility and of circumstance).
In this context a redistribution mechanism is a balanced budget condi-
tion that states that sum of all individuals income under each profile $i$
is equal to the sum of redistribution functions for the same profile and
trait. More axiomatic characterizations are provided by Bossert, relying on reasonable axioms like equal distribution for equal relevant traits, individual monotonicity of relevant traits, additive responsiveness w.r.t. irrelevant traits.

If it is the Bossert case the study of environments in which society is interested in equalizing opportunity for income rather than a more general advantage (production behavior being not responsive to the tax system), Flauerbaey studies a situation in which individuals care about welfare not about income only (individuals being responsible for their preferences but not for their handicaps). He finds axioms on individuals’ preferences and transfers capturing essential requisites of equality of opportunity, that lead him to impossibility results. Solutions to this problem are found by the weakening of axioms.

9 Conclusions

We have examined throughout the first four chapters different hypotheses on the type of society’s ordering rules, the definition and representativeness of individuals’ preferences, the measurement of individuals’ utility, assumptions under certainty or uncertainty regarding their orderings and manage to find some aggregation rules that aimed to maximize social welfare. The non-optimistic result of Arrovian impossibility was broken by the introduction of utility functions. Hypotheses on the degree of measurability and comparability of utility functions in the society allowed us to find out some of the most utilized social welfare functionals, that gave us some measure of utility maximizing social rule. More realistic hypothesis over individual interactions were tested by Nash resulting in bargaining processes, but without
the possibility of deriving some plausible welfare conclusions (due mainly
to the *non—cooperative* realistic nature of the economy). By redefining
appropriately economic environments and vNM-obedient utility functions
the stronger welfarism axiom has been defined. Nevertheless, justice still re-
mains ‘an ideal bargaining outcome’, as Gibbard calls it (1991), in the sense
that bargaining aims to find the optimal allocation of resources, without
considering the a-priori inequality of persons. Heterogeneity in individual’s
preferences must be explained not only in terms of their actual ordering
of alternatives but also accounting for *what actually imposed their choice
system*, that may be either the political regime under which they were born,
their social status, genealogy, etc. In my view, after the birth, individual
have some different and some common preferences that are comparable up
to some level, for example individuals may be willing maximize some general
utility function $v^h = u(j) + \epsilon^h$ where the $h$ superscript states for individual-
characteristic terms whereas the term without superscript states for utility
common to all individuals. In this case, $u(j)$ may be some vector contain-
ing variables that are suspected to explain individual’s common utility. In
this case population would be divided into *cells* having similar objectively
homogeneous characteristics (age, real income, education, geographical lo-
cation etc.) and appropriate hypotheses would have to be tested on each
cell of individuals. A social welfare functional in this case may account for
an aggregation rule over cells of individuals. A new issue would then arise
probably when one will try to associate a weighting rule to each class of in-
dividuals into the final aggregation. Supposing that even this problem will
be overcome, it still remains one big-questioning issue to answer to: *what
about the souls’ preferences a-priori* ...

The following chapters provided us with different and revolutionary ap-
approaches to criticize welfarism. The Amartya Sen’s attack on welfarism based on the consideration that some individual’s welfare were offensive, opened new doors to a further introspective analysis of the welfare concept. Dworkin and Scanlon’s critiques to welfarism are based on the view that welfarism must not retain individuals responsible for certain choices following from tastes (into which they identify themselves). On the same idea, Arneson and Cohen agree that any egalitarian theory must allow differences in advantage (intended as welfare), while libertarians and Lockean have focused on the element of responsibility in distributive justice. Nevertheless, there still are areas that require improvements such as finding a deeper articulation of ’advantage’ (even if Sen proposes a proxy for it, namely capability and Arneson proposes opportunity for welfare, they do not succeed to respond to explain problems of truncated conceptions for welfare formed as response to poor opportunities). When this problem is solved, there must be defined an aggregation of observable and measurable characteristics. The implementation of equality of opportunity through advantage as proposed throughout the previous sections, represents a tool that help us to derive a just allocation of resources, once the problems defining advantage and circumstances are solved. Impossibility theorems are statements that no uncompromising solutions exists embodying all conceptual requirements of a theory, but ‘such theorems do not preclude in general the existence of acceptable compromises’ as Roemer states.

To conclude, I retain that the wisdom found in some still applicable proverbs might inspire further introspective analysis on the fundamental concepts of equality, opportunity and fairness:

- 'The beginning of reform is not so much to equalize property as to train the noble sort of natures not to desire more, and to prevent the lower
from getting more.’ *Aristotle BC 384-322;*

- 'Equality of opportunity is an equal opportunity to prove unequal talents.' *Viscount Samuel;*

- 'A fair request should be followed by the deed in silence.' *Dante (Alighieri) 1265-1321.*

10 References

- Roemer E.J. *Theories of Distributive Justice.* Harvard University Press. 1996