Intertemporal General Equilibrium Model With
Imperfect Competition For The Evaluation Of European
Integration

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September 24, 2007
1. Introduction

This paper presents a state of the art intertemporal multicountry multisectoral general equilibrium application, with trade and production, imperfect competition and increasing returns to scale in the non-competitive firms. The focus in on the description of the mathematical structure of the model, on calibration technique and computational considerations. The model provides an investigation of European trade integration effects on welfare and employment.

The world economy consists of six regions: Great Britain (GB), the Federal Republic of Germany (Gr), France (Fr), Italy (It), the rest of the Europe (RE) and the rest of the world (ROW)\(^1\). There are nine sectors of production for each county. Four of these are assumed perfectly competitive: food, beverage and tobacco; agriculture and primary products; other manufacturing industries (textile, wood, paper, metallurgy, minerals); transports and service. In this sectors, commodities are treated as differentiated in demand by their geographical origin (i.e. countries are linked by an Armington system). The other five industries are non-competitive: pharmaceutical products; chemistry other than pharmaceutical products; motor vehicles; office machinery; other machinery and transport materials. In the latter sectors, firms are assumed to be symmetric within national boundaries, they operate with fixed primary factor costs and therefore face increasing returns to scale in production. The symmetry assumption implies that imperfectly competitive domestic firms within a sector have the same cost structure and market shares, and consequently charge the same price even if the goods are imperfectly substitutes. They have no monopsony power on any market for inputs, primary or intermediate. Each individualist oligopolist produces a different good. In the short run, the market structure is fixed (i.e., the number of oligopolists remains constant) and so firms may experience non-zero profits. In the long run, however, Chamberlinian entry and exit condition vanishes these rents. The competitive game between oligopolistic firms is assumed to be Bertrand or Cournot-Nash.

Final demand decisions are made in each country by a single representative household, competitive, infinitely lived and utility maximizing. The domestic household owns all the countries’ primary factors, labor and physical capital, which it rents to domestic firms only, at the same competitive prices regardless of the sector. The representative households decide the optimal level of investment and consumption consistent with their intertemporal budget constraints. They can borrow or lend on international markets. Leisure and population growth variables are not considered in this model.

\(^1\) I refer explicitly to the theoretical structure of Mercenier’s models (1995b, 2002)
In the initial equilibrium, national markets within Europe are segmented: because of nontariff barriers that prevent consumers from cross-border arbitraging, as norms, government procure policies, security regulation, non-competitive firms behave as price-discriminating oligopolists. The policy experiment is modelled as the elimination of the possibility for oligopolistic firms to price discriminate between client countries within Europe. The hypothesis is performed under two alternative labor assumptions. The first scenario assumes flexible wages with constant employment, so there are competitive labor markets and a vertical labor supply curve. Instead in the second scenario, wages are held fixed in terms of the consumer price index and the labor supply is horizontal, so the employment is entirely determined by the firms. None of these two extreme scenarios is realistic, they nevertheless provide a range of estimates that would presumably include the predictions made with a more realistic assumption.

The scheme of the paper is the following: in the next section the model is described in detail in its static and dynamic structure; section 3 introduces trade policy experiment and presents the welfare criterion by which this is to be evaluated; calibration and computational considerations are made in section 4; some conclusions and possible further experiments are discussed in section 5.

2. The model

2.1 Notation

Sectors of activity are marked by indices $s$ and $t$, with $S$ representing the set of all industries, so that $s,t = 1,\ldots, S$. The set is partitioned into the subset of competitive, constant return to scale sectors, denoted $C$, and the subset of non-competitive, increasing return to scale industries, denoted $\bar{C}$. Even if $C$ also denotes aggregate consumption, no confusion can arise. Countries are identified by indices $i$ and $j$, with $i,j = 1,\ldots, W=EU \cup ROW$, with the first subset represents the European Union and the $ROW$ represents the rest of the world. To keep track of the trade flows the first two indices denote, respectively, the country and the industry supplying the good and, when appropriate, the next two the client country and industry. So, for example the subscript $isjt$ says that the firm of country $i$ and sector $s$ sells its product in the country $j$ to the sector $t$. 

2.2 Dynamic structure

As we said, in each country there is a single representative household. Labor is in fixed supply, the competitive prices of labor and capital factor are respectively \( w \) and \( r \) (for notational convenience the country subscript is neglected in this subsection). The decisions variables of the household are consumption (\( C \)) and investment (\( I \)), and its intertemporal problem is to maximize

\[
\int_0^\infty e^{-\rho t} \frac{C(t)^{1-\gamma}}{1-\gamma} dt, \quad \text{(eq.1)}
\]

subject to

\[
\dot{K}(t) = I(t) - \delta K(t), \quad \text{(eq.2)}
\]

and subject to

\[
\int_0^\infty e^{-\rho t} \left[ p_c(t)C(t) + p_I(t)I(t) \right] \leq \int_0^\infty e^{-\rho t} \left[ w(t)L(t) + r(t)K(t) + \sum_s \pi_s(t) \right] dt + F(0), \quad \text{(eq.3)}
\]

\( K(0), F(0) \) given

Eq.2 accounts for capital accumulation with depreciation, eq.3 is the intertemporal budget constraint. It says that the sum of discounted stream of consumption and investment expenditures cannot exceed the discounted sum of revenues earned from primary factor owners plus initial of foreign assets \( F(0) \). The term \( \sum_s \pi_s(t) \) accounts for the possibility of supranormal profits in the short run in the non-competitive markets. All countries have the same constant discount rates \( \rho \).

2.2 Households

Each country is represented by a family. It’s possible to break household ‘s decision into a consumer and an investor choice problem because of the separability assumptions on preferences and investment technologies. The domestic consumer values products of competitive industries from different countries as imperfect substitutes (the Armington assumption, 1969), while he treats as specific each good produced by individual firms operating in the non-competitive industries (the Dixit-Stiglitz specification, 1977). There are two-level utility function. The first level, with Cobb-Douglas preference, combines consumption goods of county \( i \) with respect to the sector \( s \) (\( c_{i,s} \)) assuming constant expenditure shares (\( \rho_s \)). The second level, with CES preferences, determines
the optimal composition of the consumption aggregates in terms of geographical origin for competitive industries or in terms of the individual firm’s product for the non-competitive sectors.

Formally the consumer’s preferences are

$$\log C_i = \sum_{s \in S} \rho_{si} \log c_{si}, \quad \sum_{s \in S} \rho_{si} = 1,$$

$$c_{si} = \left[ \sum_{j \in W} \delta_{jsi} c_{js}^{\sigma_{js}^{-1}} \right]^{\sigma_{si} / \sigma_{js}^{-1}}, \quad s \in C, \quad (eq.4)$$

$$c_{si} = \left[ \sum_{j \in W} n_{ji} \delta_{JSi} c_{jsi}^{\sigma_{jsi}^{-1}} \right]^{\sigma_{si} / \sigma_{jsi}^{-1}}, \quad s \in \bar{C},$$

where $\delta_{jsi}$ are share parameters, $\sigma_{js}$ the Armington substitution elasticities, $\sigma_{jsi}^f$ the Dixit-Stiglitz differentiation elasticities and $n_{ji}$ denotes the number of symmetric oligopolists operating in country $j$, sector $s$. Observe that when, $s \in \bar{C}$, it represents the sales of a single representative firms. The interpretation of the two elasticities $\sigma_{js}$, $\sigma_{jsi}^f$ is therefore very different: the latter will typically be larger than the former. For goods that are non traded, $\delta_{jsi} = 0, \forall i \neq j$.

The consumer maximizes eq.4 respect to $c_{jsi}$, subject to:

$$p_{ci} C_i \geq \sum_{j \in W} \left\{ \sum_{s \in S} p_{jsi} c_{jsi} + \sum_{s \in S} n_{ji} p_{jsi} c_{jsi} \right\}, \quad (eq.5)$$

where $p_{jsi}$ is the price to which the firm of country $j$ of the sector $s$ sells its product to the representative consumer of country $i$. The term on the left side is aggregate consumption expenditures at current prices, it results from the intertemporal decision of the household.

The investor’s problem is to determine the optimal composition of the domestic investment good; for this, the investor the following expression with respect to $I_{jsi}$:
\[
\log I_s = \sum_{s \in S} \omega_s \log I_{si}, \quad \sum_{s \in S} \omega_s = 1,
\]

\[
I_{si} = \left[ \sum_{j \in W} \delta^j_{\mu_s} \frac{\sigma^{j-1}}{\sigma^j} \right] \frac{I_{\mu_j}}{\sigma^j}, \quad s \in C,
\]

(eq.6)

\[
I_{\bar{s}i} = \left[ \sum_{j \in W} n_{\bar{s}j} \delta^{j}_{\bar{s}j} \frac{\sigma^{j-1}}{\sigma^j} \right] \frac{I_{\mu_{\bar{s}j}}}{\sigma^j}, \quad s \in \bar{C},
\]

subject to

\[
p_B I_s \geq \sum_{j \in W} \left\{ \sum_{s \in S} p_{\mu_s} I_{\mu_j} + \sum_{s \in \bar{S}} n_{\mu_{\bar{s}j}} \frac{p_{\mu_{\bar{s}j}} I_{\mu_{\bar{s}j}}}{\sigma_{s \mu_j}} \right\},
\]

(eq.7)

where \( p_{\mu_j} \) are prices that investors take as given, and the term on the left side of the inequality results from the intertemporal decision of the household (i.e., aggregate investment expenditures at current prices). Observe that the share parameters \( \delta^j_{\mu_s} \) and \( \delta^j_{\mu_{\bar{s}j}} \) in eq.4 and eq.6 are specific to each decision problem, so that price responsiveness of the two final demand components will differ, even if the consumer and investor are assumed to have the same substitution and differentiation elasticities (\( \sigma_s \), \( \sigma_{s}^{j} \)) since no econometric information is available on potential differences.

2.3 Firms in the competitive industries

In competitive industries, the representative firms of country \( i \), sector \( s \), operate with a Cobb-Douglas constant returns to scale technologies, combining variable capital and labor as well intermediate inputs. Material inputs are introduced in the production function in a way similar to the way consumption goods are treated in the preference of household: with an Armington specification for goods produced by competitive industries and with an Ethier (1982) specification (i.e., with product differentiation at the firm level) in the imperfectly competitive sectors.

Thus, the firm resolves the following minimization problem with respect \( x_{\mu_{ji}}, L_{\mu_j}^v \) and \( K_{v_j}^v \).
\[
\min \sum_{i,j \in W} \left( \sum_{m \in L} p_{jm} x_{jim} + \sum_{r \in S} n_{jr} p_{jm} x_{jris} \right) + w_i L_{is}^v + r_i K_{is}^v \quad \text{(eq.8)}
\]
subject to
\[
\log Q_{is} \leq \alpha_{Lis} \log L_{is}^v + \alpha_{Kis} \log K_{is}^v + \sum_{r \in S} \alpha_{nis} \log x_{nis}
\]
\[
x_{jis} = \left[ \sum_{j \in W} \beta_{jis} x_{jis}^{\sigma^{-1}} \right]^{\frac{\sigma_j}{\sigma_j - 1}} \quad , \quad t \in C, \quad \text{(eq.9)}
\]
\[
x_{jis} = \left[ \sum_{j \in W} n_j \beta_{jis} x_{jis}^{\sigma^{-1}_j} \right]^{\frac{\sigma_j^*}{\sigma_j^* - 1}} \quad , \quad t \in \bar{C},
\]
where \(\alpha\) and \(\beta\) are share parameters with \(\alpha_{Lis} + \alpha_{Kis} + \sum_{r \in S} \alpha_{nis} = 1\)
\[
\beta_{jis} = 0 \ \forall j \neq i \text{ if } t \text{ is non-traded, and the } \sigma_j \text{ and } \sigma_j^* \text{ have the same interpretation as the } \sigma_j \text{ and } \sigma_j^* \text{ in eq.4 and eq.6.}
\]

For a given level of output \(Q_{is}\), the second side of inequality of eq.8 is equal to \(Q_{is} v_{is}\), where \(v_{is}\) is the variable unit cost or marginal cost. It can be interpreted as the variation of the firm’s optimal cost function with respect to an infinitesimal variation of firm’s output (i.e., shadow price).

Cost minimization implies marginal cost pricing \((p_{ij} = v_{is})\) and zero profits \((\pi_{is} = 0)\) in the competitive sectors.

2.4 Firms in non-competitive industries

Non competitive industries have a Cobb-Douglas increasing returns to scale production function. In addition to variable costs associated with technological constraints similar to eq.9, the individual firm in county \(i\) , sector \(s\), has fixed primary costs. Thus, the relationship between variable unit cost and total unit cost or average cost \((V_{is})\) becomes:
\[
V_{is} = v_{is} + \frac{w_i L_{is}^v + r_i K_{is}^v}{Q_{is}} \quad s \in \bar{C}
\]
where \(Q_{is}, L_{is}^v, K_{is}^v\) are, respectively the individual firm’s output, fixed labor and fixed capital.

The oligopolistic firm maximizes its profits in country \(j\), \(\pi_{ij}\).
\[ \pi_{ij} = p_{is} z_{isj}(p) - (v_{is} z_{isj}(p) + fx) \]

where \( z_{isj}(p) \) is the amount of good produced by the firm \( s \) sector and sold to country \( j \), 
\( fx \) are fixed costs

In the Bertrand case of non cooperative game, the price-strategy which maximizes the profits yields:

\[ \frac{p_{ij} - v_{is}}{p_{is}} = -\frac{\partial \log z_{isj}}{\partial p_{ij}} \quad s \in C, j \in W \quad \text{(eq.10)} \]

Alternatively, in the Cournot case of non cooperative game, the oligopolistic firm’s profits in country \( j \), \( \pi_{ij} \) and the quantity-strategy which maximizes the profits yields:

\[ \pi_{ij} = p_{ij}(z) z_{isj} - (v_{is} z_{isj} + fx) \]

\[ \frac{p_{ij} - v_{is}}{p_{is}} = -\frac{\partial \log p_{ij}}{\partial z_{isj}} \quad s \in C, j \in W \quad \text{(eq.11)} \]

Eq.10 and eq.11 are the usual Lerner’s equations and the right side of the equalities are the elasticities with respect to the price and quantity. The firm is endowed with the knowledge of preference (eq.4) and technologies (eq.6, eq.9) of its clients. It then performs a partial equilibrium profit maximization calculation assuming that in each country, each individual client’s current expenditure on the whole industry is unaffected by its own strategic action\(^2\) so that

\[ \frac{\partial p_{ij} p_{ij} C_{ij}}{\partial z_{isj}} = 0 \quad j \in W, \]

\[ \frac{\partial \alpha_{isj} p_{ij} I_{ij}}{\partial z_{isj}} = 0 \quad j \in W, \quad \text{(eq.12)} \]

\[ \frac{\partial \alpha_{isj} v_{is} Q_{is}}{\partial z_{isj}} = 0 \quad j \in W, t \in S \]

\(^2\) The instantaneous general equilibrium adopted is a compromise in terms of informational requirements between the primitive conjectural Bertrand/Cournot –Nash-Walras equilibrium of Negishi (1961) and the objective Bertrand/Cournot –Nash-Walras equilibrium introduced by Gabszewicz and Vial (1972). In their maximization the firms neglect the feedback effect of their decisions on their profits via income (the “Ford effect”) and via input-output multipliers (the “Nikaido effect”). This partial equilibrium assumption simplifies very much the computations. It has also been advocated in the theoretical literature (Hart 1985) to avoid the non-existence problems highlighted by Roberts and Sonnenschein (1977) and Dierker and Grodal (1986). Thus, oligopolistic firms are assumed to make their strategic decisions with systematic errors.
The definitions of oligopolistic industry profits and supply are the following:

\[
\pi_{is} = n_{is} \left( \sum_{j \in W} p_{isj} z_{isj} - V_{is} Q_{is} \right), \quad s \in \bar{C} \quad \text{(eq. 13)}
\]

\[
Q_{is} = \sum_{j \in W} z_{isj}, \quad \text{(eq. 14)}
\]

2.5 Static equilibrium conditions

The instantaneous general equilibrium is defined as a static allocation, supported by a vector of prices \((p_{isj}, w_i, r_i)\), consistent with the intertemporal constraints and choices (eq.1-eq.3), such that

- Consumers maximize eq.4 subject to eq.5;
- Investors maximize eq.6 subject to eq.7;
- Firms maximize eq.8 subject to eq.9;
- Oligopolistic firms set prices according to eq.10, eq.11 and eq.12 and satisfy the resulting demand so that:

\[
z_{isj} = c_{isj} + I_{isj} + \sum_{r \in S} x_{isjr}, \quad s \in \bar{C}, i, j \in W \quad \text{(eq. 15)}
\]

Supply equals demand in each competitive market:

\[
Q_{is} = \sum_{j \in W} \left( c_{isj} + I_{isj} + \sum_{r \in S} x_{isjr} \right), \quad s \in C, i \in W \quad \text{(eq. 16)}
\]

\[
K_i = \sum_{s \in C} K_{is} + \sum_{s \in C} n_{is} \left( K_{is}^* + K_{ir}^* \right), \quad i, j \in W \quad \text{(eq. 17)}
\]

\[
L_i = \sum_{s \in C} L_{is} + \sum_{s \in C} n_{is} \left( L_{is}^* + L_{ir}^* \right), \quad i, j \in W \quad \text{(eq. 18)}
\]

Industry concentration \(n_{is} > 1\), \(s \in \bar{C}, i \in W\) adjusts with inertia to the existence of nonnegative oligopoly rents so that, in the long run, these rents are null. The process of entry and exit is implemented in the following way:

\[
n_{is}(0) \text{ given}, n_{is}(\infty) \text{ such that } \pi_{is}(\infty) = 0
\]

\[
n_{is}(t) = \theta \left( n_{is}(\infty) - n_{is}(0) \right), \quad 0 < \theta < 1
\]
\( n_a \) is a real rather than an integer variable. That can appear not realistic especially in a market strongly concentrated. Nevertheless, it must be interpreted as an index of product variety rather than the number of real firms in the market because the symmetric firms are abstract objects.

The first period ROW wage rate is chosen as the numeraire\(^3\).

2.6 The oligopolistic mark-up

The difficulty here is to keep track of individual’s firms variable. Define \( \mathbf{P}_j \) as the vector on market \( j \):

\[
\mathbf{P}_j = \left( p_{ij}^1, \ldots, p_{ij}^n, p_{ij}^f, \ldots, p_{ij}^n, \ldots, p_{ij}^f, \ldots, p_{ij}^n \right),
\]

where \( p_{ij}^f \) is the price charged by firm \( f \) of country \( i \) on market \( j \). The sector index is not considered in order to not complicate the notation needlessly. Define in a similar way \( \mathbf{Z}_j, \mathbf{C}_j, \mathbf{I}_j, \mathbf{X}_\mu \), as the vectors, respectively, of sales of firm \( f \) of country \( i \) in market \( j \), consumption of representative household of country \( j \) of good produced by firm \( f \) of country \( i \), investment of representative household of country \( j \) of good produced by firm \( f \) of country \( i \), and inputs demand by firm \( f \) of country \( j \), sectors \( \mu \). In market \( j \), firms face a demand system that, according to eq. 12 and eq.15, can be written in a matricial form as:

\[
\mathbf{Z}_j = \mathbf{C}_j (\mathbf{P}_j (\mathbf{Z}_j)) + \mathbf{I}_j (\mathbf{P}_j (\mathbf{Z}_j)) + \sum_{\mu} \mathbf{X}_{\mu} (\mathbf{P}_j (\mathbf{Z}_j)) \quad (\text{eq.19})
\]

Total differentiation yields:

\[
\mathbf{dZ}_j = \left[ \frac{\partial \mathbf{C}_j}{\partial \mathbf{P}_j} + \frac{\partial \mathbf{I}_j}{\partial \mathbf{P}_j} + \sum_{\mu} \frac{\partial \mathbf{X}_{\mu}}{\partial \mathbf{P}_j} \right] \frac{\partial \mathbf{P}_j}{\partial \mathbf{Z}_j}, \quad (\text{eq.20})
\]

where \( \frac{\partial}{\partial .} \) are matrices of partial derivatives. Let \( \hat{\mathbf{P}}_j \) be a diagonal matrix with the elements of \( \mathbf{P}_j \) on the diagonal, and define \( \hat{\mathbf{Z}}_j, \hat{\mathbf{C}}_j, \hat{\mathbf{I}}_j, \hat{\mathbf{X}}_{\mu} \) in a similar way. Let \( \mathbf{E}(\mathbf{C}_j, \mathbf{P}_j) = \frac{\partial \mathbf{C}_j}{\partial \mathbf{P}_j} \hat{\mathbf{P}}_j \hat{\mathbf{C}}_j^{-1} \) be a matrix of

\(^3\) In the general equilibrium with imperfect competition the question of the numeraire is very important (Gabszewicz and Vial, 1972) because its choice influences the simulation’s effects on the final results (e.g., on welfare). If we disregard theoretical issues, a practical way to avoid this numeraire problem is to choose a normalization rule that involves only competitive prices.
cross/own-elasticities of consumption with respect to price (equivalent procedure holds for \( Z_j, I_j, X_j \)), then eq.20 can be easily be re-written as:

\[
\frac{dZ_j}{dZ} = \left[ E(C_j,P_j)\dot{C}_j + E(I_j,P_j)\dot{I}_j + \sum_j E(X_j,P_j)\dot{X}_j \right]E(P_j,Z_j)dZ_j \quad \text{(eq.21)}
\]

Non cooperative behavior implies that firm \( f \) solve the system with \( dz_{ij}^f = 1 \) and all other elements of vector \( dZ \), set to zero. This yields the values of the right-side of eq.10 and eq.11 for firm \( f \). Conceptually, the computation of an equilibrium requires solving one such system for each firm \( f \in i \) in all markets \( j \). The cost of such a calculation would be prohibitive without the assumption of symmetry between domestic firms.

3. The policy simulation and evaluation of welfare

3.1 The trade experiment

The experiment consists to simulate Europe’s move to a single market by forcing firms to switch from their initial segmented market pricing strategy according with eq.10 and eq.11 to an integrated market strategy determined from their average EU-wide monopoly power.

Thus, the fundamental equation in non competitive markets becomes:

\[
\frac{p_{ij} - v_s}{p_{ij}} = -\lambda \frac{\partial \log p_{ij}}{\partial z_{ij}} - (1 - \lambda) \frac{\partial \log p_{ijEU}}{\partial z_{ijEU}} \quad s \in \bar{C}, j \in EU \quad \text{(eq.22)}
\]

with \( \lambda = 1 \) in the calibration. The simulation sets \( \lambda = 0 \). The elasticity on the right side is evaluated using EU-aggregated demand\(^4\).

The presence of non tariff barriers confer to firms the power to price discriminate between national markets. Firms are forced to set the same price in European market. Because non tariff barriers are essentially unobservable, they are treated as latent variable.

What can be expected from the trade integration experiment in terms of welfare? Firms are thought to charge prices in their domestic market, in which they usually hold the largest share. The conjecture is that the consumer prices decrease relatively to factor prices and that European consumers will be better off with the completion of the European single market. In addition, the

\(^4\) System of eq.21 remains essentially unchanged (market \( j \) now represents the aggregate EU market), but the price elasticities are now weighted averages of those of individual countries.
new pricing rule could reduce the level of profits forcing some firms to exit from the market because of Chamberlin exit/entry condition. So, the smaller number of surviving firms can produce on a larger scale with lower average costs. The positive outcome in terms for the consumer could be offset by two opposite effects. In fact, exit of the firms from an industry means reduced product diversity. This has a direct welfare costs, since consumers are endowed with love of variety type of preferences a la Dixit-Stiglitz. Furthermore, diversity in available intermediate goods affects production efficiency in all sectors: everything else equal, exit of firms increases variable unit costs in all other sectors competitive and non competitive.

3.2 The welfare criterion

Central to any normative analysis is the measure of welfare, which is now made precise.

Let \( \hat{C}(t) \) be the reference stream of consumption (benchmark equilibrium) and \( C(t) \) the corresponding time profile computed after the implementation as from date \( t=0 \) of trade policy simulation (counter factual equilibrium).

The welfare gain is determined from the following utility indifference condition:

\[
\int_0^\infty e^{-\rho t} \frac{\hat{C}(t)(1+\phi)}{1-\gamma} dt = \int_0^\infty e^{-\rho t} \frac{C(t)}{1-\gamma} dt. \tag{eq.23}
\]

that is, the welfare gain resulting from the policy change is equivalent from the perspective of the representative household to increase the reference consumption profile by \( \phi \) per cent. The measure of \( \phi \) accounts for both transitional and long-term effects of the policy on the household’s well being, putting relatively low weight on the latter because of discount rate \( \rho \). It is sometimes useful to restrict the welfare analysis to steady state effects, in particular when making comparisons with prediction from static models. Thus, let \( \lim_{t \to \infty} \hat{C}(t) = \hat{C}_{ss} \) and \( \lim_{t \to \infty} C(t) = C_{ss} \) be, respectively, the steady state consumption values before and after the shock and plug these constant values in the utility indifference condition to obtain:

\[
\hat{C}_{ss} (1+\phi_{ss}) = C_{ss}, \tag{eq.24}
\]
where $\phi_{ss}$ is the equivalent variation welfare measure, most frequently used in static applied general equilibrium analysis. With the equivalent variation, the point of reference is the old equilibrium because the prices are the prices of initial equilibrium.

4. Calibration and computational consideration

The database includes bilateral trade flows, separate input-output tables for domestic and imported inputs, final demands by type and sectoral origin, production and labor earnings figures. All are collected from standard publications. When necessary, consistency among the sources is ensured by using a RAS (row and column sum procedure). The number of symmetric firms in non-competitive sectors is inferred from Herfindahl industry concentration indices. Note that, because of the hypothesis of symmetry among firms within the sector, it is sufficient to calculate the reciprocal of the index for the single firm to obtain $n_s$.

Concerning the competitive side, the literature includes several sources of econometric estimates of Armington elasticities, $\sigma_s$. So, the calibration is easy. Conversely, a crucial issue in GE models with imperfect competition is the calibration of initial markups and returns to scale in the oligopolistic industries. Since no reliable estimates on product differentiation, $(\sigma_s^*, \sigma_s^f)$, returns to scale, price-costs margins in oligopolistic industries are available, the strategy is to put exogenously reasonable values for $\sigma_s^*, \sigma_s^f$, and then determine jointly the base-year price system and scale elasticities consistent with the data base and the optimal price-discriminating Cournot-Nash behavior of non-competitive firms.

The elasticities on the right side of the pricing equation (eq.10, eq.11), that is $\frac{\partial \log p_{sj}}{\partial z_{ej}}$, depend on the substitution elasticities $(\sigma_s^*, \sigma_s^f)$, on the number of firms($n_s$) in country $i$ sector $s$, and on the market shares (i.e., $\theta_{ij}$) the exporting country has in the client market $j$. Denote $\tilde{e}_{sj}$ the current price trade flows as supplied by the database. The market shares $\theta_{ij}$ are ratios of $\tilde{e}_{sj}$ and expenditures terms $(\rho_{sj} p_{cj}, \omega_{sj} p_{ij}, \alpha_{si} \rho_{sj} L_{ij})$. These expenditures are exogenous to the firm for the assumption of eq.12. Furthermore, they are known set so that the $\theta_{ij}$ may be treated as parameters in the calibration. For calibration purposes, the elasticities can therefore be written in a convenient compact form as
\[-\frac{\partial \log p_{isj}}{\partial \zeta_{isj}} = E_{isj}(\tilde{e}_{isj}, n_s, \sigma_s, \sigma_f), \quad s \in \mathcal{C} \quad (eq.25)\]

where $E_{isj}(.)$ denotes a function of which we know the form and the parameter values. Substituting eq.25 in the Lerner formula eq.11, we can obtain

\[
p_{isj} = \frac{1}{\nu_{is} - E_{isj}(\tilde{e}_{isj}, n_s, \sigma_s, \sigma_f)}, \quad s \in \mathcal{C} \quad (eq.26)\]

so for a given (as yet unknown) level of the variable unit cost $\nu_{is}$, the prices charged by firms in each national market may be computed from the data exogenously. To calculate $\nu_{is}$, define $\bar{p}_{is}$ as the average selling price of the firm operating in country $i$; then, by definition $\bar{p}_{is}$ satisfies

\[
\bar{p}_{is} = \sum_{j \in \mathcal{W}} \tilde{e}_{isj} = \sum_{j \in \mathcal{W}} e_{isj} / p_{isj}, \quad s \in \mathcal{C} \quad (eq.27)\]

With $\bar{p}_{is}$ fixed at unity by a normalization, eq.26 and eq.27 jointly determine the variable unit costs $\nu_{is}$ and the segmented market price system, consistent with the data set, with preferences and with the competitive game assumed to prevail in the base year. The assumption of zero profits determines average costs: $V_{is} = \bar{p}_{is}$. The fixed costs can be computed from the following expression:

\[
V_{is} = \nu_{is} + \frac{w_i L_i^f + r_i K_i^f}{Q_i} \quad s \in \mathcal{C}
\]

\[
w_i L_i^f + r_i K_i^f = \nu_{is} Q_i \left[ \frac{\nu_{is}}{\nu_{is}} - 1 \right] \quad s \in \mathcal{C}
\]

Due to the lack of reliable data on the composition of fixed costs, the assumption is that fixed costs have the same share of capital and labor inputs.

An other important issue is the calibration in the dynamic model. Before to introduce the procedure used, we note that the intertemporal budget constraints (eq.3) can be re-written in the following differential form:
\[
\dot{F}(t) = \rho F(t) + w(t)L(t) + r(t)K(t) + \sum_s \pi_s(t) - \left[p_c(t)C(t) + p_I(I(t))\right], \quad \text{(eq.28)}
\]

\[
F(0) \quad \text{given} \quad \lim_{t \to \infty} e^{-\rho t} F(t) = 0
\]

where, for notational convenience, country subscripts is neglected. Thus, I make use of results by Mercenier and Michel (1994) on dynamic aggregation. The first results is based on two fundamental propositions. The first one says that there exists a sequence of discount factors \( \alpha_n \) for which stationary solution of the continuous time optimisation problem is a stationary solution of the discrete time problem. This sequence is unique within the choice of \( \alpha_n > 0 \), and it is defined by the following recurrence relation:

\[
\alpha_{n+1} = \frac{\alpha_n}{1 + \rho \Delta_n},
\]

with \( 0 \leq n \leq N - 2 \),

where \( \Delta_n = t_{n+1} - t_n \) are intervals of time and \( t_n(n=0,...,N) \) are possibly unequally spaced dates

The second proposition says that the stationary solution of the infinite horizon continuous time problem is also the constant solution of the finite horizon discrete time problem with terminal state value function equal to

\[
\frac{1}{\rho} \left[ \frac{C(t_N)}{1 - \gamma} \right] \quad \text{and the discount factor} \quad \beta_n = \alpha_{N-1}, \quad \text{provided that}
\]

proposition 1 holds for \( n=0,1,...,N-2 \).

The second results is more empirical and concerns the formula used for temporal aggregation, that is to generate the sample date. The rule of thumb is the following:

\[
t_n = \frac{1}{N} \log \left( \frac{n}{N+1} \right)
\]

Now, we can write the finite horizon discrete time approximation to the individual household’s intertemporal choice problem:

\[
Max \sum_{n=0}^{N-1} \alpha_n \Delta_n \left[ \frac{C(t_n)}{1 - \gamma} \right] + \beta_n \frac{1}{\rho} \left[ \frac{C(t_N)}{1 - \gamma} \right], \quad \text{(eq.29)}
\]

subject to

\[
F(t_{n+1}) - \rho F(t_n) = \Delta_n \left[ \rho F(t_n) + w(t_n)L(t_n) + r(t_n)K(t_n) + \sum_s \pi_s(t_n) - p_c(t_n)C(t_n) - p_I(I(t_n)) \right],
\]

\[
K(t_{n+1}) - K(t_n) = \Delta_n \left[ I(t_n) - \delta K(t_n) \right], \quad F(t_0), K(t_0) \quad \text{given}
\]
If the world economy is assumed initially in steady state, these results make the calibration straightforward using the following first order conditions:

\[
\frac{C(t_{n-1})}{C(t_n)} = \frac{p_c(t_{n-1})}{p_c(t_n)}, \quad 0 \leq n \leq N
\]

\[
p_l(t_{n-1}) = \frac{1}{1 + \rho \Delta_n} \left[ \Delta_n r(t_n) + (1 - \Delta_n) p_l(t_n) \right], \quad 0 \leq n \leq N
\]

\[
p_l(t_N) = \frac{1}{\rho} \left[ r(t_N) - \delta p_l(t_N) \right].
\]

In the time aggregated framework, the welfare criterion becomes:

\[
\sum_{n=0}^{N-1} \alpha_n \Delta_n \left[ \hat{C}(t_n)(1 + \phi)^{-\gamma} \right] + \frac{1}{\rho} \sum_{n=0}^{N-1} \beta_n \left[ \hat{C}(t_n)(1 + \phi)^{-\gamma} \right] = \sum_{n=0}^{N-1} \alpha_n \Delta_n \frac{C(t_n)^{1-\gamma}}{1 - \gamma} + \frac{1}{\rho} \sum_{n=0}^{N-1} \beta_n \frac{C(t_n)^{1-\gamma}}{1 - \gamma},
\]

where \( \hat{C}(t_n) \) and \( C(t_n), \ n = 0, \ldots, N \), denote respectively, the benchmark and counterfactual equilibrium profiles of aggregate consumption.

The procedure adopted with dynamic discrete aggregation is to exogenize oligopolistic markups and solve for the intertemporal equilibrium allocations, prices, and industry structures. Using these newly computed prices and market shares, the optimal markups are upgraded. Then, Gauss-Seidel algorithm is iterated until convergence to a fixed point.

5. Conclusions

5.1 The results of trade integration policy

Concerning the results of trade policy experiment, I refer to the works of Mercenier (1995b, 2002). In the first model (1995b), which is not intertemporal, there are two equilibria. Thus, it should be emphasized that nonuniqueness may be well the general rule rather than the exception in GE models with imperfect competition and that if case of multiple equilibria are not encountered, maybe, it has more to do with the limitations of our numerical abilities and techniques than with the properties of the models.

Nevertheless, the existence in the second model of Mercenier (2002) of a unique equilibrium can be explained by the different hypothesis about factors and factors owners. In fact labor, capital, and the households who own them, don’t move internationally. Instead in the first
model (1995), the multiplicity depended, among other things, on mobility assumption about the factors.

The second model uses a grid dates with \( t_0 = 1, t_1 = 5, t_2 = 11, t_3 = 20, t_4 = 35 \). It provides a strong evidence in favour of the intertemporal approach. In fact, looking at the graph 1, that shows the solutions of the intertemporal pattern of aggregate consumption computed for different number of dates and the same solution for the static version\(^5\) of the model, the importance of the intertemporal substitution is very clear. It is optimal for consumers to trade present for future consumption. Clearly, a static model will miss important aspects of the structural adjustments effects of trade policy.

\[\text{Graph.1} \quad \text{Sensitivity of computed path of consumption to alternative choices of time grid}\]

\(^5\) Investment is held fixed at its base year level equal to the steady state flow of capital depreciation. The stock of foreign assets initially held by each household is constrained to remain constant.
The results for the industry structure\(^6\), (Mercenier reports the example for the British pharmaceutical example), after the implementation of the new trade policy, show that at the first date \((t_0 = 1)\), the unit variable cost has increased with respect to the initial segmented Europe-market equilibrium, due to higher wages (+2.88%) and capital rentals (+2.86%) as well as higher prices of most materials input. Also the average price with respect to numeraire\(^7\) in Europe has increased but less than the variable unit cost and the average unit cost. This means that firm’s profits are decreased and some of them are forced to exit from the market. So, in the long run post-integration equilibrium there will be a smaller number of firms (7% lower). The surviving firms will produce at a lower average cost, because of increasing returns to scale, and so also the prices will reduce again.

The results on welfare and employment of the trade policy experiment are very interesting (Mercenier, 2002). The first scenario assumes flexible wages with constant employment. Table 1 reports the welfare consequences of the trade experiment for members countries of the EU. The static and dynamic versions of the model have been simulated so that the contribution of capital can be evaluated. As the comparisons between the first two columns indicate, accounting for growth increases the disparity of welfare gains across counties and confirm that the use of a static model for policy analysis could be misleading. As one expects, taking the dynamic model in the intermediate dates makes the gains of welfare weaker (column 3).

The previous results were generated with the assumption of competitive labor markets and vertical labor supply curves. So, no job creation is allowed for and labor productivity gains are absorbed only by real wages increases. Of course, this hypothesis is not realistic.

So, a new scenario is considered. Now, European wages are tied to consumer price indices during the first five years, with employment determined by firms, labor supply being horizontal; the wages adjusts in the long run so as to maintain the employed labor force at its \(t_2 = 11\) years level. The welfare results are reported in Table 1. We see that the welfare gains have, on average, approximately tripled when compared with the flexible-wage/fixed supply case. All European countries unambiguously benefit from the trade integration. Employment rises between 0.75% and 2.5%, depending on the country considered. The reason behind this is clear enough: by forcing down the average price charged by firms within the EU, the integration policy reduces cost of living indices of European consumers with respect to the numeraire. Wage indexation, therefore, implies that European wages are reduced relative to the ROW labor costs without any loss in purchasing power of workers. The increase in the external competitiveness of the EU helps European producers gain markets shares within as well as outside Europe, raise the output, and, so, move further down their average cost curves.

---

\(^6\) The assumption here is flexible wages

\(^7\) Recall that numeraire is wage of the rest of the world (ROW)
Table 1
Welfare gains from European trade integration

<table>
<thead>
<tr>
<th></th>
<th>Static model % Equivalent variation</th>
<th>Dynamic model % Equivalent variation at the steady state</th>
<th>Dynamic model % Equivalent variation after a period</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wages flexible</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Gb$</td>
<td>0.85</td>
<td>1.08</td>
<td>0.74</td>
</tr>
<tr>
<td>$Gr$</td>
<td>0.19</td>
<td>0.64</td>
<td>0.39</td>
</tr>
<tr>
<td>$Fr$</td>
<td>0.69</td>
<td>0.84</td>
<td>0.55</td>
</tr>
<tr>
<td>$It$</td>
<td>0.50</td>
<td>0.86</td>
<td>0.59</td>
</tr>
<tr>
<td>$RE$</td>
<td>0.19</td>
<td>0.02</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>Wages indexed in the first five years</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Gb$</td>
<td>3.41</td>
<td>2.43</td>
<td></td>
</tr>
<tr>
<td>$Gr$</td>
<td>2.27</td>
<td>1.60</td>
<td></td>
</tr>
<tr>
<td>$Fr$</td>
<td>2.14</td>
<td>1.44</td>
<td></td>
</tr>
<tr>
<td>$It$</td>
<td>2.64</td>
<td>1.83</td>
<td></td>
</tr>
<tr>
<td>$RE$</td>
<td>0.86</td>
<td>0.70</td>
<td></td>
</tr>
</tbody>
</table>

5.2 Possible further experiments

Even if this kind of model is used for the analysis of trade policies, I think that its theoretical structure remain valid to evaluate the effects on welfare of European fiscal integration. In particular, it could be possible to analyse the impact of value-added tax (VAT) harmonization. VAT is the European tax par excellence; despite of many efforts to create a common regime within Europe about its application and its rates, substantial differences continue to this day across countries (look at table 2). Following Cavalletti (Fossati, 1991), value-added tax should be treated as *ad valorem* tax on final consumption. Thus, it need to calculate the aliquot weighing down on the single good. We can compute the VAT rates as ratio between the total amount of VAT paid by sector $s$ of county $i$ and the value added (production net of intermediate inputs) of sector $s$ of county $i$. 
The formula used is the following:

\[ \tau_{si} = \frac{VAT_{si}}{VA_{si}} \quad s \in S, \ i \in EU \]

Then, it suffices to insert it in the eq.5, the parameter \( \tau_{si} \) in the following way:

\[ p_c C_i \geq \sum_{j \in W} \left\{ \sum_{s \in S} (1 + \tau_{si}) p_{ji} c_{ji} + \sum_{s \in S} n_{ji} (1 + \tau_{si}) p_{ji} c_{ji} \right\} \quad j, i \in EU \quad (eq.30) \]

The last expression states that the representative consumer of country \( i \in EU \) pays the aliquot of her/his country even if the product comes from country \( j \). This is consistent with the fiscal principle, still in force in European Union, that a domestic firm, which buys a product from a foreign firm, pays according to the domestic aliquot\(^8\) and, so, the final consumer will pay according with the domestic aliquot too. Unfortunately, Eq.30 doesn’t account for the goods bought by a person of country \( i \) in the country \( j \), for which she/he pays according with the aliquot of country \( j \). This can represent an important distortion.

The role of government is to fix the rates and consume the proceed, according to this equation:

\[ p_c G_i = \sum_{j \in W} \left\{ \sum_{s \in S} \tau_{si} p_{ji} c_{ji} + \sum_{s \in S} n_{ji} \tau_{si} p_{ji} c_{ji} \right\} \]

The calibration is simple enough. It suffices to set the values of \( \tau_{si} \), computed from the national accounts at the benchmark equilibrium and, then, impose the following condition at the counterfactual equilibrium:

\[ \tau_{si} = \tau_{sEU} \quad \forall i \in EU \]

where \( \tau_{sEU} \) could be an average value of the rate.

The welfare equivalent variation could be calculated starting the segmented Europe market equilibrium or starting the integrated Europe market equilibrium, that is with \( \lambda = 0 \) or \( \lambda = 1 \).

---

\(^8\) We must say that there are programs on examination to reform this principle and make VAT paid by the domestic firms, that buy goods of firms of other European countries, according to the aliquot of the latter ones. But these programs are difficult to apply so far.
<table>
<thead>
<tr>
<th>Country</th>
<th>Minimum Aliquot</th>
<th>Reduced Aliquot</th>
<th>Normal Aliquot</th>
<th>Special Aliquot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>1</td>
<td>6</td>
<td>21</td>
<td>12.5</td>
</tr>
<tr>
<td>Denmark</td>
<td>-</td>
<td>-</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>-</td>
<td>7</td>
<td>16</td>
<td>-</td>
</tr>
<tr>
<td>Greece</td>
<td>4</td>
<td>8</td>
<td>18</td>
<td>-</td>
</tr>
<tr>
<td>Spain</td>
<td>4</td>
<td>7</td>
<td>16</td>
<td>-</td>
</tr>
<tr>
<td>France</td>
<td>2.1</td>
<td>5.5</td>
<td>20.6</td>
<td>-</td>
</tr>
<tr>
<td>Ireland</td>
<td>4</td>
<td>12.5</td>
<td>21</td>
<td>12</td>
</tr>
<tr>
<td>Italy</td>
<td>4</td>
<td>10</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>3</td>
<td>6</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-</td>
<td>6</td>
<td>17.5</td>
<td>-</td>
</tr>
<tr>
<td>Austria</td>
<td>-</td>
<td>10/12</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>Portugal</td>
<td>-</td>
<td>5/12</td>
<td>17</td>
<td>-</td>
</tr>
<tr>
<td>Finland</td>
<td>-</td>
<td>8/17</td>
<td>22</td>
<td>-</td>
</tr>
<tr>
<td>Sweden</td>
<td>-</td>
<td>6/12</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-</td>
<td>5</td>
<td>17.5</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 2*

*Value-added tax aliquots for EU15 countries in force from May 1st 1999*

*Source: DG, fiscalità e unione doganale.*
Bibliography


Mercenier J. and B. Akitoby, 1993, “On intertemporal general-equilibrium reallocation effects of Europe’s move to a single market”, Institute for empirical macroeconomics working paper 87, Federal Reserve Bank of Minneapolis, Minneapolis, MN.


