Productivities, Preferences and Inequality of Well-Being*

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1. Introductory Remarks

The US versus the EU: Preferences or productivity?

Stylized economy

Individual well-being

- Utility
- Stochastic dominance over joint distributions of consumption and leisure
- Consumption

Comparisons of distributions of individual well-being

Modifications of the distribution of productivities and changes in preferences

A List of questions

- Do less dispersed productivities among the population give rise to less consumption inequality?
- If not, then is it possible to identify those restrictions to be placed on the utility function that guarantee that consumption inequality decreases when productivities are more concentrated among the population?
- Assuming productivities are given, which modifications of the preferences would lead to more equally distributed consumption levels between the individuals?
- How do the distribution of productivities and changes in the preferences interact when determining the distribution of consumption?

2. Notation and Preliminary Definitions

2.1. The Stylized Economy

- Preference ordering $\succsim$ over $X := \{(c, \ell) \mid c > 0 \text{ and } 0 < \ell < H\}$
- Utility function: $u(c, \ell)$
- Gross income: $z = g(\ell; w, m) = w\ell + m \quad (w > 0 \text{ and } m \geqslant 0)$
- Personalised utility function: $U(c, z; w, m) := u\left(c, \dfrac{z - m}{w}\right)$
  - continuous and differentiable
  - increasing in consumption and decreasing in gross income
  - Spence-Mirrlees condition

$$MRS(c, z; w, m) := -\frac{U_z(c, z; w, m)}{U_c(c, z; w, m)} \text{ decreasing in } w,\ \forall\ (c, z),\ \forall\ m. \tag{2.1}$$

2.2. The "Aid Thyself Heaven Help You" Equilibria

Agent (U, w, m) solves

$$P(U, w, m) \qquad (c, z) \ \max \ U(c, z; w, m) \ \text{ s.t. } c \leqslant z \ \text{ and } \ \frac{z - m}{w} < H$$

and we get

$$\text{(2.2a)} \qquad Z(\mathbf{w}, m) := (Z(w_1, m), \ldots, Z(w_n, m)),$$

$$\text{(2.2b)} \qquad C(\mathbf{w}, m) := (C(w_1, m), \ldots, C(w_n, m)),$$

$$\text{(2.2c)} \qquad L(\mathbf{w}, m) := (L(w_1, m), \ldots, L(w_n, m)),$$

where $L(w, m) = (Z(w, m) - m)/w$.

Beware

- No taxation: $c = z$.
- From now on no exogenous income: $m = 0$.

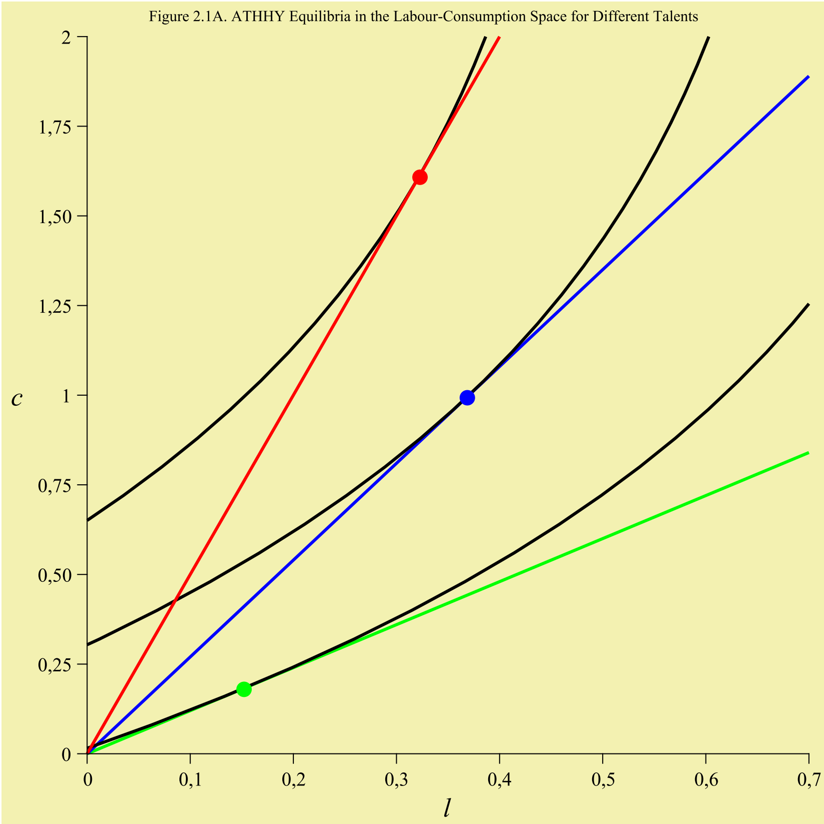
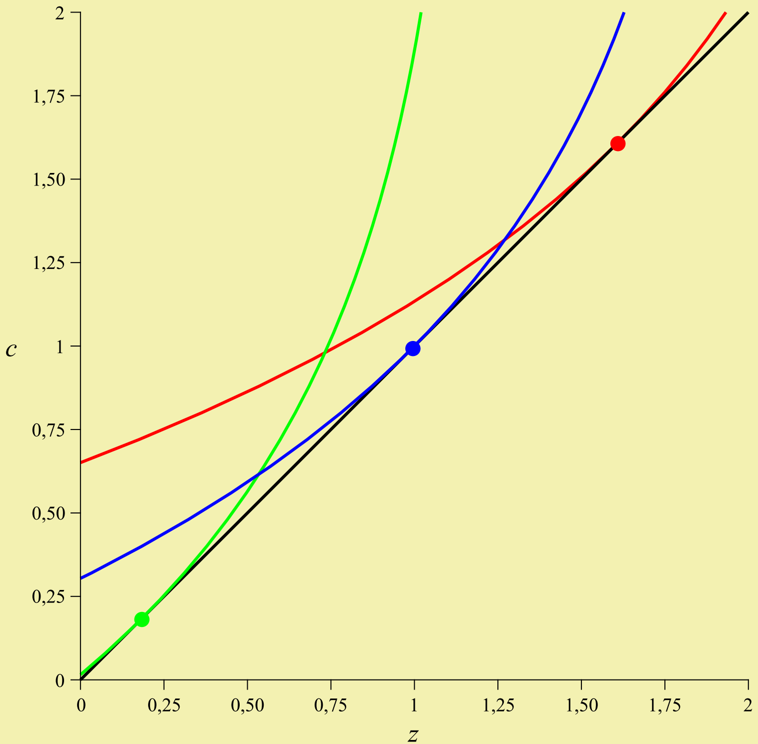


Figure 2.1A. ATHHY Equilibria in the Labour-Consumption Space for Different Talents
c
2
1,75
1,50
1,25
1
0,75
0,50
0,25
0
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
l

Figure 2.1B. ATHHY Equilibria in the Income-Consumption Space for Different Talents



3. Modifications in the Distribution of Productivities

How to capture modifications in the distribution of productivities $\mathbf{w} = (w_1, \ldots, w_n)$?

DEFINITION 3.1. Given two distributions of productivities $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ *is less dispersed than* $\mathbf{w}^\circ$, which we write $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$, if and only if

$$w^*_{(j)}/w^*_{(i)} \leqslant w^\circ_{(j)}/w^\circ_{(i)},\ \forall\, i = 1, 2, \ldots, j-1,\ \forall\, j = 2, 3, \ldots, n, \tag{3.1}$$

where $w^\circ_{(1)} \leqslant w^\circ_{(2)} \leqslant \cdots \leqslant w^\circ_{(n)}$ and $w^*_{(1)} \leqslant w^*_{(2)} \leqslant \cdots \leqslant w^*_{(n)}$.

Definition 3.2. Given two distributions of talent $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ is obtained from $\mathbf{w}^\circ$ by means of a *uniform proportional progressive transfer* if there exists $\lambda, \xi > 1$ and two individuals i, j $(1 \leqslant i < j \leqslant n)$ such that:

(3.2a) $$w^*_h = \lambda\, w^\circ_h,\ \forall\, h \in \{1, 2, \ldots, i\};\ \ w^*_h = w^\circ_h/\xi,\ \forall\, h \in \{j, j+1 \ldots, n\};$$

(3.2b) $$\lambda\,(w^\circ_1 \times \cdots \times w^\circ_i) = (w^\circ_j \times \cdots \times w^\circ_n)/\xi;$$

(3.2c) $$w^\circ_h = w^*_h,\ \forall\ h \in \{i+1 \ldots, j-1\};\ \text{and}$$

(3.2d) $$(w^*_k - w^*_h)(w^\circ_k - w^\circ_h) \geqslant 0,\ \forall\, h \neq k.$$

Equivalently, we will say that $\mathbf{w}^\circ$ results from $\mathbf{w}^*$ by means of a *uniform proportional regressive transfer*.

Does not modify the **geometric mean**: $\gamma(\mathbf{w}) := \sqrt[n]{w_1 \times \cdots \times w_n}$

DEFINITION 3.3. Given two distributions of productivities $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ *is more efficiently distributed than* $\mathbf{w}^\circ$, which we write $\mathbf{w}^* \geq_{ME} \mathbf{w}^\circ$, if and only if

$$w^*_{(i)} \geqslant w^\circ_{(i)}, \ \forall\, i = 1, 2, \ldots, n. \tag{3.3}$$

DEFINITION 3.4. Given two distributions of productivities $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ *is more efficiently and less dispersed than* $\mathbf{w}^\circ$, which we write $\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$, if and only if

$$\mathbf{w}^* \geq_{ME} \mathbf{w}^\circ \ \text{ and } \ \mathbf{w}^* \geq_{LD} \mathbf{w}^\circ. \tag{3.4}$$

DEFINITION 3.5. Given two distributions of productivities $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ *is less efficiently and less dispersed than* $\mathbf{w}^\circ$, which we write $\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$, if and only if

$$\mathbf{w}^* \geq_{LE} \mathbf{w}^\circ \ \text{ and } \ \mathbf{w}^* \geq_{LD} \mathbf{w}^\circ. \tag{3.5}$$

4. More Equally Distributed Consumption Levels

What do we mean by saying that $\mathbf{c}^* = (c_1^*, \ldots, c_n^*)$ is more equal than $\mathbf{c}^\circ = (c_1^\circ, \ldots, c_n^\circ)$?

DEFINITION 4.1. Given two consumption distributions $\mathbf{c}^*, \mathbf{c}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{c}^*$ *relative Lorenz dominates* $\mathbf{c}^\circ$, which we write $\mathbf{c}^* \geq_{RL} \mathbf{c}^\circ$, if and only if

$$RL\left(\frac{k}{n}; \mathbf{c}^*\right) \geq RL\left(\frac{k}{n}; \mathbf{c}^\circ\right), \ \forall\, k = 1, 2, \ldots, (n-1), \tag{4.1}$$

where

$$RL\left(\frac{k}{n}; \mathbf{c}\right) := \frac{1}{n} \sum_{j=1}^{k} \frac{c_{(j)}}{\mu(\mathbf{c})}, \ \forall\ k = 1, 2, \ldots, n, \tag{4.2}$$

$$c_{(1)} \leqslant c_{(2)} \leqslant \ldots \leqslant c_{(n)} \ \text{ and} \tag{4.3}$$

$$\mu(\mathbf{c}) := \frac{1}{n} \sum_{i=1}^{n} c_i. \tag{4.4}$$

4.1. Identical Preferences and Different Distributions of Productivities

Example 4.1. Choose

$$\mathbf{w}^{\circ} = (1.80, 3.24); \quad \mathbf{w}^{*} = (1.50, 2.50);$$

$$u(c, \ell) = u^{2}(c, \ell) = c - e^{\ell}.$$

Observe that $w_1^* < w_1^\circ$, $w_2^* < w_2^\circ$ and $w_2^\circ / w_1^\circ = 1.80 > 1.66 = w_2^* / w_1^*$, hence

$$\mathbf{w}^{*} >_{LELD} \mathbf{w}^{\circ}.$$

At the ATHHY equilibrium, we have $\mathbf{c}^\circ = (1.058, 3.808)$ and $\mathbf{c}^* = (0.608, 2.290)$, which implies

$$c_2^\circ / c_1^\circ = 3.600 < 3.766 = c_2^* / c_1^*,$$

thus

$$C(\mathbf{w}^{\circ}, 0) >_{RL} C(\mathbf{w}^{*}, 0).$$

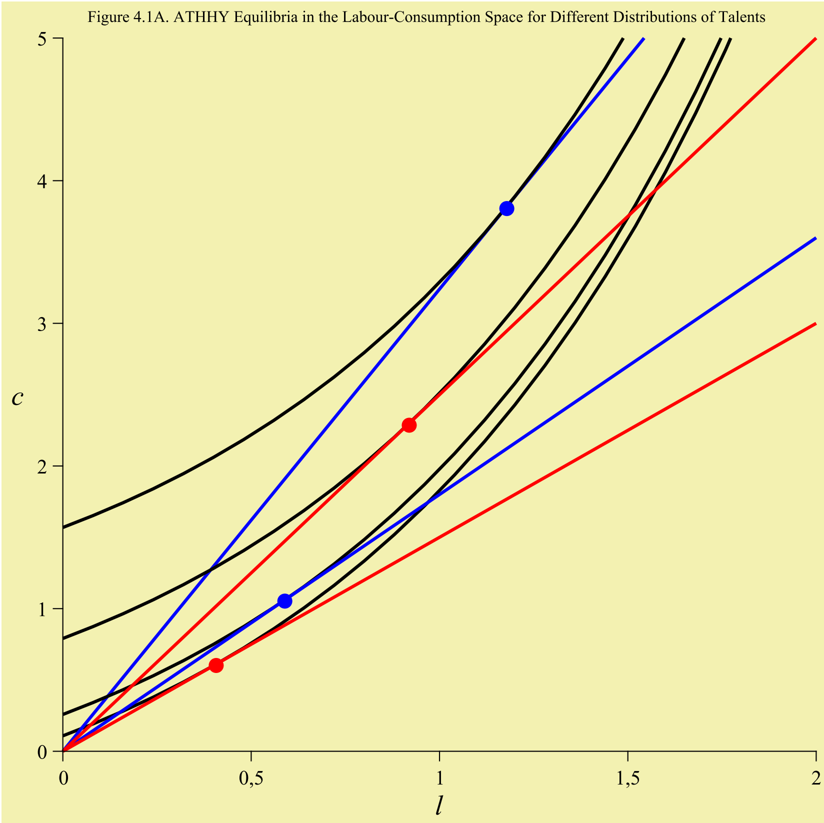


Figure 4.1A. ATHHY Equilibria in the Labour-Consumption Space for Different Distributions of Talents
c
l
0
1
2
3
4
5
0,5
1,5

EXAMPLE 4.2. Choose

$$\mathbf{w}^\circ = (0.50, 2.00); \quad \mathbf{w}^* = (0.60, 2.39);$$

$$u(c, \ell) = u^3(c, \ell) = \ln c - \frac{1}{c} - \ell.$$

Observe that $w_1^* > w_1^\circ$, $w_2^* > w_2^\circ$ and $w_2^\circ / w_1^\circ = 4.00 > 3.98 = w_2^* / w_1^*$, hence

$$\mathbf{w}^* >_{MELD} \mathbf{w}^\circ.$$

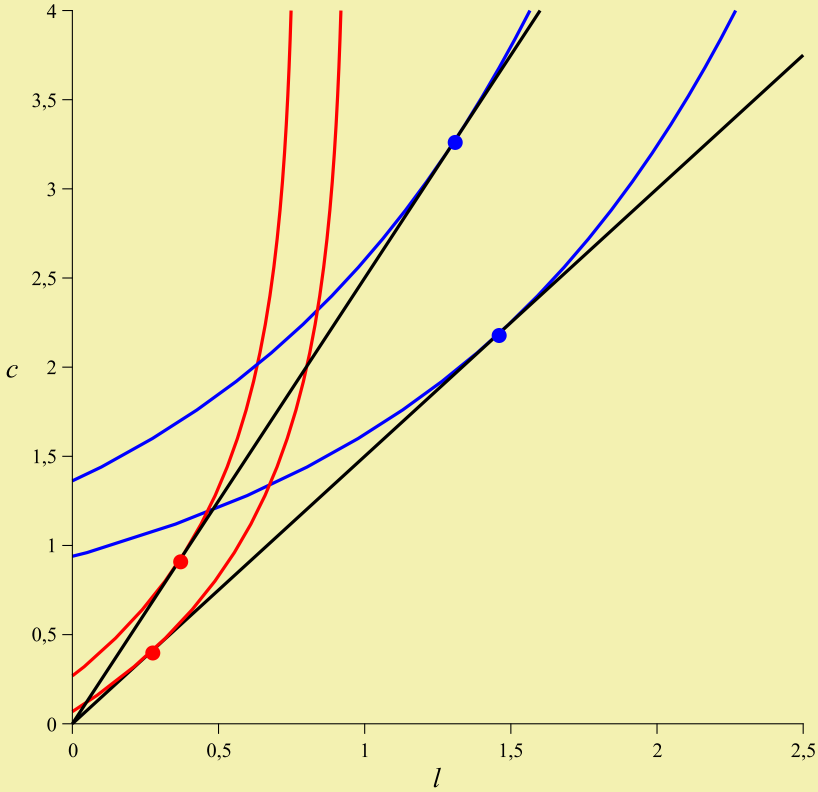
At the ATHHY equilibrium, we have $\mathbf{c}^\circ = (1.000, 2.732)$ and $\mathbf{c}^* = (1.130, 3.148)$, which implies

$$c_2^\circ / c_1^\circ = 2.732 < 2.785 = c_2^* / c_1^*,$$

thus

$$C(\mathbf{w}^\circ, 0) >_{RL} C(\mathbf{w}^*, 0).$$

Figure 4.3A. ATHHY Equilibria in the Labour-Consumption Space for Different Preference Orderings



Have you thought of providing an example where $\gamma(\mathbf{w}^*) = \gamma(\mathbf{w}^\circ)$? Yes, but I haven't done yet!

Given $J \in \{LD, MELD, LELD\}$, we want to identify the class $\mathscr{C}_1(J, RL)$ of consumption functions such that

$$P_1(J,RL) \qquad \forall\, \mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++} : \begin{array}{ccc} \mathbf{w}^* & \xrightarrow{C} & \mathbf{c}^* \\ {\scriptstyle \geq_J}\downarrow & & \downarrow{\scriptstyle \geq_{RL}} \\ \mathbf{w}^\circ & \xrightarrow[C]{} & \mathbf{c}^\circ \end{array} \quad \text{iff} \quad C \in \mathscr{C}_1(J,RL) \subseteq \mathscr{C},$$

where $\mathscr{C}$ is the set of admissible consumption functions.

Proposition 4.1. *The following two statements are equivalent:*

(a) *For all* $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$; $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ \Longrightarrow C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$.

(b) $[0 \leqslant] \dfrac{C'(w)\,w}{C(w)} \left[= 1 + \dfrac{L'(w)\,w}{L(w)} \right]$ *is constant in* w, *for all* $w > 0$.

$$\frac{C(\lambda w^*)}{C(w^*)} = \frac{C(\lambda w^\circ)}{C(w^\circ)}, \ \forall\ \lambda \geqslant 1, \ \forall\ w^*, w^\circ, \lambda w^*, \lambda w^\circ \in \mathbb{R}_{++} \tag{4.5}$$

a functional equation whose solution is

$$C(w) = \beta w^\eta \ (\beta > 0, \eta > 0), \ \forall\ w \in \mathbb{R}_{++} \tag{4.6}$$

Proposition 4.2. *The following two statements are equivalent:*

(a) *For all* $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$; $\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ \Longrightarrow C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$.

(b) $[0 \leqslant] \dfrac{C'(w)\, w}{C(w)} \left[= 1 + \dfrac{L'(w)\, w}{L(w)}\right]$ *is non-increasing in* w, *for all* $w > 0$.

(4.7)
$$\frac{C(\lambda w^*)}{C(w^*)} \leqslant \frac{C(\lambda w^\circ)}{C(w^\circ)}, \ \forall\ w^*, w^\circ \in \mathbb{R}_{++}, \ \forall\ \lambda \geqslant 1$$

(4.8)
$$\eta(C', w) \geqslant \eta(C, w) - 1 = \eta(L, w), \ \forall\ w \in \mathbb{R}_{++}$$

Proposition 4.3. *The following two statements are equivalent:*

(a) *For all* $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$; $\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ \Longrightarrow C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$.

(b) $[0 \leqslant] \dfrac{C'(w)\,w}{C(w)} \left[= 1 + \dfrac{L'(w)\,w}{L(w)}\right]$ *is non-decreasing in* w, *for all* $w > 0$.

(4.9) $$\frac{C(\lambda w^*)}{C(w^*)} \geqslant \frac{C(\lambda w^\circ)}{C(w^\circ)},\ \forall\, w^*, w^\circ \in \mathbb{R}_{++},\ \forall\, \lambda \geqslant 1$$

(4.10) $$\eta(C', w) \leqslant \eta(C, w) - 1 = \eta(L, w) = \eta(L, w),\ \forall\, w \in \mathbb{R}_{++}$$

Table 4.1: Propositions 4.1, 4.2 and 4.3 in a glance

	$C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$	
$\mathbf{w}^* \geq_J \mathbf{w}^\circ$	Consumption Function	Utility Function
$\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$	$C(w) = \beta w^\eta \ (\beta, \eta > 0)$	$u^1(c,\ell) = c - \frac{\ell^2}{2}$ $u^{10}(c,\ell) = -(\ell+3)e^{-\frac{c-3}{\ell+3}-1}$
$\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$	$\frac{C'(w)w}{C(w)} \downarrow w$	$u^2(c,\ell) = c - e^\ell \ (1 < w < \infty)$ $u^{11}(c,\ell) = -(\ell+3)e^{-\frac{c-2}{\ell+3}-1} \ (\frac{1}{3} < w < \infty)$
$\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$	$\frac{C'(w)w}{C(w)} \uparrow w$	$u^3(c,\ell) = \ln c - \frac{1}{c} - \ell$ $u^{12}(c,\ell) = -(\ell+3)e^{-\frac{c-4}{\ell+3}-1}$

4.2. Different Preferences and Identical Distributions of Productivities

Example 4.3. Choose $\tilde{\mathbf{w}} = (1.50, 2.50)$,

$$u^\circ(c,\ell) = u^3(c,\ell) = \ln c - \frac{1}{c} - \ell \quad \text{and}$$

$$u^*(c,\ell) = u^6(c,\ell) = -e^{-c} - \ell.$$

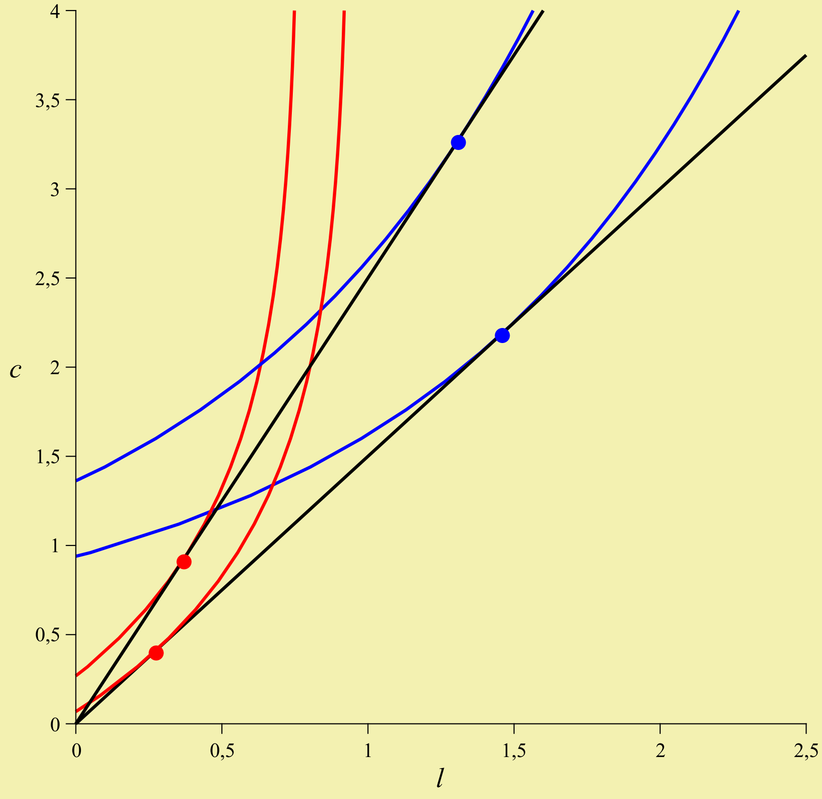
At the ATHHY equilibrium, we have $\tilde{\mathbf{c}}^\circ = (2.186, 3.265)$ and $\tilde{\mathbf{c}}^* = (0.405, 0.916)$, which implies

$$\tilde{c}_2^\circ/\tilde{c}_1^\circ = 1.493 < 2.259 = \tilde{c}_2^*/\tilde{c}_1^*,$$

thus

$$C(\tilde{\mathbf{w}}^\circ, 0) >_{RL} C(\tilde{\mathbf{w}}^*, 0).$$

Figure 4.3A. ATHHY Equilibria in the Labour-Consumption Space for Different Preference Orderings



Example 4.4. Choose $\hat{\mathbf{w}} = (3.50, 5.00)$,

$$u^{\circ}(c,\ell) = u^{3}(c,\ell) = \ln c - \frac{1}{c} - \ell \quad \text{and}$$

$$u^{*}(c,\ell) = u^{6}(c,\ell) = -e^{-c} - \ell.$$

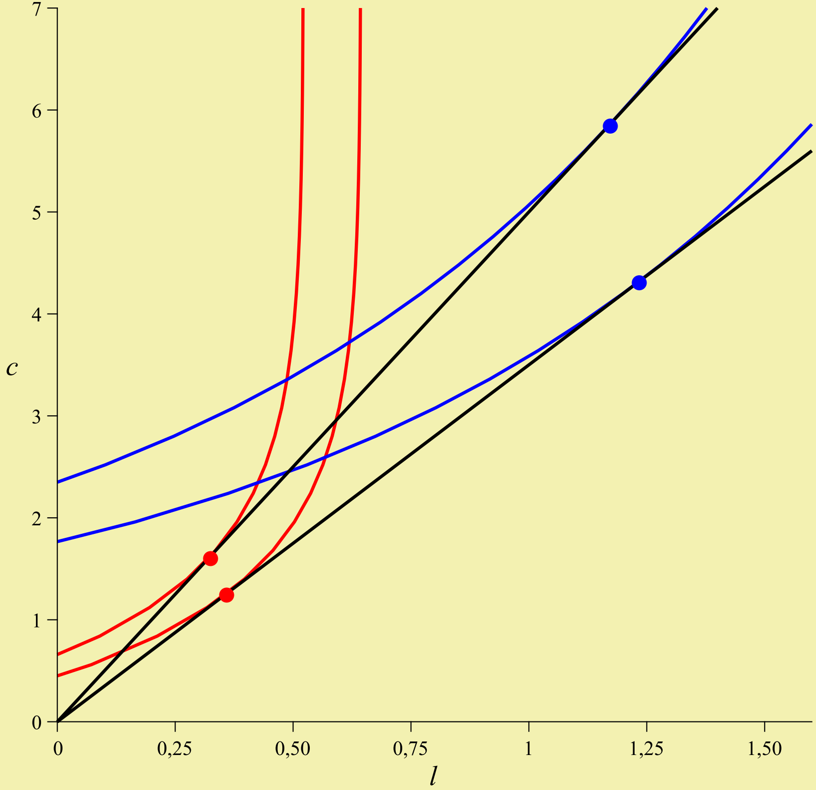
At the ATHHY equilibrium, we have $\hat{\mathbf{c}}^{\circ} = (4.311, 5.854)$ and $\hat{\mathbf{c}}^{*} = (1.252, 1.609)$, which implies

$$\hat{c}^{\circ}_2/\hat{c}^{\circ}_1 = 1.357 > 1.284 = \hat{c}^{*}_2/\hat{c}^{*}_1,$$

thus

$$C(\hat{\mathbf{w}}^{*}, 0) >_{RL} C(\hat{\mathbf{w}}^{\circ}, 0).$$

Figure 4.4A. ATHHY Equilibria in the Labour-Consumption Space for Different Preference Orderings



We want to identify the set $\mathbb{C}_2\,(RL)$ of couples of consumption functions (C^*, C°) such that

$$P_2(RL) \qquad \forall\ \mathbf{w} \in \mathbb{R}^n_{++} :\ \mathbf{w} \xrightarrow{C^*} \mathbf{c}^*,\quad \mathbf{w} \xrightarrow{C^\circ} \mathbf{c}^\circ,\quad \mathbf{c}^* \geq_{RL} \mathbf{c}^\circ \ \text{ iff } \ (C^*, C^\circ) \in \mathbb{C}_2\,(RL) \subseteq \mathscr{C} \times \mathscr{C}$$

Proposition 4.4. *The following two statements are equivalent:*

(a) *For all* $\mathbf{w} \in \mathbb{R}^n_{++}$; $C^*(\mathbf{w}) \geq_{RL} C^\circ(\mathbf{w})$.

(b) $[0 \leqslant] \dfrac{C^{*\prime}(w)\, w}{C^*(w)} \leqslant \dfrac{C^{\circ\prime}(w)\, w}{C^\circ(w)}$, *for all* $w > 0$.

(4.11)
$$\frac{C^*(\lambda w)}{C^*(w)} \leqslant \frac{C^\circ(\lambda w)}{C^\circ(w)}, \ \forall\, w \in \mathbb{R}_{++}, \ \forall\, \lambda \geqslant 1.$$

(4.12)
$$1 + \frac{L^{*\prime}(w)\, w}{L^*(w)} \leqslant 1 + \frac{L^{\circ\prime}(w)\, w}{L^\circ(w)}, \text{ for all } w > 0$$

Table 4.2: Proposition 4.4 in a glance

$C^* (\mathbf{w}) \geq_{RL} C^\circ (\mathbf{w})$	
CONSUMPTION FUNCTIONS	UTILITY FUNCTIONS $u^*(c,\ell)$ AND $u^\circ(c,\ell)$
$\dfrac{C^{*\prime}(w)\,w}{C^*(w)} \leqslant \dfrac{C^{\circ\prime}(w)\,w}{C^\circ(w)}$	$u^1(c,\ell) = c - \dfrac{\ell^2}{2}$; $\quad u^8(c,\ell) = 2\sqrt{c} - \ell$
	$u^3(c,\ell) = \ln c - \dfrac{1}{c} - \ell$; $\quad u^9(c,\ell) = \frac{5}{2}c - \ell e^{-\frac{1}{\ell}} - \displaystyle\int_1^\infty \frac{e^{-t\ell}}{t}dt$

4.3. Different Preferences and Different Distributions of Productivities

EXAMPLE 4.5. Choose

$$\mathbf{w}^{\circ} = (1.50, 3.00); \quad \mathbf{w}^{*} = (1.35, 2.60);$$

$$u^{\circ}(c,\ell) = u^{3}(c,\ell) = \ln c - \frac{1}{c} - \ell \text{ and}$$

$$u^{*}(c,\ell) = u^{6}(c,\ell) = -e^{-c} - \ell.$$

Observe that $w_1^* < w_1^{\circ}$, $w_2^* < w_2^{\circ}$ and $w_2^{\circ}/w_1^{\circ} = 2.000 > 1.925 = w_2^*/w_1^*$, hence

$$\mathbf{w}^{*} >_{LELD} \mathbf{w}^{\circ}.$$

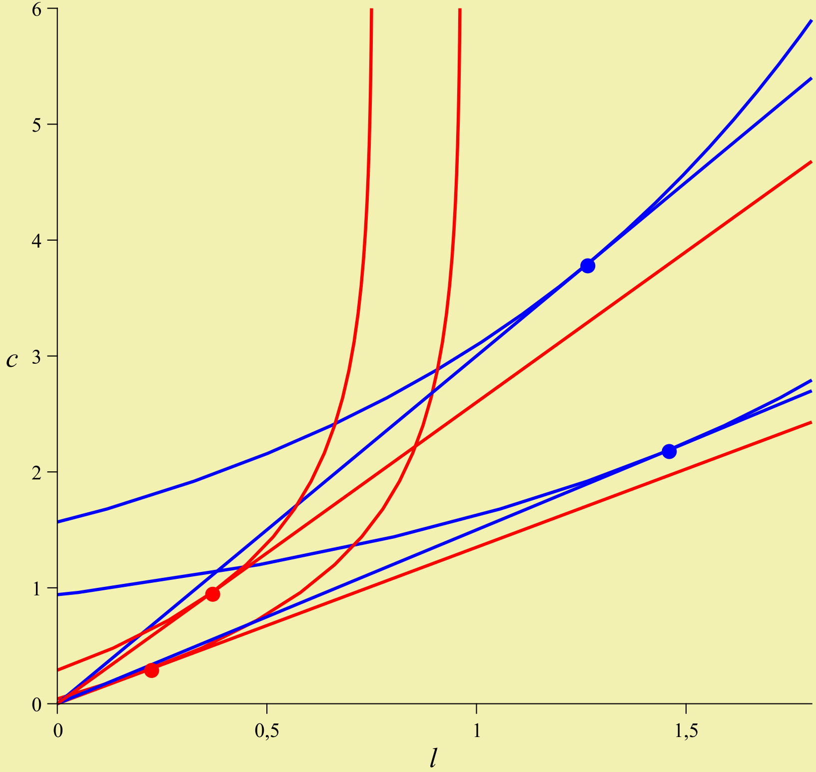
At the ATHHY equilibrium, we have $\mathbf{c}^{\circ} = (2.186, 3.791)$ and $\mathbf{c}^{*} = (0.300, 0.955)$, which implies

$$c_2^{\circ}/c_1^{\circ} = 1.734 < 3.183 = c_2^*/c_1^*,$$

thus

$$C(\mathbf{w}^{\circ}, 0) >_{RL} C(\mathbf{w}^{*}, 0).$$

Figure 4.5A. ATHHY Equilibria in the Labour-Consumption Space for Different Distributions of Talents and Preference Orderings



Example 4.6. Choose

$$\mathbf{w}^\circ = (3.50, 4.50); \quad \mathbf{w}^* = (4.30, 5.40);$$

$$u^\circ(c,\ell) = u^3(c,\ell) = \ln c - \frac{1}{c} - \ell \text{ and}$$

$$u^*(c,\ell) = u^6(c,\ell) = -e^{-c} - \ell.$$

Observe that $w_1^* > w_1^\circ$, $w_2^* > w_2^\circ$ and $w_2^\circ/w_1^\circ = 1.285 > 1.255 = w_2^*/w_1^*$, hence

$$\mathbf{w}^* >_{MELD} \mathbf{w}^\circ.$$

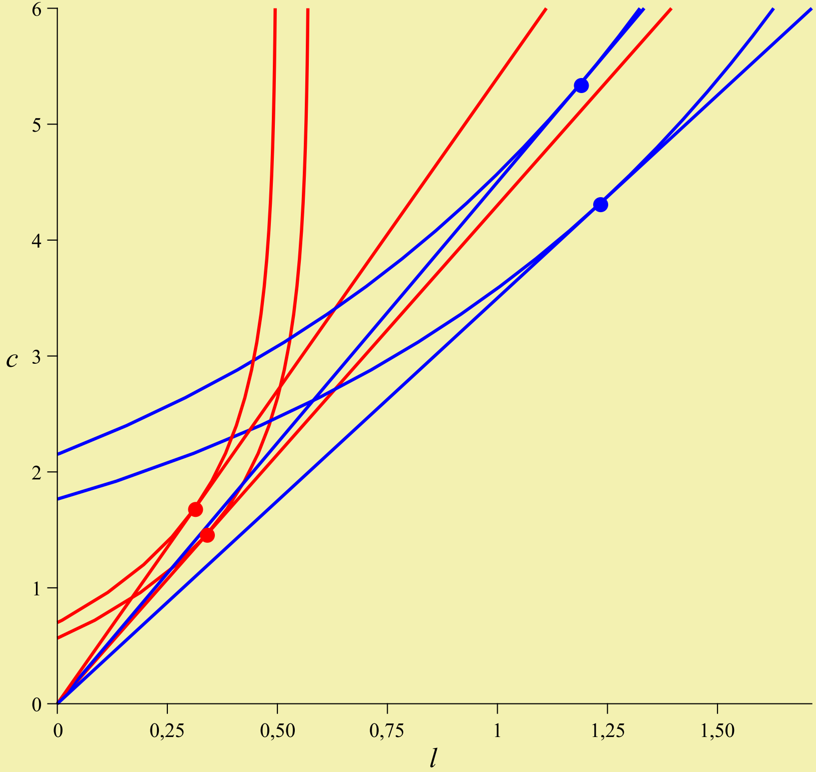
At the ATHHY equilibrium, we have $\mathbf{c}^\circ = (4.311, 5.342)$ and $\mathbf{c}^* = (1.458, 1.686)$, which implies

$$c_2^\circ/c_1^\circ = 1.239 < 1.156 = c_2^*/c_1^*,$$

thus

$$C(\mathbf{w}^*, 0) >_{RL} C(\mathbf{w}^\circ, 0).$$

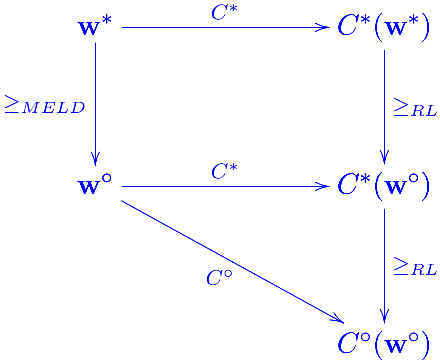
Figure 4.6A. ATHHY Equilibria in the Labour-Consumption Space for Different Distributions of Talents and Preference Orderings

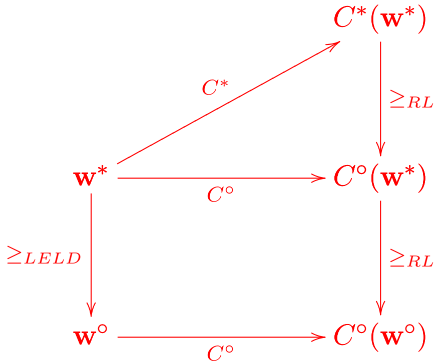


Given $J \in \{LD, MELD, LELD\}$, we want to identify the set $\mathbb{C}_3\,(J, RL)$ of couples of consumption functions (C^*, C°) such that

$$P_3(J, RL) \qquad \forall\ \mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++} : \begin{array}{ccc} \mathbf{w}^* & \xrightarrow{C^*} & \mathbf{c}^* \\ \geq_J \downarrow & & \downarrow \geq_{RL} \\ \mathbf{w}^\circ & \xrightarrow[C^\circ]{} & \mathbf{c}^\circ \end{array} \ \text{iff}\ (C^*, C^\circ) \in \mathbb{C}_3\,(J, RL) \subseteq \mathscr{C} \times \mathscr{C}$$

Exploiting Propositions 4.2 and 4.3 on the one hand, and Proposition 4.4 on the other hand, observe that, for $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$ whenever $\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$ or $\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$, it is sufficient that:





$$\frac{C^{*\prime}(w)\,w}{C^*(w)} \downarrow w$$

$$\frac{C^{*\prime}(w)\,w}{C^*(w)} \leqslant \frac{C^{\circ\prime}(w)w}{C^\circ(w)} \ \forall\ w$$

$$\frac{C^{*\prime}(w)\,w}{C^*(w)} \leqslant \frac{C^{\circ\prime}(w)w}{C^\circ(w)} \ \forall\ w$$

$$\frac{C^{\circ\prime}(w)\,w}{C^\circ(w)} \uparrow w$$

Proposition 4.5. *The following two statements are equivalent:*

(a) For all $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$; $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ \Longrightarrow C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$.

(b) There exists H *verifying* $\dfrac{H'(w)\,w}{H(w)}$ *constant in* w *such that*

$$[\,0 \leqslant\,]\ \frac{C^{*\prime}(w)\,w}{C^*(w)} \leqslant \frac{H'(w)\,w}{H(w)} \leqslant \frac{C^{\circ\prime}(w)w}{C^\circ(w)},\ \forall\ w > 0.$$

Proposition 4.6. *The following two statements are equivalent:*

(a) *For all* $\mathbf{w}^{*}, \mathbf{w}^{\circ} \in \mathbb{R}^{n}_{++}$; $\mathbf{w}^{*} \geq_{MELD} \mathbf{w}^{\circ} \Longrightarrow C^{*}\left(\mathbf{w}^{*}\right) \geq_{RL} C^{\circ}\left(\mathbf{w}^{\circ}\right)$.

(b) *There exists* H *verifying* $\dfrac{H'(w)\,w}{H(w)}$ *non-increasing in* w *such that*

$$[\,0 \leqslant\,]\,\frac{C^{*\prime}(w)\,w}{C^{*}(w)} \leqslant \frac{H'(w)\,w}{H(w)} \leqslant \frac{C^{\circ\prime}(w)w}{C^{\circ}(w)},\ \forall\ w > 0.$$

Proposition 4.7. *The following two statements are equivalent:*

(a) For all $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$; $\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ \Longrightarrow C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$.

(b) There exists H *verifying* $\dfrac{H'(w)\,w}{H(w)}$ *non-decreasing in* w *such that*

$$[\,0 \leqslant\,]\ \frac{C^{*\prime}(w)\,w}{C^*(w)} \leqslant \frac{H'(w)\,w}{H(w)} \leqslant \frac{C^{\circ\prime}(w)w}{C^\circ(w)},\ \forall\, w > 0.$$

Table 4.3: Propositions 4.5, 4.6 and 4.7 in a glance

	$C^* (\mathbf{w}^*) \geq_{RL} C^\circ (\mathbf{w}^\circ)$	
$\mathbf{w}^* \geq_J \mathbf{w}^\circ$	CONSUMPTION FUNCTION	UTILITY FUNCTIONS $u^*(c,\ell)$ AND $u^\circ(c,\ell)$
$\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$	$\frac{C^{*\prime}(w)\,w}{C^*(w)} \leqslant \frac{H'(w)\,w}{H(w)} \leqslant \frac{C^{\circ\prime}(w)w}{C^\circ(w)}$	$u^4(c,\ell) = \ln c - \ell;\quad u^5(c,\ell) = 2\sqrt{c} - \ell$
	$H(w) = \beta w^\eta\ (\beta, \eta > 0)$	$u^3(c,\ell) = \ln c - \frac{1}{c} - \ell;\quad u^2(c,\ell) = c - e^\ell\ (w > 1)$
$\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$	$\frac{C^{*\prime}(w)\,w}{C^*(w)} \leqslant \frac{H'(w)\,w}{H(w)} \leqslant \frac{C^{\circ\prime}(w)w}{C^\circ(w)}$	$u^4(c,\ell) = \ln c - \ell;\quad u^5(c,\ell) = 2\sqrt{c} - \ell$
	$\frac{H'(w)w}{H(w)} \downarrow w$	$u^4(c,\ell) = \ln c - \ell;\quad u^2(c,\ell) = c - e^\ell\ (w > 1)$
$\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$	$\frac{C^{*\prime}(w)\,w}{C^*(w)} \leqslant \frac{H'(w)\,w}{H(w)} \leqslant \frac{C^{\circ\prime}(w)w}{C^\circ(w)}$	$u^4(c,\ell) = \ln c - \ell;\quad u^5(c,\ell) = 2\sqrt{c} - \ell$
	$\frac{H'(w)w}{H(w)} \uparrow w$	$u^3(c,\ell) = \ln c - \frac{1}{c} - \ell;\quad u^2(c,\ell) = c - e^\ell\ (w > 1)$

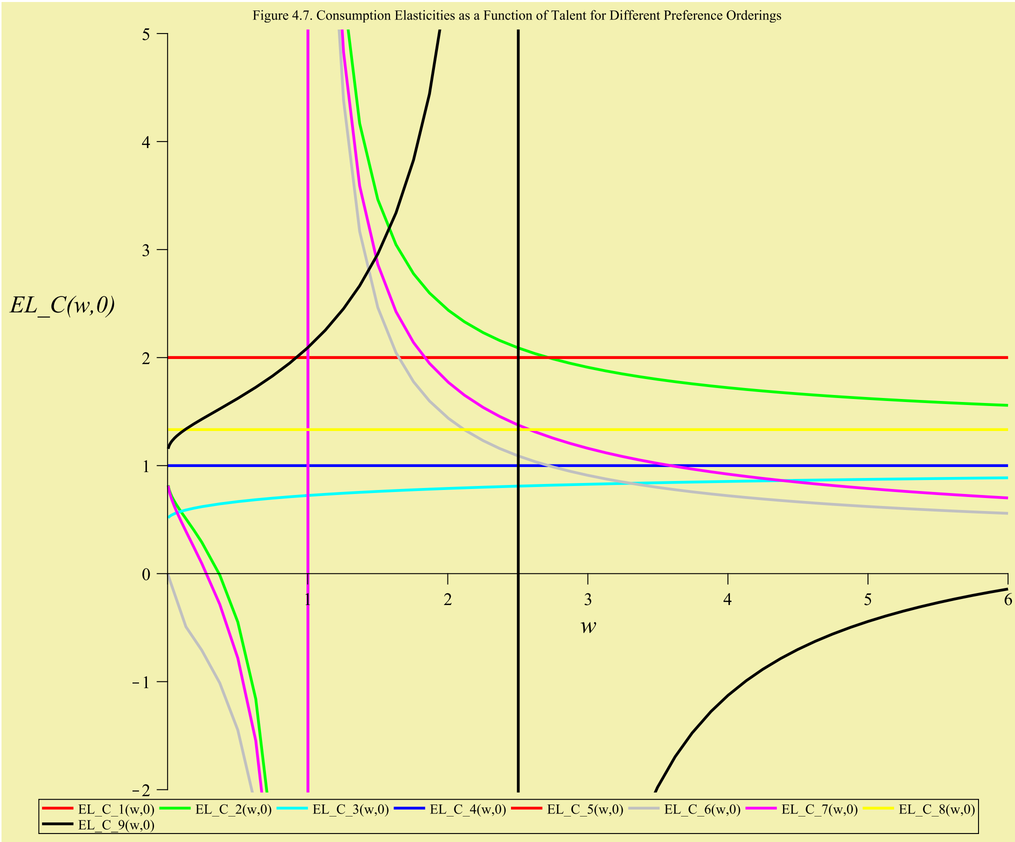


Figure 4.7. Consumption Elasticities as a Function of Talent for Different Preference Orderings
EL_C(w,0)
w
5
4
3
2
1
0
-1
-2
1
2
3
4
5
6
EL_C_1(w,0)
EL_C_2(w,0)
EL_C_3(w,0)
EL_C_4(w,0)
EL_C_5(w,0)
EL_C_6(w,0)
EL_C_7(w,0)
EL_C_8(w,0)
EL_C_9(w,0)

Table 4.4: List of the utility functions used in Figure 4.7

$u^1(c,\ell) = c - \frac{\ell^2}{2}$
$u^2(c,\ell) = c - e^{\ell} \quad (w > 1)$
$u^3(c,\ell) = \ln c - \frac{1}{c} - \ell$
$u^4(c,\ell) = \ln c - \ell$
$u^5(c,\ell) = 2\sqrt{c} - \ell$
$u^6(c,\ell) = -e^{-c} - \ell \quad (w > 1)$
$u^7(c,\ell) = -e^{-c} - e^{\ell} \quad (w > 1)$
$u^8(c,\ell) = 2\sqrt{c} - \frac{\ell^2}{2} \quad (w > 1)$
$u^9(c,\ell) = \frac{5}{2}c - \ell e^{-\frac{1}{\ell}} - \int_1^{\infty} \frac{e^{-t\ell}}{t} dt \quad (0 < w < 1)$

Table 4.5: Stern's utility functions

$$u(c,\ell) = \frac{\ell - b}{\chi} e^{\frac{\chi(c+a)}{\ell-b}-1} \qquad \left(a = \frac{\rho}{\chi} - \frac{\xi}{\chi^2};\ b = \frac{\xi}{\chi};\ \chi = -1;\ \xi = 3\right)$$

$$u^{10}(c,\ell) = u(c,\ell) \quad (\rho = +1)$$

$$u^{11}(c,\ell) = u(c,\ell) \quad (\rho = 0)$$

$$u^{12}(c,\ell) = u(c,\ell) \quad (\rho = -1)$$

Table 4.6: Preston and Walker's utility function

$$u(c,\ell) = \beta\left(\frac{\beta c + \gamma - \ell}{\alpha - \beta\ell}\right) - \ln\left(\frac{\alpha - \beta\ell}{\beta^2}\right) \qquad (\alpha = \alpha;\ \beta = \beta;\ \gamma = \gamma)$$

5. Quasilinear Preferences

Letting $u(c,\ell) = v(c) - \ell$, where v is increasing and strictly concave, we get $\boxed{C(w,m) = v'^{-1}(1/w)}$ and the elasticity

$$\eta(C,w) \equiv \frac{C'(w)\,w}{C(w)} = \frac{1}{\eta(C^{-1},c)} = \frac{\dfrac{1}{v'(c)}}{-\dfrac{v''(c)\,c}{v'(c)^2}} = \frac{-1}{\dfrac{v''(c)\,c}{v'(c)}}. \tag{5.1}$$

Upon differentiating (5.1) and since consumption increases with productivity, we deduce that

$$\eta_w(C,w) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \;\text{ iff }\; \frac{v''(c)\,c}{v'(c)} - \frac{v'''(c)\,c}{v''(c)} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1. \tag{5.2}$$

Conditions (5.2) are reminiscent of the notions of decreasing, constant and increasing relative risk aversion where the difference between relative risk aversion and relative prudence plays a crucial role.

Table 5.1: Consumption relative inequality and the properties of the utility function

	Consumption Function Elasticity	Consumption Utility Function
A. Changes in the distribution of productivities		
$\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ \Longrightarrow C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$	$\eta_w(C, w) \geqslant 0$	$\frac{v'''(c)\,c}{v''(c)} - \frac{v''(c)\,c}{v'(c)} \leqslant 1$
$\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ \Longrightarrow C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$	$\eta_w(C, w) \leqslant 0$	$\frac{v'''(c)\,c}{v''(c)} - \frac{v''(c)\,c}{v'(c)} \geqslant 1$
B. Changes in preferences		
$C^*(\mathbf{w}) \geq_{RL} C^\circ(\mathbf{w})$	$\eta(C^*, w) \geqslant \eta(C^\circ, w)$	$-\frac{v^{*\prime\prime}(c)\,c}{v^{*\prime}(c)} \leqslant -\frac{v^{\circ\prime\prime}(c)\,c}{v^{\circ\prime}(c)}$
$C^\circ(\mathbf{w}) \geq_{RL} C^*(\mathbf{w})$	$\eta(C^*, w) \leqslant \eta(C^\circ, w)$	$-\frac{v^{*\prime\prime}(c)\,c}{v^{*\prime}(c)} \geqslant -\frac{v^{\circ\prime\prime}(c)\,c}{v^{\circ\prime}(c)}$

6. Limitations, Open Questions and Further Work

- Absolute Lorenz dominance: $\xi(C, w) := C'(w)\, w$.
- Implications for the structure of preferences of restrictions on the consumption function[s] in the general case.
- Unambiguous welfare improvements: generalised Lorenz dominance and $\xi(C, w) := C'(w)\, w$.
- No taxation: we observe choices under taxation constraint.
- Modifications in the joint distribution of talent and exogenous income: multidimensional approach and infra-modular consumption functions.
- Possibility that the society's members have distinct preferences.