Robust multidimensional normative appraisal

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Object of the talk :

- Foundations of **multidimensional** normative evaluation.
- **Reference:** One-dimensional normative evaluation: comparing distributions of a single attribute (income) between a given number of households.
- n individuals (households) identical in every respect other than the considered attribute (income)
- $y = (y_1,\ldots,y_n)$ an income distribution
- $y_{(.)} = (y_{(1)},\ldots,y_{(n)})$ ordered permutation of y
- **Q:** When are we « sure » that income distribution y is « normatively better » than income distribution z **?**

One-dimensional normative dominance (4 equivalent **1st order** answers (ordinal information on the attribute only)

- 1: When all utilitarian planners who assume that individuals transform income into well-being by the same **increasing utility function** would rank y above z
- 2: When $y_{(.)}$ has been obtained from $z_{(.)}$ by giving from outside **income increments** to some (or all) individuals.
- 3: When the **number (fraction) of poor** is lower in y than in z for every poverty line.
- 4: When $y_{(.)}$ **component-wise dominates** $z_{(.)}$

One-dimensional normative dominance (4 equivalent **2nd order** answers (cardinal information on the attribute)

- 1: When all utilitarian social planners who assume that individuals transform income into well-being by the same **increasing and concave utility function** would rank y above z.
- 2: When $y_{(.)}$ has been obtained from $z_{(.)}$ by a finite sequence of **Pigou-Dalton transfers and/or increments.**
- 3: When **poverty gap** is lower in y than in z for all poverty lines.
- 4: When the (generalized) **Lorenz curve** associated to y is nowhere below and sometime above that of z.

These equivalences are nice because they connect together:

- An **explicit** and **robust ethical foundation** (utilitarian (actually even larger, see Gravel & Moyes (SCW 2013) unanimity over a plausible class of individual utility functions)
- **Elementary transformations** (Pigou-Dalton transfers, increments) that identify clearly the nature of the normative improvements that are at stake.
- **Empirically implementable** criteria (Lorenz, Poverty dominance) that can be used in practice to perform normative evaluation.

Normative dominance (1)

- Very often expressed in terms of the (ethically contentious) utilitarian doctrine.

- Utilitarian dominance: state a is normatively better than state b if : $\sum_{i=1}^{n} U_i(a) \geq \sum_{i=1}^{n} U_i(b)$

- holds for every profile of utility functions $<U_i>^n_{i=1}$ in a class C.

- Welfarist dominance: state a is normatively better than state b if $W(U_1(a),\ldots.,U_n(a)) \geq W(U_1(b),\ldots.,U_n(b))$

holds for every profile of utility functions $<U_i>^n_{i=1}$ in some classe C and every symmetric social welfare function W: $\mathbb{R}^n \to \mathbb{R}$ in some class $\mathcal{W}$.

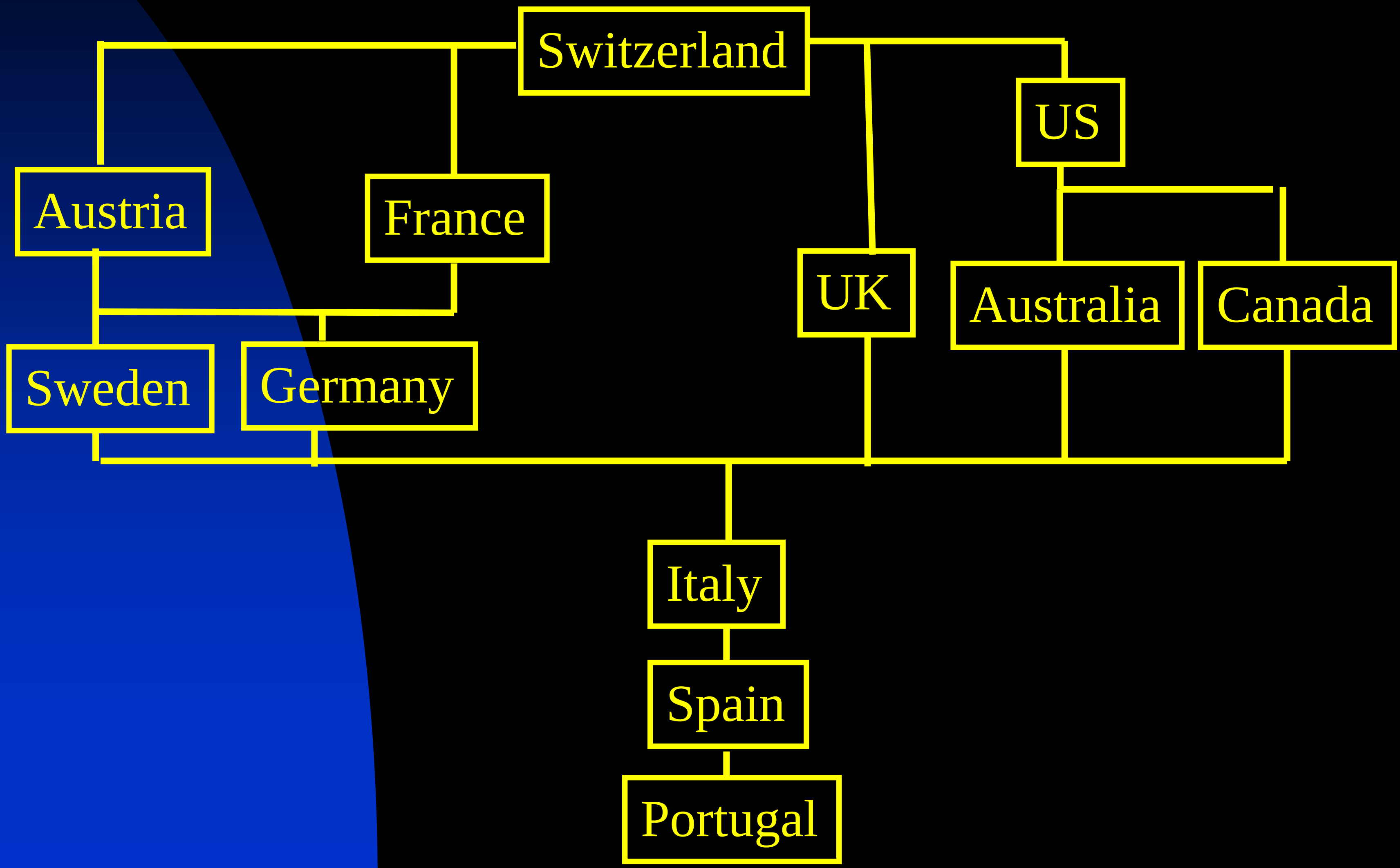
Normative dominance (2)

- Gravel and Moyes (2013): If the class $\mathcal{C}$ of profiles of utility functions is closed under certain functional operations, then welfarist dominance (over some class $\mathcal{W}$ of symmetric welfare function) and utilitarian dominance coincide.
- Example1: If $\mathcal{W}$ is the class of increasing and symmetric social welfare functions, then the closedness of $\mathcal{C}$ under the composition of any utility functions with an increasing function $f: \mathbb{R} \to \mathbb{R}$ is sufficient for the coincidence of Welfarist and Utilitarian unanimity.
- Example 2: If $\mathcal{W}$ is the class of increasing and Schur-concave social welfare function, then the closedness of $\mathcal{C}$ under the composition of every utility functions with any increasing and concave transformation is sufficient for the coincidence of Welfarist and Utilitarian unanimity.

Dominance equivalence theorems

- Can be extended to distributions involving different number of attribute holders (individuals, households) (Dalton principle of population)
- Very useful to compare countries, evaluate policies, etc.
- Generate incomplete (but robust) rankings (incompleteness is the price to pay for robustness; incompleteness decreases with the order of dominance).
- can be completed by using more ethically contentious indices (who are then asked to be consistent with the dominance orders)
- Despite their incompleteness they can be quite discriminatory (ex: ranking 11 OECD countries (Gravel, Moyes & Tarroux based on their distribution of disposable (2009))

Robust ranking of countries by disposable income



Object of the research surveyed here:

- To establish analogous foundations for comparing distributions of several attributes (income, health status, need categories, access to public goods, exposure to risk, etc.) (to find a “lost paradise” according to Trannoy (2004).
- Many results deal with two attributes only.
- The only analogue of the dominance equivalence results that we are aware of (Gravel and Moyes (2012)) deals with two attributes, one cardinally measurable, the other only ordinally so).
- Focuses on approaches that establish at least an equivalence between normative dominance and an empirically implementable criterion (exit: Koshevoy (1995;1998), Koshevoy & Mosler, Muller & Scarsini (2012) and several others (sorry for them)! etc.

Two-dimensional normative dominance (basic framework)

- 2 attributes (say income -possibly cardinally measurable) and health (ordinal))
- n individuals
- Situation: $(x;a) = (x_1,a_1,\ldots, x_n,a_n,) \in A \subset \mathbb{N}^{2n}_+$ with A bounded.
- x_i : amount of income accruing to i.
- a_i : ability level of i.
- **Q:** When can situation $(x;a)$ be considered unambiguously better than situation $(x';a')$?

Answer 1: Utilitarian (welfarist ?) dominance at the 1st order.

- $\mathcal{U}_0$: class of all (twice differentiable) functions U that verify $U_i \geq 0$.(Lehman (1955), Østerdal (2010)).
- $\mathcal{U}_1^-$: class of all (twice differentiable) functions U that verify $U_i \geq 0$ for i=1,2 & $U_{12} \leq 0$. (Hadar & Russell (1974), Atkinson & Bourguignon (1982))
- $\mathcal{U}_1^+$: class of all (twice differentiable) functions U that verify $U_i \geq 0$ & $U_{12} \geq 0$.(Levy & Paroush (1974), Atkinson & Bourguignon (1982)).
- Utilitarianism = welfarism under this class (if W is increasing)

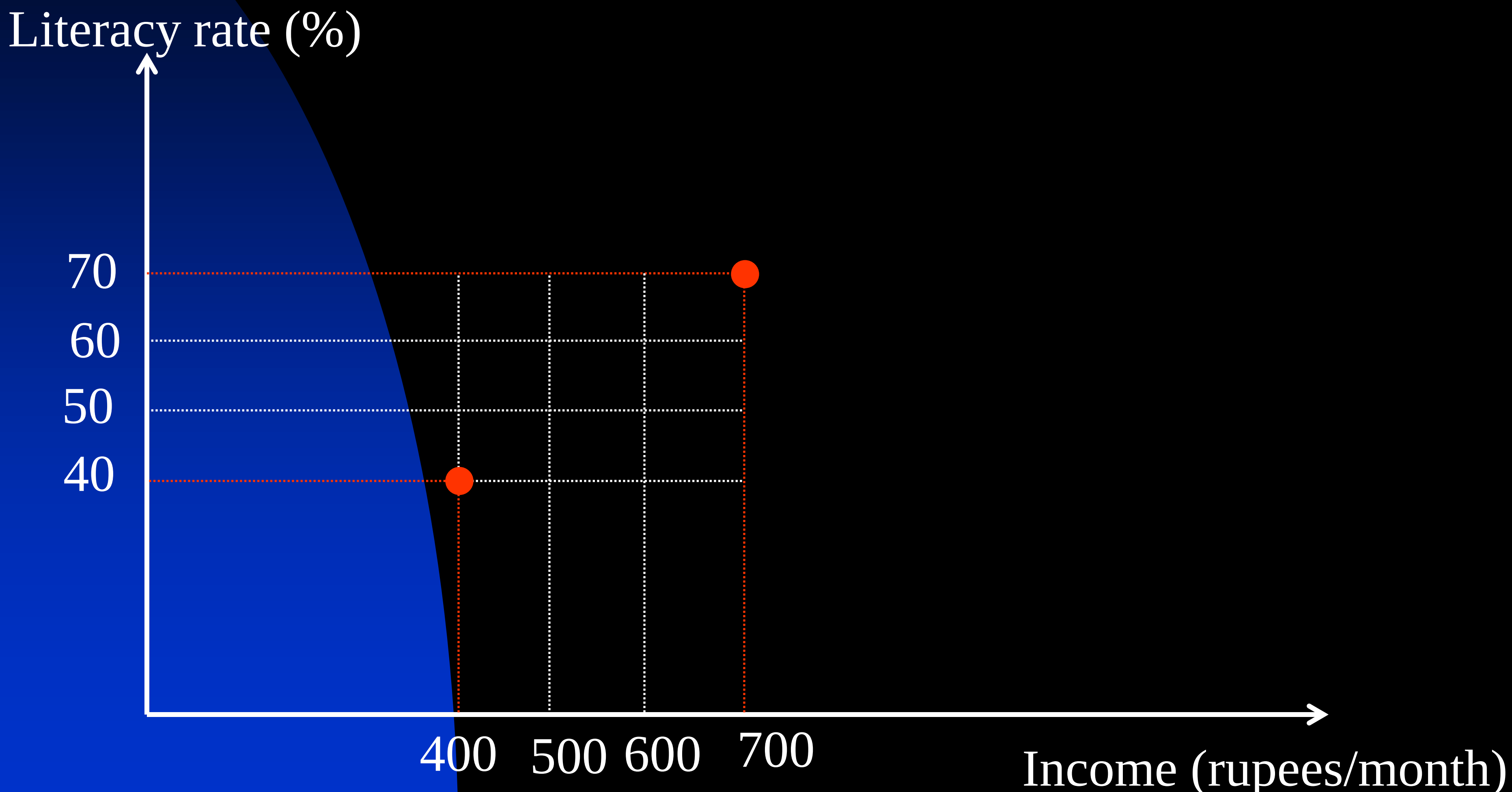
Answer 1': Utilitarian (welfarist ?) dominance at the 2nd (?) order.

- $\mathcal{U}^2_0$: class of all (twice differentiable) functions U that verify $U_i \geq 0$ and $U_{11} \leq 0$ (not studied).
- $\mathcal{U}_2^-$: class of all (twice differentiable) functions U that verify $U_i \geq 0$ for $i = 1,2$, $U_{12} \leq 0$ and $U_{11} \leq 0$ (Bourguignon (1989), Gravel & Moyes (2012)).
- $\mathcal{U}_2^+$: class of all (twice differentiable) functions U that verify $U_i \geq 0$ & $U_{12} \geq 0$ $U_{11} \leq 0$ (not studied, can be studied from Gravel & Moyes (2012)).
- Welfarism (if G is increasing, symmetric and Schur-Concave) = utilitarianism for the class $\mathcal{U}^2_0$ and $\mathcal{U}_2^-$ but not for the class $\mathcal{U}_2^+$

Answer 2: elementary transformations

- 1st order: When $(x;a)$ has been obtained from $(x';a')$ by a finite sequence of increments (of either or both attributes (improving transfer of mass between density (Østerdal (2010).
- 1st order (bis): When $(x;a)$ has been obtained from $(x';a')$ by a finite sequence of favorable (for $\mathcal{U}_1^-$) (correlation reducing) permutations.
- 2^{nd} (?) order. When $(x;a)$ has been obtained from $(x';a')$ by a finite sequence of favorable permutations and/or Pigou-Dalton transfers of income (between individuals with the same ability).

A Favourable Permutation ?



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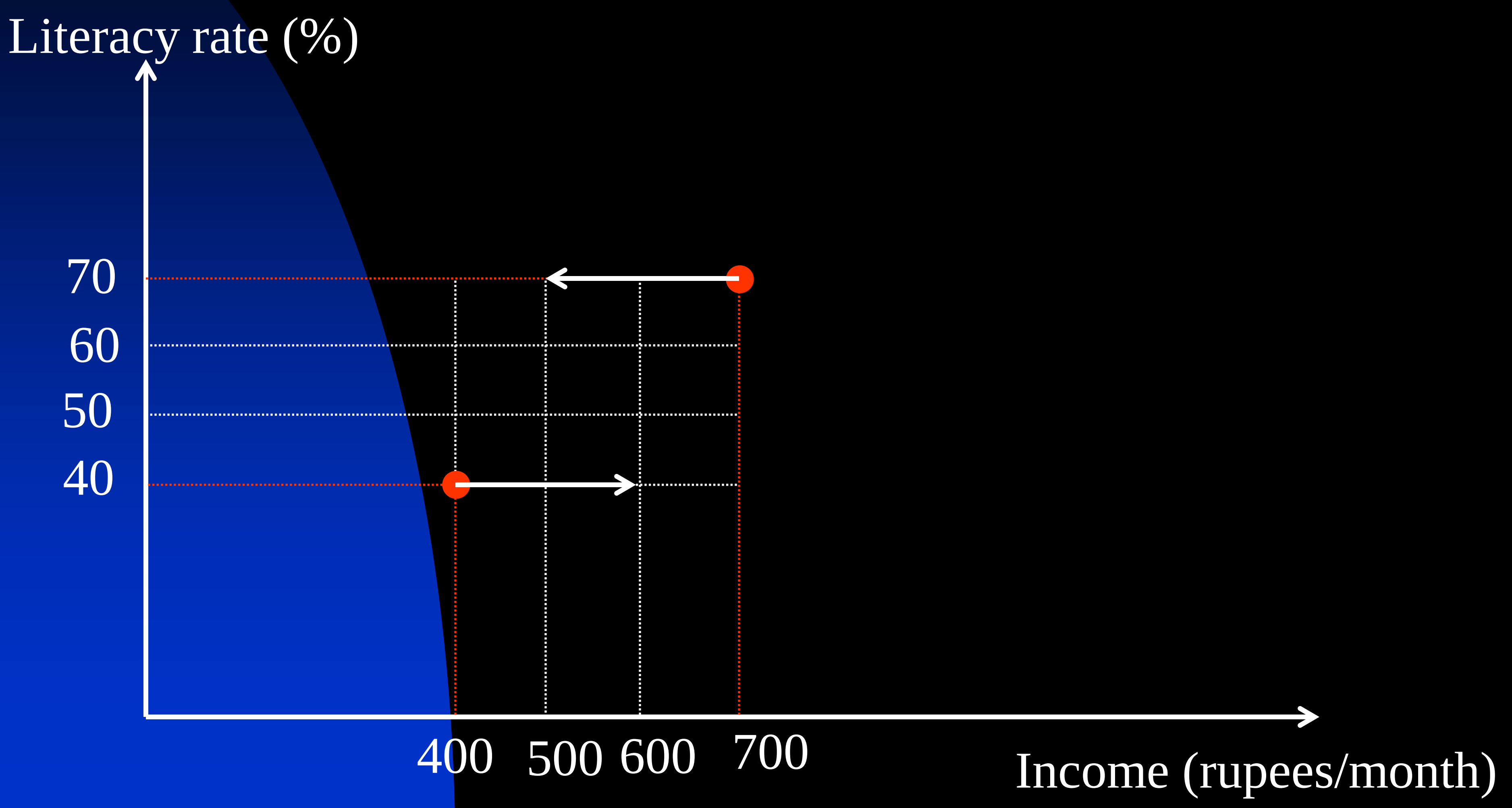

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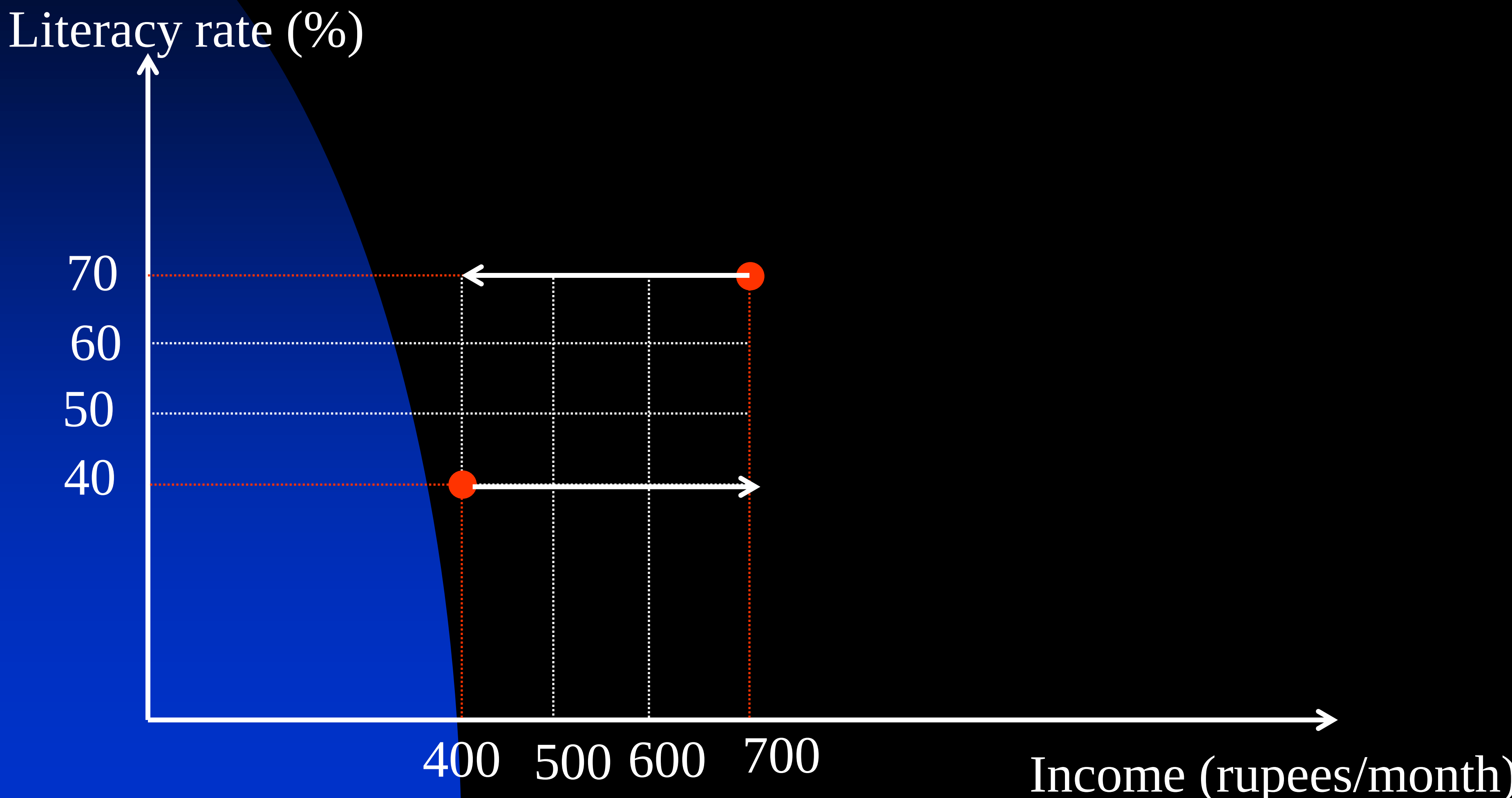

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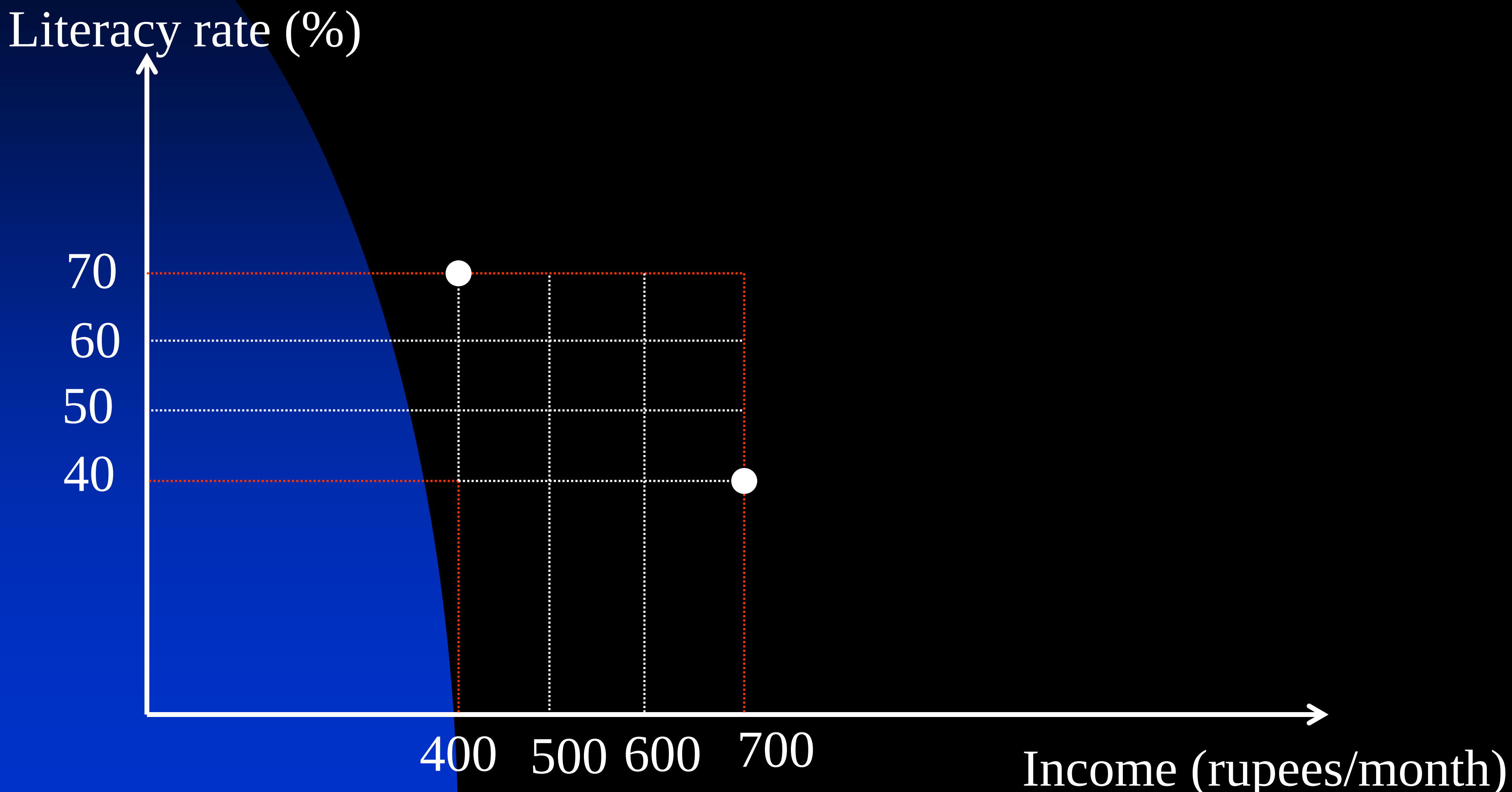

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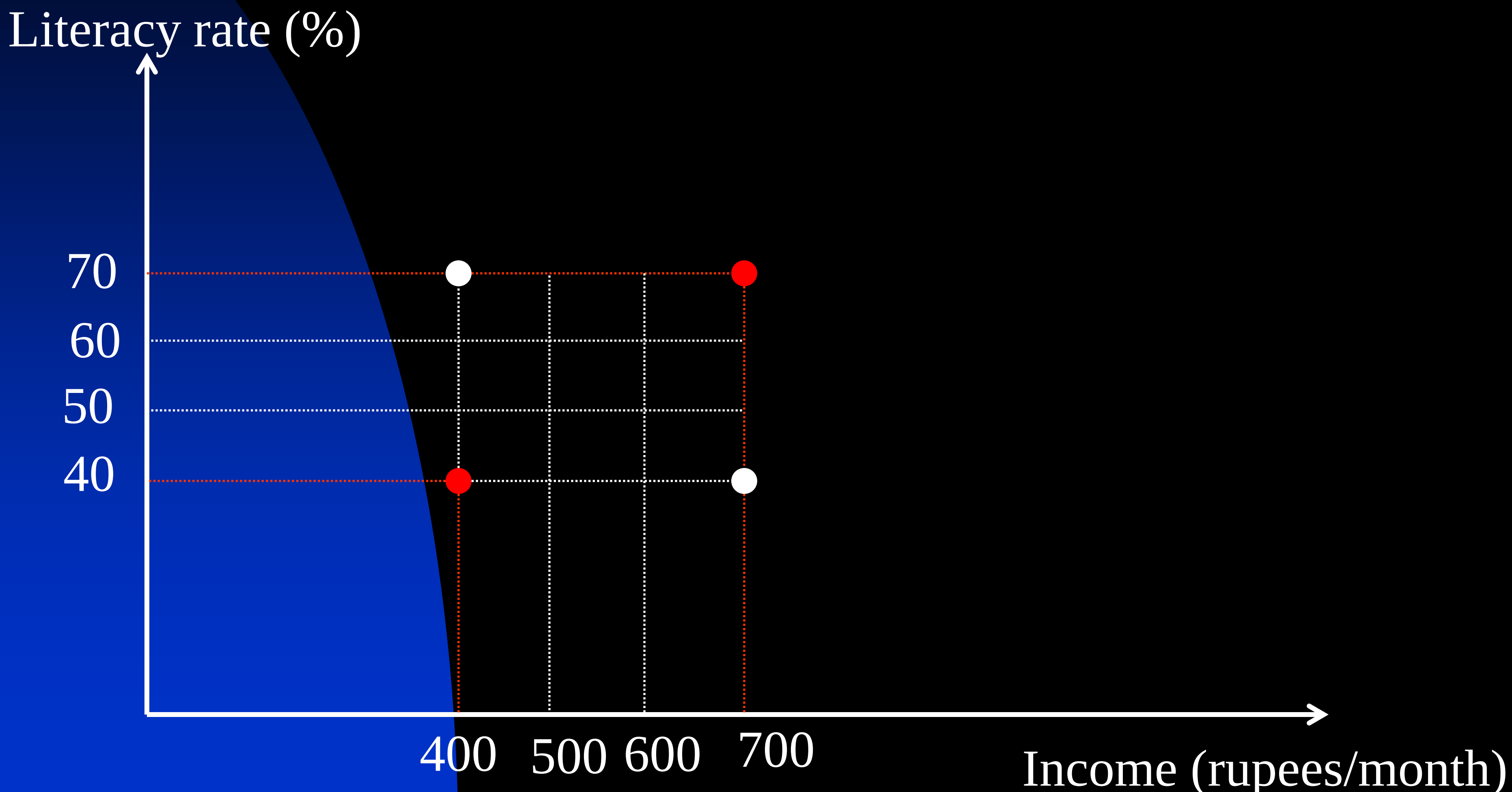
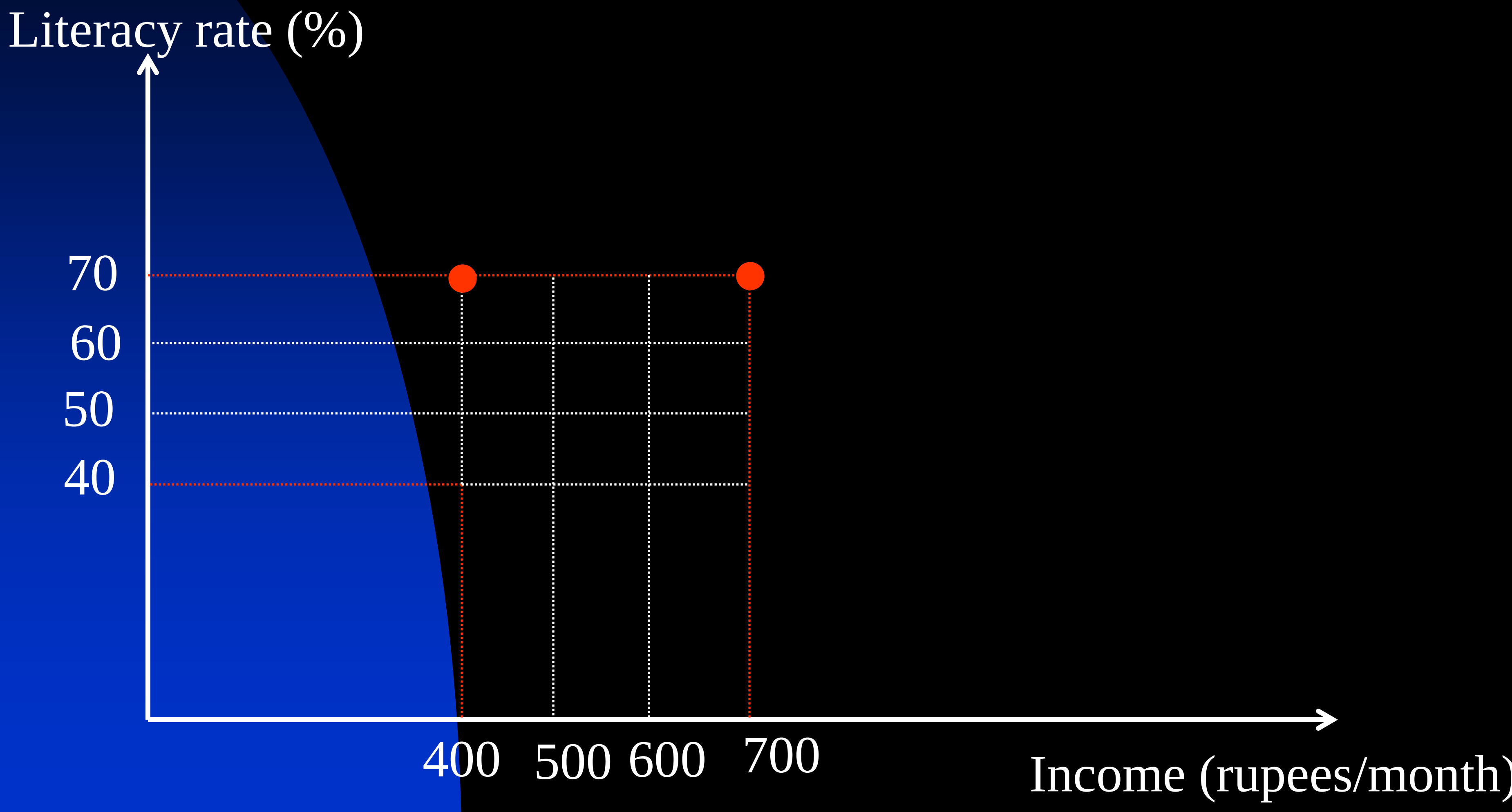


A Pigou-Dalton Transfer of income ?



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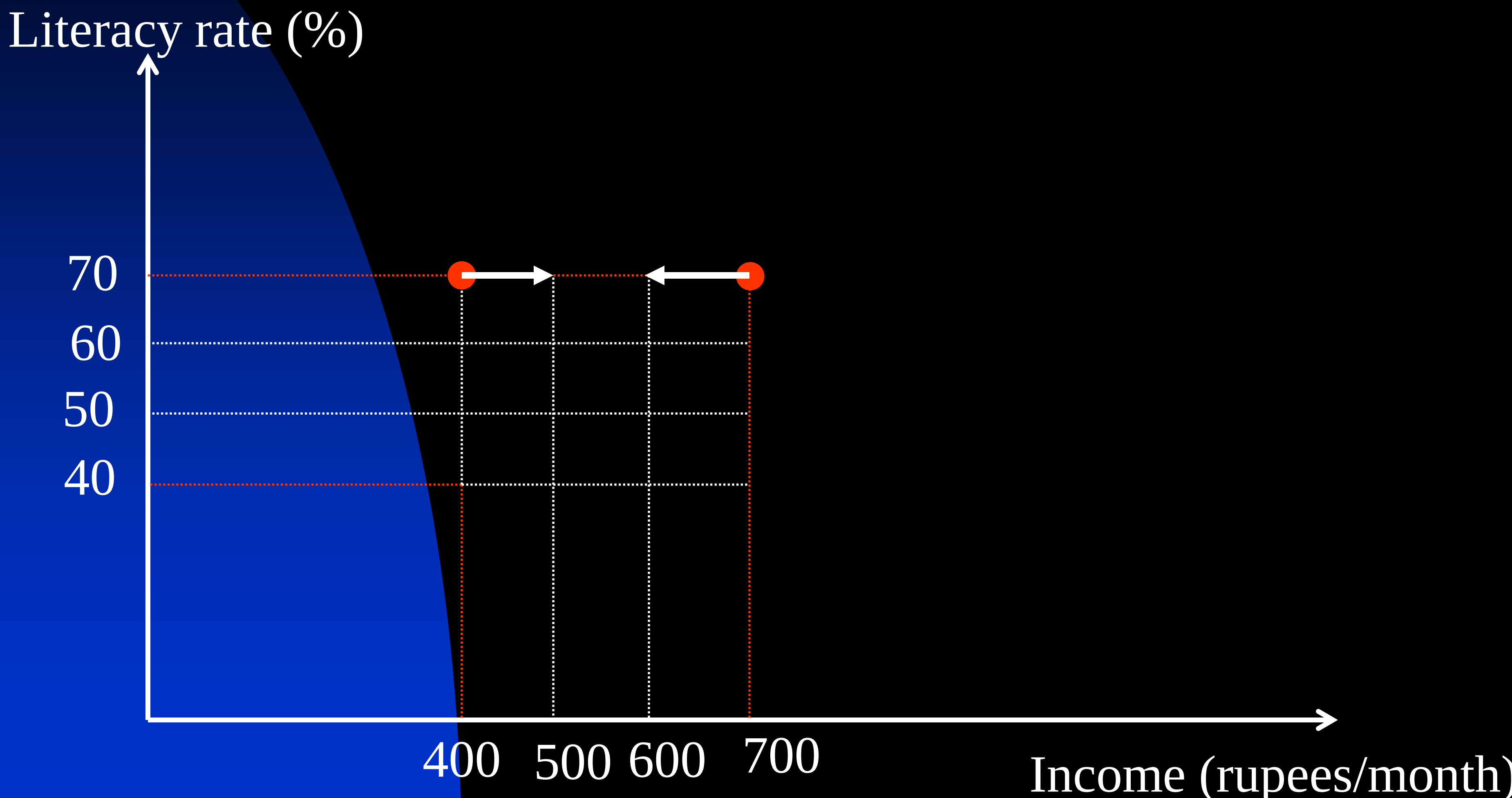

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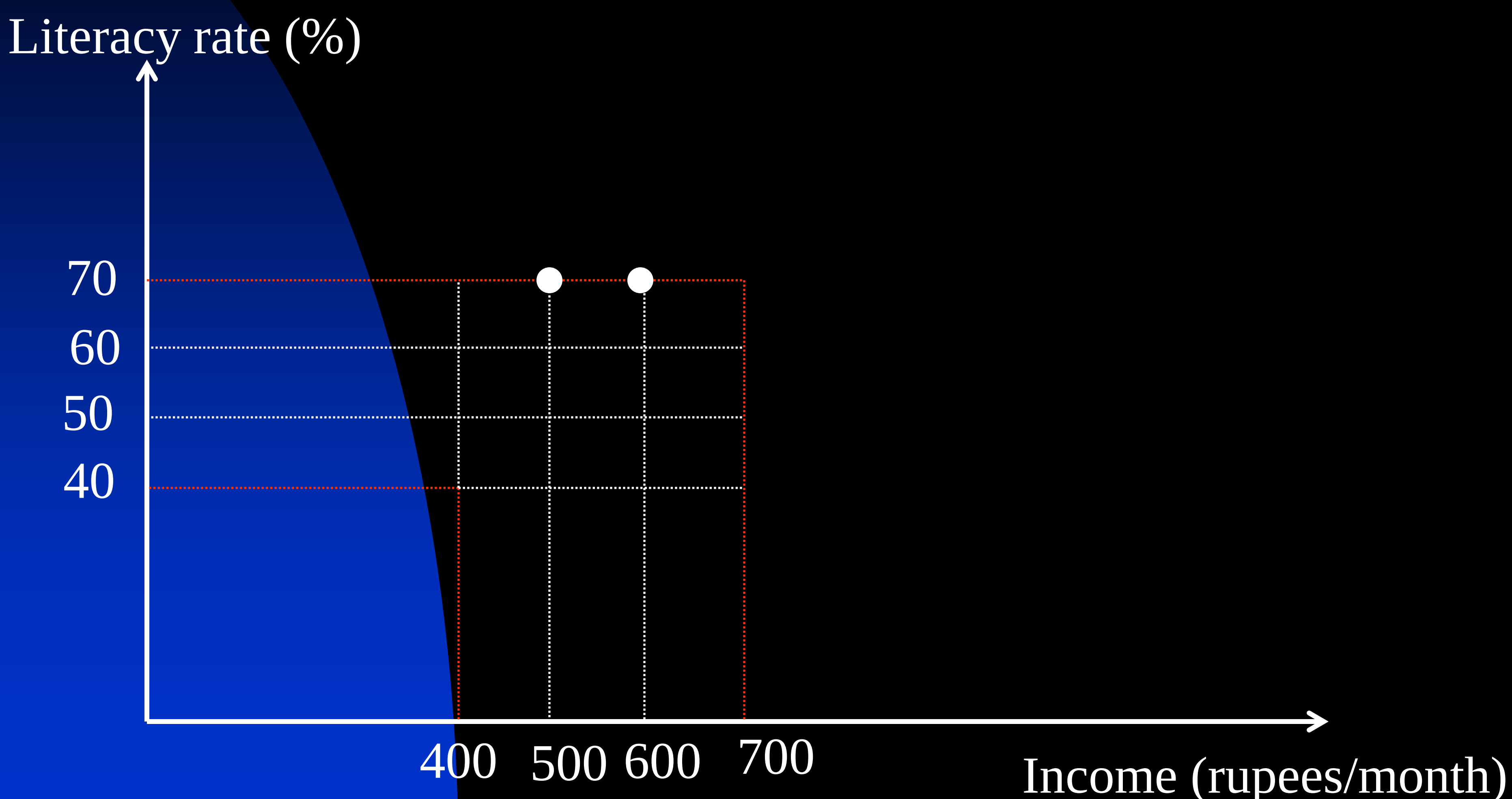

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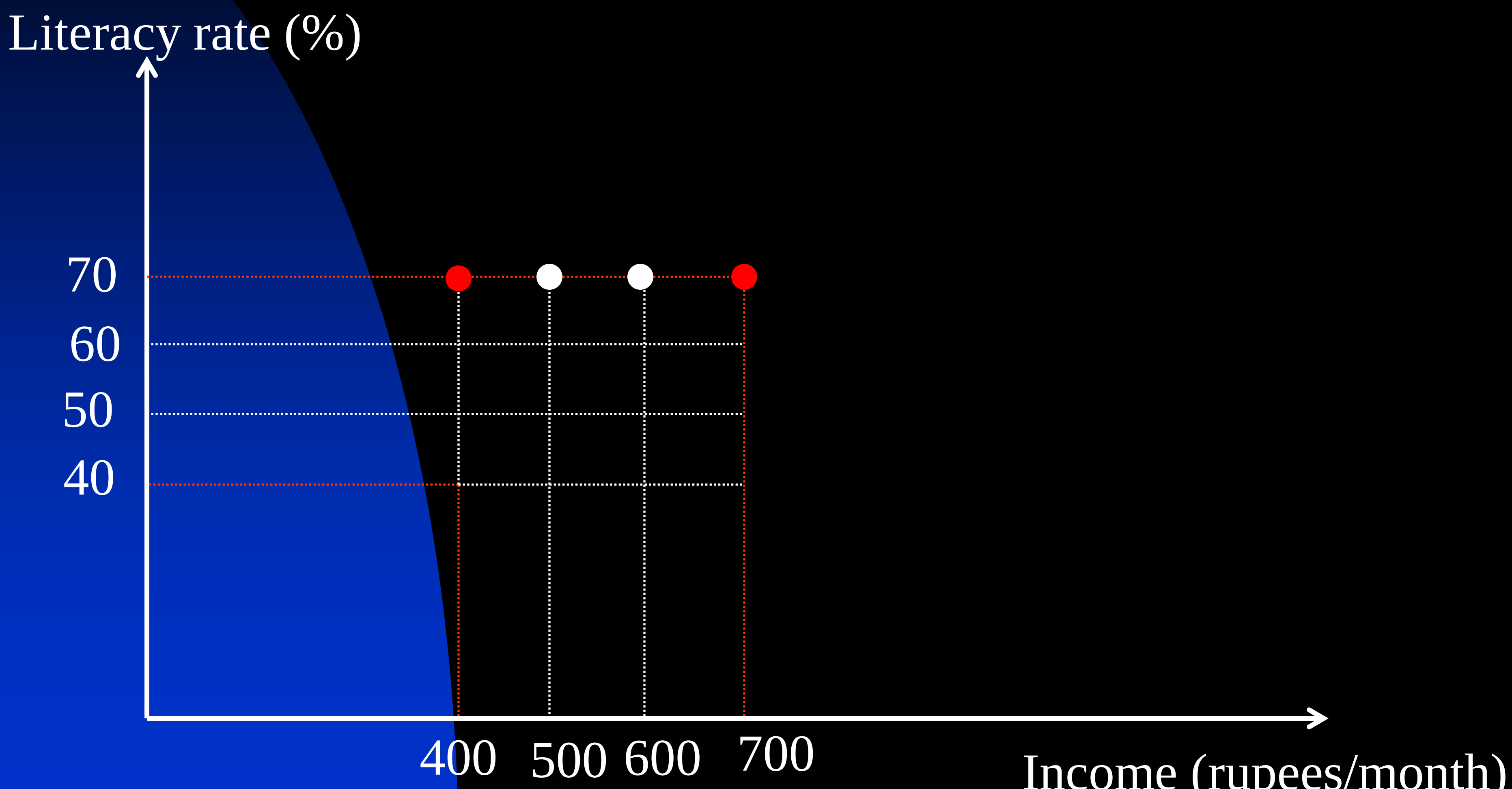
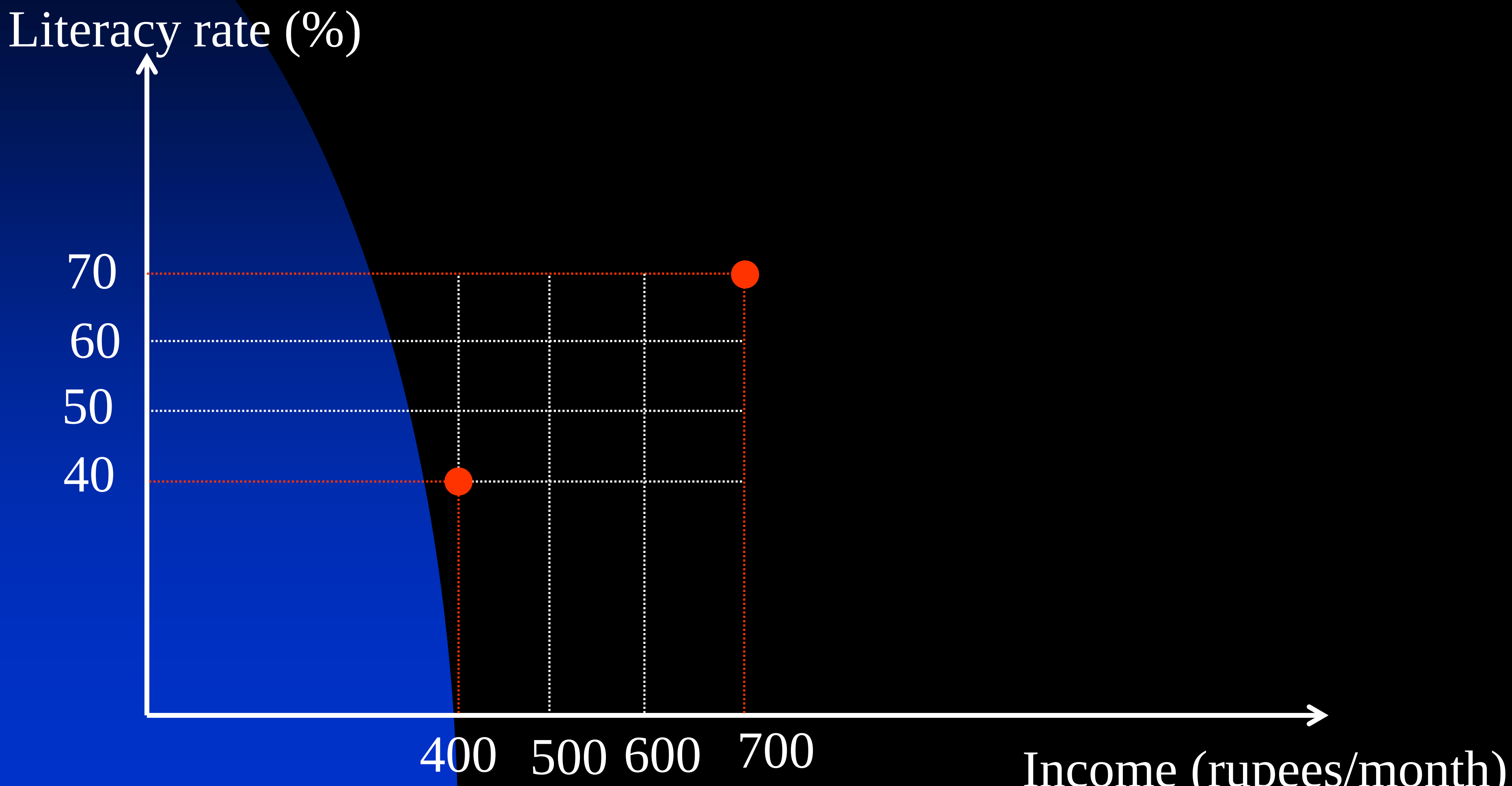


Another transfer principle: Between-Type Progressive Transfer (BTPT) of income



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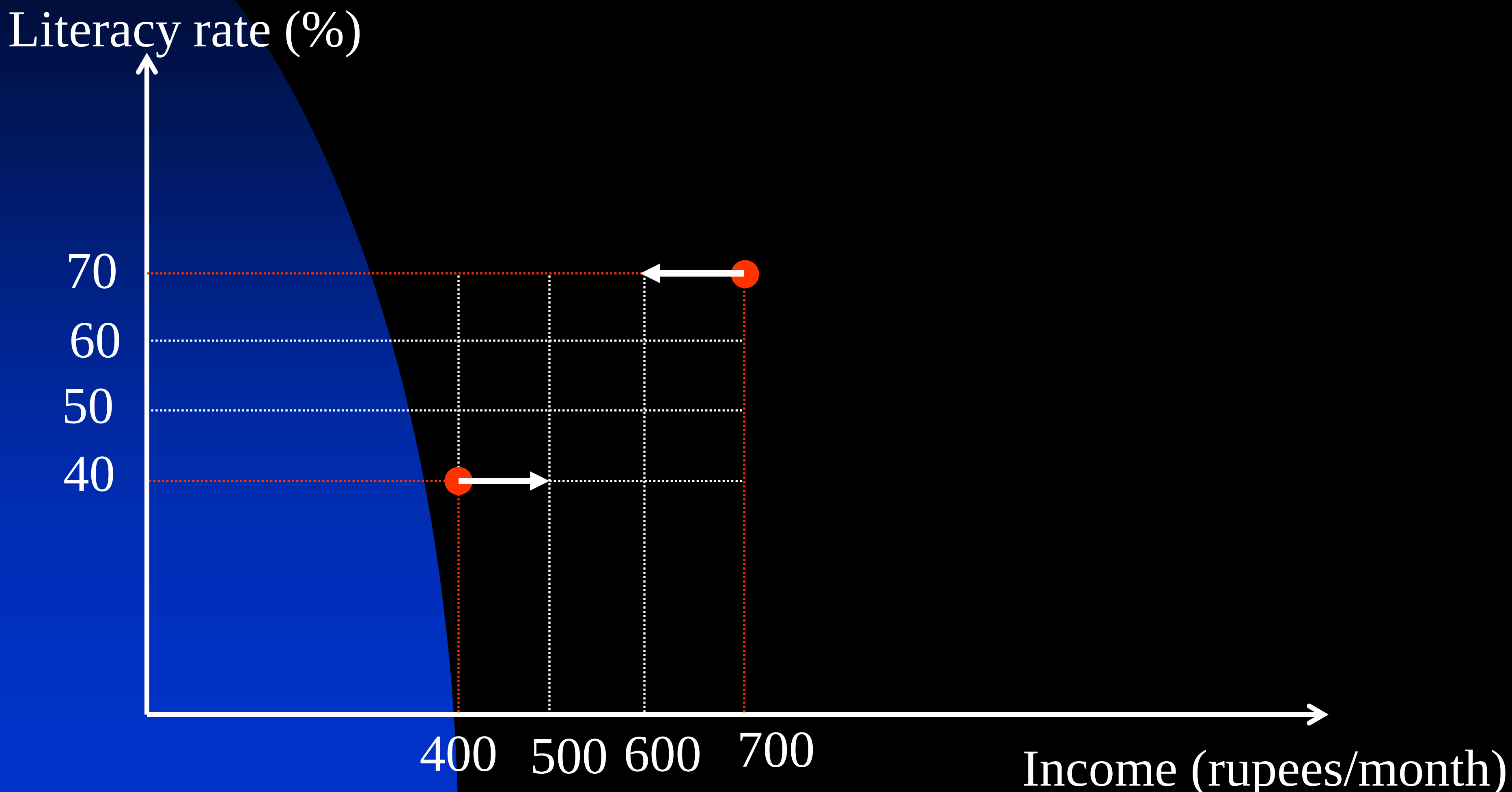

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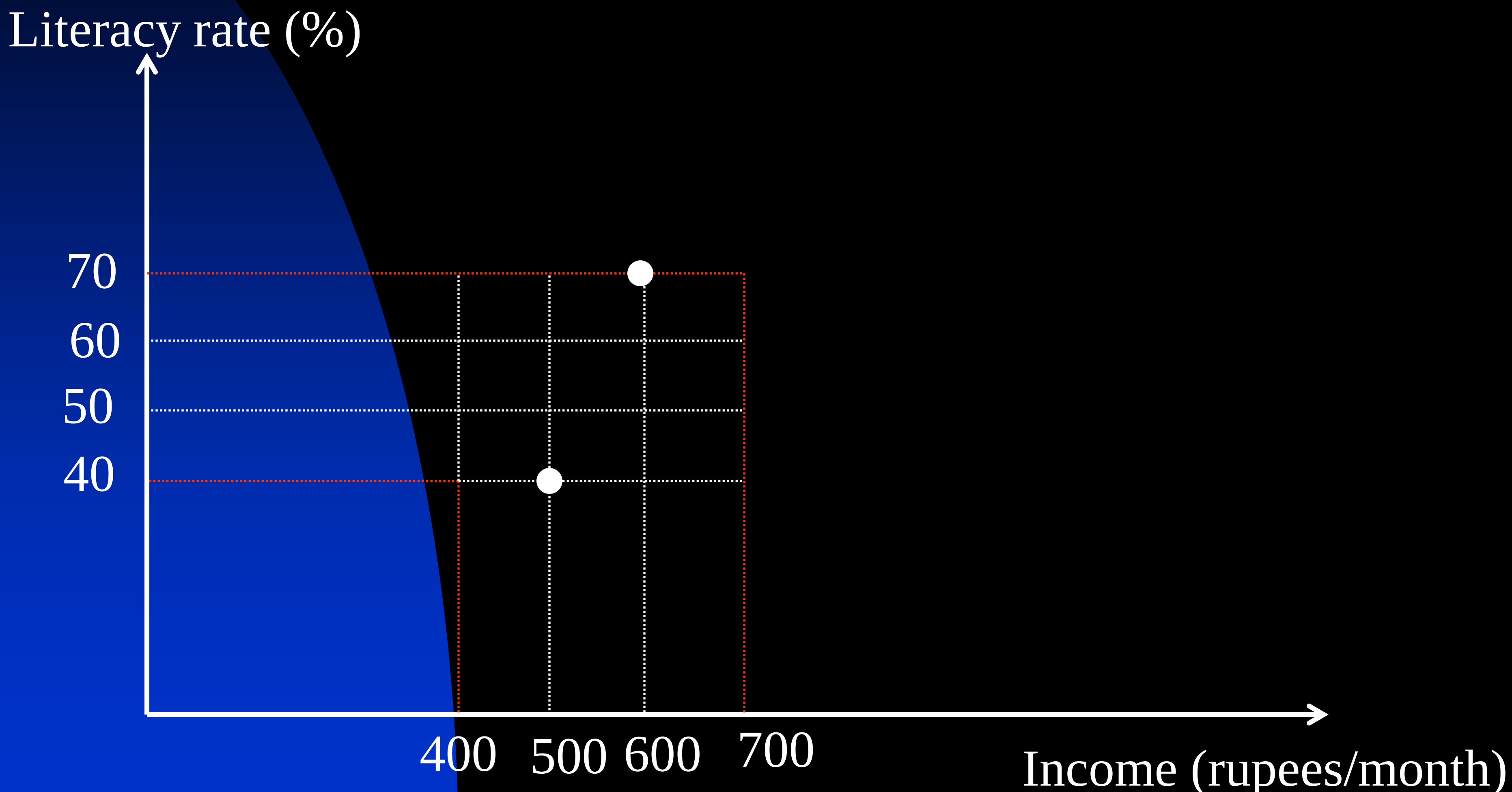

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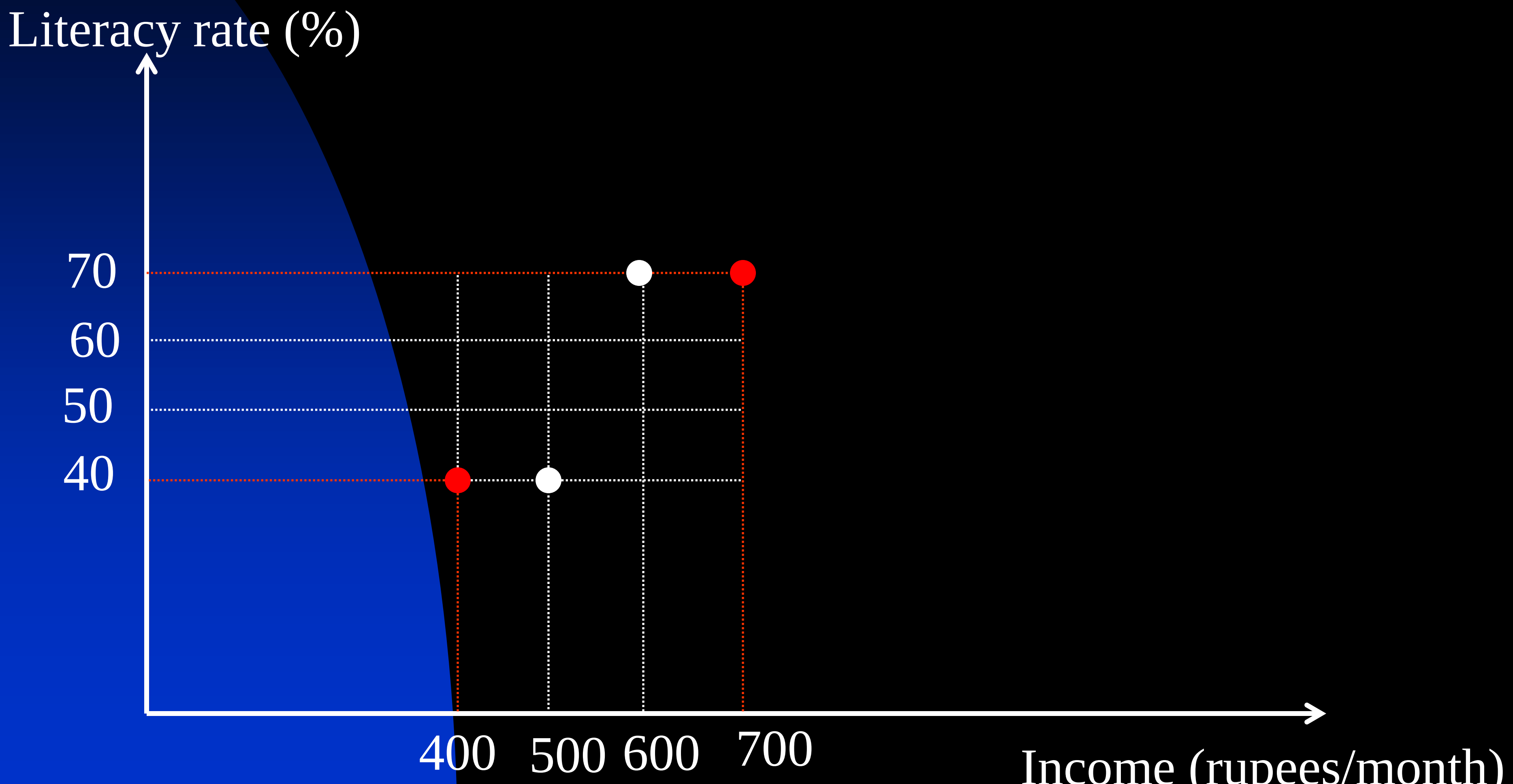
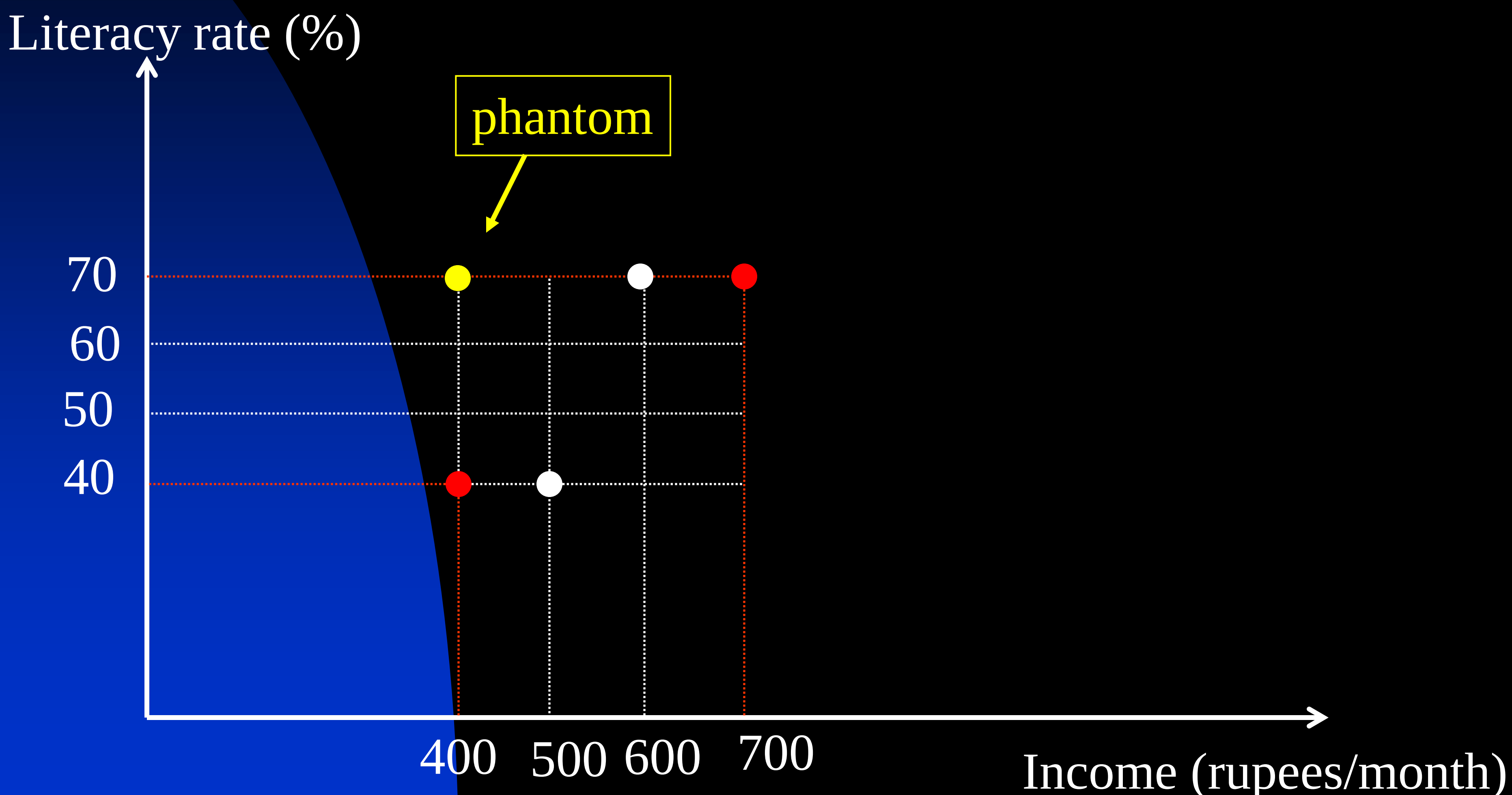


Some remarks on BTPT

- BTPT is considered good by Utilitarian (and welfarist) dominance over the class $\mathcal{U}^{-}_{2}$
- We don't know whether Utilitarian/welfarist dominance over $\mathcal{U}^{-}_{2}$ imply the possibility of going from the dominated situation to the dominant one by a finite sequence of BTPT of income.
- Provided that some dummy (phantoms) individuals can be added to distributions, any BTPT of attribute 1 can be broken up into Pigou-Dalton transfers of good 1 and favorable permutations.

Decomposing a BTPT with a phantom ?



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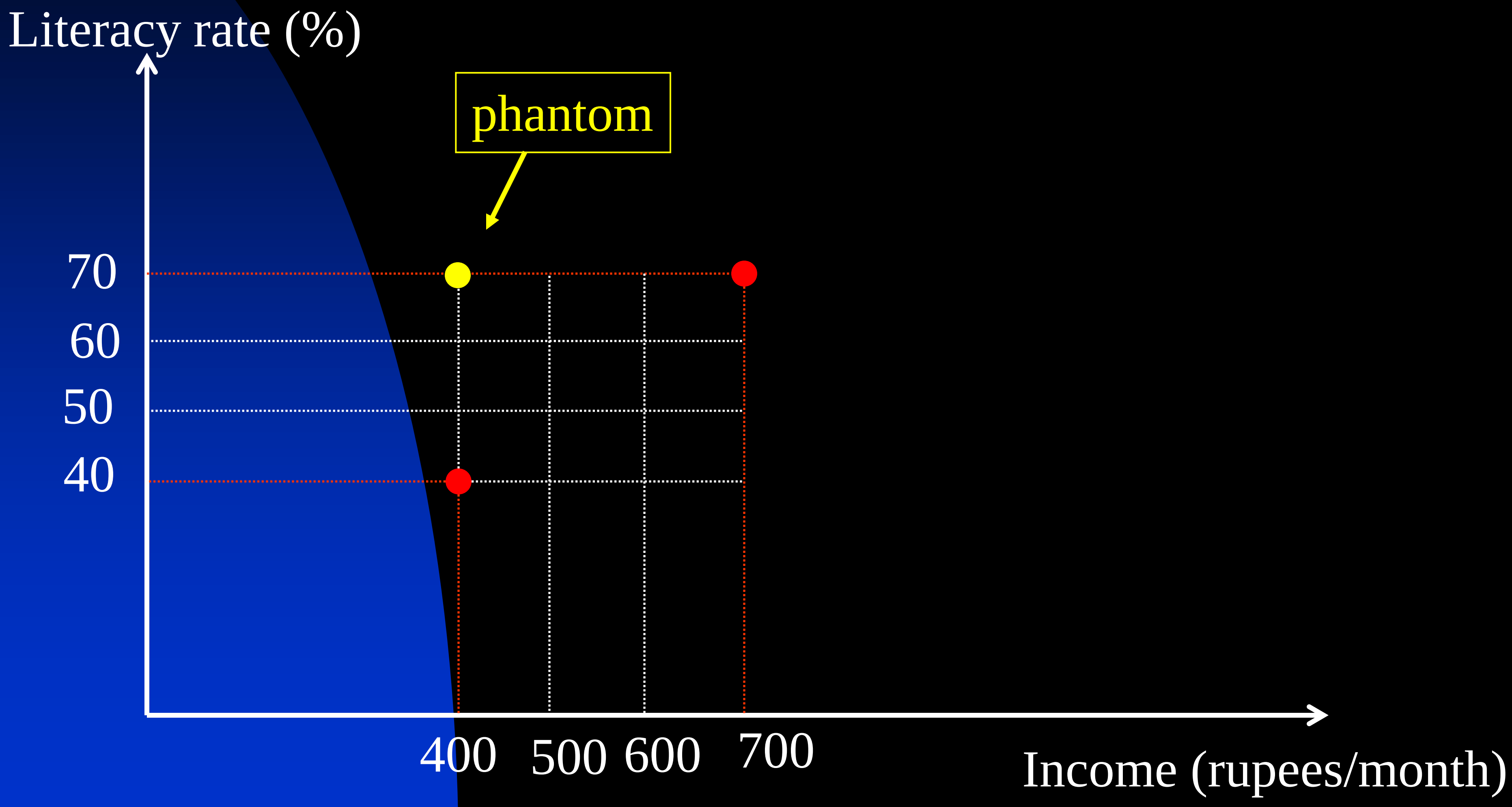

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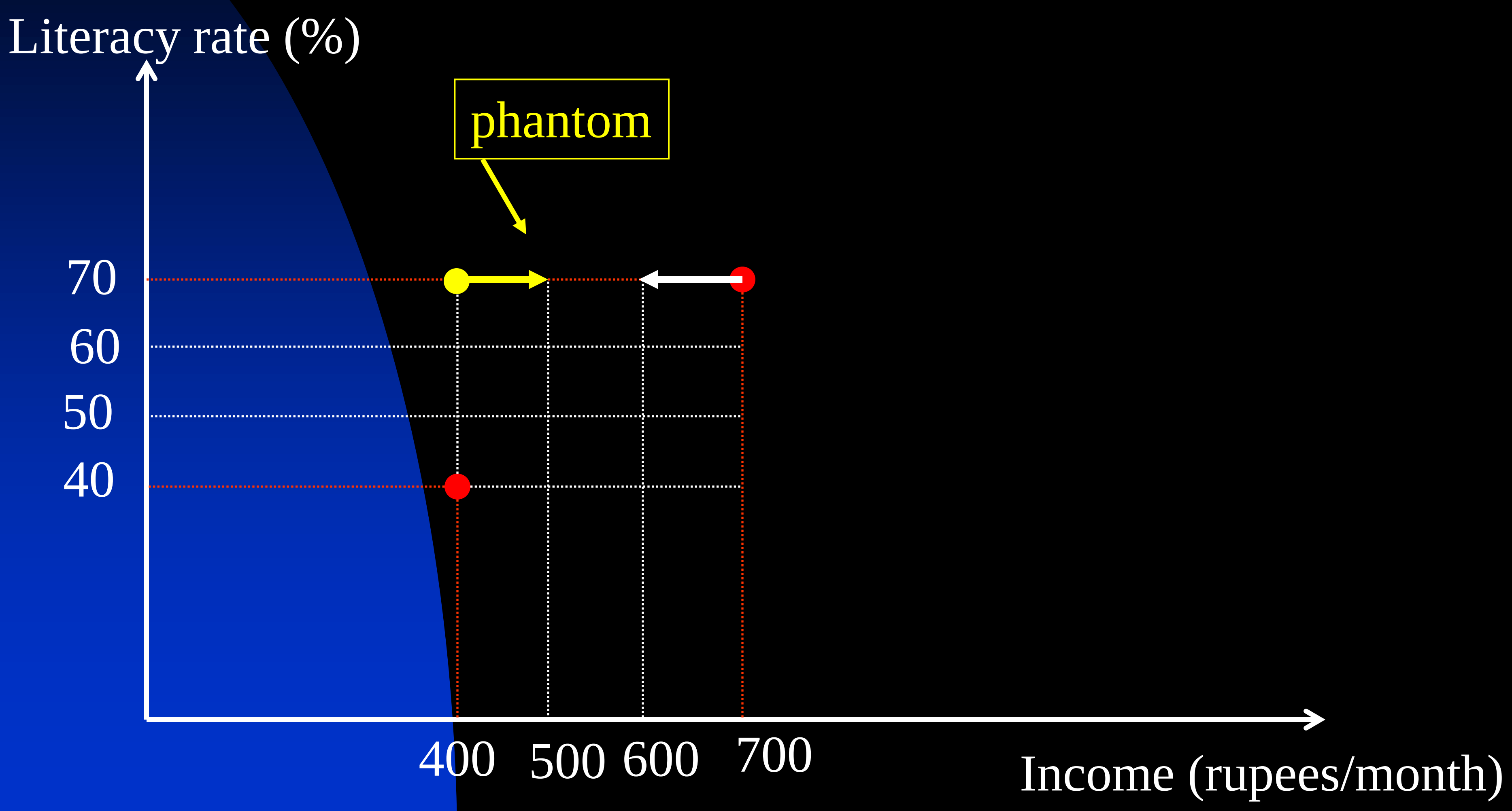

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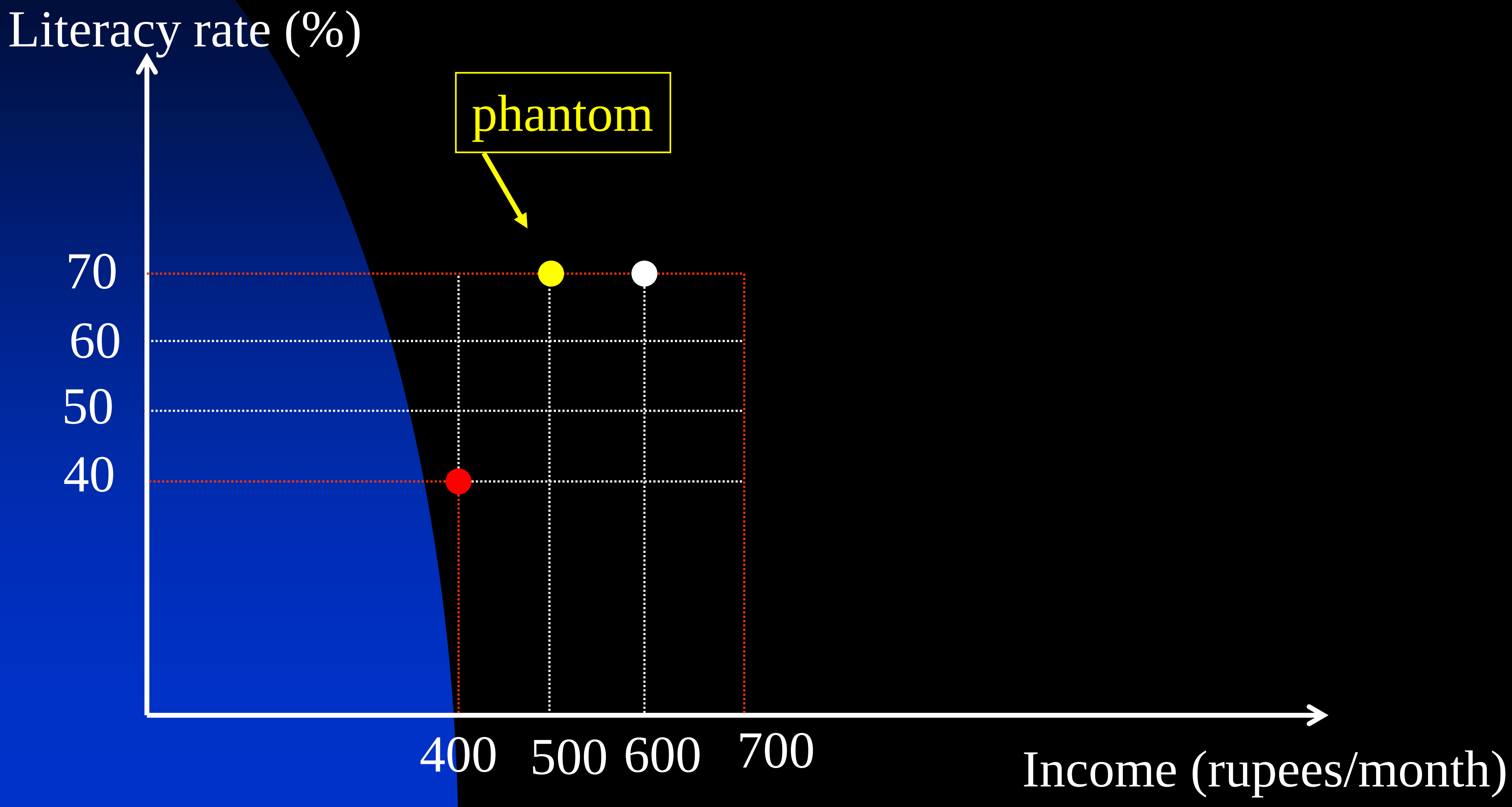

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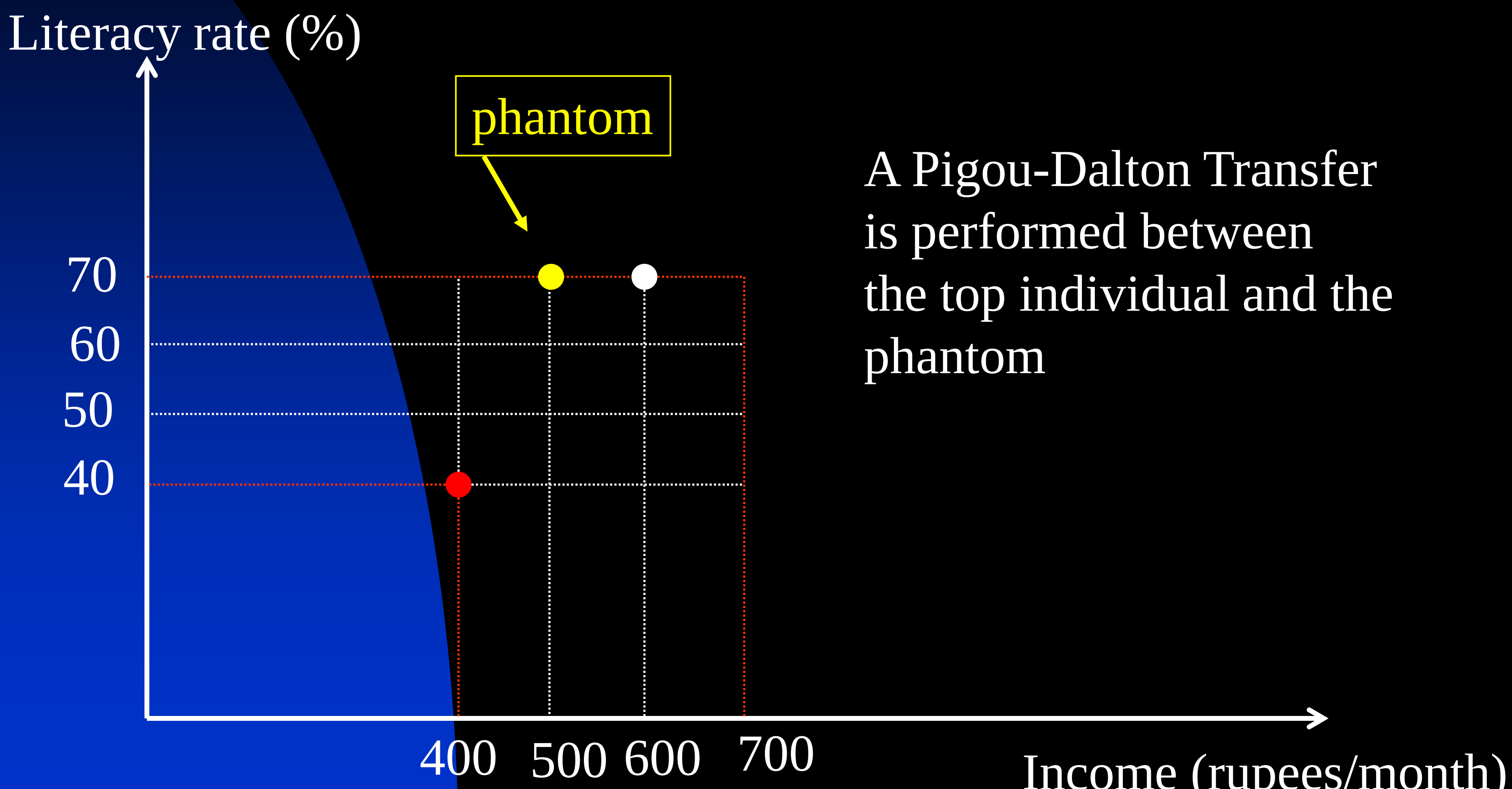
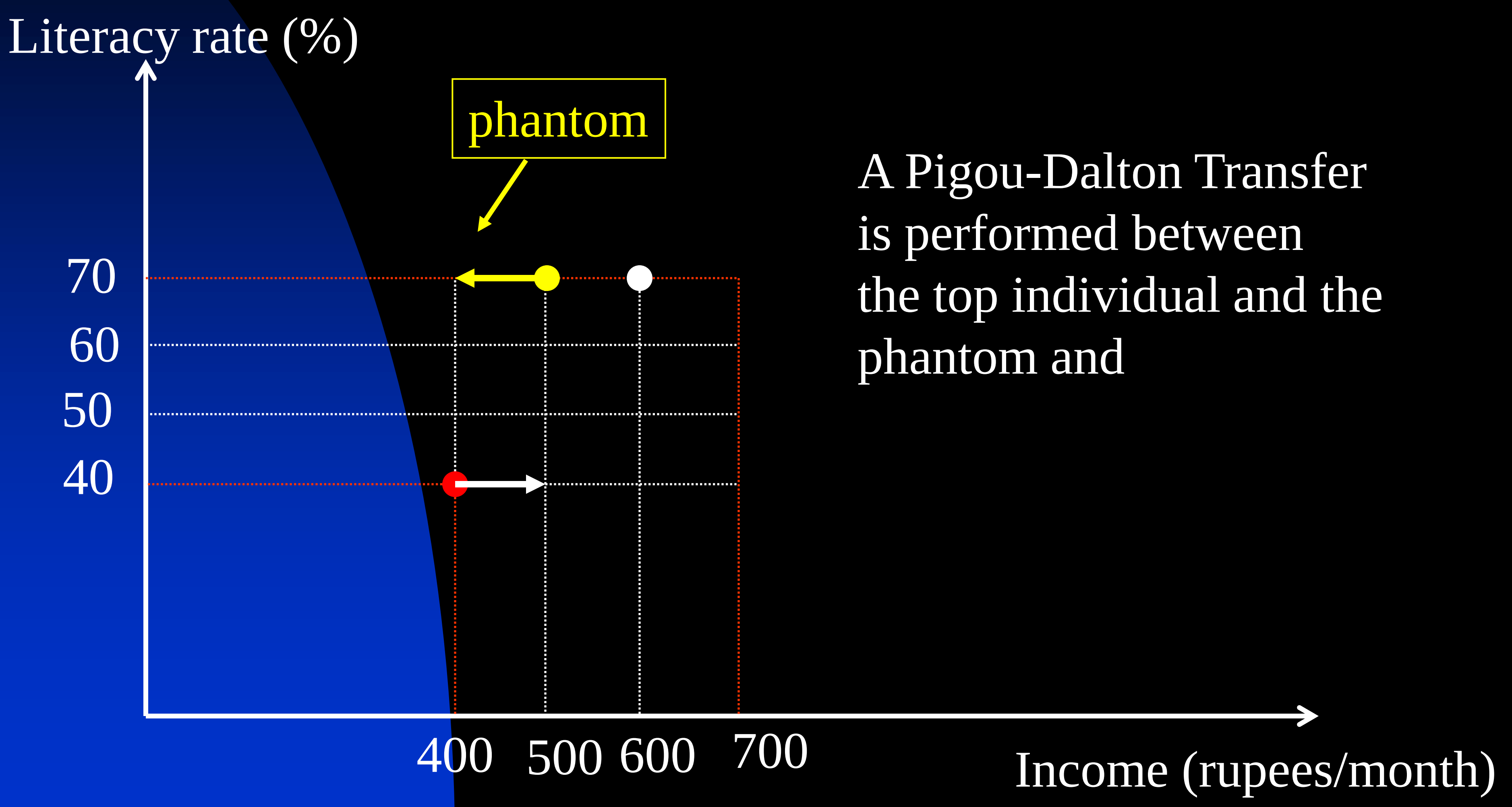


A Pigou-Dalton Transfer is performed between the top individual and the phantom

Decomposing a BTPT with a phantom ?



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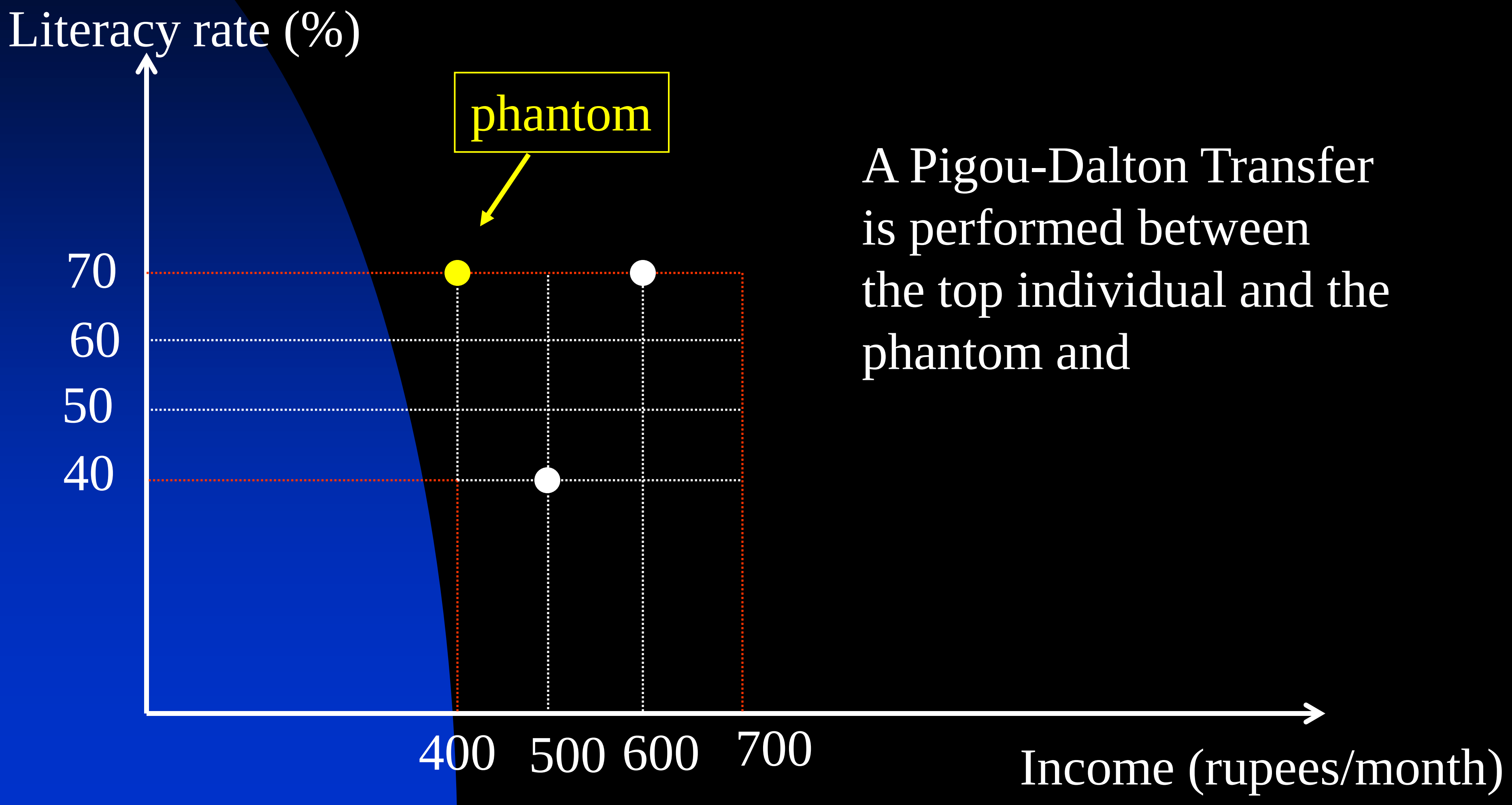
Decomposing a BTPT with a phantom ?



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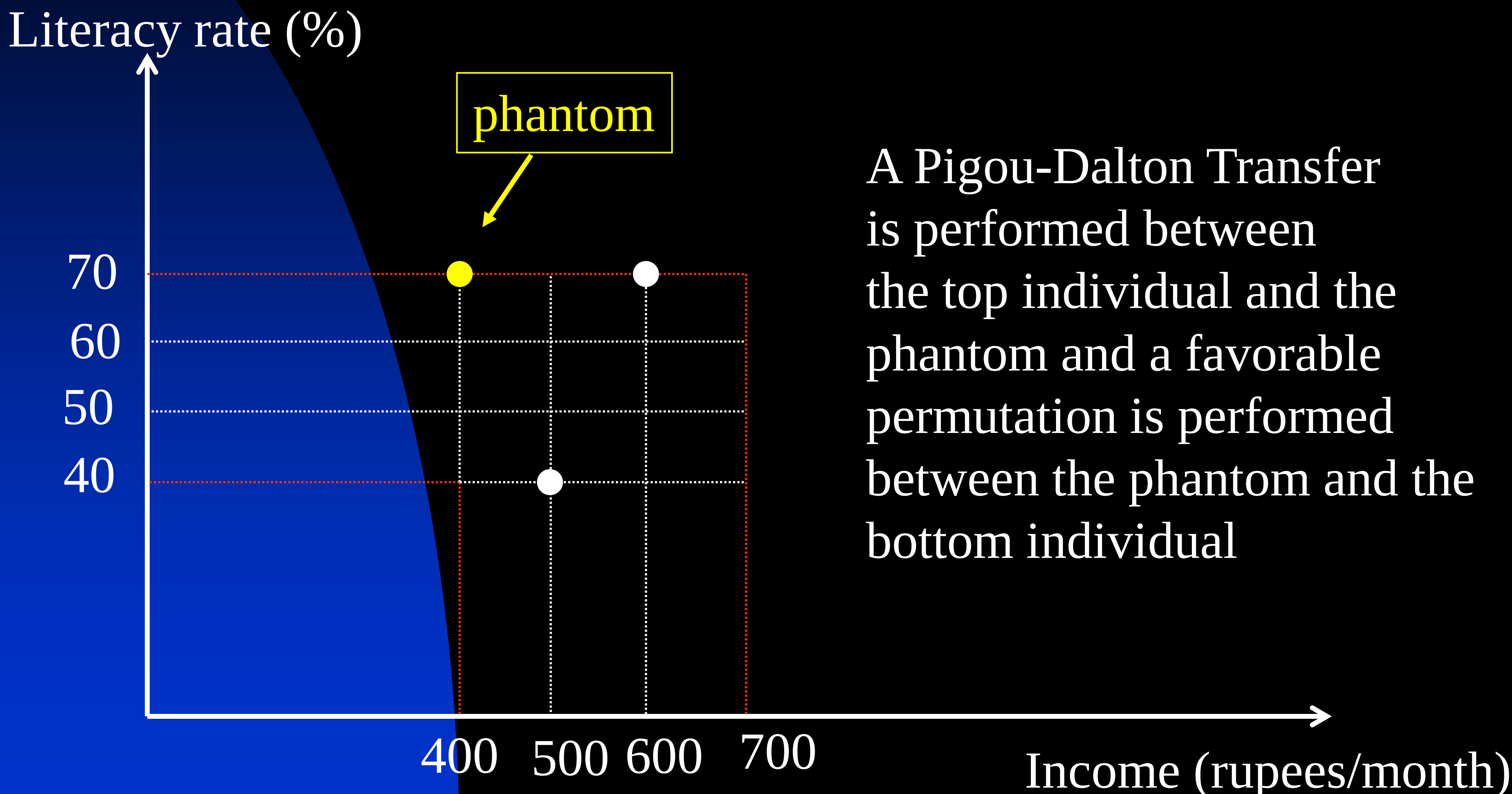
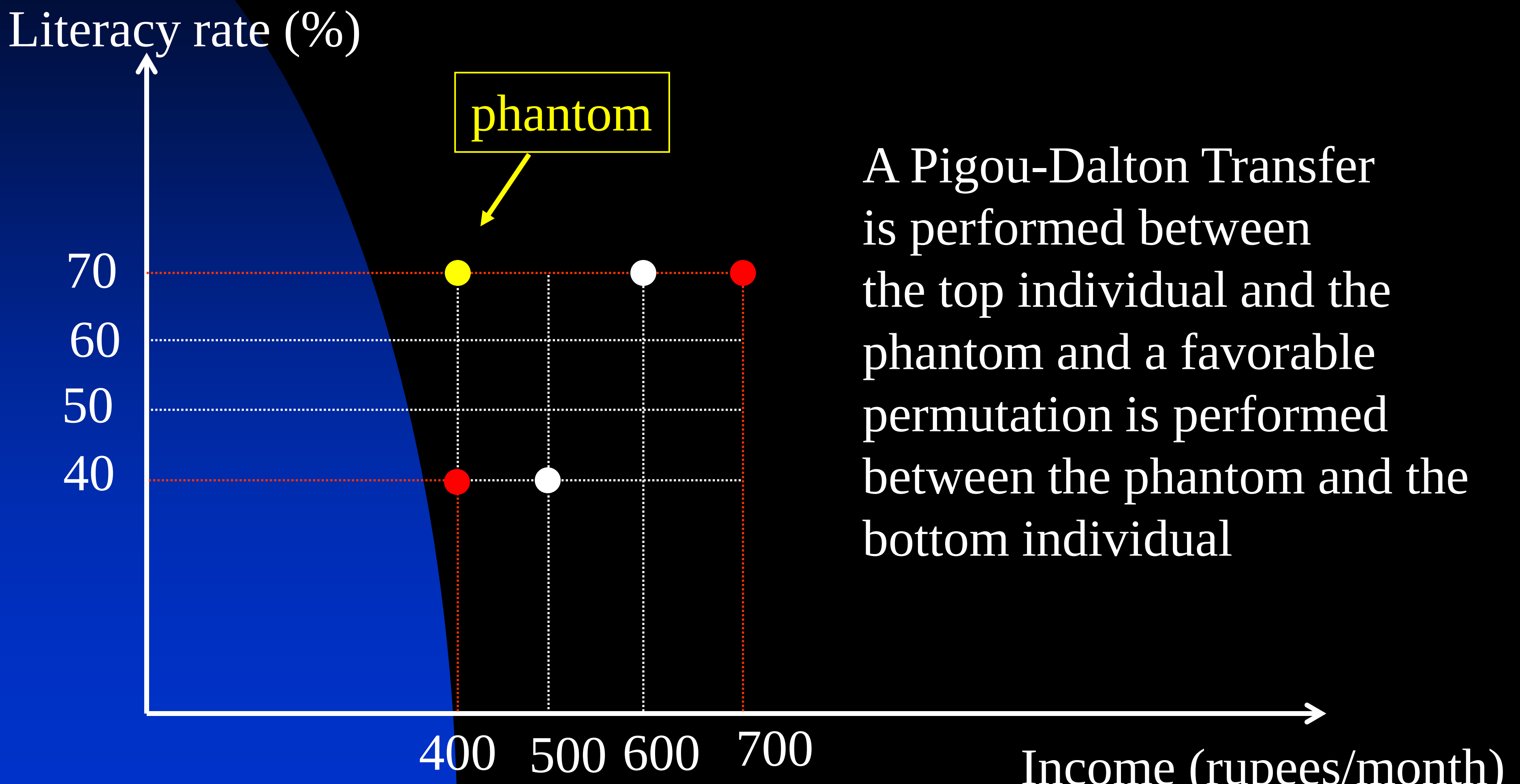
Decomposing a BTPT with a phantom ?



A Pigou-Dalton Transfer is performed between the top individual and the phantom and a favorable permutation is performed between the phantom and the bottom individual

Decomposing a BTPT with a phantom ?



A Pigou-Dalton Transfer is performed between the top individual and the phantom and a favorable permutation is performed between the phantom and the bottom individual

Implementable criteria: robust poverty evaluation.

- How can one define poverty in a multi dimensional setting ?
- Any poverty evaluation requires two steps:
- 1 a criterion for identifying the poor.
- 2 A numerical evaluation of poverty given this identification (fraction of poor, etc.)
- What is not completely obvious in the multiple dimensional setting is step 1.
- A basic feature of any criterion for identifying the population of the poor: If someone is poor, than anyone with less attribute is also poor.
- In the one dimensional setting, the sets of individuals whose income is below some number (poverty line) are the only sets of individuals that satisfy this property.

Answer no 3: implementable criteria (1)

- A possible set of poor in the (two-dimensional) setting is any finite set $S \subset \mathbb{N}^2_+$ of attributes bundles such that if $(s_1,s_2) \in S$ and $(s_1',s_2') \leq (s_1,s_2)$, then $(s_1',s_2') \in S$. (comprensive sets)
- 1st order answer 1) When the number of individuals who are poor is lower in $(\boldsymbol{x};\boldsymbol{a})$ than in $(\boldsymbol{x}';\boldsymbol{a}')$ for any possible set of poor S.

Answer no 3: implementable criteria (2)

- 1st order (bis): when the number of individuals that are simultaneously poor the two dimensions is lower in ($\boldsymbol{x}$;$\boldsymbol{a}$) than in ($\boldsymbol{x}$';$\boldsymbol{a}$') for any pair of poverty lines (one such line for every attribute) (intersection definition of poverty)
- 1st order (ter): when, given any pair of poverty lines (one such line for every attribute) the number of individuals that are poor in at least one dimension is lower in ($\boldsymbol{x}$;$\boldsymbol{a}$) than in ($\boldsymbol{x}$';$\boldsymbol{a}$'). (union definition of poverty)

Answer no 3: implementable criteria (3)

- 2nd order: when the poverty gap in attribute 1 is lower in (x;a) than in (x';a') for all poverty lines that are non-increasing with respect to attribute 2 (Bourguignon 1989).

$$\sum_{i=1}^{n} \max(z(a_i) - x_i, 0) \leq \sum_{i=1}^{n} \max(z(a_i^{'}) - x_i^{'}, 0)$$

$$\text{for all non-increasing functions } z$$

Theorem 1: (Østerdal (2010) & Gravel & Moyes (2013): the following statements are equivalent:

- (x,a) welfare (or utilitarian) dominates (x',a') y for the class $\mathcal{U}_o$.
- One can go from (x',a') to (x,a) by a finite sequence of increments of income and/or abilities.
- For any comprehensive set of poor $S \subset \mathbb{N}^2_+$ the number of individual who are in this set is smaller in $(\boldsymbol{x},\boldsymbol{a})$ than in $(\boldsymbol{x}',\boldsymbol{a}')$.

Theorem 2. (Gravel & Moyes (2012) the following statements are equivalent:

- (x,a) welfare (or utilitarian) dominates (x',a') for the class $\mathcal{U}^{-}_{1}$.
- One can go from (x',a') to (x,a) by a finite sequence of favorable permutations and/or increment of income and/or ability and/or permutation of individual situations.
- There is a lower number of individuals who are simultaneously poor in the two attributes in (x,a) than in (x',a') for all pair of poverty lines.

Theorem 2: the following statements are equivalent

- (x,a) welfare (or utilitarian) dominates (x',a') for the class $\mathcal{U}_2$.
- One can go from (x',a') augmented by a distribution of the two attributes between a finite population of phantoms to (x,a) augmented by the same distribution of attributes among the same population of phantoms by a finite sequence of favorable permutations and/or Pigou-Dalton Transfers of attribute 1 and/or increments of income and/or ability and/or permutation of individual situations.
- (x,a) dominates (x',a') for the Bourguignon criterion.

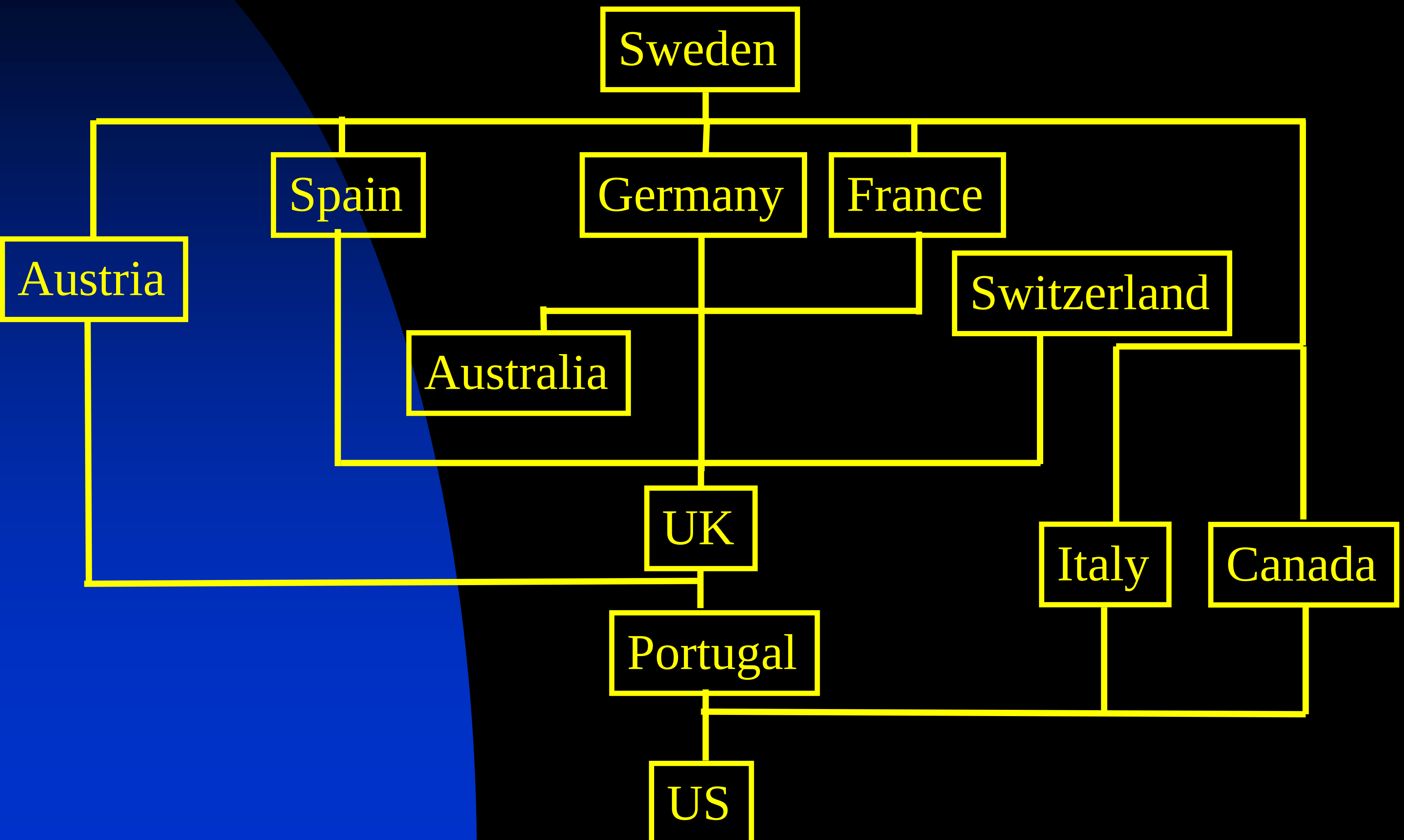
These results generalize easily

- To distributions involving different numbers of individuals (Dalton principle of population), if utilitarianism is replaced by average utilitarianism.

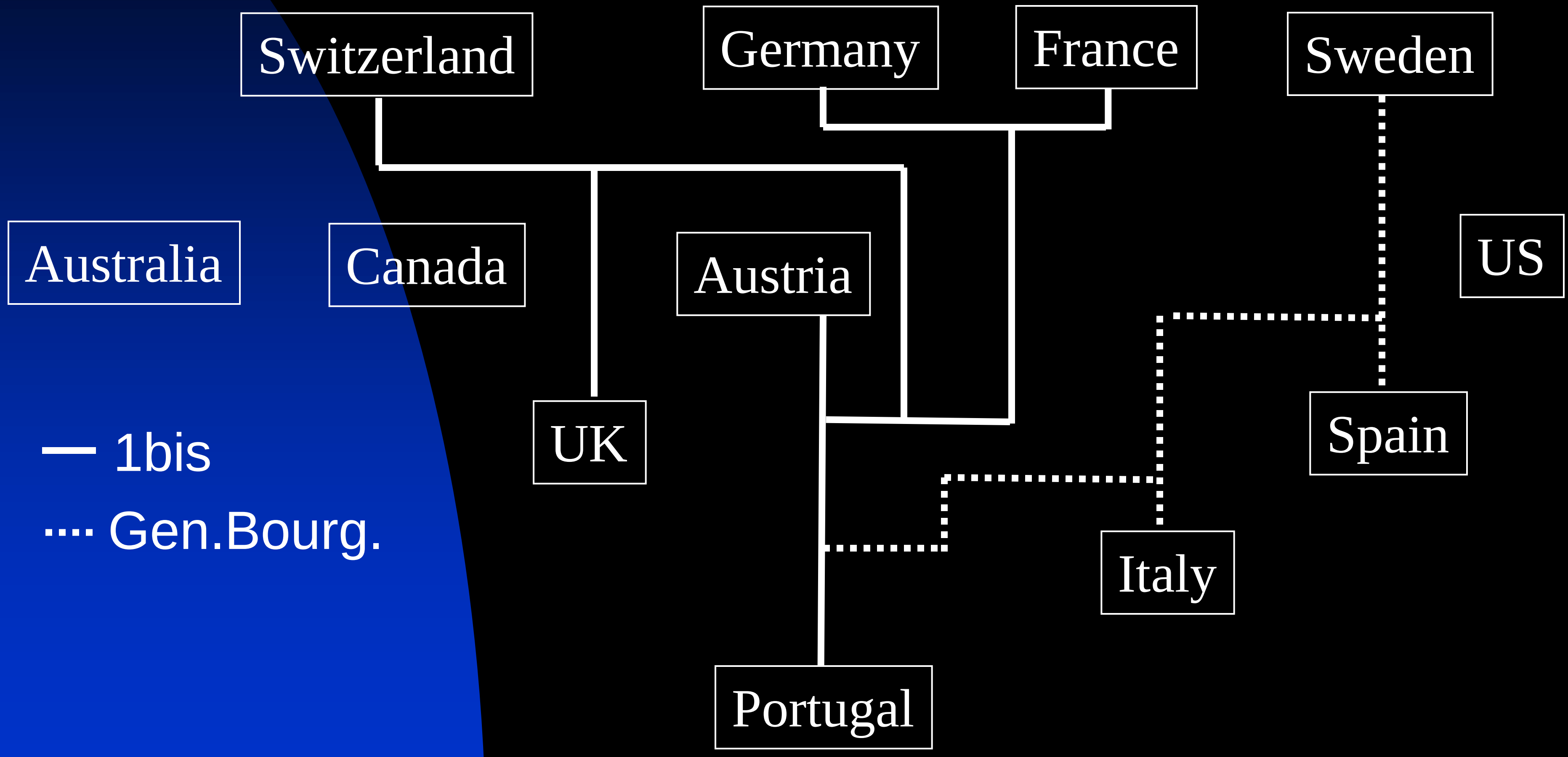
Illustration: OECD countries

- Household disposable income (as before)
- infant mortality rate (number of death before the age of 1 by thousand of birth) in the region of residence of the households (interpreted as an ordinal index of quality of the health service)
- To make things comparable between countries of different sizes, we have chosen large regions (not more than five or six per country).
- Here is the picture.

Robust ranking of OECD countries by infant mortality (1st order)



Two dimensional dominance



k-dimensional normative dominance

- k attributes.
- n individuals.
- Distribution: $x = (x_1^1, x_2^1, \ldots, x_k^1, \ldots, x_1^n, \ldots, x_k^n) \in N_+^{nk}$
- x^i_j = amount of attribute j accruing to household i
- **Q**: When are we « sure » that a distribution of k attributes y is normatively better than a corresponding distribution z ?
- **A**: (first order 1: Osderdal (2010) theorem is valid for any number of dimensions)

k-dimensional normative dominance: 2 equivalent « 1st order » answers

- (1) when y Utilitarian dominates z for the class C^1_- of all selfish and identical utility functions satisfying: $(-1)^{\#H}\, U_{h_1 h_2 \ldots h_{\#H}}(a) \leq 0$, for every $a \in \mathbb{R}_+^k$ and $H = \{h_1, \ldots, h_{\#H}\} \subseteq \{1, \ldots, k\}$
- (2) when the **number (fraction) of individuals who are simultaneously poor in all attributes** is lower in y than in z for all vectors of poverty lines (one line for every attribute).

Remarks on this equivalence (1)

- Established in Gravel & Mukhopadhyay (2010) for k attributes.
- Hadar and Russel (1974) provides one direction of it for the general k dimensional case, Atkinson and Bourguignon (1982) provides also the same direction for the 2-dimensional case.
- N.B. Utilitarian dominance is equivalent to welfarist one (for the class $\mathbb{W}$ of symmetric and increasing social welfare function).
- Elementary transformations ? (some insight from a combination of Decancq (2012), Osterdal (2010))

Remarks on this equivalence (2)

- Alternative poverty criterion: Union: you are poor if you are below at least one poverty line.
- Conjecture in the k dimensional case (proved in the two dimensional case in Levy & Paroush (1974) and (in one direction) in Atkinson & Bourguignon (1982)): The two following statements are equivalent:
- (1) y Utilitarian dominates z for the class C^1_+ of all selfish and identical utility functions satisfying: $U_{h_1 h_2 \ldots h_{\#H}}(a) \geq 0$, for every $a \in \mathbb{R}_+^k$ and $H = \{h_1, \ldots, h_{\#H}\} \subseteq \{1, \ldots, k\}$
- (2) The **number (fraction) of individuals who are simultaneously poor in at least one attribute** is lower in y than in z for all vectors of poverty lines (one line for every attribute).

k-dimensional normative dominance: 2 equivalent « 2nd order » answers

- (1) when y Utilitarian dominates z for the class C^2_- of all selfish and identical utility functions in C^1_- who satisfy(in addition): $(-1)^{\#H\cup J}U_{h_1\ldots h_{\#H}\, j_1\ldots j_{\#J}}(a) \leq 0$, for every $a \in \mathbb{R}_+^k$, $H = \{h_1,\ldots,h_{\#H}\} \subseteq \{1,\ldots,k\}$ and $J = \{j_1,\ldots,j_{\#J}\} \subseteq \{1,\ldots,k\}$ such that $H \cap J = \emptyset$.
- (2) when, for all subsets of attributes, the **product of poverty gaps in all attributes in the subset** is lower in y than in z for every vector of poverty lines (one line per attribute).

Remarks on this equivalence

- Established in Gravel & Mukhopadhyay (2010) for k attributes.
- Hadar & Russel (1974) provides one direction of it for the general k dimensional case, Atkinson and Bourguignon (1982) provides also the same direction for the 2-dimensional case and Anderson (JEI, 2008) has provided the proof for one direction in the three –dimensional case.
- This (truly ?) 2nd order dominance requires of course cardinally meaningful measure of each attribute.
- N.B. No coincidence of welfarist and utilitarian dominance here.

Empirical illustration: India (Gravel & Mukhopadhyay)

- Purpose: Appraising the impact, on the distribution of well-being, of the spectacular growth episode that has taken place in India since the end of the eighties
- Claim that the growth has been accompanied by a raise in inequalities

Data (1)

- NSS data on households consumption expenditures and district of residence (rounds 43 (1987-1988), 52 (1995-1996) and 58 (2001-2002)).
- Census data on literacy (fraction of the district population above 7 years old who is literate) for the years 1981, 1991 and 2001).
- Census data (for the same census years) on district under 5 infant mortality (number of children who die before the age of 5 per thoushand births). Rates are calculated by the International Institute for Population Science.
- National Crime Record Bureau for district data on crime (number of murders, attempted murders and rapes per million individuals) (years 1988, 1996 and 2002).

Data (2)

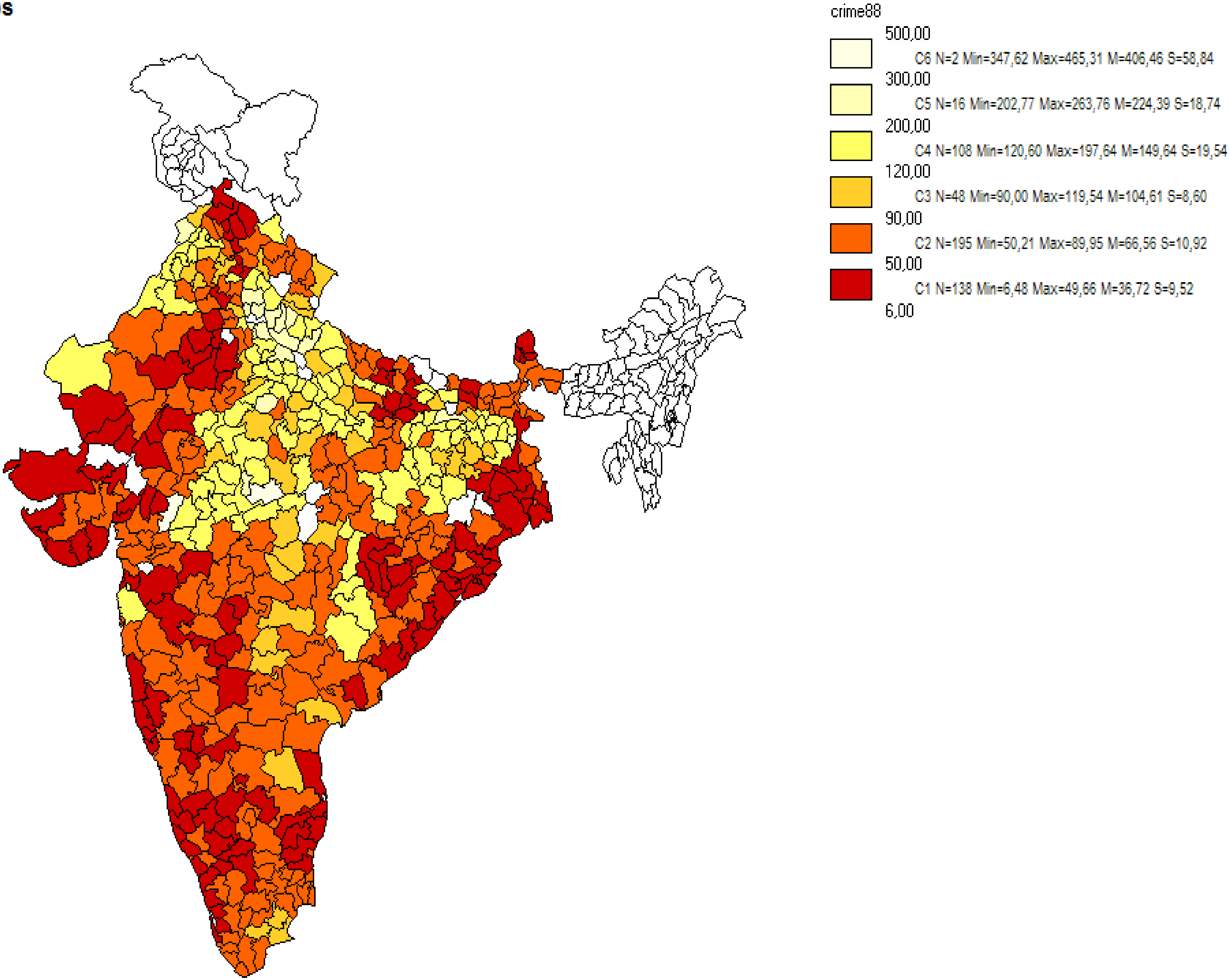
- Households figures have been converted into individual one using OECD equivalence scales (square root of household size).
- Consumption is expressed in 2002 rupees (Urban non-manual employees price index for urban, agricultural labourers price index for rural, Deaton (2005) Fisher price index for comparison\pooling urban-rural)

Empirical implementation

- Statistical inference is performed using Bishop and Fornby (1999) Union-Intersection approach
- Statistical inference is performed on a discretization of the interval of observed values of consumption based on the median value of each 100 rupees subinterval (rounding off to the nearest percentile has been done for district variables)
- Involves the verification of about 180 000 inequalities!!

Some geographical trends

Maps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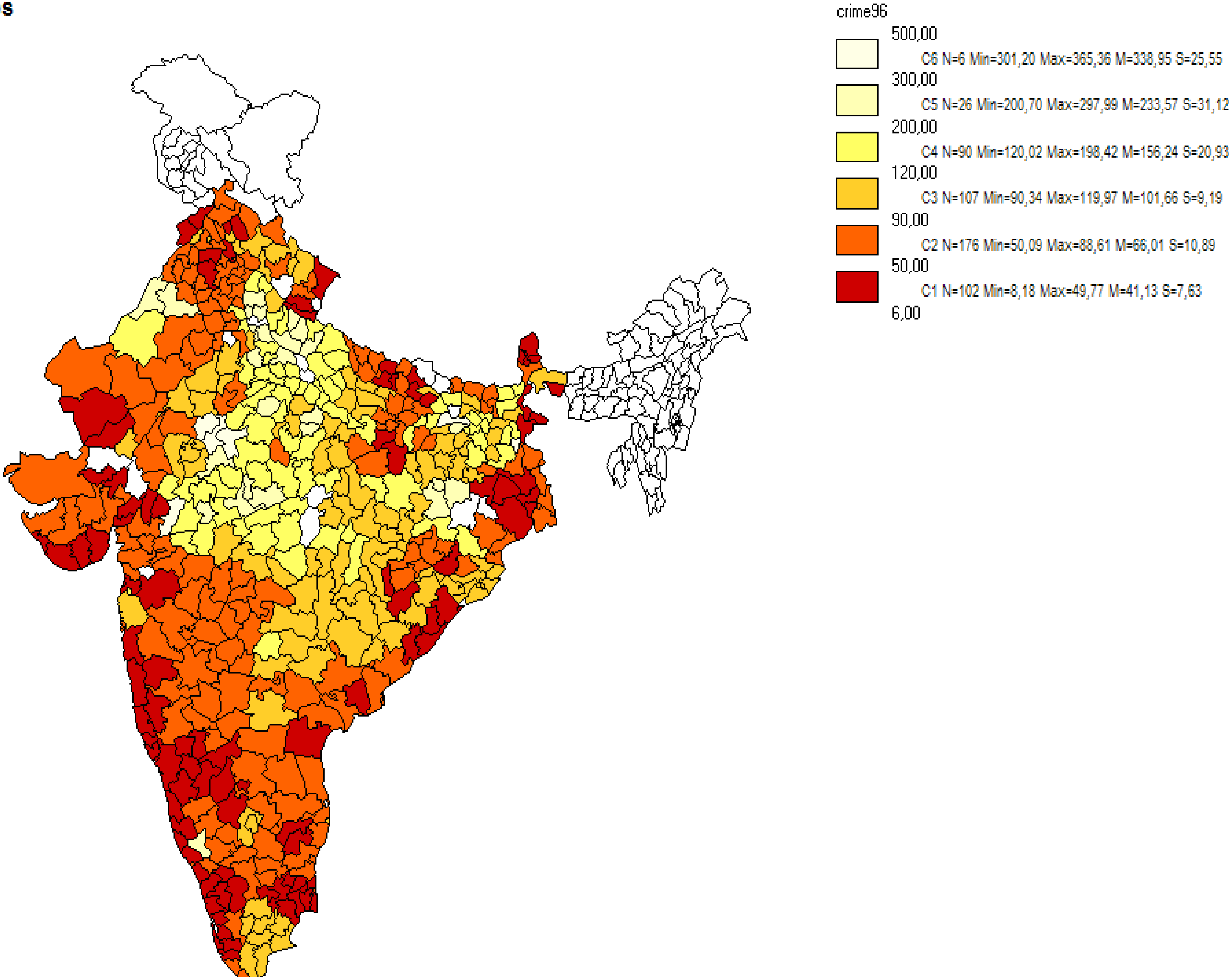
500,00
C6 N=2 Min=347,62 Max=465,31 M=406,46 S=58,84
300,00
C5 N=16 Min=202,77 Max=263,76 M=224,39 S=18,74
200,00
C4 N=108 Min=120,60 Max=197,64 M=149,64 S=19,54
120,00
C3 N=48 Min=90,00 Max=119,54 M=104,61 S=8,60
90,00
C2 N=195 Min=50,21 Max=89,95 M=66,56 S=10,92
50,00
C1 N=138 Min=6,48 Max=49,66 M=36,72 S=9,52
6,00

Maps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300,00
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200,00
C4 N=90 Min=120,02 Max=198,42 M=156,24 S=20,93
120,00
C3 N=107 Min=90,34 Max=119,97 M=101,66 S=9,19
90,00
C2 N=176 Min=50,09 Max=88,61 M=66,01 S=10,89
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C1 N=102 Min=8,18 Max=49,77 M=41,13 S=7,63
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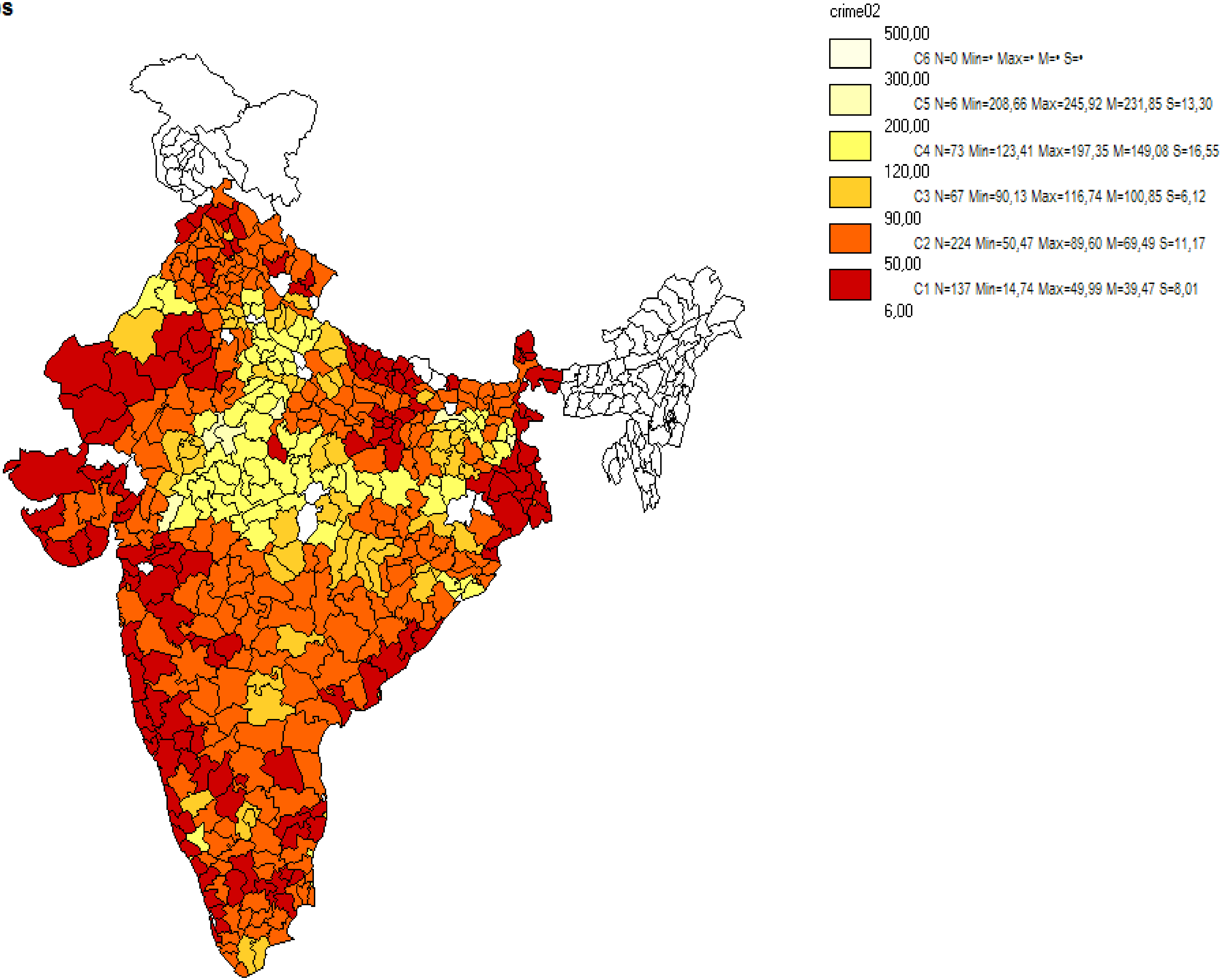
Maps


crime02
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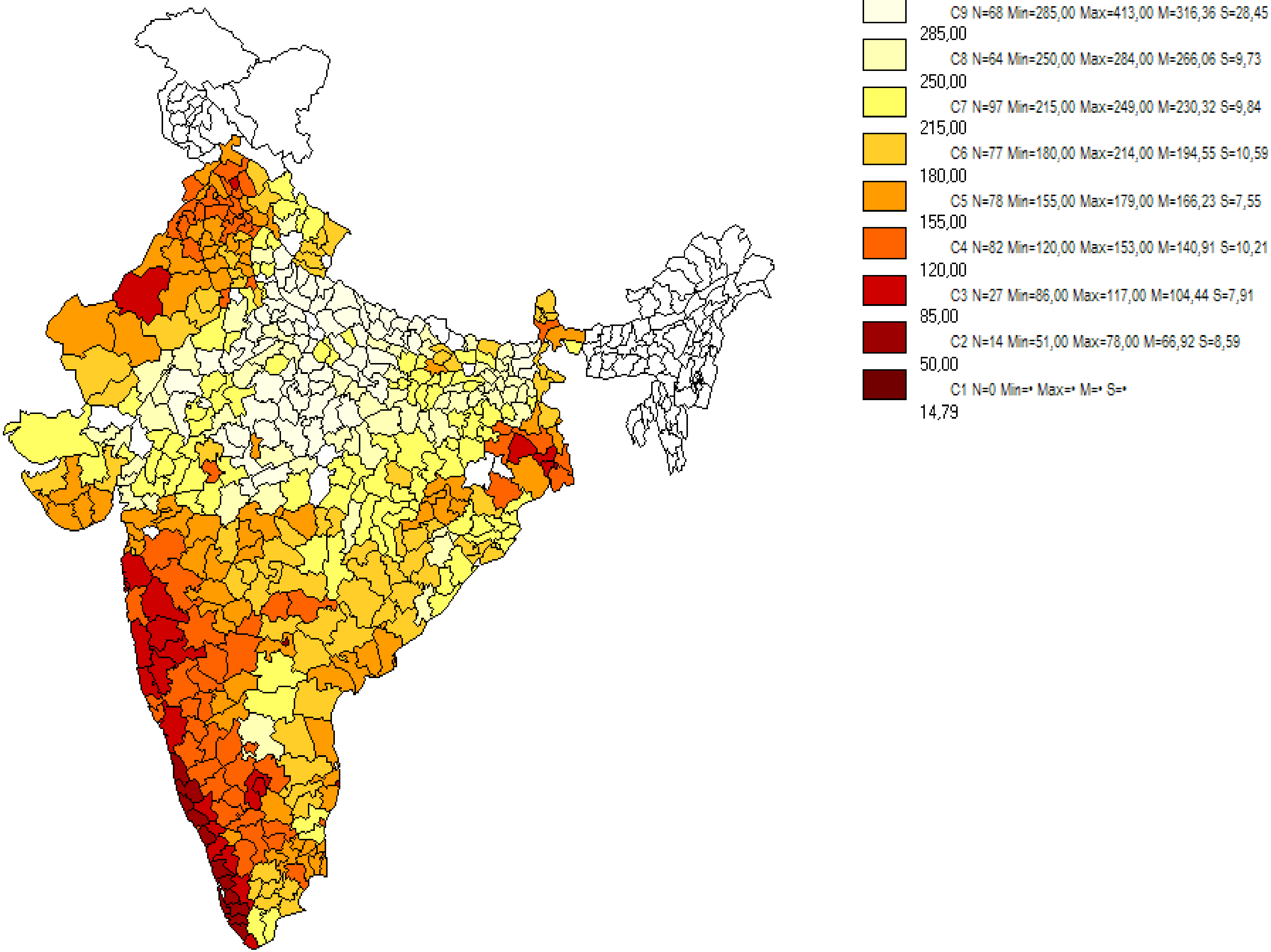
C4 N=73 Min=123,41 Max=197,35 M=149,08 S=16,55
120,00
C3 N=67 Min=90,13 Max=116,74 M=100,85 S=6,12
90,00
C2 N=224 Min=50,47 Max=89,60 M=69,49 S=11,17
50,00
C1 N=137 Min=14,74 Max=49,99 M=39,47 S=8,01
6,00

Maps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250,00
C7 N=97 Min=215,00 Max=249,00 M=230,32 S=9,84
215,00
C6 N=77 Min=180,00 Max=214,00 M=194,55 S=10,59
180,00
C5 N=78 Min=155,00 Max=179,00 M=166,23 S=7,55
155,00
C4 N=82 Min=120,00 Max=153,00 M=140,91 S=10,21
120,00
C3 N=27 Min=86,00 Max=117,00 M=104,44 S=7,91
85,00
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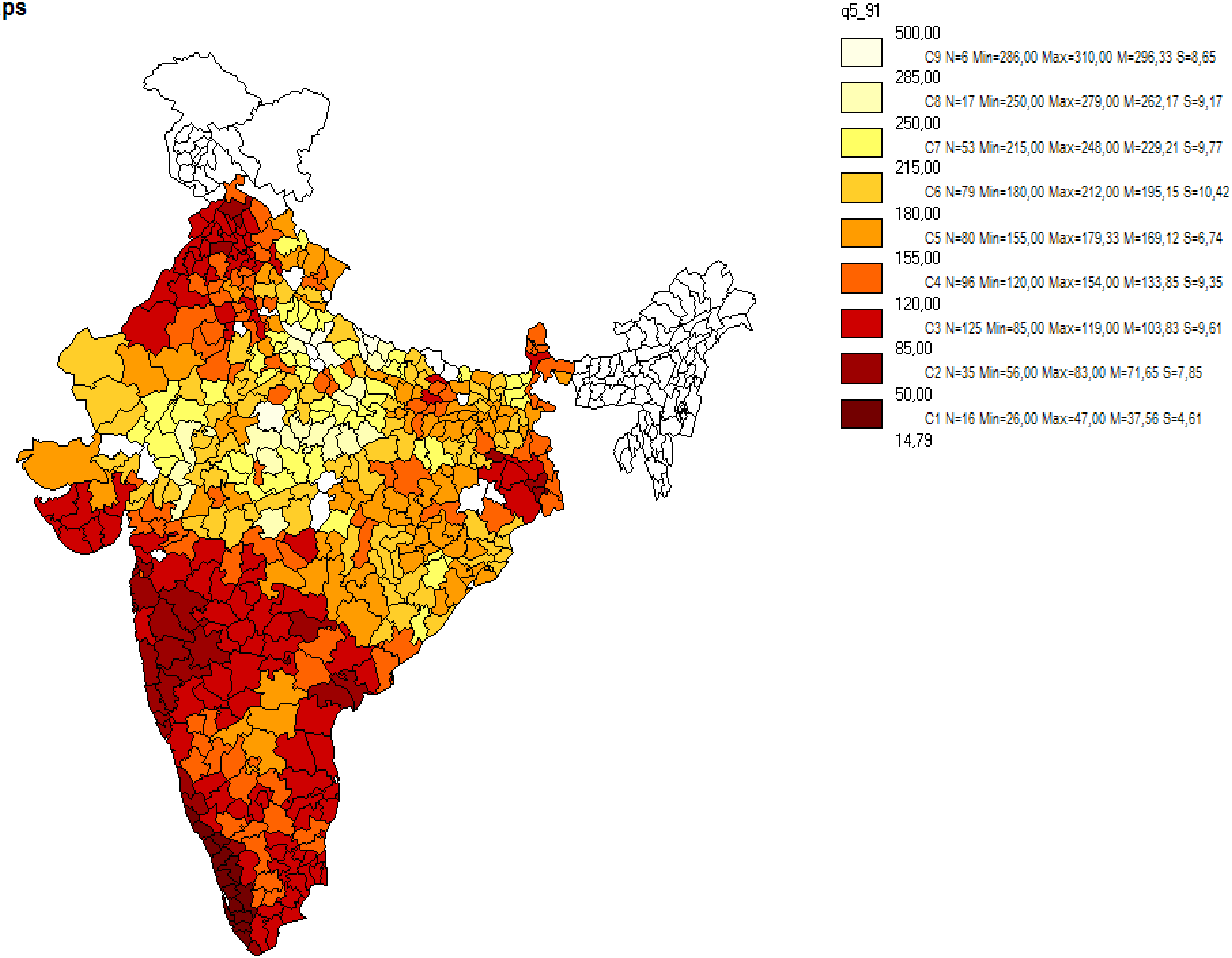
50,00
C1 N=0 Min=• Max=• M=• S=•
14,79

Maps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285,00
C8 N=17 Min=250,00 Max=279,00 M=262,17 S=9,17
250,00
C7 N=53 Min=215,00 Max=248,00 M=229,21 S=9,77
215,00
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180,00
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155,00
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C3 N=125 Min=85,00 Max=119,00 M=103,83 S=9,61
85,00
C2 N=35 Min=56,00 Max=83,00 M=71,65 S=7,85
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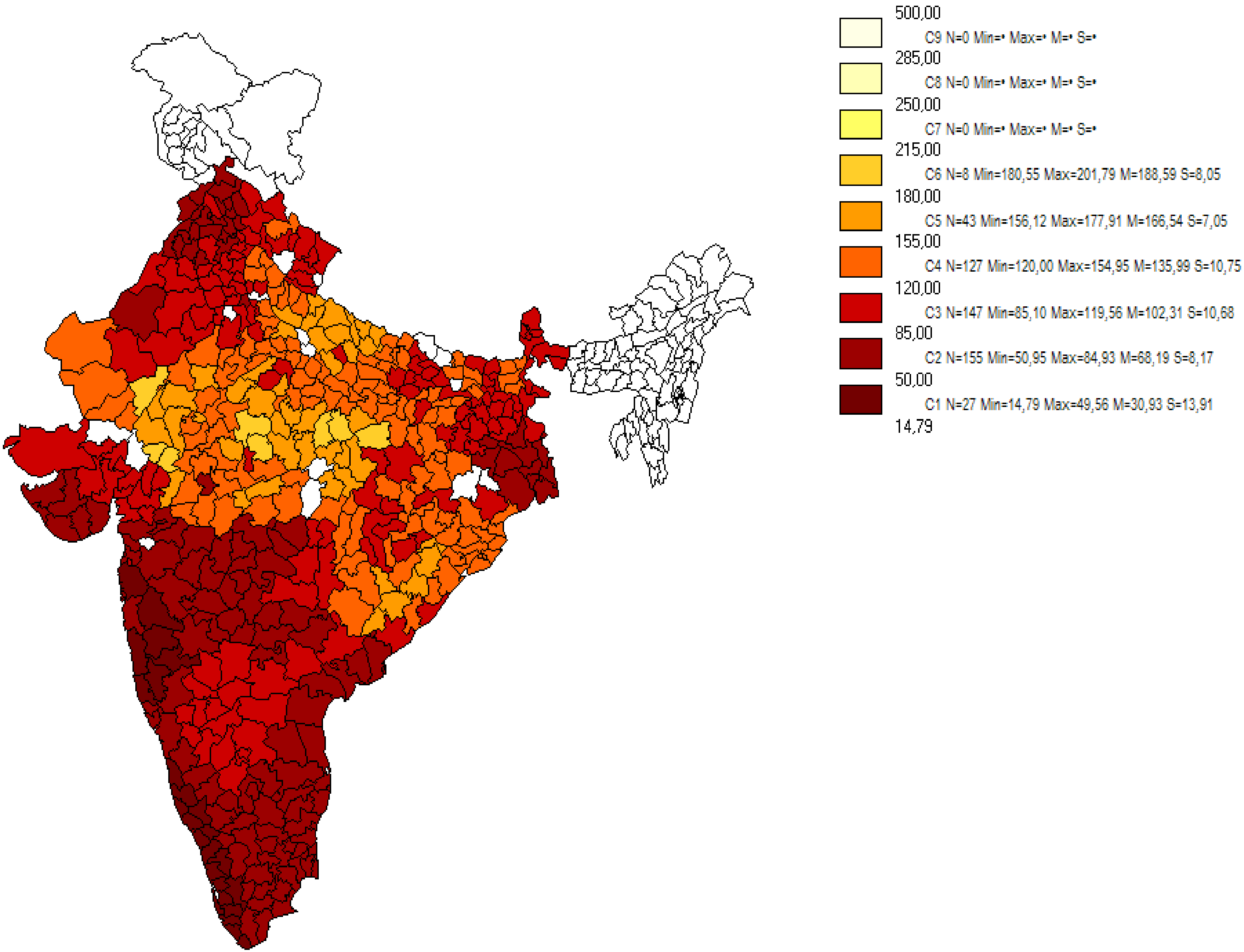
C1 N=16 Min=26,00 Max=47,00 M=37,56 S=4,61
14,79

Maps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C8 N=0 Min=• Max=• M=• S=•
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C6 N=8 Min=180,55 Max=201,79 M=188,59 S=8,05
180,00
C5 N=43 Min=156,12 Max=177,91 M=166,54 S=7,05
155,00
C4 N=127 Min=120,00 Max=154,95 M=135,99 S=10,75
120,00
C3 N=147 Min=85,10 Max=119,56 M=102,31 S=10,68
85,00
C2 N=155 Min=50,95 Max=84,93 M=68,19 S=8,17
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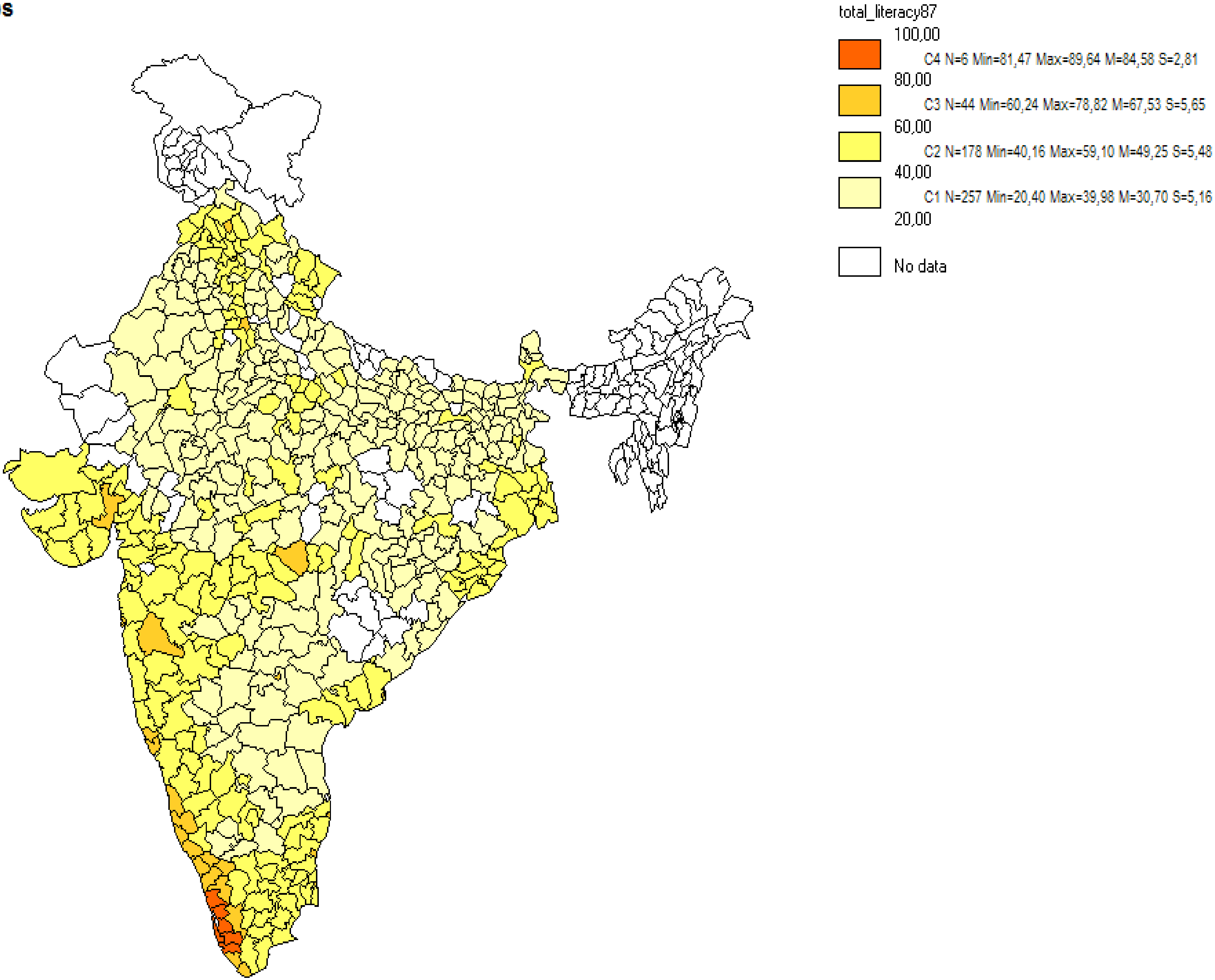
14,79

Maps



total_literacy87
100,00
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60,00
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40,00
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20,00
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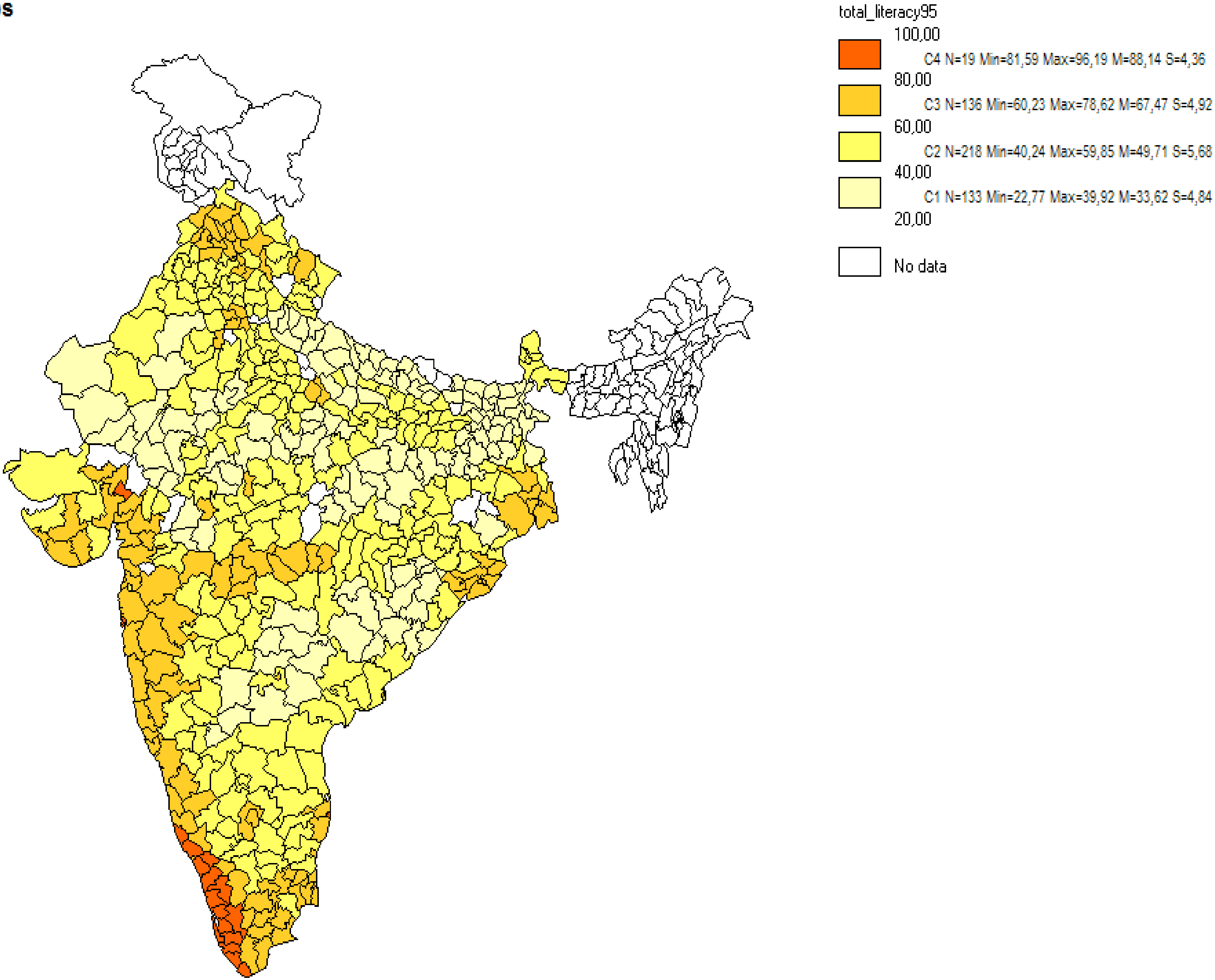
Maps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80,00
C3 N=136 Min=60,23 Max=78,62 M=67,47 S=4,92
60,00
C2 N=218 Min=40,24 Max=59,85 M=49,71 S=5,68
40,00
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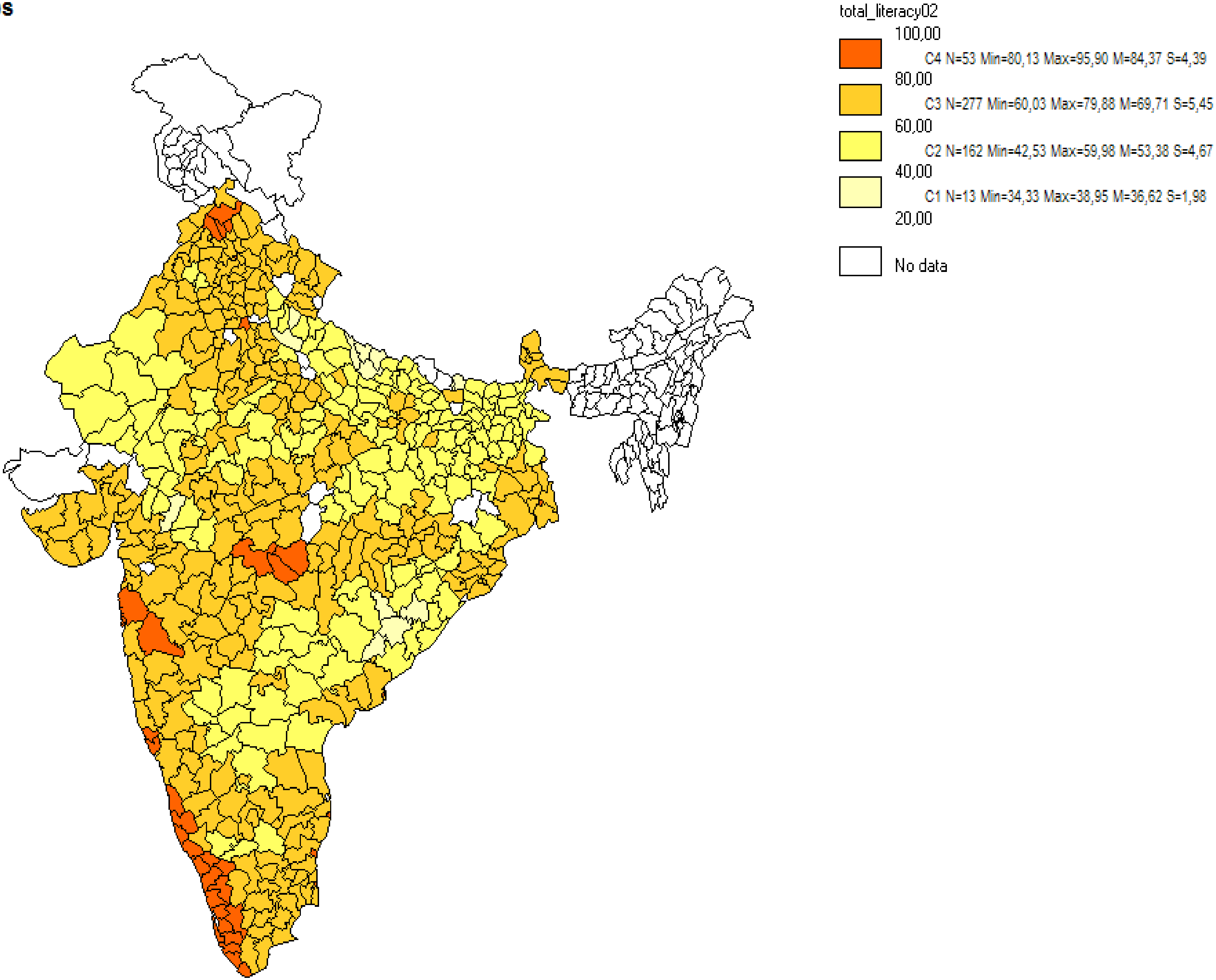
20,00
No data

Maps


total_literacy02
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80,00
C3 N=277 Min=60,03 Max=79,88 M=69,71 S=5,45
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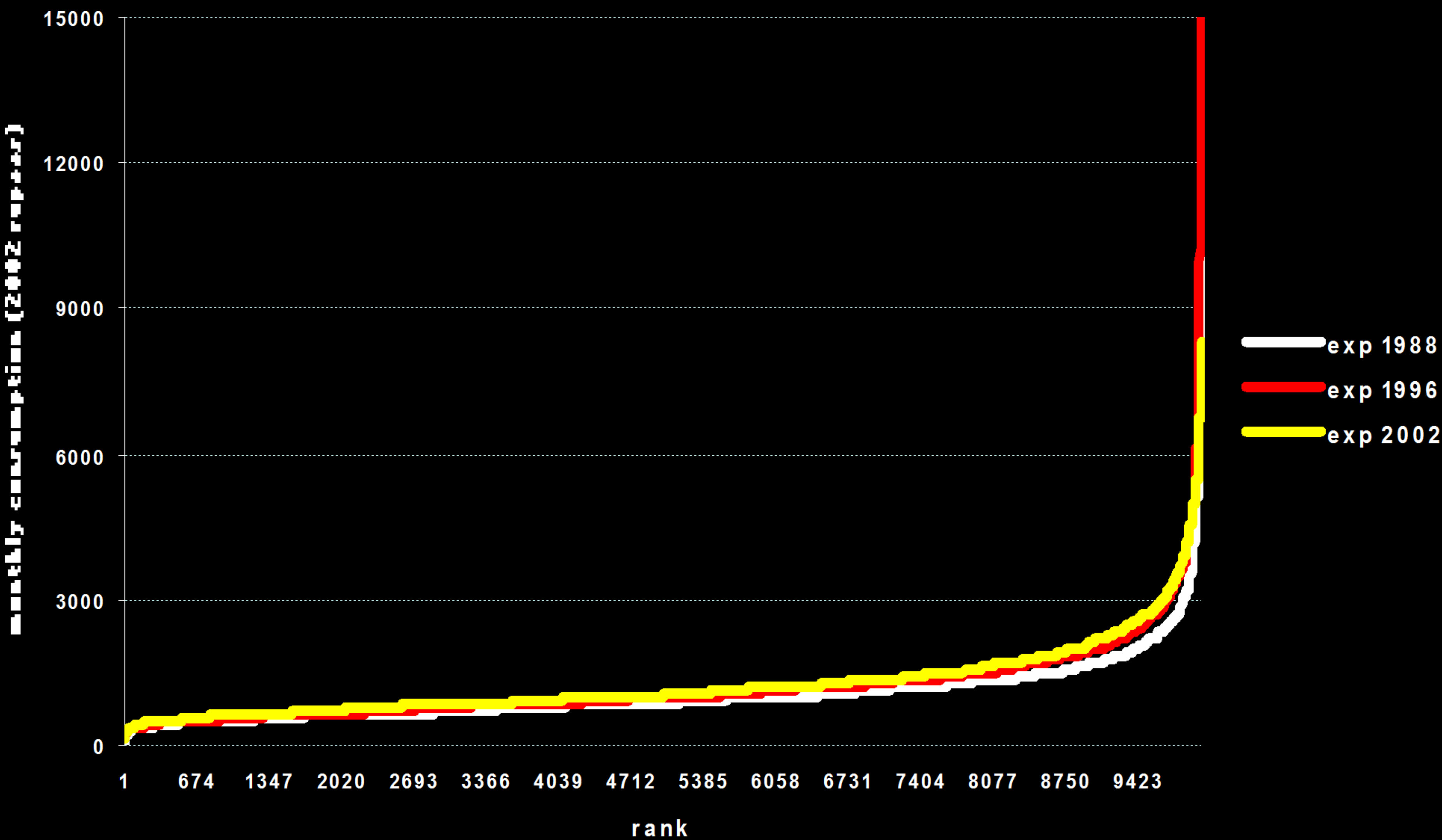
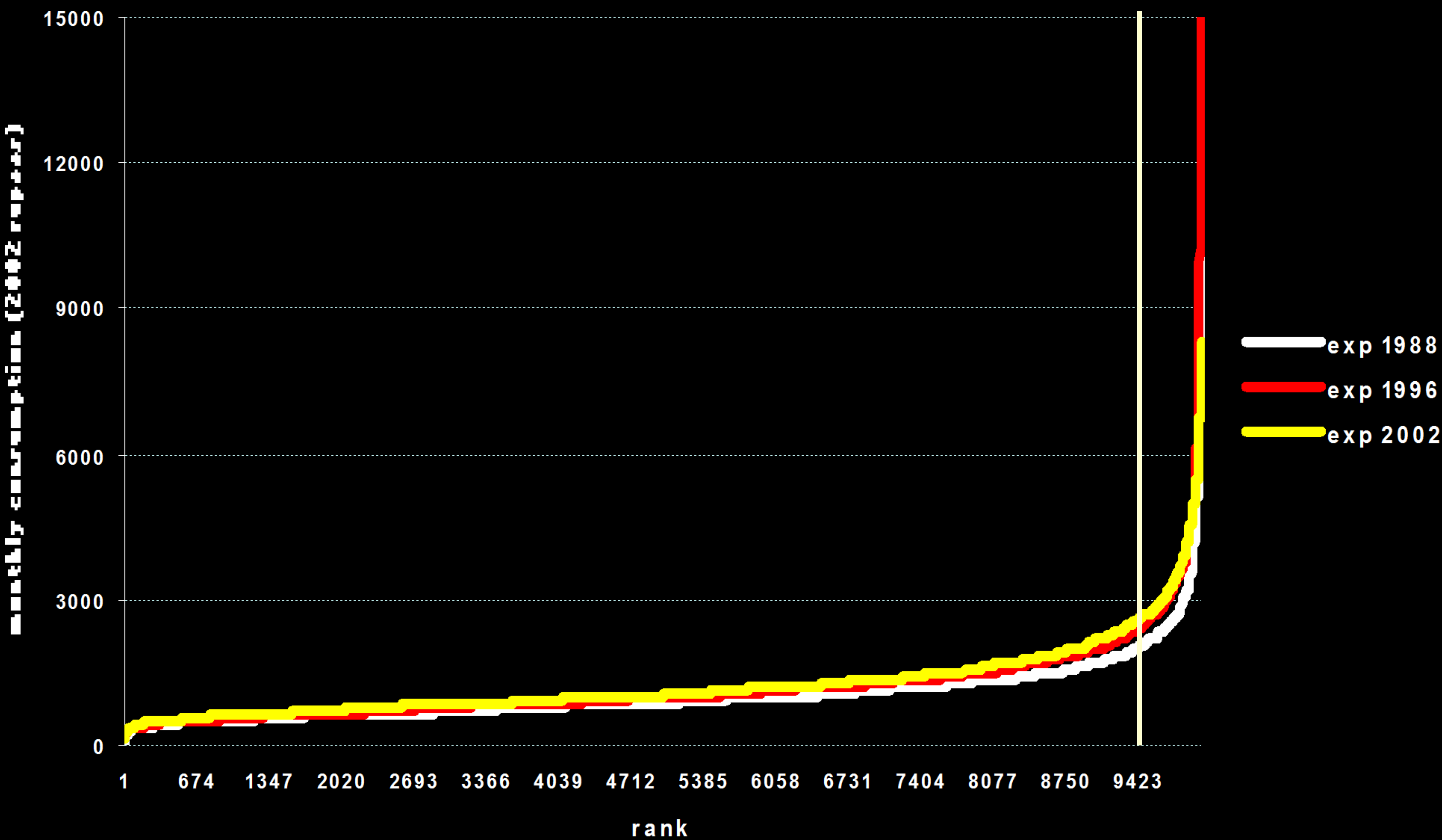
C2 N=162 Min=42,53 Max=59,98 M=53,38 S=4,67
40,00
C1 N=13 Min=34,33 Max=38,95 M=36,62 S=1,98
20,00
No data

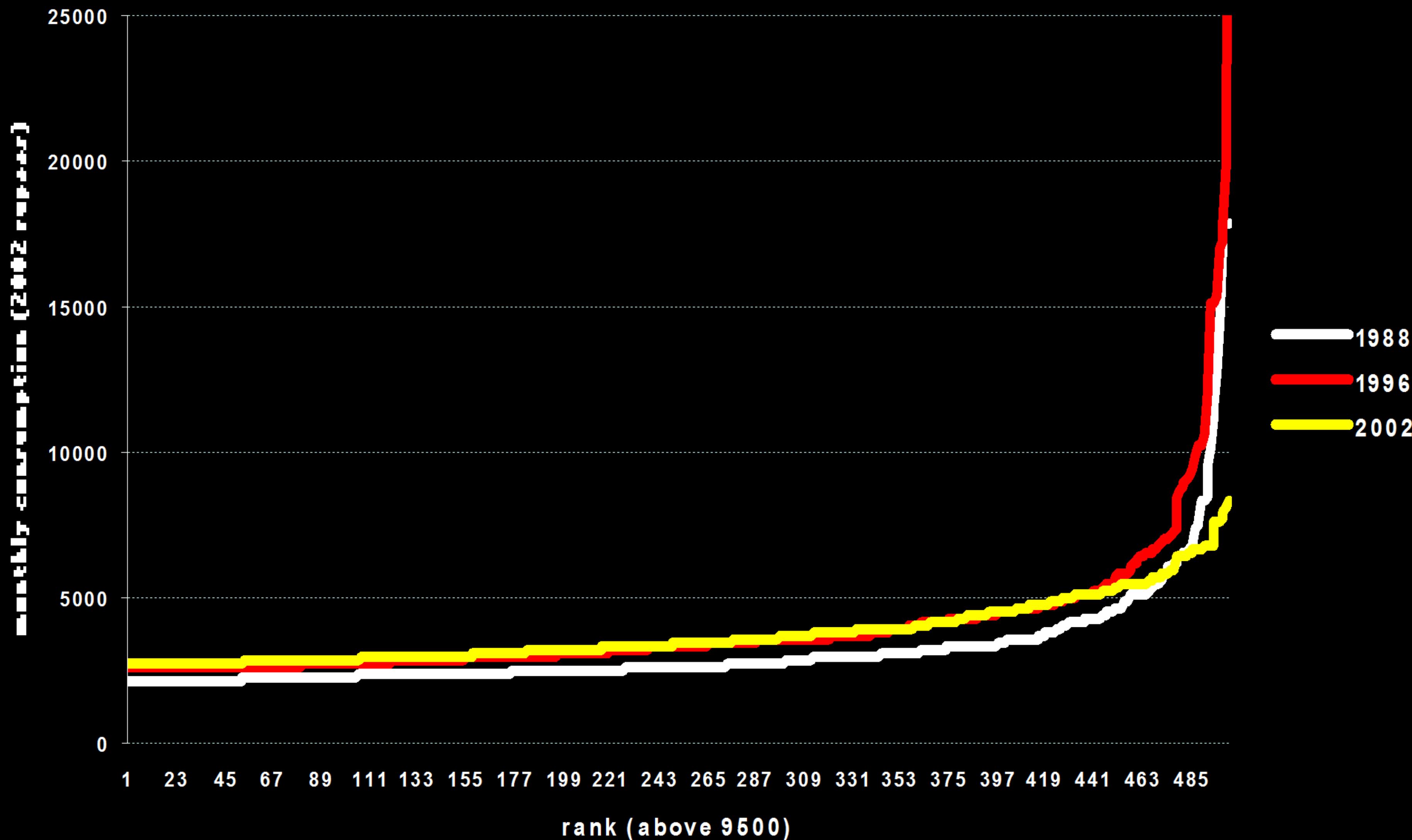
Ordered consumption vectors (all India)



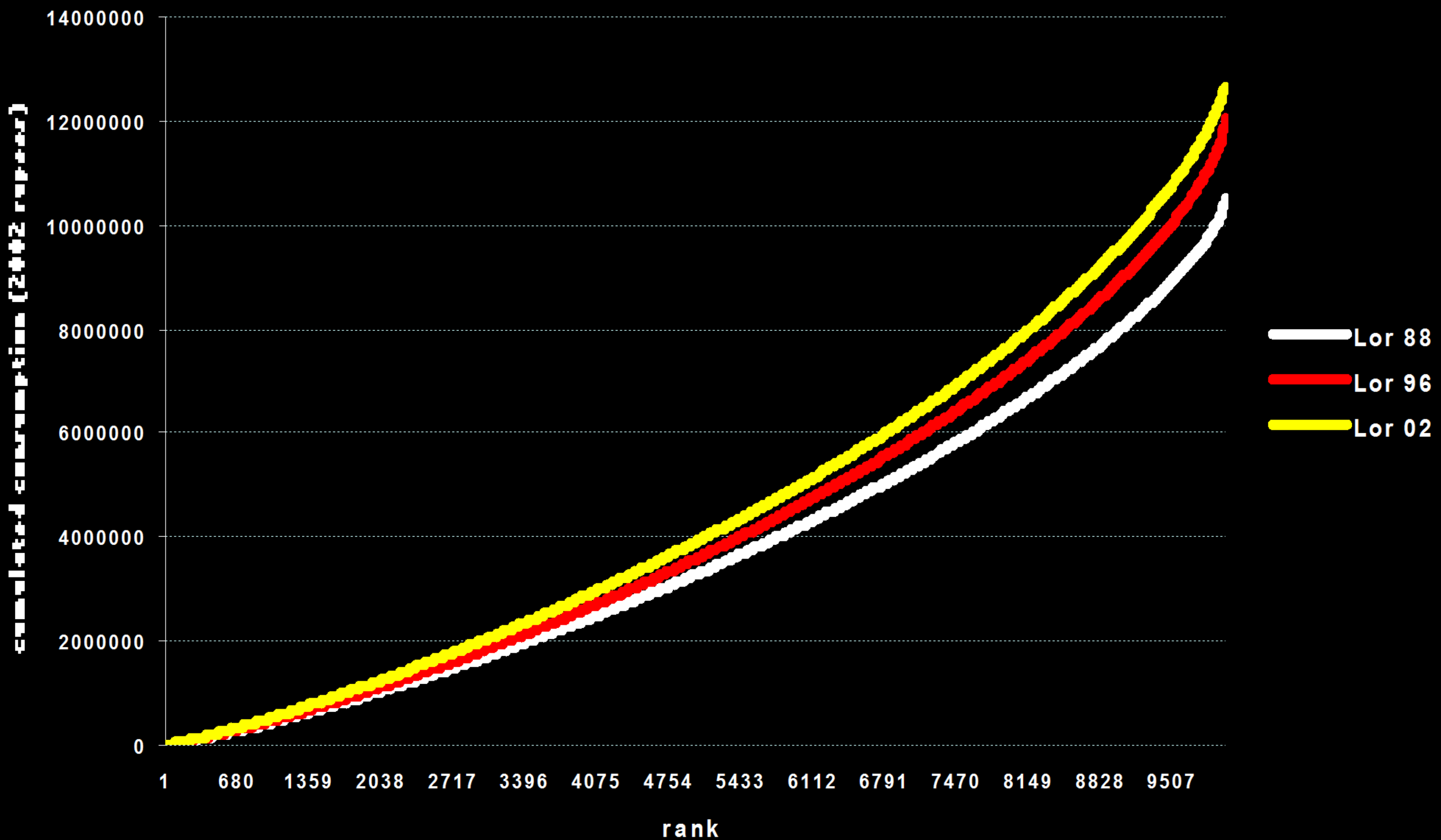
Ordered consumption vectors (all India)



Ordered consumption vectors (upper part)



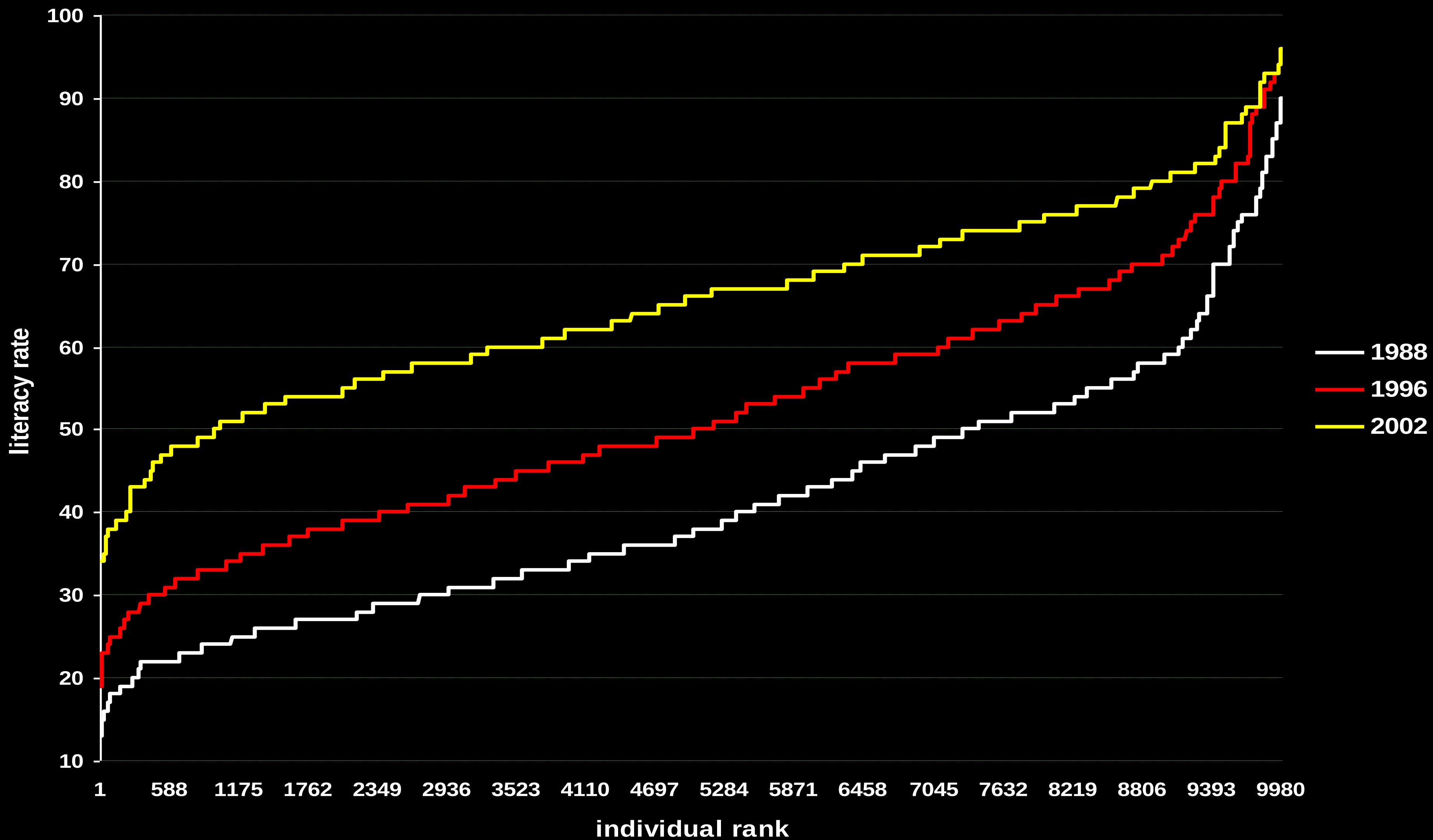
Lorenz curves (all India)



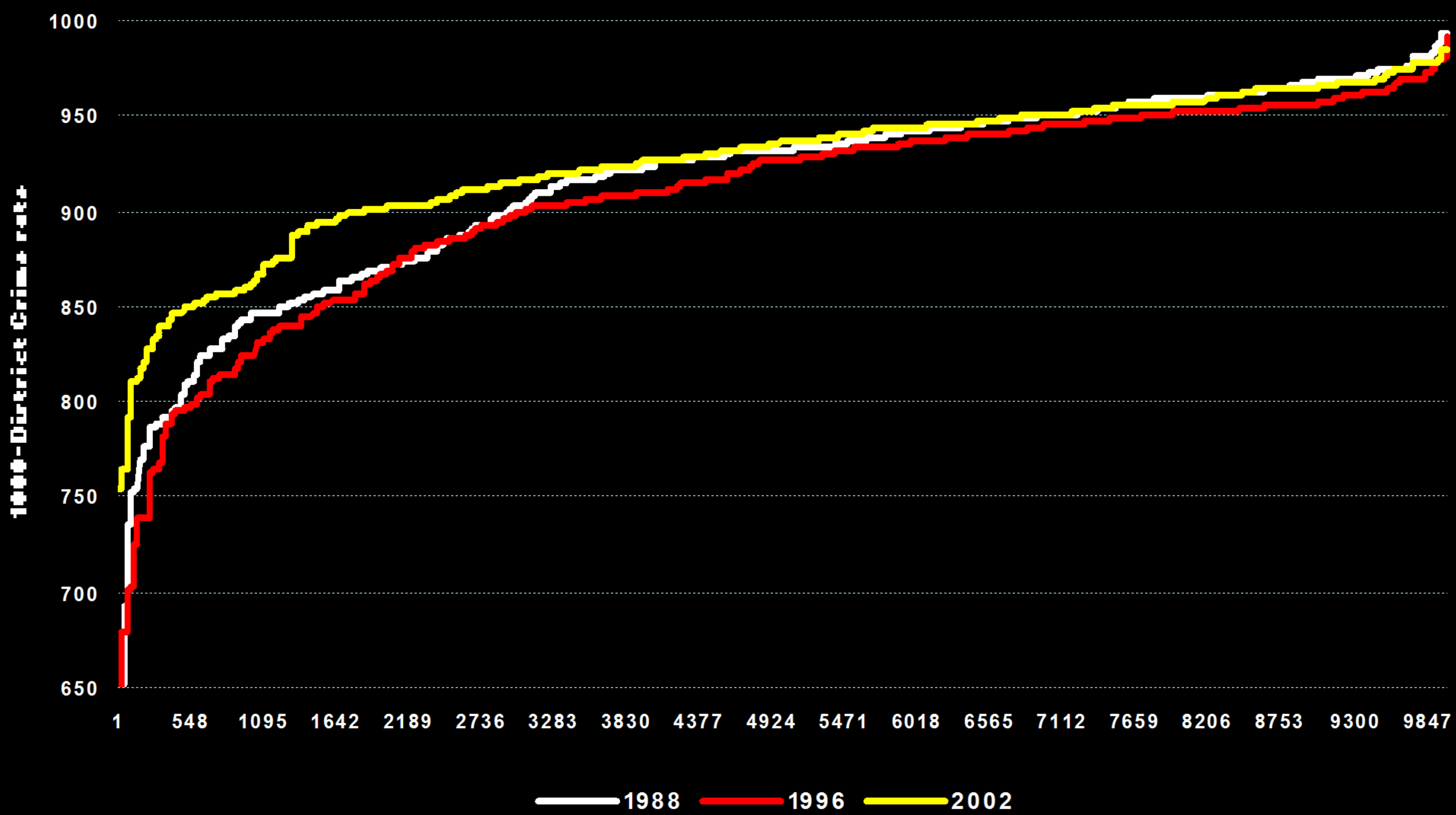
Clear verdict :

- If individual well-being depends upon consumption only, all utility inequality averse welfarist planners would agree to say that India is better off now than in 1996 (or than 1986)
- Poverty gap has gone down in India over the three periods no matter what the definition of the poverty line is (this is true also for headcount poverty for all « reasonable » poverty lines)
- Where do the controversy about increasing inequalities comes from ?

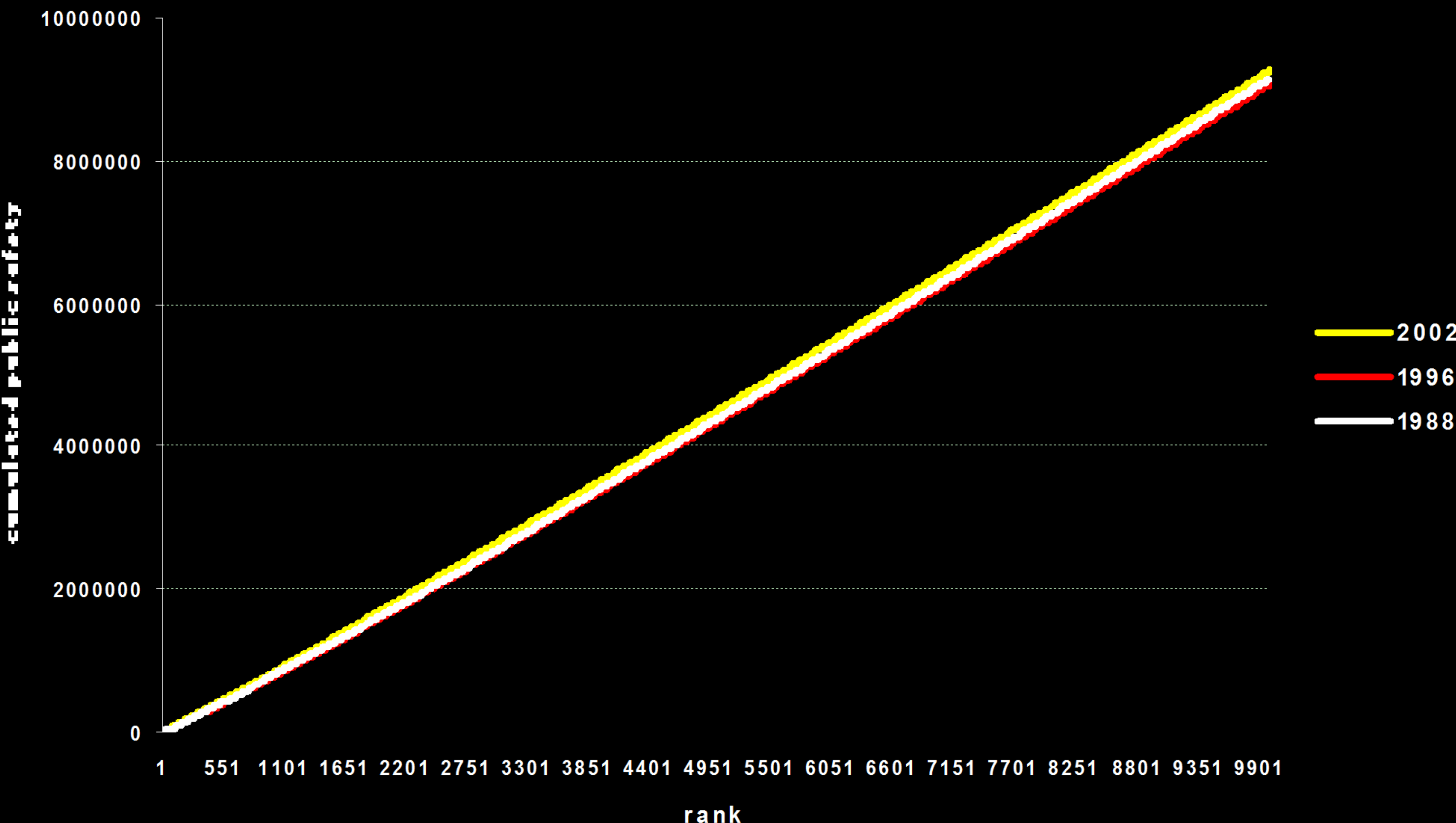
Ordered vectors of district literacy rates



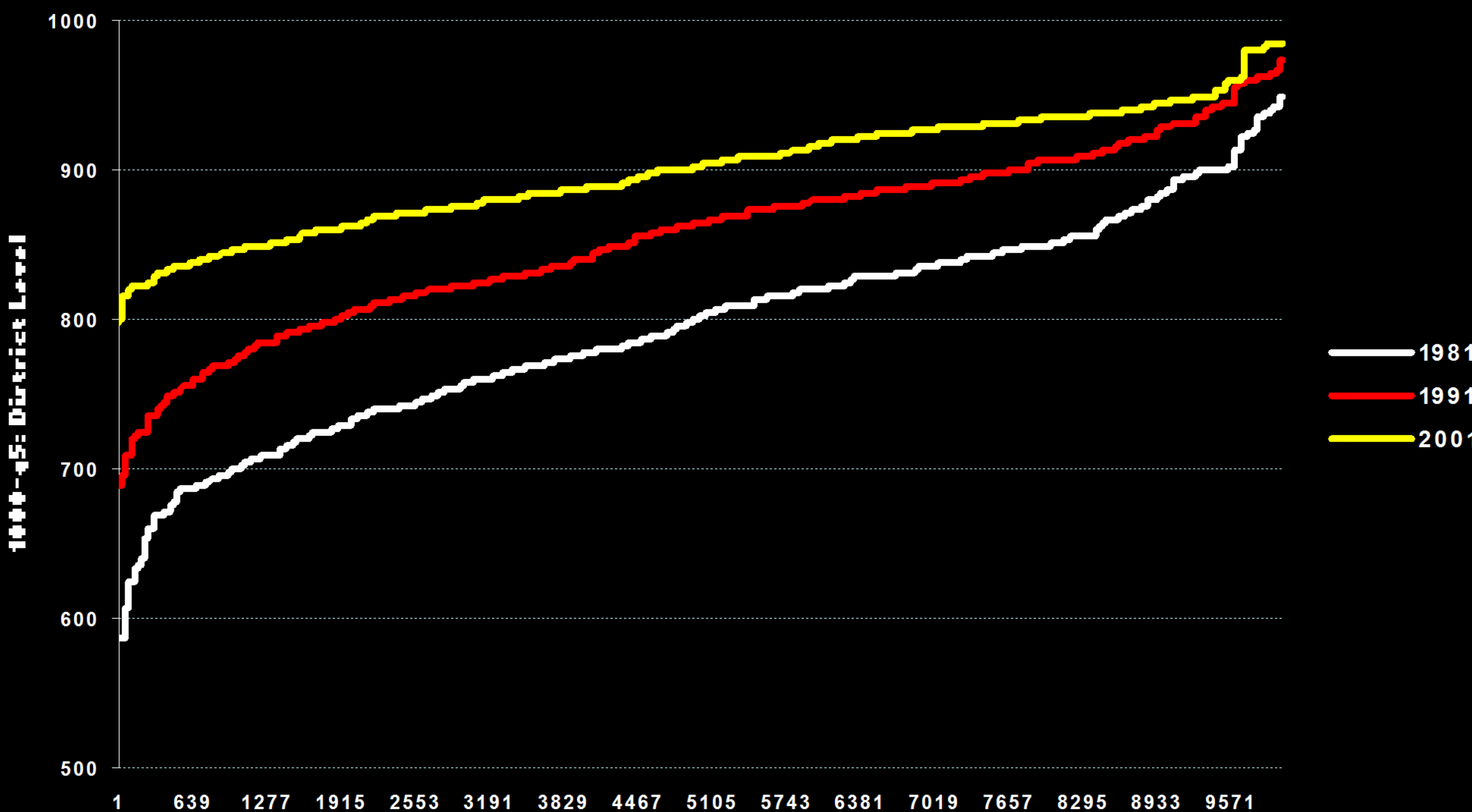
Public safety ordered vectors



Public safety Lorenz curve (make sense ?)



Protection against infant mortality risk



Four-dimensional comparisons ?

- Compare the joint distributions of all four attributes
- Multidimensional Dominance at a given order requires one-dimensional dominance at the same order in all dimension in isolation
- This suggests that the behavior of crime will prevent dominance at the first order
- Careful with graphical intuition. Needs to resort to statistical inference (check if inequalities in either direction are statistically significant)

Four-dimensional comparisons ?

- 2002 dominates 1996 and 1988 at the 2nd order for all four variables
- No dominance in either direction between 1996 and 1988

Three-dimensional comparisons (without crime)

- 2002 dominates 1996 and 1996 dominates 1988 for the three other attributes at the first order
- Hence the observed crossing of the consumption ordered vectors turned out to be non-significant

Comments on Robust multidimensional normative evaluation

- Complexity increases with the number of dimensions.
- Solution: put more structure on the problem.
- Exploit the specific nature of the « attributes » that are considered.
- Good example: risk
- Infant mortality: risk of loosing one's kid. Literacy rate: probability that the average person that you encounter does not know how to read. Crime rate: probability of being the victim of a criminal act.
- Various aspect of life involves risk.
- Why not developping dominance analysis for comparing socially risky situations (Fleurbaey (2010) ?

A formal setting (Gravel & Tarroux (2013) (1)

- n individuals (households) indexed by $i \in N$.
- Each individual can fall in l mutually exclusive states, indexed by $j \in \Omega$
- States are ordered from the worst (being murdered) to the best (no criminal aggression).
- Every Individual receives a pecuniary consequence (income) in every state in which he/she can fall (may or not depend upon the state)
- As before, we assume that all possible income levels are taken from some finite set $I = \{0,1,\ldots,m\}$ (incomes are measured in cents and there is a maximum amount of cents, m, that someone can hope to get).
- A SRS: a lottery p on the set $X = (\Omega \times I)^n$ of all n-dimensional vectors of state-income pairs (one such pair for every individual).
- $p(j_1,y_1,\ldots,j_n,y_n)$ joint probability of observing individual i falling in state j_i and receiving income y_i in that state.

A formal setting (Gravel and Tarroux (2013) (2)

- $\pi^{pi}_j(y)$ probability that i be in state j with income y in the SRS p.
- Individual i has a VNM preference ordering $\succsim_i$ on the set of all lotteries on X
- Individuals are selfish (they care only about their state and what they get in it) and – up to a permutation of their states – identical.
- This means that for every state j, there is a function U_j: $I \rightarrow \mathbb{R}$ such that, for every individual i:

$$p \succsim_i q \Leftrightarrow \sum_{j\in\Omega}\sum_{y\in I}\pi_j^{pi}(y)U_j(y) \geq \sum_{j\in\Omega}\sum_{y\in I}\pi_j^{qi}(y)U_j(y)$$

A formal setting (Gravel & Tarroux (2013) (3)

- SRS are compared by an anonymous VNM social ordering $\succsim$ that satisfies the weak Pareto principle.
- This means that for every two lotteries p and q, $p \succsim_i q$ for all i implies $p \succsim q$ and
- $p \succ_i q$ for all i implies $p \succ q$.
- By virtue of a version of Harsanyi (1955)'s aggregation theorem due to Weymark (1993), this implies that $\succsim$ can be written as

$$p \succsim q \Leftrightarrow \sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in I}\pi_j^{pi}(y)U_j(y) \geq \sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in I}\pi_j^{qi}(y)U_j(y)$$

A new notion of Normative dominance

- SRS ◘ normatively dominates SRS ❑ for a class **U** of ● state-dependant utility functions ✞$_{ɚ}$:I ! **R** (for ɚ 2 Ω), denoted ◘ %$_{U}$ ❑, if for all combinations of ● functions ✞$_{ɚ}$ in the class, one has:

$$\sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in I}\pi_j^{pi}(y)U_j(y) \geq \sum_{i\in N}\sum_{j\in\Omega}\sum_{y\in I}\pi_i^{qi}(y)U_j(y)$$

Comments on this dominance

- Thanks to the famous Harsanyi-Sen debate, it is not utilitarian.
- A dominance of a SRS over another is a ranking that is agreed upon by a large spectrum of anonymous and Pareto inclusive VNM social orderings.
- This interpretation rides however on the ex ante approach to social decision making under uncertainty and the VNM Morgenstern properties imposed on the social planner (remember Diamond (1967) critique).

Class of utility functions (1)

- **U^1** = the set of functions U_j (for $j \in \{1,\dots,l-1\}$) satisfying $U_j(y+1) - U_j(y) \geq U_{j+1}(y+1) - U_{j+1}(y) \geq 0$ for every $y \in \{0,\dots,m-1\}$
- Marginal utility of income is positive and decreasing with respect to the states
- Plausible ? (see e.g. Viscusi and Evans (1990): people with preferences like this would over choose to over insure themselves when facing actuarially fair health insurance contracts)
- Alternative class (Gravel & Moyes (2013b):
- **$U^{1'}$** = the set of functions U_j (for $j \in \{1,\dots,l-1\}$) satisfying $U_{j+1}(y+1) - U_{j+1}(y) \geq U_j(y+1) - U_j(y) \geq 0$ for every $y \in \{0,\dots,m-1\}$

Class of utility functions (2)

- **U^2 = U^1** $\cap$ the set of functions ✟$_{ℯ𝓇}$ (for ℯ𝓇 **2** {1,…,*l*-1}), satisfying: 0 ≥ ✟$_{ℯ𝓇+🗁}$(*y*+2) -2✟$_{ℯ𝓇+🗁}$(*y*+1)+ ✟$_{ℯ𝓇+🗁}$(*y*) ≥ ✟$_{ℯ𝓇}$(*y*+2) -2✟$_{ℯ𝓇}$(*y*+1)+ ✟$_{ℯ𝓇}$(*y*) for every $y \in \{0,\ldots,m\text{-}2\}$.
- Absolute risk aversion (as measured by the Arrow-Pratt Coefficient) is decreasing with the state)
- Alternative class (Gravel & Moyes (2013b):
- **U$^{2'}$ = U$^{1'}$** $\cap$ the set of functions ✟$_{ℯ𝓇}$ (for ℯ𝓇 **2** {1,…,*l*-1}), satisfying: 0 ≥ ✟$_{ℯ𝓇}$(*y*+2) -2✟$_{ℯ𝓇}$(*y*+1)+ ✟$_{ℯ𝓇}$(*y*) ≥ ✟$_{ℯ𝓇+1}$(*y*+2) -2✟$_{ℯ𝓇+1}$(*y*+1)+ ✟$_{ℯ𝓇+1}$(*y*) for every $y \in \{0,\ldots,m\text{-}2\}$.

Implementable criteria (1)

- Sequential Expected Headcount Poverty (SEHP) dominance: SRS □ dominates SRS □ for the SEHP criterion if for every poverty line $t \in I$, and every state $k \in \Omega$, one has:

$$\sum_{i \in N} \sum_{j \leq k} \sum_{y \leq t} \pi_{ij}^{p}(y) \leq \sum_{i \in N} \sum_{j \leq k} \sum_{y \leq t} \pi_{ij}^{q}(y)$$

Implementable criteria (2)

- Sequential Expected Poverty Gap (SEPG) dominance: SRS □ dominates SRS □ for the SEPG criterion if, for every poverty line $t \in I$, and every state $k \in \Omega$, one has:

$$\sum_{i \in N} \sum_{j \le k} \sum_{y \le t} \pi_{ij}^{p}(y) \max[t - y, 0] \le \sum_{i \in N} \sum_{j \le k} \sum_{y \le t} \pi_{ij}^{q}(y) \max[t - y, 0]$$

And, for every state k, one has in addition:

$$\sum_{i \in N} \sum_{j \le k} \sum_{y \le m} \pi_{ij}^{p}(y) \le \sum_{i \in N} \sum_{j \le k} \sum_{y \le m} \pi_{ij}^{q}(y)$$

Results

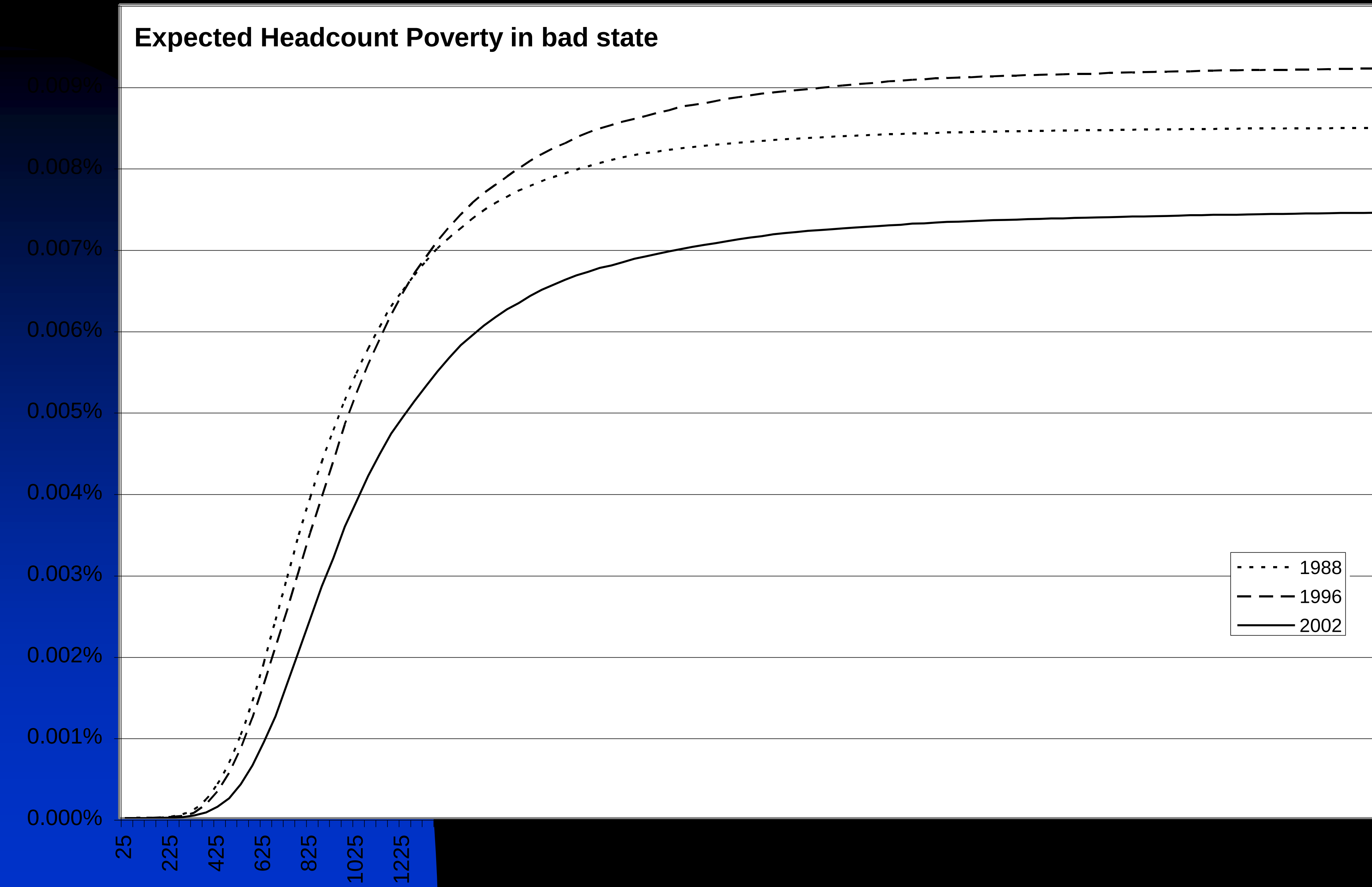
- Theorem 1: SRS □ dominates SRS □ for the SEHP criterion if and only if □ normatively dominates SRS □ for all utility functions † in **U¹**.
- Theorem 2: SRS □ dominates SRS □ for the SEPG criterion if and only if □ normatively dominates SRS □ for all utility functions † in **U²**.

Comments on the results

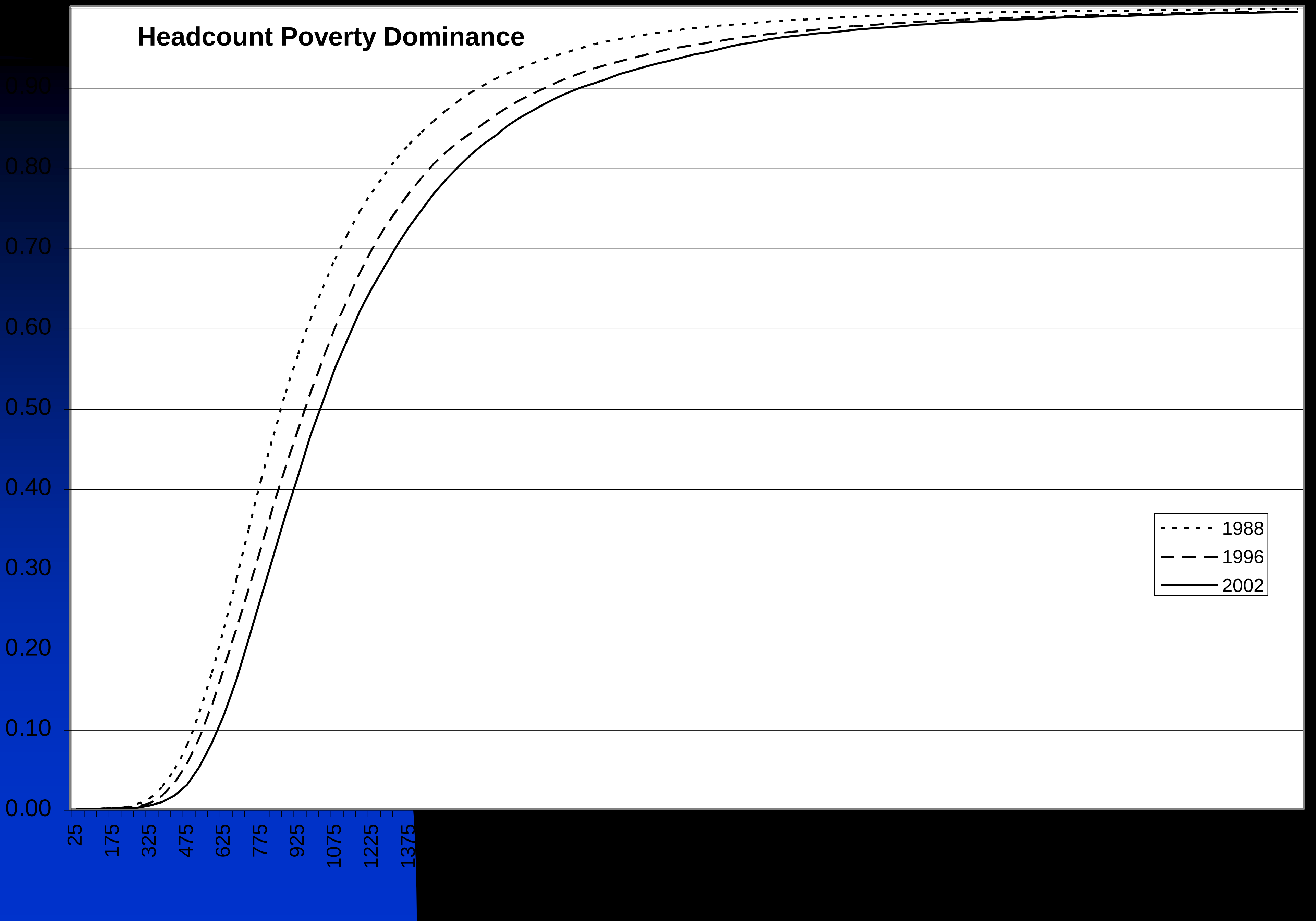
- Proof: discrete (Abel) integration by part for sufficiency, and remark that every use of the SEHP and/or SEPG criterion corresponds to a (degenerate) utility function in the class U^1 and/or U^2 respectively for necessity.
- These results can be seen as generalizations of more traditional two-dimensional dominance results (state of nature is an ordinal variable, and income a cardinal one).
- SEPG is analogous to Jenkins & Lambert (1993) (in fact Bazen and Moyes (2003))
- **Important difference:** here individuals have different probabilities of being in the same state.

Indian empirical implementation (1)

- NSS data on households consumption expenditures and district of residence (rounds 43 (1987-1988), 52 (1995-1996) and 58 (2001-2002))
- National Crime Record Bureau for district data on crime (number of murders, attempted murders and rapes per million individuals) (years 1988, 1996 and 2002).



Expected Headcount Poverty in bad state
0.009%
0.008%
0.007%
0.006%
0.005%
0.004%
0.003%
0.002%
0.001%
0.000%
25
225
425
625
825
1025
1225
1988
1996
2002


Headcount Poverty Dominance
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
25
175
325
475
625
775
925
1075
1225
1375
1988
1996
2002

Main lesson

- The allocation of risk exposure to violent crime was better in 2002 than in either 1996 or 1988 for the SEHP criterion.
- Remember that such a clear-cut verdict could not be obtained with standard multidimensional dominance criterion à la Atkinson-Bourguignon (at least not at the 1st order).
- Hence, using the risk structure helps!
- No conclusion for comparing 1988 and 1996 at the 1st order.

France-US comparisons of labour market related risk (1)

- Discussions about the extent to which job insecurity has increased in the nineties, and how continental Europe and US compare on that front (see e.g. Gottschalk & Moffit (1999) or Farber (2004) for US evidence on this and Givord & Maurin (2004) for French one.
- These discussions are conducted with average figures, and do not look at the distribution of these risks of unemployment among workers.
- Risk of involuntary loosing one's job, or of staying involuntary unemployed is a clearly important one.
- Moreover, it is not clear what is the ordering of the states (e.g. unemployed vs employed).
- Let us do a quick France – US comparison using our criteria.

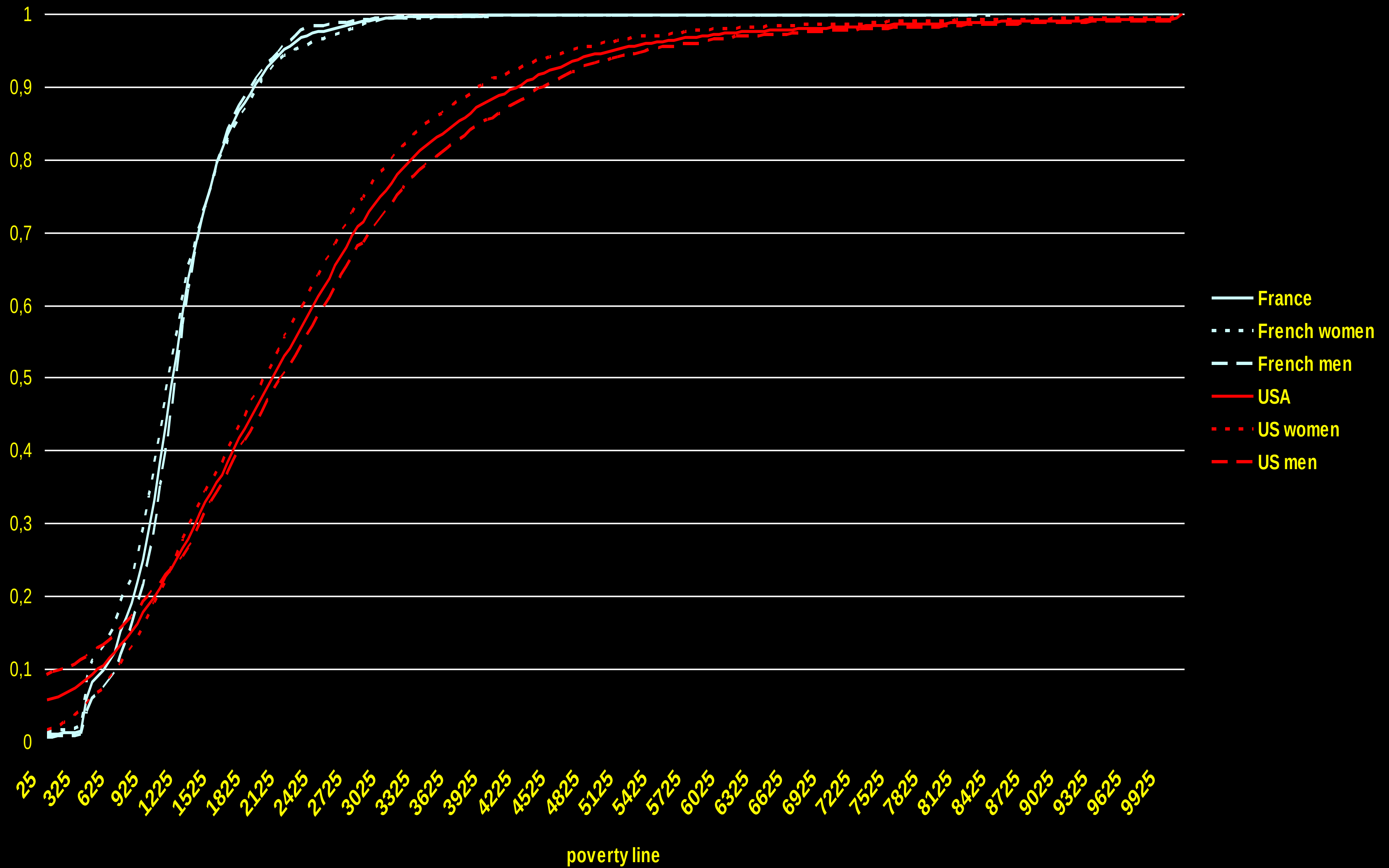
France-US comparisons of labour market related risk (2)

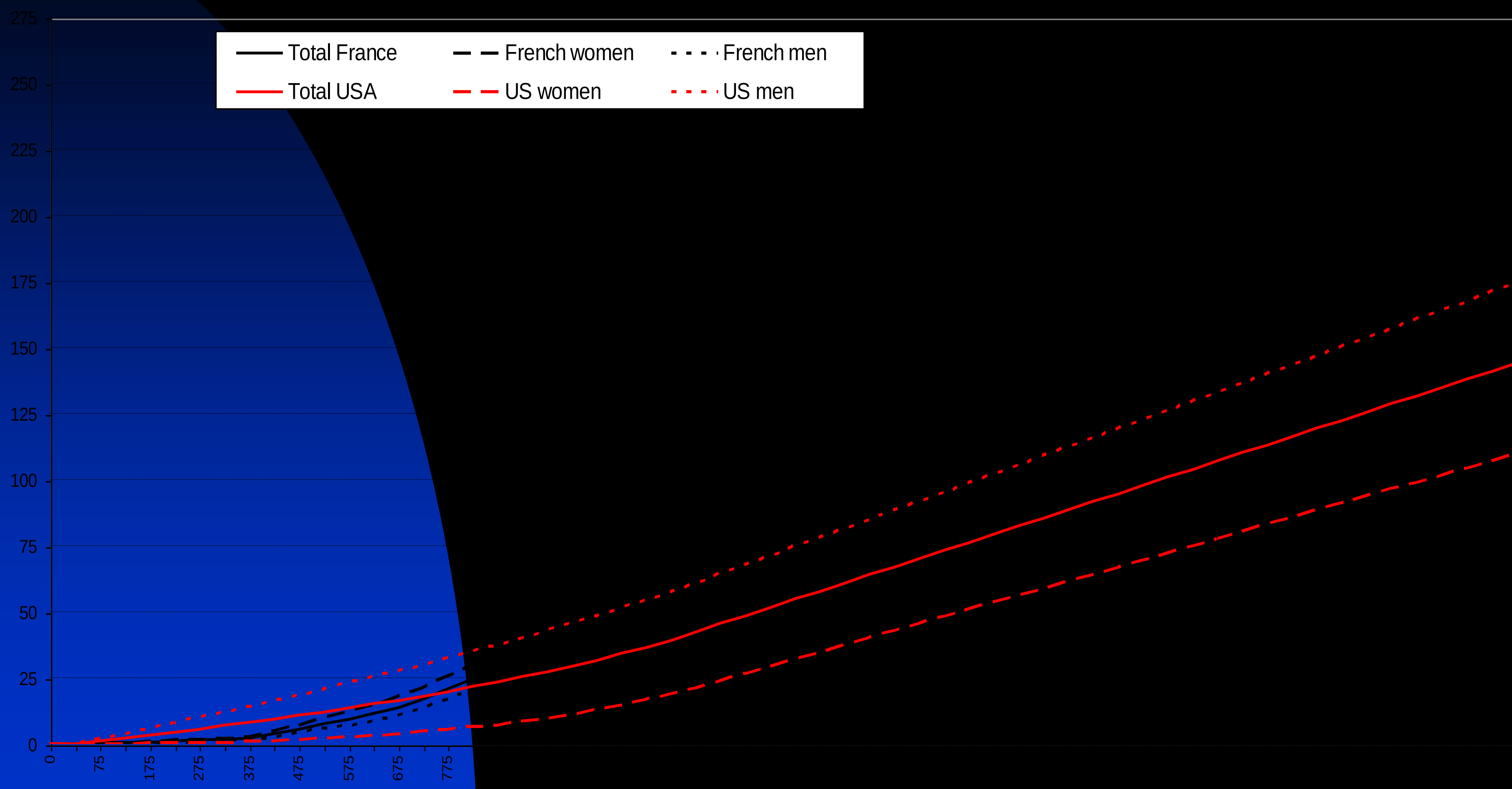
- French Labour Force Survey (FLFS)) for France, US Current Population Survey-March Supplement (CPS-MS) for both 2003 and 2004. The LFS contains 50,524 respondents
- Focus on single individuals without children (employed and unemployed), 6,953 individuals in France, 7 523 in the US.
- The employment trajectories of these individuals enable us to assign to several groups of them (formed on the basis of observable characteristics) a probability of being unemployed in 2004.
- These probabilities mean different things for different people.

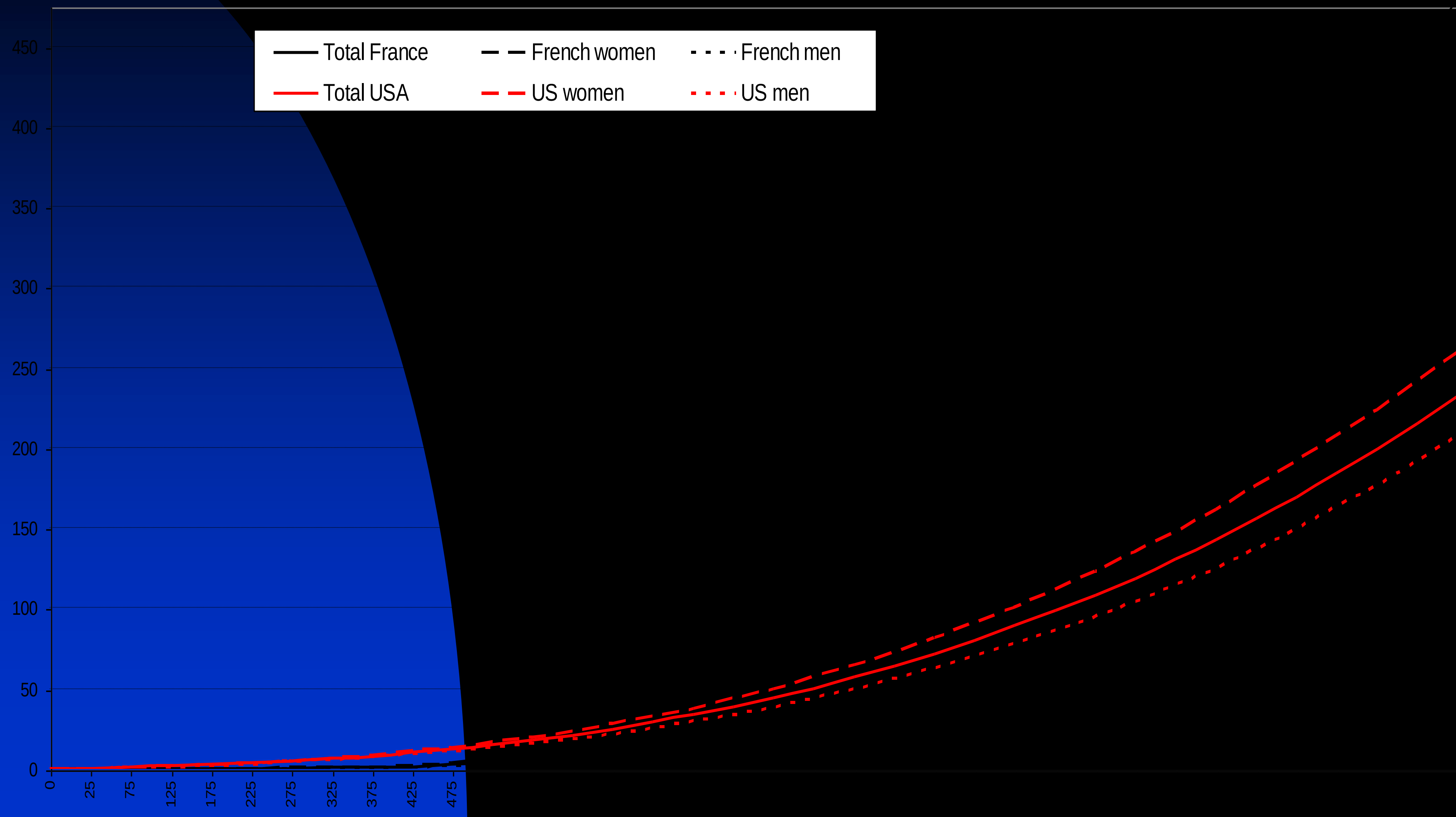
France-US comparisons of labour market related risk (3)

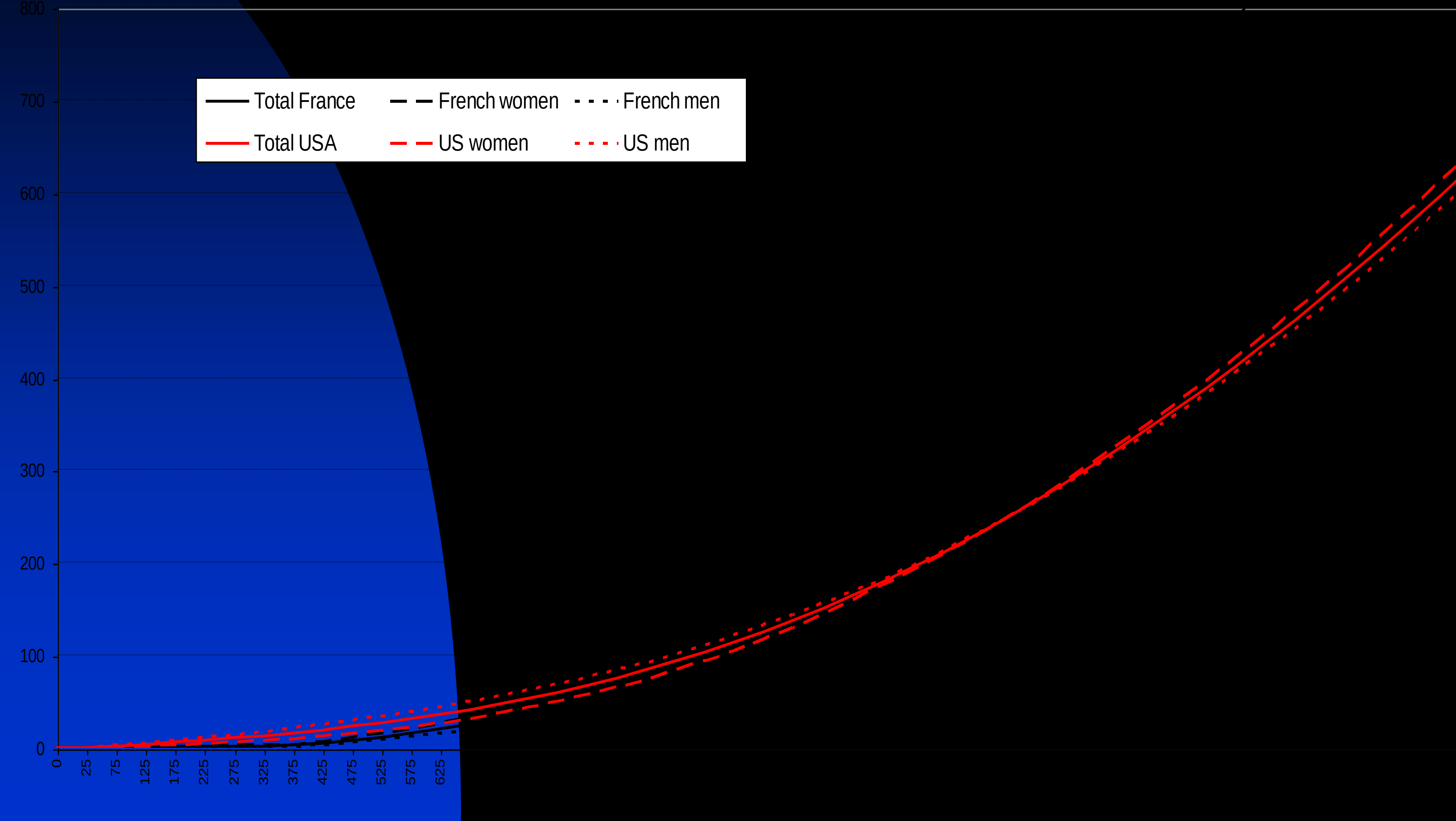
- For an employed individual in 2003, it is the probability of becoming (involuntarily) unemployed in 2004 (38 groups of employed individuals in 2003 were formed)
- For an unemployed individual in 2003, it is the probability of staying in that situation in 2004 (10 groups of unemployed individuals were formed).
- In each of the two states, we assigned to each individual his/her net earning (gross earning minus taxes plus transfers).
- Transfers: Unemployment insurance (in the two countries) + RMI (in France, because transfer payments in the US are not given to single adult households).
- As unemployed individuals do not work, we have imputed a wage to them based on the estimation, on the sample of working individuals, of a wage equation (we have corrected for the sample selection bias using a Heckman procedure)

Standard headcount poverty curves




Total France
French women
French men
Total USA
US women
US men
275
250
225
200
175
150
125
100
75
50
25
0
0
75
175
275
375
475
575
675
775


Total France
French women
French men
Total USA
US women
US men
0
50
100
150
200
250
300
350
400
450
0
25
75
125
175
225
275
325
375
425
475


Total France
French women
French men
Total USA
US women
US men
800
700
600
500
400
300
200
100
0
0
25
75
125
175
225
275
325
375
425
475
525
575
625

Comparison results (statistical inference)

Comparison	Result (SEPG dominance)
France-US (unemployment bad)	?
France-US (employment bad)	?
France-US (state independant)	?
French male-female (unemployment bad)	?
French male-female (employment bad)	male
French male-female (state independant)	male
US male-female (unemployment bad)	?
US male-female (employment bad)	?
US male-female (state independant)	?

Main lesson

- Non comparability of France and US (comes mainly from the veto power of the very poor who are better treated in France, despite the fact that their probability of being unemployed is higher).
- Men have better exposure to risk of job lost than women in France (if employment is bad, or if state independance)
- No such gender dominance is observed in the US.
- Hence US appears to offer a better « equality of opportunities » of exposure to job protection between men and women than the US!!! (???)

Conclusion

- Multidimensional Robust normative analysis is not done enough.
- Results are difficult to obtain, but they are coming.
- These tools are also highly appropriate to do mobility analysis, analysis of policy involving time, etc.
- The dominance analysis is flexible (see Bazen and Moyes (2012) work on Elitism).
- Complexity increases with the dimensions and the order of dominance.