

Social Heterogeneity, Conflicts and Secession”

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Abstract

Reading List for my lectures at the Third Winter School on Inequality and Collective Welfare Theory ”**Identity and Social Cohesion**”, January 8-11, 2008, Alba di Canazei (Italy)

The main purpose of these notes is to explore through a limited sample of papers the following question. Given a description of a social environment (with a special attention paid to the differences among the agents acting in this environment) and the ultimate goal of this group/organization/society (selection of a policy, choice of a social alternative,...), what is the relationship between the degree of heterogeneity displayed by the social profile and the degree of conflict/instability in the decision-making process. To what extent, is it true that wasteful activities like lobbying and violent conflicts are more likely to prevail in social environments displaying an important heterogeneity. This question as well as closely related questions have been addressed by many authors in various fields (economics, sociology, political science, ...) and from different complementary perspectives (analytical, empirical, decriptive,...). It should be clear that these brief notes do not pretend, of course, to cover this large topic but simply to offer a specific point of view largely based upon the works of the presenter on secessions and coalition formation. This short lecture will be organized as follows.

1. Introductory Remarks on Social Heterogeneity.
2. A Model of Lobbying.
3. A Model of Nation Formation and Secession.

Social heterogeneity

The first task is to derive a description of social heterogeneity . Given a space of characteristics Θ (which can be qualitative (ethnicity, religion, gender,...) or quantitative (income, geographic location, ideology,...), a pattern of social heterogeneity or a social environment is formally defined as a measure over Θ . Each individual or unit is described as a point in that space. The measure can be discrete or continuous. When it is discrete, the measure consists of K clusters where each cluster k is populated by n_k units with characteristics $\theta_k \in \Theta$ i.e. a social pattern or environment is a K -tuple $(n_k, \theta_k)_{1 \leq k \leq K}$; the total number of units in the population is $n = \sum_{1 \leq k \leq K} n_k$. When it is continuous, it is typically described by its density f or its cumulative F . The set of social environments is denoted by \mathcal{E} . In some cases, the space of characteristics will be endowed with a metric and we will be then in position to measure the "social distance" between two groups and more generally to benefit from a geometric description of the societal data as a set of points in a metric space. In some cases (a list appeared at the very end of this file), the social situation will be summarized by a data consisting exclusively of the matrix of pairwise distances between characteristics. This will happen for instance when the space of characteristics and the space of outcomes are both equal to some multidimensional Euclidean space and the preference of each agent

is circular with his/her individual characteristic as the center as in the celebrated spatial model in political science.

The second task is define the social institutions and mechanisms leading to the selection of a social outcome. The terminology is broad enough to accomodate a variety of different situations. It can refer to be a particular dimension of the current public policy (taxation, public expenditures,...) of the group under scrutiny or to a set of social or political activities taking place within the society like lobbying and political activism. It can also refer to any social conflict like a civil war or a secession as extreme forms of violent resolution of conflicts of opinions and interests among individuals and groups in the population. The set of social outcomes is denoted by O .

We will examine the mapping $E : \mathcal{E} \rightarrow O$ which maps any social environment into a social outcome. Given a set of social institutions/mechanisms and a pattern of preferences of the actors over the feasible social outcomes, the mapping E reflects the combined influence of the intrinsic features of the institutions and the behavioral strategic responses of the actors. We will assume that the basic preferences of the actors are determined by their individual characteristics. However, if the composition of the group is part of the social outcome, then the ultimate payoff of an actor may depend also upon the characteristics of the other members of the group.

A Model of Lobbying

We first examine the influence of social heterogeneity in the case of a model of lobbying due to Esteban and Ray (1999). Lobbying is a social activity which is often considered to be wasteful as the resources and efforts spent in such activities could be reallocated towards productive tasks. The social cost of lobbying is therefore not negligible but is certainly much lower than the social cost attached to a violent conflict like a civil war. Lobbying may arise as soon as a conflict of interests/preferences appear among the components of the society. There are K groups with n_k units in each group n_k units. They normalize the size of the population to 1 i.e. $\sum_{1 \leq l \leq K} n_k = 1$. The set of social outcomes O consists of K possible issues : one favorite issue for each group and issue k is identified as the issue most preferred by group k . Here, a social outcome is thought of as a pure public good. Define u_{kl} as the utility derived by a member of group k if issue l is chosen by society. From what precedes, $u_{kk} > u_{kl}$ for all k, l with $k \neq l$. They assume that agents can influence the outcome by allocating resources to lobbying. Denote by r_{kl} the resources expended by a typical individual of group k in support of outcome l : the total amount of resources spend by group k is $n_k \sum_{1 \leq l \leq K} r_{kl} = n_k r_k$ where r_k is the total amount of resources spent per group

member. Then, the total resources devoted to lobbying by society is $R \equiv \sum_{1 \leq l \leq K} n_l r_l$.

They use R as a measure of societal conflict.

Resources are assumed to be acquired at a cost : $c(r)$ denotes the individual cost of supplying r . If we denote by p_l the probability that issue l will be chosen, the expected utility of a member of group k who expends resources r_k is given by

$$\sum_{1 \leq l \leq K} p_l u_{kl} - c(r_k)$$

They assume that the probabilities are determined according to the following formula :

$$p_k = s_k \equiv \frac{\sum_{1 \leq l \leq K} n_l r_l u_{lk}}{R} \text{ for all } k = 1, \dots, K$$

This formula is of course very special but except for that, the model is general enough to accomodate different environments through the specification of the matrix $(u_{kl})_{1 \leq k, l \leq K}$.

Esteban and Ray ignore free-rider problems within each group and assume (in their analysis) that no group expends resources on outcomes other than its preferred position. Under some convexity assumption on the function c , they prove that this non cooperative game has a Nash equilibrium. Defining

$$v_{kl} = u_{lk} - u_{kl}$$

they prove that a vector of resources (r_1^*, \dots, r_K^*) is an (interior) equilibrium iff :

$$\sum_{1 \leq l \leq K} s_k s_l v_{kl} = c'(r_k) r_k \text{ for all } k = 1, \dots, K$$

They also demonstrate that if $c'''(r) \geq 0$ for all r , then the equilibrium is unique. During the lectures, I will provide illustrations of the equilibrium through examples taken from their paper. In the case where the cost function is iso-elastic i.e. $c(r) = \alpha^{-1} r^\alpha$ for some $\alpha > 1$, the equilibrium equations can be rewritten as :

$$\sum_{1 \leq l \leq K} s_k s_l v_{kl} = r_k^\alpha = s_k^\alpha \left(\frac{R}{n_k} \right)^\alpha \text{ for all } k = 1, \dots, K$$

If we denote by W the $K \times K$ matrix with generic element $w_{kl} \equiv n_k^\alpha v_{kl}$, by s^β the vector $(s_1^\beta, \dots, s_K^\beta)$ and write $\lambda \equiv R^\alpha$, the equilibrium equations can be written in matrix form as :

$$W s = \lambda s^{\alpha-1}$$

the solution to this system is not generally unique but we know from their result on uniqueness that it is the case if $\alpha \geq 2$. When $\alpha = 2$, it reduces to a Frobenius theorem. The equilibrium resource shares are the entries in the positive eigenvector of the matrix :

$$W = \begin{pmatrix} n_1^2 v_{11} & n_1^2 v_{12} & \cdot & \cdot & \cdot & n_1^2 v_{1K} \\ n_2^2 v_{21} & n_2^2 v_{22} & \cdot & \cdot & \cdot & n_2^2 v_{2K} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ n_K^2 v_{K1} & n_K^2 v_{K2} & \cdot & \cdot & \cdot & n_K^2 v_{KK} \end{pmatrix}$$

while R^2 is the unique positive eigenvalue of this matrix.

Distributional Characteristics and Level of Conflict

Esteban and Ray examine the impact of two features of population distributions: the distribution of the group sizes over the different groups in the population and the profile of group distances. Here, the social environment E is fully described by the vector (n_1, \dots, n_K) and the matrix $(v_{kl})_{1 \leq k, l \leq K}$. Their first proposition is the following

Proposition Under the assumption $c'''(r) \geq 0$ for all $r \geq 0$, if E and E' are two social environments such that $n_k = n'_k$ for all k and $v'_{kl} \geq v_{kl}$ for all k, l with strict inequality for some k and l , then $R' > R$.

They turn then to the question of population distribution over a given set of groups and group distances and emphasizes bimodality : conflict has tendency to increase when there are two similar-sized, opposed groups in society. They prove that this is indeed the case under the assumption that **intergroup antagonism is symmetric** i.e. $v_{kl} = v_{lk}$. In the case where $K = 2$, the equilibrium equations simplify to :

$$\frac{c'(r_1)r_1}{c'(r_2)r_2} = \frac{v_{12}}{v_{21}}$$

This equation tells us that the amount of individual effort contributed by one group relative to the other depends on $\frac{v_{12}}{v_{21}}$ but not on the sizes of the respective groups. Under symmetry i.e. $v_{12} = v_{21} = v$, we obtain $r_1 = r_2 = R$. The societal conflict is then given by the condition :

$$Rc'(R) = vn_1n_2$$

It follows that R is maximal when $n_1 = n_2 = \frac{1}{2}$. More generally, they prove the following.

Proposition Under the assumptions $c'''(r) \geq 0$ for all $r \geq 0$ and $v_{kl} = v_{lk}$ for all k and l , there exists a symmetric bimodal distribution which yields at least as much conflict as any other distribution.

During the lectures, we will examine through examples the importance of the conditions leading to the conflict-maximality of bimodal distributions.

Conflict and Distribution in Contests

They proceed to a detailed examination of a class of social environments known as contests. They are defined by the assumption $v_{kl} = v_{kj}$ for all k and all $l, j \neq k$. Without loss of generality, they set $v_{kk} = 0$ and $v_{kl} = 1$ for all k and l such $k \neq l$. The equilibrium equations can be restated as :

$$\sum_{l \neq k} s_l = \frac{c'(r_k)R}{n_k} \text{ for all } k$$

or equivalently

$$z_k(1 - s_k) = c'\left(\frac{s_k}{z_k}\right) \text{ for all } k$$

where $z_k \equiv \frac{n_k}{R}$. If we regard, for the moment, z_k as an exogenous variable, the above equation uniquely defines s_k as some function h of z_k with h increasing and twice continuously differentiable.. The social environment is truly described here by the vector (n_1, \dots, n_K) but we observe that the vector (z_1, \dots, z_K) only differs from it through a scaling factor R . With h in place, the equilibrium value of R satisfies :

$$\sum_{1 \leq k \leq K} h\left(\frac{n_k}{R}\right) = 1$$

When is conflict maximized in such case ? The case where $K=2$ is analysed as before but the case $K \geq 3$ is more complicated. From a repeated application of population shifts across pairs of groups, we could expect that the uniform distribution is a conflict maximizer but the matters are more complex than this simple conjecture. During the talk, I will explain why **the uniform distribution is not a global maximizer of conflict, while it is a local maximizer of conflict.**

Activism

It is of interest to ask how the intensity of lobbying of a particular group might vary with group size. An index of intensity is the share of resources contributed by a particular group relative to its numerical strength in the population. The ratio $\frac{s}{n}$ is obviously equal to $\frac{r}{R}$ the per capita contribution made by a group relative to the mean. Patterns of lobbying intensity are related closely to questions of extremism or moderation within a society. Such concepts are without meaning unless some metric is assigned across groups, so that extremism would then refer to a situation where radical groups lobby more for their preferred outcomes. Alternatively a situation is moderate if centrist groups are the more vocal. It is worth noting that these notions are useful in understanding situations where the true distribution of societal characteristics may be exaggerated or hidden by its publicly observed conflicts.

Esteban and Ray prove that larger groups always lobby more intensively than smaller groups. They offer a more complete investigation of this question in the case where the metric model is described by a line. They consider a symmetric distribution on three groups : two groups of size m (called the radicals) situated on either side of a middle class of size $1 - 2m$. The loss is given by a to the centrist group should either of the radicals win, and by a to the radicals should the middle class win. However, should a radical group win, the loss to the other radical group is given by b with $b > a$. It follows from the assumed symmetry that the two radicals contribute equal shares s so that the centrist contributes $1 - 2s$. Here, a situation is extremist if $s > m$ and moderate otherwise. Esteban and Ray prove the following :

Proposition Define $m^* = \frac{a}{b+2a}$. Then a situation is extremist if $m > m^*$ but is moderate if $m < m^*$. The critical value m^* lies in the interval $(0, \frac{1}{3})$

I will illustrate on the board this proposition. The point worth noting is that an already equal society (with a relatively small share of radical groups) will display an even greater degree of moderation in its decision-making, compared to its population distribution. On the other hand, once the population share of the radicals crosses a critical magnitude, the radicals contribute more than their population share, leading to a situation that looks more conflictual than the underlying population warrants. Note that extremism manifests itself when each of the radical groups has strictly lower population than the middle class (the limit case arises in the case of a contest i.e. when $b = a$). When alienation becomes convex i.e. $b > 2a$, extremism manifests itself even if the total population of radicals is less than the middle class (the limit case arises in the case of linear alienation i.e. when $b = 2a$). I will comment the links with the Olson's thesis during the lectures.

Esteban and Ray also provide few insights on the relationships between the measure of societal conflict used in this paper and polarization. In this lobbying setting, their main

measure of polarization (we will cross the road of this measure several times during the lectures) is given by :

$$P = \sum_{1 \leq k \leq K} \sum_{1 \leq l \leq K} n_k^{1+\gamma} n_l v_{kl}$$

where γ is a strictly positive parameter. In the iso-elastic case, they obtain:

$$R^\alpha = \sum_{1 \leq k \leq K} \sum_{1 \leq l \leq K} \left(\frac{n_k}{s_k} \right)^\alpha s_l^2 s_l v_{kl}$$

If $s_k = n_k$ for all k , then $R^\alpha = P$ for the specific case $\gamma = 1$. In some sense, it is the behavioral nature of conflict, which forces $s_k \neq n_k$ in specific circumstances, which makes the measure of societal conflict depart from polarization in a significant way.

Complements

This analysis of the influence of societal characteristics on lobbying or any other form of societal conflict can be adjusted and/or extended in various directions. I will allude to some of them during the lectures.

- In a long series of papers, Esteban and Ray have provided several analytical models of class and ethnic conflicts. In particular, the role of within group heterogeneity is examined carefully. This dimension is also one of the main focus of the analysis of lobbying provided by Le Bretion and Salanié (2003) who examine the support to several Olsonian claims like for instance the impact of the within-group inequality on the severity of the free-riding problem. To some extent, a large portion of the Olson's program can be viewed as an attempt to evaluate the impact of the exogenous characteristics of a group on the efficiency of its social action(s).

- The special influence function used here is quite controversial. It would be nice to explore the question above in the context of the popular common agency model of lobbying. The social environment is the same as in Esteban and Ray and there is an exact characterization of the rent received by the common agent at equilibrium.

A Model of Nation Formation

We consider a nation whose citizens have preferences over a unidimensional policy space I , given by the interval $I = [0, 1]$. We adopt a spatial interpretation of I , where a policy choice is represented by the location of the government. Each citizen's preferences are single-peaked and are identified with her ideal point in I . The distribution of all ideal points is

given by a cumulative distribution function F , defined over the space I . For the most part, we assume that the distribution gives rise to piecewise continuous density function f . We denote by $\lambda(S) = \int_S f(t)dt$ the induced measure on subsets S of I . The total mass of I is equal to 1, i.e., $\lambda(I) = 1$.

The nation is organized either in one unified country, represented by the entire interval I , or it can be partitioned into several smaller countries S_1, S_2, \dots, S_K . We do not impose restrictions on country formation, and, in principle, every group of individuals S can form a country. Only for simplicity we require that each country consists of a union of a finite number of intervals with a positive mass.

Each country chooses a policy from the issue space I . If an individual t is a citizen of country S , whose government chooses a location $p \in I$, then the disutility or “transportation” cost incurred by that individual, $d(t, p)$, is determined by the distance between t and the location of the government:

$$d(t, p) = |t - p|.$$

Each country S has to cover the cost of public goods $G(S)$ which we simply call *government cost*. For simplicity, we assume that the government costs are independent of S , i.e., $G(S) = g$, where g is a positive constant.

Thus, the “aggregate” cost of country S is $g + D(S)$, where

$$D(S) = \min_{p \in I} \int_S d(t, p) f(t) dt$$

is the transportation cost of the citizens of S . It is easy to verify that the total transportation cost in the region S is minimized when the government selects its median location m that satisfies:

$$\int_{S \cap [0, m]} f(t) dt = \int_{S \cap [m, 1]} f(t) dt.$$

That is, if $M(S)$ is the set of median locations of country S , then for every $m \in M(S)$ we have

$$D(S) = \int_S d(t, m) f(t) dt.$$

Since the transportation cost incurred by a citizen is represented by the distance between her location and the policy chosen by the country to which she belongs, **it again points out to the conflict between heterogeneity and increasing returns to size. On the one hand, a larger country would require a smaller per capita contribution towards the fixed component of the government costs g . On the other hand, a larger and more heterogeneous country may face a larger mass of dissatisfied citizens residing far away from the capital.**

We now introduce the notion of a *cost allocation*, that determines the monetary contribution of each individual t towards the cost of government:

For every set $S \subset I$, a measurable function x , defined on the S , is called an S -cost allocation if it satisfies the budget constraint:

$$\int_S x(t)f(t)dt = g.$$

The I -cost allocations will simply be referred as cost allocations.

There is a large domain of cost allocations that provide a different degree of cost burden on the citizens of the country. For our discussion below, however, the special role will be played by the *laissez-faire* allocation that provides no compensation from one citizen to another and simply divides the total government cost equally among all citizens of the country. That is,

The S -cost allocation x_S^* , defined by $x_S^*(t) = \frac{g}{\lambda(S)}$ for all $t \in S$, is called the *laissez faire* S -allocation. Obviously, the *laissez faire* I -allocation, or simply, the *laissez faire* allocation x^* is given by $x^*(t) = g$ for all $t \in I$.

Efficiency and Laissez Faire Stability

The problem of efficiency reduces to the following two questions:

- should the nation stay united or be divided into several smaller countries,
- where the governments should be located.

Our discussion provides a simple answer to the choice of the governments' locations: if a country S is formed, then its government should be located at one of its medians, and so the country incurs $D(S)$ in transportation costs. To examine the question whether the country should be united, we introduce the following condition that formally defines, when, from the perspective of a social planner, it is inefficient to break up the existing country I into several smaller countries. This would imply that the aggregate transportation and monetary costs of all citizens of I are minimized if the country is not partitioned into several parts.

The country I is *efficient* if for every partition $P = (S_1, \dots, S_K)$ of I we have:

$$D(I) + g \leq \sum_{k=1}^K [D(S_k) + g].$$

The following result is straightforward:

Proposition : **There is a cut-off value of government costs g^e such that the country I is efficient if and only if $g \geq g^e$.**

This result is quite intuitive: to justify the superiority of the country I on pure efficiency grounds, it has to be the case that the advantages of economies of scale are sufficient to

compensate for the heterogeneity of preferences across citizens. Thus, high government costs would make it inefficient to break up a united country.

The next result formulates the *Minimal Division Principle* and is important for analysis of efficiency of the existing country. This principle asserts that for *any distribution of citizens' ideal points* the test of efficiency of country I has to be verified only against its break-up into two connected countries. That is, if country I is inefficient, then there exists a point $t \in I$ such that the break-up of I into $[0, t]$ and $[t, 1]$ would decrease the total cost in the country I , i.e.,

$$D(I) + g > D([0, t]) + g + D([t, 1]) + g,$$

or

$$g < D(I) - (D([0, t]) + D([t, 1])).$$

Thus, we have

Minimal Division Principle : The cut-off value of government costs g^e is given by

$$g^e = \text{Sup}_{t \in I} \{D(I) - D([0, t]) - D([t, 1])\}.$$

We now turn to the issue of stability. We assume that decisions on location of the government and the mechanism of financing its cost result from democratic process and are made via majority voting. The median voter theorem implies that the government of S is situated at the location of its median citizen. Following Alesina and Spolaore (1997), we assume at this point that the cost allocation chosen by the government provides no compensation to any group of citizens. This is precisely the *laissez faire* cost allocation, according to which the burden of the government cost is shared equally across the country and each citizen in country S pays a tax equal to $\frac{g}{\lambda(S)}$.

Since the focus of this section is *laissez faire* allocations, the notion of stability we examine here is that of *laissez faire stability*. The country would be stable in this regard if there is no region S such that every resident of S would prefer the *laissez faire* allocation in S over the *laissez faire* allocation in the united country I . The country I is *laissez faire stable* if it is immune against any threat of secession.

The region S is *prone to secession* if there exists a median $m \in M(S)$ such that:

$$d(t, m(I)) + g > d(t, m) + \frac{g}{\lambda(S)} \text{ for all } t \in S.$$

The country I is *laissez faire stable* if there is no region S prone to secession. That is, there exists a median $m(I) \in M(I)$ such that for every S and every $m \in M(S)$ there is $t \in S$

such that

$$d(t, m(I)) + g \leq d(t, m) + \frac{g}{\lambda(S)}.$$

The proposition below states that a country is stable if and only if economies of scale are large enough:

Proposition : There is a cut-off value of government costs g_{lf}^s such that the country is laissez faire stable if and only if $g \geq g_{lf}^s$.

Conditions for Efficiency and Laissez Faire Stability

To determine the range of government costs that ensure efficiency and laissez faire stability we will first identify the domain of cumulative distribution functions to be used in our analysis. First, we require

(Symmetry) The density function f is symmetric with respect to the center, i.e., $f(t) = f(1 - t)$ for all $t \in I$.

This assumption is quite standard. It implies that the geographical center $\frac{1}{2}$ of the country is also the median location in I .

To introduce the second assumption, we need some additional notation. For each $t \in I$, let L_t and R_t be the sets of citizens to the left and right of the point t , respectively, i.e., $L_t = [0, t]$ and $R_t = [t, 1]$. For the sets L_t and R_t denote by $L(t)$ and $R(t)$ their respective sets of median locations. We assume hereafter that they admit nondecreasing and continuous selections, which are differentiable except possibly in a finite number of points. In what follows, these selections will be denoted respectively l and r and by their derivatives by l' and r' . It is useful to mention that the symmetry of the distribution guarantees that for every selection l of L there exists a selection r of R such that for every $t \in I$

$$r(t) + l(1 - t) = 1.$$

For simplicity we assume that the set $L(1)$ consists of a single point. Therefore for any selections l and r , we will also have : $l(0) = 0$, $l(1) = \frac{1}{2}$, $r(0) = \frac{1}{2}$ and $r(1) = 1$. We assume:

(Gradually Escalating Median GEM) There exists a selection l such that $l'(t) \leq 1$ on the interval $[0, 1]$.

We denote by \mathcal{F} the set of distribution functions satisfying *SY* and *GEM*.

Assumption *GEM* implies that if we increase the length of the interval $L_t = [0, t]$ by a small positive number δ , then the median of the interval $L_{t+\delta} = [0, t + \delta]$ moves to the right by the increment smaller than δ . Obviously, if the distribution is symmetric the selection r of R , given by (3), satisfies also $r'(t) \leq 1$.

The class of distribution functions satisfying *GEM* contains the family of *log-concave functions*. That class, in turn, includes “truncated” versions of a large number of well-known distributions, such as the uniform, the normal and the exponential, among many others. There are, in addition, distribution functions that are not log-concave but nevertheless satisfy the GEM assumption, such as some classes of bimodal distributions (Le Breton and Weber (2003)).

We now turn to derivations of the threshold values g^e and g_{if}^s defined in the preceding section.

Proposition If the distribution F belongs to the class \mathcal{F} , then

$$(i) \quad g^e = \frac{1}{2} - 4 \int_{l(\frac{1}{2})}^{\frac{1}{2}} tf(t)dt,$$

and

$$(ii) \quad g_{if}^s = \text{Sup}_{0 \leq s < t \leq \frac{1}{2}} \frac{F(t) - F(s)}{1 - F(t) + F(s)} \left(\frac{1}{2} - 2t + m(s, t) \right),$$

where $m(s, t)$ denotes the median of the interval $[s, t]$.¹

The reason for computational complexity of the last formula is that, since a threat of secession should be countered for all intervals $[s, t]$, the supremum is taken over two parameters, s and t . If, however, the threat of secession is limited to the most distant regions (intervals $[0, t]$), the formula for g_{if}^s can be substantially simplified. Formally, we say that the property of *significance of distant regions - (SDR)* is satisfied if for any region S , which is prone to secession, the most distant region containing the same mass of citizens $\lambda(S)$ is also prone to secession. If SDR holds, the supremum in expression (ii) of Proposition 4.1 is attained when s is equal to zero. A close inspection of the expression for g_{if}^s also shows that the range of parameter t can be narrowed to $[0, t^*]$, where $t^* < \frac{1}{2}$ is the unique solution of the equation $\frac{1}{2} - 2t + l(t) = 0$, and so individuals close to the center will never be involved in secessions. Thus,

$$g_{if}^s = \text{Sup}_{0 \leq t \leq t^*} \frac{F(t)}{1 - F(t)} \left(\frac{1}{2} - 2t + l(t) \right).$$

This simplified formula cannot always be applied within the class \mathcal{F} . As an example will demonstrate, a major threat of secession may emerge from regions in the center rather than from those on the margins, contrary to the SDR property. To conclude the section, we provide a sufficient condition on distribution functions to satisfy the SDR property:

¹The formula for g_{if}^s remains valid for the distributions that satisfy *SY* but not *GEM*. Note also that the major vehicle in derivation of g^e and g_{if}^s is Proposition 3.3 that does not rely on the symmetry of the distribution. Thus, similar formulas could also be derived for continuous distributions that are not necessarily symmetric. However, since this extension would have no impact on our basic conclusions, we have chosen to limit our presentation to the symmetric case.

Proposition : SDR holds for any symmetric distribution function whose positive density f satisfies $f(t) \geq 2f(m(s, t))$ for all s, t with $0 \leq s \leq t \leq \frac{1}{2}$.

Polarization and Reconciliation of Efficiency and Stability

We now examine the issue of reconciliation of efficiency and stability. The first question to be addressed is whether efficiency of the country I would guarantee its laissez faire stability. In our framework it comes down to the comparison of the cut-off values g^e and g_{lf}^s . If g^e is greater than g_{lf}^s , then the efficiency of country I would imply its laissez faire stability. However, if the opposite inequality holds, there is a range of government costs that yields the efficiency but not the laissez faire stability of I . The objective of this section is to demonstrate that there is no unambiguous answer to this question. What is even more important is that the relationship between efficiency and laissez faire stability crucially depends on the polarization of the citizens' preferences in the country I . We shall demonstrate that efficient countries are laissez faire stable as long as the heterogeneity of nation's citizens does not generate an excessive degree of polarization of their preferences.

To formally address the issue of polarization one would require a definition of polarization index. Esteban and Ray (1994) and Foster and Wolfson (1994) (see also Wang and Tsui (2000)) suggest general principles and offer some particular families of indices of polarization. However, since they deal with finite distributions, there is some adjustment to be made for using these indices in the continuum framework. In this paper we adopt the index of polarization, proposed by Alesina, Baqir and Easterly (1999), that represents the "median distance to the median". In our framework it simply amounts to $\frac{1}{2} - l(\frac{1}{2})$, i.e., the distance between the median of the entire distribution and the median of its left (or right) half. This index has some desirable properties and is consistent with the first degree of stochastic dominance shifts. In particular, a symmetric shift of mass of citizens away from the center towards the margins, 0 and 1, would obviously move the median of the left half of the distribution, $l(\frac{1}{2})$, further to the left and increase the degree of polarization.

Let a be a parameter, satisfying $0 < a \leq \frac{1}{3}$. For every a , let F_a be the symmetric probability distribution on $[0, 1]$ whose density f_a is defined as follows:

$$f_a(t) = \begin{cases} \frac{1}{3a} & \text{if } t \in [0, a] \cup [\frac{1-a}{2}, \frac{1+a}{2}] \cup [1-a, 1] \\ 0 & \text{if } t \in (a, \frac{1-a}{2}) \cup (\frac{1+a}{2}, 1-a). \end{cases}$$

Straightforward computations lead to the following median sets of the distribution F_a on the

interval $[0, t]$:

$$L_a(t) = \begin{cases} \left\{ \frac{t}{2} \right\} & \text{if } t \in [0, a] \\ \left\{ \frac{a}{2} \right\} & \text{if } t \in (a, \frac{1-a}{2}) \\ \left\{ \frac{t}{2} - \frac{1}{4} + \frac{3a}{4} \right\} & \text{if } t \in [\frac{1-a}{2}, \frac{1+a}{2}] \\ \left[a, \frac{1-a}{2} \right] & \text{if } t \in (\frac{1+a}{2}, 1-a) \\ \left\{ \frac{t}{2} \right\} & \text{if } t \in [1-a, 1]. \end{cases}$$

The piecewise differentiable function $l_a(t)$ defined below represents a selection from $L_a(t)$:

$$l_a(t) = \begin{cases} \frac{t}{2} & \text{if } t \in [0, a] \\ \frac{a}{2} & \text{if } t \in (a, \frac{1-a}{2}) \\ \frac{t}{2} - \frac{1}{4} + \frac{3a}{4} & \text{if } t \in [\frac{1-a}{2}, \frac{1+a}{2}] \\ t + \frac{a-1}{2} & \text{if } t \in (\frac{1+a}{2}, 1-a) \\ \frac{t}{2} & \text{if } t \in [1-a, 1]. \end{cases}$$

Since $l'_a(t) \leq 1$ for all $t \in [0, 1]$, it follows that for all $a \in (0, \frac{1}{3}]$, F_a satisfies GEM.

Insert Figure 1 here

Note that in this example, the country consists of three groups, located at the intervals $[0, a]$, $[\frac{1-a}{2}, \frac{1+a}{2}]$, and $[1-a, 1]$. If $a = \frac{1}{3}$, the distribution F_a is simply uniform on the entire interval $[0, 1]$. However, if the parameter a decreases, the ‘‘homogeneity’’ within each of three groups raises, whereas intra-group ‘‘heterogeneity gap’’ widens. This, according to Esteban and Ray (1994), would lead to an increased degree of polarization within the country.

Indeed, the polarization index we use, denoted by γ , yields the following expression for every distribution function F_a :

$$\gamma_a(I) = \frac{1}{2} - \frac{3a}{4}.$$

Thus, the degree of polarization is a decreasing function of the parameter a , and within the class of functions F_a , the index is lowest when the distribution is uniform at $a = \frac{1}{3}$. This observation allows us to formulate our result concerning the relationship between efficiency, laissez faire stability and the degree of polarization:

Proposition : There exists a critical value $a^* \in (0, \frac{1}{3})$ such that:

whenever parameter a belongs to the range $[a^*, \frac{1}{3}]$, the value of g^e is greater or equal to g_{lf}^s ;

whenever parameter a belongs to the range $(0, a^*)$, the value of g^e is lower than that of g_{lf}^s .

This proposition has several important implications. It states that, within a class of distribution functions satisfying assumptions *SY* and *GEM*, there is a critical degree of polarization below which efficiency yields laissez faire stability. However, if a degree of polarization is above this cut-off value, then efficiency does not imply laissez faire stability and a redistribution is needed to eliminate possible secession threats. In that respect, we reinforce the necessity of transfers as a secession preventing policy.

Voting on Transfers

Now we turn to the study of compensation schemes that may emerge under the political mechanism of majority voting and relate it to the issue of polarization of citizens' preferences discussed in the previous section.

We assume that both decisions on the location of the government and the transfer scheme are decided by a majority voting in the country. For the sake of simplicity, let us further assume that, within each possible region S , only linear compensation schemes are considered and we restrict our attention to the S -cost allocation of the type

$$x(t) = \mu - \alpha |t - p|,$$

where p denotes the location of government, μ and α are parameters satisfying $\mu > 0, 0 \leq \alpha \leq 1$. Under this specification, the choice of a high value of α corresponds to a high compensation for those located far away from p . In particular, $\alpha = 1$ guarantees every citizen the full compensation for the disutility of distance that results in Rawlsian allocation. On the other hand, $\alpha = 0$ provides no compensation at all and gives rise to laissez faire allocation. The budget balance constraint $\int_S x(t) f(t) dt = g$ implies that

$$\mu(\alpha, S) = \frac{g}{\lambda(S)} + \alpha \hat{d}(S, p),$$

where

$$\hat{d}(S, p) = \frac{\int_S |t - p| f(t) dt}{\lambda(S)}$$

is the average distance of residents of region S to location p . Thus, considering compensation schemes, citizens of the country S , in effect, select a single parameter, the degree of equalization α . This is a two-dimensional voting problem and, as in Alesina and Spolaore (1997) and Alesina, Baqir and Easterly (1999), we avoid the issue of existence of an equilibrium by assuming that the voting is sequential. The citizens first vote on the compensation scheme and then on the location of the government. A voting equilibrium will be denoted by $(p^*(S), \alpha^*(S))$.

Note that for any choice of the policies α and p in region S within I , the total cost of citizen $t \in S$ is:

$$(1 - \alpha) |t - p| + \frac{g}{\lambda(S)} + \alpha \hat{d}(S, p).$$

It will sometimes be convenient to present the cost as

$$\alpha(\hat{d}(S, p) - |t - p|) + |t - p| + \frac{g}{\lambda(S)}.$$

With this new formulation, all citizens whose distance from p is more than the country average $\hat{d}(S, p)$, have single peaked preferences over α with a peak at 1, whereas all citizens whose distance from p is less than the country average, have single peaked preferences over α with a peak at 0.

To state our first result concerning the voting equilibrium, we again utilize the polarization index γ representing the median distance to the median. To simplify our discussion, let us assume that for every country S , its median $m(S)$ is uniquely defined. Then for every S we have:

$$\hat{d}(S, m(S)) = \frac{D(S)}{\lambda(S)},$$

where, to recall, $D(S)$ is the minimal aggregate transportation cost within S .

For every S , we define the polarization index $\gamma(S)$ representing the median distance to the median $m(S)$. That is, each of the two inequalities

$$|t - m(S)| \leq \gamma(S) \text{ and } |t - m(S)| \geq \gamma(S)$$

holds for at least 50% of the citizens of S .

The next proposition characterizes the voting equilibrium in S . It states that the location of the government at S is always chosen at its median. As far as the compensation scheme is considered, the solution is “bang-bang”: if the median distance to the median, $\gamma(S)$ is smaller than the average distance from the median, $\hat{d}(S, m(S))$ the equilibrium scheme entails no compensation and, in fact, results in the laissez faire allocation; but if the median distance to the median is larger than the average distance from the median, the unique equilibrium compensation scheme entails full compensation and generates the Rawlsian allocation. Thus, countries whose degree of polarization exceeds the value $\hat{d}(S, m(S))$, sustain full compensation as the only political equilibrium.

Proposition : Assume that $\hat{d}(S, m(S)) \neq \gamma(S)$. Then there exists a unique voting equilibrium defined by:

$$p^*(S) = m(S)$$

and

$$\alpha^*(S) = \begin{cases} 0 & \text{if } \hat{d}(S, m(S)) > \gamma(S) \\ 1 & \text{if } \hat{d}(S, m(S)) < \gamma(S). \end{cases}$$

Note also that in Proposition 6.1 we do not examine the situation when $\hat{d}(S, m(S)) = \gamma(S)$. In this case we have a tie between two extreme solutions $\alpha = 0$ and $\alpha = 1$. (This happens, for instance, when F is the uniform distribution.) In this case, one may introduce a small administrative cost of public funds that would break the tie, as in Alesina and Spolaore (1997), in favor of the laissez faire allocation.

Let us determine now the equilibrium compensation scheme for the entire nation. In the case where the function f is symmetric, it is easy to verify that $\alpha^*(I) = 1$ if and only if

$$2 \int_0^{\frac{1}{2}} tf(t)dt > l\left(\frac{1}{2}\right),$$

and the country votes for full compensation if the mean of the distribution on $[0, \frac{1}{2}]$ is greater than its median on that interval. Whether this is indeed the case, obviously depends on the type of the distribution function and, as the following two examples of the distribution functions from the class \mathcal{F} indicate, the above inequality may or may not hold.

Example : Consider a quadratic cumulative distribution function, whose density $f(t)$ is given by

$$f(t) = \begin{cases} 4t & \text{if } t \leq \frac{1}{2} \\ 4 - 4t & \text{if } t \geq \frac{1}{2}. \end{cases}$$

Then $2 \int_0^{\frac{1}{2}} tf(t)dt = \frac{1}{3}$, whereas $l(\frac{1}{2}) = \frac{1}{\sqrt{8}}$. Thus, the inequality is violated and the laissez fare allocation $\alpha^*(I) = 0$ is the voting equilibrium.

Example : Consider the bimodal cumulative function, whose density $f(t)$ is given by

$$f(t) = \begin{cases} \frac{4}{3} - \frac{4}{3}t & \text{if } t \leq \frac{1}{2} \\ \frac{4}{3}t & \text{if } t \geq \frac{1}{2}. \end{cases}$$

Then $2 \int_0^{\frac{1}{2}} tf(t)dt = \frac{2}{9}$ whereas $l(\frac{1}{2}) = \frac{4-\sqrt{10}}{4} \approx 0.21$. Thus, (6) holds and therefore the full compensation scheme $\alpha^*(I) = 1$ emerges as the only political equilibrium.

Since the equilibrium notion in this section is different from that in Section 3, the definition and analysis of stability there should be modified. Indeed, it should take into account the compensation schemes $\alpha^*(I)$ in the country I and $\alpha^*(S)$ for any country S that contemplates a threat of secession.

The region S is *p-prone to secession* if, given voting equilibria in I and S , all citizens of S would be better off if S secedes, i.e., the following inequality

$$(1 - \alpha^*(S))|t - m(S)| + \frac{g + \alpha^*(S)D(S)}{\lambda(S)} < (1 - \alpha^*(I))|t - m(I)| + g + \alpha^*(I)D(I).$$

holds for all $t \in S$. The country I is *politically stable* if there is no region *p-prone to secession*.

Proposition : Assume that $D(I) \neq \gamma(I)$. Then there is a cut-off value of the government costs, denoted g^p , such that the country is politically stable if and only if $g \geq g^p$.

It is natural to study a link between efficiency and political stability. The following example exhibits a country that is efficient but is politically unstable.

Example : Consider the following symmetric distribution on $[0, 1]$:

region S_1 - $\frac{3}{10}$ of the total population located in 0,
region S_2 - $\frac{4}{10}$ of the total population located in $\frac{1}{2}$,
region S_3 - $\frac{3}{10}$ of the total population located in 1.

Proposition 4.1 yields $g^e = \frac{3}{20}$. Turn now to the issue of secession. For the country I , $m(I) = \frac{1}{2}$ and since $\gamma(I) = \frac{1}{2}$ and $D(I) = \int_I |t - m(I)| f(t) dt = \frac{3}{10}$, we have $\alpha^*(I) = 1$.

For three regions S_1, S_2 , and S_3 , the choice of α is irrelevant since there would be no transportation costs in the case of their secession. For regions $S_1 \cup S_2$ and $S_2 \cup S_3$, the median is located at $\frac{1}{2}$ and, obviously, $\alpha^*(S) = 0$. For region $S_1 \cup S_3$, α^* is equal to zero and the average distance to the median is $\frac{1}{2}$. Then country I is politically stable if the following four inequalities holds:

$$\begin{aligned} g + \frac{3}{10} &\leq \frac{10g}{3} \text{ (for regions } S_1 \text{ or } S_3), \\ g + \frac{3}{10} &\leq \frac{10g}{4} \text{ (for region } S_2), \\ g + \frac{3}{10} &\leq \frac{10g}{7} + \frac{1}{2} \text{ (for regions } S_1 \cup S_2 \text{ or } S_2 \cup S_3), \\ g + \frac{3}{10} &\leq \frac{10g}{6} + \frac{1}{2} \text{ (for region } S_1 \cup S_3). \end{aligned}$$

These inequalities are satisfied if $g \geq g^p = \frac{1}{5}$. Since $g^p > g^e$, it follows that the country I can be efficient without being politically stable.

It is instructive to point out that in this example a threat of secession comes from the center S_2 . Indeed, if joined by the central region S_2 , the more distant regions S_1 or S_3 would have sufficient number of citizens to enforce full equalization through the voting mechanism.

We will show now that efficiency can be restored through the transfer mechanism. Consider a symmetric cost allocation that assigns the total cost x to every citizen in regions S_1 and S_3 and y to every citizen in region S_2 . To be immune to the threat of secession, it has to satisfy the following list of inequalities:

$$\begin{aligned} x + \frac{2y}{3} &= \frac{10g}{6} + \frac{1}{2} \text{ (budget constraint),} \\ x &\leq \frac{10g}{3} \text{ (secession threat of regions } S_1 \text{ or } S_3), \\ y &\leq \frac{10g}{4} \text{ (secession threat of region } S_2), \\ x + \frac{4y}{3} &\leq \frac{10g}{3} + \frac{1}{2} \text{ (secession threat of regions } S_1 \cup S_2 \text{ or } S_2 \cup S_3), \\ x &\leq \frac{10g}{6} + \frac{1}{2} \text{ (secession threat of region } S_1 \cup S_3). \end{aligned}$$

From the first and the last inequalities we obtain $y \geq 0$. That is, the citizens in S_2 must make a nonnegative contribution. Since $x = \frac{10g}{6} + \frac{1}{2} - \frac{2y}{3}$, the three intermediate inequalities yield

$$\text{Max}(0, \frac{3}{4} - \frac{5g}{2}) \leq y \leq \frac{5g}{2}.$$

We can see that a solution to the inequality above exists if and only if g is greater or equal to the value $\frac{3}{20} = g^e$, the point where the two lines intersect. This demonstrates that we can restore efficiency by using transfers. Figure 2 identifies the values of y that allow prevention of secession threats. It is interesting to note that moderate effects of economies of scale (g is close to g^e) yield a narrow range of secession-proof compensation schemes.

An Example with a Threat of Secession from the Center

Let x, y be positive numbers with $x < y < \frac{1}{2}$. Consider the following distribution, which, obviously, is not log-concave:

region S_1 - 12% of the population is in 0,

region S_2 - 12% of the population is in x ,

region S_3 - 20% of the population is in y ,

region S_4 - 12% of the population is in $\frac{1}{2}$,

region S_5 - 20% of the population is in $\frac{1}{2} - y$,

region S_6 - 12% of the population is in $\frac{1}{2} - x$,

region S_7 - 12% of the population is in 1.

If we consider only the intervals with the endpoint at 0, the cut-off value of g is the lowest value of g for which the regions S_1 , $S_1 \cup S_2$ and $S_1 \cup S_2 \cup S_3$ are not prone to secession. Thus, the following three inequalities hold:

$$\frac{100}{12}g \geq g + \frac{1}{2}, \quad \frac{100}{24}g \geq g + \frac{1}{2} - x, \quad \frac{100}{44}g + y - x \geq g + \frac{1}{2} - y.$$

After rearranging the terms, we obtain

$$g \geq \text{Max}\left(\frac{3}{44}, \frac{3}{19} - \frac{6}{19}x, \frac{11}{28} - \frac{22}{28}(2y - x)\right). \quad (1)$$

On the other hand, the region consisting of S_2 and S_3 is not prone to secession if

$$g \geq \frac{4}{17} - \frac{8}{17}y. \quad (2)$$

Now choose $g = 0.157$ and $y = 0.164$. Then inequality (2) is violated. However, it is easy to verify that inequality (1) is strict as long as $0.003 < x < 0.027$. Thus, for these parameter values, the region $S_2 \cup S_3$ is prone to secession whereas the regions containing S_1 are not.

The intuition here is clear. The individuals located in 0 wish to join prone to secession region $S_2 \cup S_3$. However, they cannot commit, once in, to maintain the location of the seceding government at y . Instead, by joining in, the region S_1 would shift the median of the new region from y to x . This shift, however, is going to be rejected by the region S_3 that would be unwilling to join the secession.

Polarization and Heterogeneity

The cutpoint $g^e \equiv g^{st}$ called hereafter the *stability threshold* or *unity index* quantifies the minimal returns to size that are sufficient to prevent credible secession threats. As we mentioned in the introduction, this threshold can be viewed as the *minimal* burden on the country which still guarantees its unity. It is quite easy to observe that the notions of stability and secession-proofness are closely linked to the cost of public good. Indeed, a high cost of public good may facilitate regional cooperation and mitigate a threat to instability posed by regions. On the other hand, a low cost of public goods could reduce incentives for economic unity and raise the intensity of secession threats.

The natural question we address in this paper is the investigation of the link between the stability threshold a degree of the country's polarization. In order to do so, in the next section we proceed with examination of polarization index.

Indices of polarization, introduced in Esteban and Ray (1994), Duclos, et al. (2004) Tsui and Wang (2000), are based on the notions of identification within one's own group and alienation towards the others. For a continuous cumulative distribution function F on $[0, 1]$, Duclos et al. (2004) have derived the following polarization index $\gamma_\alpha(F)$:

$$\gamma_\alpha(F) = \int_0^1 \int_0^1 |x - y| f(x)^{1+\alpha} f(y) dx dy,$$

where f is the density function of F , and the parameter α satisfies $0.25 \leq \alpha \leq 1$. If F is a discrete distribution supported on the set $\{x_0, \dots, x_n\}$, and p_i is the probability of x_i , the index (derived by Esteban and Ray (1994)) is given by

$$\gamma_\alpha(F) = \sum_{i=0}^n \sum_{j=0}^n p_i^{1+\alpha} p_j |x_i - x_j|,$$

where the parameter α belongs to the interval $[0, \alpha^*]$, where $\alpha^* \approx 1.6$. To cover both infinite and finite cases, we assume throughout the rest of the paper that $0.25 \leq \alpha \leq 1$, so that both (1) and (2) hold.

As alluded before, our analysis of conflicts will be performed under the assumption that citizens' ideal points form several disjoint clusters (that represent geographical regions or

groups with similar political views). This will highlight the following two attributes of conflict situations (in addition to the existence of clusters). The first is heterogeneity of preferences within clusters, which represents conflicts within each region or group. The second is reflected by the number of distinct groups within the society, when a smaller (but greater than one) number of clusters represents a higher degree of confrontation. In order to focus solely on these two factors and eliminate other effects, we shall consider a family of step distribution functions with the support over a finite number of equal intervals (clusters). We shall also assume complete uniformity of the distribution of citizens' ideal points within each cluster. Thus, all distributions in our class \mathcal{F} will be characterized by two parameters, the number of clusters, n and their length, a .

Formally, let an integer $n \geq 2$ and the parameter $a \in (0, \frac{1}{n}]$ be given. Consider a function $f_{n,a}$ on the unit interval $[0, 1]$:

$$f_{n,a}(t) = \begin{cases} \frac{1}{na} & \text{if } t \in [j\frac{1-a}{n-1}, j\frac{1-a}{n-1} + a] \text{ for } j = 0, 1, \dots, n-1 \\ 0 & \text{otherwise.} \end{cases}$$

That is, $f_{n,a}$ is the density function of the distribution which is supported and uniform on the n intervals of length a , removed from each other by the same distance. Denote the corresponding distribution by $F_{n,a}$. We also introduce $\{F_{n,0}\}$ for $n \geq 2$, which is a discrete limiting distribution of $\{F_{n,a}\}$ for $a \in (0, \frac{1}{n}]$. That is, $F_{n,0}$ is supported, and is uniform, on the finite set that consists of n equidistant points $\{0, \frac{1}{n-1}, \frac{2}{n-1}, \dots, \frac{n-1}{n-1} = 1\}$.

Now, as in Duclos et al. (2004), let $0.25 \leq \alpha \leq 1$, and denote $\gamma_\alpha(n, a) \equiv \gamma_\alpha(F_{n,a})$. We have the following expression for the polarization index:

$$\gamma_\alpha(n, a) = \begin{cases} \left(\frac{1}{na}\right)^\alpha \frac{n+1-na}{3n}, & \text{if } a > 0 \\ \left(\frac{1}{n}\right)^\alpha \frac{n+1}{3n}, & \text{if } a = 0. \end{cases}$$

Obviously, the distribution $F_{n,a}$ becomes less polarized when a or n increase: The polarization index $\gamma_\alpha(n, a)$ declines in each of its two variables.

According to our interpretation, the dependence of $\gamma_\alpha(n, a)$ on a describes the fixed-clusters polarization effect (*FCPE*), while its dependence on n reflects the variable-clusters polarization effect (*VCPE*). Thus, both effects reduce the polarization index.

It is worth pointing out that the index γ_α exhibits discontinuity in the transition from continuous distributions $F_{n,a}$ for $a > 0$ to $F_{n,0}$: $\lim_{a \searrow 0} \gamma_\alpha(n, a) = \infty$. The reason is that according to this index, discrete distributions are infinitely more polarized than continuous ones (due to the presence of infinitely dense clusters in former). The index still allows comparisons of discrete distributions $\{F_{n,0}\}$, via (??), but they belong to a different (higher) league of polarization when it comes to comparing them with continuous distributions $\{F_{n,a}\}$.

The index should not therefore be used for comparisons across these two subfamilies of distributions, but only for comparisons inside each subfamily.

The Linkage between the Stability Threshold and Polarization

We study how the stability threshold reacts to changes in polarization. First, we explicitly calculate the stability threshold for the distributions in our class. For every function $F_{n,a} \in \mathcal{F}$ we shall use a notation $g^{st}(n, a)$ instead of $g^{st}(F_{n,a})$.

Proposition : For $n \geq 2, a \in [0, \frac{1}{n}]$, the stability threshold $g^{st}(n, a)$ is given by:

$$g^{st}(n, a) = \frac{1}{8} \left(1 + (1 - an) \frac{1 + \frac{4}{n} (\lfloor \frac{n+2}{4} \rfloor - \lfloor \frac{n+1}{4} \rfloor)}{2^{\lfloor \frac{n-1}{2} \rfloor} + 1} \right),$$

where $\lfloor x \rfloor$ stands for the *integer part* of x , i.e., for the largest integer that does not exceed x .

We now turn to our conclusions:

- Proposition :** (i) The stability threshold is positively correlated with FCPE. That is, the increase in a for fixed n reduces both the polarization index and the stability threshold.
- (ii) The link between the stability threshold and VCPE is ambiguous. That is, while an increase in n reduces the polarization index $\gamma_\alpha(n, a)$, it does not necessarily reduce, or increase, the stability threshold $g^{st}(n, a)$ for fixed a .
- (iii) The VCPE is strong enough to make the combined effect of FCPE and VCPE on the stability threshold ambiguous as well. That is, even if both n and a increase, thereby reducing the polarization index $\gamma_\alpha(n, a)$, this does not necessarily reduce, or increase, the stability threshold $g^{st}(n, a)$.

Thus, in general, the relationship between polarization and stability is not monotone. According to the above proposition, the stability threshold of $F_{n,a}$ decreases with the increase of a (and the implied fall in the distribution's polarization) for fixed n , but, in general, is not monotone in $\gamma(n, a)$ for a given a . To illustrate this point, consider the finite case $a = 0$ and denote by $S(n, t)$ the set of citizens that for given n are located at t . As it is shown in the proof of the Proposition, the first deviation from monotone decline of $g^{st}(n, 0)$ in n occurs when $n = 6$. Indeed, $g^{st}(6, 0) = \frac{1}{6} > g^{st}(5, 0) = \frac{3}{20}$. The reason for this spike, alluded to in the introduction, can be explained as follows. When $n = 5$, the “central cluster” $S(5, \frac{1}{2})$ (which does not exist when $n = 6$) makes secessions difficult. Indeed, in the united country scenario the optimally chosen government location is also at the center. The existence of a relatively big central cluster (which incurs zero transportation cost) has a mitigating effect on the aggregate transportation cost burden. And, if we consider the sets $S(5, 0) \cup S(5, \frac{1}{4})$ or $S(5, \frac{3}{4}) \cup S(5, 1)$, none has a “central block” with zero transportation cost. This means that

these regions would incur quite high transportation costs in the case of secession. However, the situation changes drastically when $n = 6$. In this scenario, there is no central cluster in I to mitigate aggregate transportation cost; on the other hand, each of the “secession-relevant” regions $S(6, 0) \cup S(6, \frac{1}{5}) \cup S(6, \frac{2}{5})$ or $S(6, \frac{3}{5}) \cup S(6, \frac{4}{5}) \cup S(6, 1)$ have central clusters which help to reduce transportation costs in the case of secession. This makes secession more likely and the country less stable for $F_{6,0}$, compared to the more polarized distribution $F_{5,0}$. (This argument can only be made for the switch from $n = 4m + 1$ to $4m + 2$ for a positive integer m . Indeed, we only observe upward jumps in $g^{st}(n, 0)$ at $n = 4m + 2$ as the proof of Proposition 4.2 will show. To see why the argument cannot be extended, consider for instance the case of $n = 3, 4$. When $n = 3$, aggregate transportation costs are mitigated by the central cluster, $S(3, \frac{1}{2})$, but the extreme clusters $S(3, 0)$ and $S(3, 1)$ would now incur no transportation costs in case of secession, and so are relatively less deterred from seceding compared to the case of $n = 5$. And when $n = 4$, there is no central cluster to mitigate the transportation cost, but $S(4, 0) \cup S(4, \frac{1}{3})$ or $S(4, \frac{2}{3}) \cup S(4, 1)$ do not have central clusters either, which reduces their incentives to secede compared to the case of $n = 6$. By Proposition 4.1, here $g^{st}(4, 0) = \frac{1}{6} = g^{st}(3, 0)$, and so indeed the less polarized country is not less stable.)

It is worthwhile to note that, for a positive fixed a , the decline of $g^{st}(n, a)$ in n is restored if the value of n is large enough (and thus polarization is low):

Proposition : For every $0 < a < 1$, there exists a value $n(a)$ such that $g^{st}(n_1, a) \leq g^{st}(n_2, a)$ whenever $n_1 > n_2 > n(a)$ and $n_1 a \leq 1$.

The Stable Number of Countries and Polarization Indices

When the government cost is low, the united nation is no longer stable. The question that we analyze now is what is the number of smaller countries that could guarantee the stability of partition of $[0, 1]$.

Denote by $\bar{K}(g, n, a)$ the maximal number of countries in a stable partition of I (when the distribution of ideal points is $f_{n,a}$ and the government cost is g), and by $\underline{K}(g, n, a)$ – the minimal number of countries. For simplicity, we will focus attention on $K(g, n, a) = \underline{K}(g, n, a)$; all our observations apply to $\bar{K}(g, n, a)$ just as well. We shall call $K(g, n, a)$ the *stable number* of countries. It is natural to ask how it is affected by the change in $\gamma_\alpha(n, a)$, the polarization degree of $f_{n,a}$.

First, it turns out that K does not, in general, behave monotonically in the polarization degree. Indeed, pick $g_0 \in (\frac{3}{20}, \frac{1}{6})$. Then, since $g^{st}(4, 0) = g^{st}(6, 0) = \frac{1}{6} > g_0$, and $g^{st}(5, 0) =$

$\frac{3}{20} < g_0$, we have

$$K(g_0, 4, 0), K(g_0, 6, 0) > 1, \text{ and } K(g_0, 5, 0) = 1.$$

Moreover, since $g^{st}(n, a)$ is continuous in a for a fixed n , for all positive and sufficiently small a_4, a_5 , and a_6

$$K(g_0, 4, a_4), K(g_0, 6, a_6) > 1, \text{ and } K(g_0, 5, a_5) = 1.$$

Consequently:

Proposition : The stable number of countries is not monotone in the polarization degree. That is, while a simultaneous increase of both n and a reduces the polarization index $\gamma_\alpha(n, a)$, it does not necessarily decrease, or increase, the stable number $K(g, n, a)$ for a given c .

The example on which this corollary is based utilizes relatively high values of g . It turns out that for low values of c the stable number *does* behave monotonically in the polarization index: it decreases with polarization, as we show below. Intuitively, this reflects the fact that in a very polarized society each cluster is relatively uniform, and hence, when separated from others, can exist as a separate and stable country even when the government cost is very low. Thus, for a wide range of low g , highly polarized I should not be split into more countries than there are clusters, which keeps the stable number bounded. However, when the society is not polarized, and its members' preferences are spread uniformly, low g necessitates a very fine partition to achieve stability, because of the wide spread of preferences.

Proposition : Given two integers $2 \leq n_1 \leq n_2$ and $0 \leq a_1 \leq a_2 \leq \frac{1}{n_2}$, there exists $g(n_1, n_2, a_1, a_2) > 0$ such that for every $0 < g \leq g(n_1, n_2, a_1, a_2)$,

Miscellanies

The above developments represent some preliminary insights on the relationships between the social heterogeneity and the social outcome in the case where the social outcome consists of a partition of the initial population into several autonomous nations with or without the implementation of compensation transfers within each of these nations. We have focused on the dependency of several efficiency and stability thresholds upon some parameters of the density describing the distribution of a one dimensional characteristic within the population. This analysis can be pursued along several different lines :

- We could examine the patterns of social heterogeneity for which there do not exist of stable coalition structures. Instability represent a strong form of chaos as no partition can accomodate the existing conflicts of interest. This possibility may arise when compensation schemes are prohibited as illustrated by the following example :

Consider a society with eight agents, located at the points $l^1 = 0$, $l^2 = l^3 = l^4 = A = \frac{1}{42}$, $l^5 = B = \frac{1}{14} = \frac{3}{42}$ and $l^6 = l^7 = l^8 = C = \frac{11}{42} - \delta$, where δ is a small positive number

In this example there are three groups of agents: agent 1 at 0 on the left, agents 2, 3, 4, 5, located in the middle at points A and B , and agents 6, 7, 8 located at C on the right. The last three should be together in the same jurisdiction; moreover, they would prefer to be together with 2, 3, 4, 5 in the jurisdiction $N \setminus \{1\}$. But if $N \setminus \{1\}$ forms, then agent 1, together with the group 6, 7, 8 would form a blocking jurisdiction. Next, if $\{1, 6, 7, 8\}$ forms, the agents 2, 3, 4, 5 would offer agent 1 to form a blocking jurisdiction $\{1, 2, 3, 4, 5\}$. But then, again, agents 6, 7, 8 together with agents 2, 3, 4, 5 are better off in the jurisdiction $N \setminus \{1\}$. Thus, there is a blocking cycle of “dividing a dollar” type game, and since the grand jurisdiction N is core stable, no core stable partition would emerge. A complete proof is following.

denote $X = \{2, 3, 4\}$, $Y = \{6, 7, 8\}$. Suppose, in negation, that there is a stable partition P .

Note first, that partition P cannot assign every member of Y the total cost of more than $\frac{1}{3}$, since otherwise Y would block P . We proceed in several steps:

Step 1. Suppose that there is a jurisdiction $S \in P$ such that $|S \cap Y| = 1$. Then $i \in Y \cap S$ contributes more than $\frac{1}{3}$. Indeed, if $|S| \leq 2$, i pays more than $\frac{1}{3}$. If $|S| \geq 3$ then $m(S) \leq B$ and i contributes at least $\frac{1}{6} + (C - B) = \frac{7}{42} + \frac{8}{42} - \delta = \frac{15}{42} - \delta > \frac{15}{45} = \frac{1}{3}$.

Step 2. Suppose that there is a jurisdiction $S' \in P$ such that $|S'| > 3$ and $|S' \cap Y| = 2$. Then both $j, k \in Y \cap S'$, $j \neq k$, contribute more than $\frac{1}{3}$. Indeed:

If $|S'| = 7$, then j and k contribute each $\frac{1}{7} + C - A = \frac{1}{7} + \frac{10}{42} - \delta > \frac{1}{3}$.

If $4 < |S'| < 7$, then $m(S') \leq B$, and j and k contribute each at least $\frac{1}{6} + C - B > \frac{1}{3}$.

If $|S'| = 4$, then j and k pay each at least $\frac{1}{4} + \frac{C-B}{2} = \frac{1}{4} + \frac{4}{42} - \frac{\delta}{2} > \frac{1}{3}$.

Step 3. All agents in Y stay in the same jurisdiction in P . If the members of Y are split among three jurisdictions, then each member of Y contributes more than $\frac{1}{3}$, and P is blocked by Y . If the members of Y are split among two jurisdictions, S and S' , then $|S' \cap Y| = 2$ and $|S \cap Y| = 1$. If either $|S'| > 3$ or $|S'| = 2$, then, according to Steps 1 and 2, all agents from Y contribute more than $\frac{1}{3}$, which is impossible.

Hence, we must have $|S'| = 3$, yielding $m(S') = C$. But in this case the jurisdiction $S' \cup Y$ would block P .

Thus, there is a jurisdiction $T \in P$ such that $Y \subset T$.

Step 4. Then $|T| > 3$, i.e, $T \neq Y$. Indeed, if $T = Y$ then contributions of members of T equal to $\frac{1}{3}$, and contributions of other agents are not lower than $\frac{1}{5}$, for there are only 5 other agents. Now, observe that $S^1 = Y \cup X \cup \{5\}$ blocks P , as the contributions of members of Y and X at S^1 are $\frac{1}{7} + C - B$ and $\frac{1}{7} + B - A$, being lower than their contributions at P , which are $\frac{1}{3}$ and $\frac{1}{5}$, respectively.

Step 5. $|T| \geq 7$. Indeed, if $4 \leq |T| \leq 6$, the contributions of agents from $N \setminus T$ are not lower than $\frac{1}{4} > \frac{2}{42} + \frac{1}{5}$, whereas $m(T) \geq \frac{B+C}{2} \geq \frac{7}{42} - \frac{\delta}{2}$, hence, contributions of members of $T \setminus Y$ are not lower than $\frac{1}{6} + \frac{4}{42} - \frac{\delta}{2} > \frac{2}{42} + \frac{1}{5}$; this means that the coalition $N \setminus Y$ would block P , for members of this coalition contribute not higher than $\frac{2}{42} + \frac{1}{5}$.

Step 6. $|T| \neq 7$. Indeed, let jurisdiction $|T|$ contain 7 members and consider an agent $i \in N \setminus T$. Since contributions of members of Y in T are not lower than $\frac{1}{7} + C - B = \frac{1}{3} - \delta > \frac{1}{4}$, and the contribution of i in P is equal to 1, it follows that $Y \cup \{i\}$ blocks P .

Step 7. T is not the grand jurisdiction. Indeed, since $\frac{1}{8} + C - B + \frac{B-A}{2} > \frac{1}{3}$, the grand jurisdiction N is blocked by Y .

Therefore, a stable partition does not exist

- Even, when stable structure exist, they may fail to display expected features. For instance, it could be the case that none of these stable structures is consecutive.

A consecutive core stable jurisdiction structure may fail to exist even if the set of core stable jurisdiction structures is nonempty.

Consider the following example with 34 agents, where $l^1 = 0$, $l^2 = \dots = l^{14} = a$, $l^{15} = l^{16} = l^{17} = b$, $l^{18} = \dots = l^{27} = c$, and $l^{28} = \dots = l^{34} = d$. The exact values of a, b, c, d are defined below. We claim that for this society, a set of core stable jurisdiction structures, while being nonempty, contains no consecutive partitions.

There are three groups of agents: agent 1, located at the far left, a large group C of 26 agents, located around the middle (but not at the same location), and a medium size group R of 7 agents located on the right. The large group C would not admit either of smaller groups, since such an admission would shift the median of C and could make some of its members worse off. At the same time the group R would accept agent 1, who, given increasing returns to scale, would be happy to join a larger group. Thus, there is a core stable partition which consists of two jurisdictions, the group in the middle C and the union of two smaller groups

$\{1\} \cup R$, while there are no consecutive core stable jurisdiction structures. A complete proof is following.

Denote $N^a = \{2, \dots, 14\}$, $N^b = \{15, 16, 17\}$, $N^c = \{18, \dots, 27\}$, $R = \{28, \dots, 34\}$, and $C = N^a \cup N^b \cup N^c$ (R for “right”, and C for “center”.) The locations a, b, c, d obtain the following values:

$$\begin{aligned} a &= \frac{1}{8}; & b &= \frac{1}{8} + \frac{2}{26 \cdot 27} + 2\varepsilon; \\ c &= \frac{1}{8} + \frac{2}{26 \cdot 27} + \frac{2}{33 \cdot 34} + 2\varepsilon + 2\delta; \\ d &= \frac{1}{8} + \frac{2}{26 \cdot 27} + \frac{2}{33 \cdot 34} + \frac{26}{33 \cdot 7} + 2\varepsilon + 2\delta - \xi, \end{aligned} \tag{3}$$

where ε, δ and ξ are very small positive numbers. We shall show that the set of core stable partitions, while nonempty, does not contain a consecutive partition.

We will utilize the following four observations:

(i) $\frac{1}{8} < \frac{13}{33 \cdot 7} + \frac{1}{14} < \frac{1}{33} + d - c < \frac{1}{7} < \frac{1}{34} + d - c + \frac{c-b}{2}$. This implies that all members of group R have the following preferences:

$$R \cup Q \succ R \cup Q' \succ R \cup C \succ R \succ R \cup C \cup \{1\}, \tag{4}$$

where Q is a nonempty jurisdiction with fewer than 7 agents, and Q' is a jurisdiction with exactly 7 agents. That is, all agents located at d , would prefer to be joined by at least one, but no more than six, other agents, to being in the jurisdiction with 7 other agents; the latter outcome is preferred to being in the jurisdiction with all other agents, excluding 1. This, in turn, is preferable to forming jurisdiction R , whereas the grand coalition is the least desired option among those listed here.

(ii) $\frac{1}{33} + c - a < \frac{1}{27}$. This implies that all members of C prefer $C \cup R$ to both $C \cup \{1\}$ and C .

(iii) $\frac{1}{26} + c - b + \frac{b-a}{2} < \frac{1}{23}$. This implies that all members of C prefer C to participating in a jurisdiction with no more than 23 agents.

(iv) $\frac{1}{26} < \frac{1}{27} + \frac{b-a}{2}$. The members of N^c prefer C to $C \cup \{1\}$, and thus, would be worse off when agent 1 joins C .

We first show that the partition \bar{P} into two jurisdictions, C and $\{1\} \cup R$, is core stable.

Observation (i) implies that no member of group R would engage in blocking within the jurisdiction whose median is to the left of d . Thus, a jurisdiction S , that contains a member of R , could block only if its median $m(S)$ is located at d , implying that $|S| < 14$. However, observation (iii) guarantees that no member of C would find it profitable.

Thus, it remains to consider possible blocking threats to \bar{P} from jurisdictions $S \subset \{1\} \cup C$. Observation (iv) implies that $\{1\} \cup C$ itself cannot block. The case $S = \{1\}$

is, obviously, impossible; hence, S contains some of agents from C . Observations (iii) and (iv) imply that $23 \leq |S| \leq 26$ and S contains both types of agents in C , those are located to the left and those are located to the right of $m(C) = \frac{a+b}{2}$. Since the contribution to finance a project in S is at least as much as in C , those agents from S to whom $m(S)$ is not closer than $m(C)$ would reject the membership in S , a contradiction which shows that \bar{P} is indeed, core stable.

Now assume, in negation, that there is a consecutive core stable partition P .

First, consider the case where there is $S \in P$ that contains C .

If $S \cap R = \emptyset$, then P is either $\{\{1\}, C, R\}$ or $\{\{1\} \cup C, R\}$. But both would be blocked by $C \cup R$, as by observation (i), group R prefers $C \cup R$ to R , and by observation (ii), C prefers $C \cup R$ to both $C \cup \{1\}$ and C .

If $S \cap R \neq \emptyset$, then S does not contain agent 1 (otherwise $m(S) \leq m(N)$, the grand coalition, and by observation (i), R would block P). But then $\{1\} \cup R$ blocks P .

Consider now the case where C is not a subset of any jurisdiction from P . Observation (iii) implies that if all jurisdictions in P contain no more than 23 agents, C would block P . Hence, there is a jurisdiction $S \in P$ with $|S| \geq 24$. Obviously, $m(S) \leq c$, because no more than 7 agents from S are located to the right of c .

Let $S \cap R \neq \emptyset$. Then, $|S| \geq 33$, since otherwise $1/|S| + 26/(33 \cdot 7) - \xi > 1/7$ and R will block P . But if $|S| \geq 33$, then consecutiveness implies that $C \subset S$, a contradiction.

Let $S \cap R = \emptyset$. Since S is consecutive, the group $(\{1\} \cup C) \setminus S$ contains two (possibly empty) consecutive groups of agents, denoted by X_l and X_r (X_l is to the left of X_r).

We claim that all the agents in R belong to the same jurisdiction T . Indeed, if it is not the case then either there exists a (unique, due to consecutiveness!) jurisdiction $Q \in P$ with $Q \setminus R \neq \emptyset$, and then $R \cup Q$ will block P , or not, in which case R itself will block P .

We now have three cases:

Case 1. $|X_l| = 3$ (hence, $X_r = \emptyset$). In this case each of two agents from $X_l \setminus \{1\}$ will be better off by joining S . The same holds for every member of S , since $m(S \cup \{i\}) = m(S) = b$, where $i \in X_l \setminus \{1\}$. Hence, P is blocked by a coalition $S \cup \{i\}$.

Case 2. $0 < |X_l| < 3$. Then, an agent 1 contributes more than $\frac{1}{2}$. At the same time if she joins a jurisdiction T , her contribution would be not higher then $\frac{1}{8} + d < \frac{1}{2}$, and, again, her migration does not affect $m(T)$. Hence, $T \cup \{1\}$ blocks P .

Case 3. $X_l = \emptyset$. Then, $X_r \neq \emptyset$, since otherwise S contains C , the case which has been already covered. Moreover, we have $m(S) = a$. If a member of X_r migrates to

jurisdiction S , she would pay less than in P (where her contribution is higher than $\frac{1}{10}$). Since $m(S) = m(S \cup \{i\}) = a$, it follows that $S \cup \{i\}$ is a blocking coalition.

Therefore, there is no consecutive core stable partition. \square

Even though a core stable structure may fail to exist, there are societies that do admit core stable (and even consecutive) jurisdiction structures. One such class is *equidistant* societies, where the distance between every two adjacent agents is the same. Formally, a society N is equidistant if there exists $l > 0$, such that $l^i - l^{i-1} = l$ for $i = 2, \dots, n$. We have

Proposition : Every equidistant society admits a core stable consecutive jurisdiction structure.

However, even in this case, there could exist a non-consecutive core stable jurisdiction structure. To show an example of an equidistant society with a core stable non-consecutive jurisdiction structure, consider eight agents $\{1, \dots, 8\}$, and their two-jurisdiction partition $P = \{\{2, 3, 4, 5\}, \{1, 6, 7, 8\}\}$. We choose the distance l between adjacent agents in such a way that (i) the lowest cost of a peripheral agent in a consecutive jurisdiction of size k is attained at $k = 4$; and (ii) agent 1 prefers jurisdiction $\{1, 6, 7, 8\}$ to staying alone. Then no group of agents can block, since in any jurisdiction, different from $\{1\}$, at least one agent, who

- We could consider Nash stability instead of "strong" stability. Contrary to core stable partitions, any Nash stable partition is stratified but like in the case of strong stability, in general, we cannot guarantee the Nash stability in our framework. The source for possible Nash instability is that by moving to another jurisdiction, an agent necessarily affects the recipient jurisdiction project's location, and, therefore, contributions of its members. This argument allows us to construct a "cycle of individual improvements" accompanied by other agents' cost increases.

Consider the following example with five agents, where $l^1 = 0$, $l^2 = l^3 = \frac{23}{30}$, $l^4 = \frac{29}{30}$ and $l^5 = \frac{191}{120}$ (see Figure 3).

We will show that, in a Nash stable partition, agents 2 and 3 belong to the same jurisdiction, and moreover, are joined by agent 4. Given the formed group $T = \{2, 3, 4\}$, agent 1 will join T only if agent 5 joins it, too. However, agent 5 would join T only if agent 1 would not. These "cat and mouse" preferences rule out the existence of a pure strategies Nash equilibrium.

- When there are several median locations, we could consider all of them as feasible options and define accordingly a quasi-hedonic group formation game. The notions of stability must be adjusted accordingly as explained in Bogomolnaia, Le Breton, Savvateev and Weber

(2007a). Many of the above results remain valid in this new setting. The notion of Nash stability is based upon the *consistent median* — *CM* rule. They demonstrate that under the *CM* rule, there always exists a Nash stable partition. They prove this result by using the potential functions approach pioneered by Rosenthal (1973), and further developed by Monderer and Shapley (1996).

- When the distribution is the uniform, we can provide a complete description of the set of strongly or Nash stable coalition structures as done in Bogomolnaia, Le Breton, Savvateev and Weber (2007b,c). In such a case, the emphasis is on the pattern of groups: can we have small groups together with large ones ? How the heterogeneity of the partition is going to reflect (reproduce) the heterogeneity of the society.

- We could also consider multidimensional descriptions of heterogeneity. This extension raises a number of complications but we can however discriminate among social environments according to some measures of stability. An examination of the continuous uniform two-dimensional setting is provided in Drèze, Le Breton, Savvateev, and Weber (2007). They show that the stability of this pattern is rather strong. In multidimensional setting, there are natural measures of the distance to strong stability. During the lectures, I will present some of them. An early contribution on this topic when there are two dimensions and relevant groups are pairs is Le Breton and Weber (1995).

- There is an enormous empirical literature on all these questions. The empirical analysis of lobbying emphasizes, among other things, the impact of the heterogeneity across the members of a special interest group (an industry, a region, an ethnic group,...) on the success of their actions. They use crude measures like the Herfindhal's concentration index to define the level of heterogeneity. The empirical analysis of violent conflicts like for instance civil wars pay also a lot of attention to the role of societal characteristics. Some representative references are provided below but the literature on ethnic conflicts and wars is huge and crosses several disciplinary fields.

Secession has also been investigated very systematically from an empirical angle. Secession is one (out of many) solution to a conflict. The recent history shows that secession is considered very seriously in many conflicts. I will not review this literature but during the presentation I will present some of the conclusions obtained by Sambanis (2000).

Pairwise Heterogeneity

In many problems the social environment E is described by a vector of weights $\mathbf{n} = (n_k)_{1 \leq k \leq K}$ and a matrix of distances $\mathbf{d} = (d_{kl})_{1 \leq k, l \leq K}$. It is sometimes completed by the list of feasible

alternatives for each possible group under the presumption that the partition of the original society into smaller subsocieties is a conceivable outcome. Given a subsociety S and a mechanism mapping M_S societal characteristics into a feasible outcome, we obtain a formal description of the influence of $E = (\mathbf{n}, \mathbf{d})$ on the final outcome. If we denote by \mathcal{P} the exogenous list of permitted partitions, a final outcome consists of a partition $\pi = (S_j)_{1 \leq j \leq J}$ in \mathcal{P} , and a feasible alternative x_S i.e. an alternative in $M_S(E_S)$.

- The lobbying model of Esteban and Ray an example of such setting where \mathcal{P} contains uniquely the all set of agents.

- The Axelrod and Bennett 's landscape model of coalition formation also belongs to this setting. The key ingredients are their matrix of binary propensities among nations and the weights of these nations. They impose to the partitions to be of size 2. They use this model to predict alliance patterns during world war 2 and standard-setting in the computer industry. Their condition on the size of permitted partitions is relaxed in Le Breton, Ortuño-Ortín and Weber (2007) and the connection between landscapes and potential games is explored.

- The celebrated Bueno de Mesquita's analysis of wars among two countries also makes an extensive use of a matrix describing the intensities of "proximity" for each pair of countries. He calculates these propensities through a measure of association for a contingency table constructed from the pattern of alliances of these two countries with all other countries. This matrix has been used by many authors. Some of these contributions are listed in the references.

- An early contribution is Le Breton and Weber (1995) who consider partitions into pairs in the case of an arbitrary space of characteristics. They obtain a complete characterization of social patterns leading to stability.

- Bogomolnaia and Jackson (2002) contains results on such social settings and Milchtaich and Winter (2002) offers a complete and detailed analysis of an important setting belonging to this family namely the case where the payoff of an agent is to his/her average distance to the other members of his/her group. Their settings are purely **hedonic** in the sense that the payoff derived from a specific group membership depends exclusively upon the social composition of the groups (no degrees of freedom to select the policy once the group is formed). This class of problems is analysed in Banerjee, Konishi and Somnez (2001).

- This matrix is also the key ingredient in the model of nation formation examined by Desmet, Le Breton, Ortuño-Ortín and Weber (2007). In the application of their model to Europe, they use the matrix of pairwise genetic distances to describe the intensity of the heterogeneities across the different groups in the European population. This data is taken from population genetics.

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