

# Risk and the Theory of Distributive Justice

*Is the veil of ignorance useful to promote egalitarianism?*

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"risk, inequality and social welfare"

## Two channels for introducing risk in the theory of distributive justice

- First channel: What are the social welfare functions which emerge when the space of alternatives are lotteries?
- A problem of aggregation of preferences
- From a social choice point of view, the set of alternatives is broader, the set of preferences is restricted
- Distributive justice means that we compare the social benefits to give more to an individual than to another one
- Well-beings are interpersonally comparable (levels or differences)
- There is a competition among individuals for a resource which has to be shared.

## The first example : The Harsanyi Aggregation Theorem (1955)

- The individual preferences as well the collective preferences obeys the VNM axioms
- Pareto Indifference
- The VNM utility of the social decision maker can be written as a weighted sum of the VNM utilities of the individuals
- Sen-Harsanyi dispute about the interpretation of the theorem for utilitarianism.
- For Sen, only a representation theorem.

## The controversy

- Many others contributions in that fields which convey the intuition that the restriction of domain implied by VNM preferences may bring good news for aggregation of preferences
- A more direct justification for some SWFs, or new SWFs such as relative Utilitarianism (Dhillon Mertens 1999)
- The original confusion imbedded in Harsanyi's contribution has been dismissed.
- Since the theory of VNM is ordinal, the information on preferences used by the social planner remains Ordinal and Non Comparable.

- Then it is impossible to provide a characterization of a SWF requiring some form of interpersonal comparison of welfare such as Utilitarianism without introducing explicitly such a comparison.
- See Weymark 1991 and 2005

## Second channel: Introducing risk through the veil of ignorance

- Justice needs impartiality (A. Smith " The theory of moral sentiments" and Kantianism, the Universalization Maxim).
- Impartiality: Judgements on collective life should not depend on individuals' identities and other particular circumstances
- A way to force impartiality: the veil of ignorance (Vickey (1945), Harsanyi (1953), Rawls (1971), Dworkin (1981b))
- They disagree on how thick the veil should be
- The individuals don't know some part of the real world.

- The veil of ignorance model produces impartiality because the soul or observer, or parties, who contemplate the world behind the veil is (are) deprived of the information that could lead to arbitrariness of the judgement.

## Distributive justice as some forme of egalitarianism

- Laisser-faire distribution of well-beings may be unequal
- Political philosophers try to argue that justice requires some form of egalitarianism
- Several ways to convey egalitarianism
- Full priority to the worst off like in the Rawlsian maximin
- Parfit (1997) : prioritarianism : worse off should be given priority over the better off with respect to resource allocation

- Translation in a welfarist setting: a SWF with strictly convex upper contour sets. A partial agreement with Lorenz ordering in terms of utilities for a subset of concave transformations of utility functions.
- The boundaries are maximin on one side and utilitarianism on the other

## The veil of ignorance and egalitarianism

- Can some kind of egalitarianism emerge from reasonings behind the veil of ignorance?
- As in maths, more applause when the result is far away from the premises.
- What is at stake is to convince more people that egalitarianism is a requirement of justice since it is derived from a thought experiment accessible to everyone, where the desire to redistribute from rich to poor is never invoked explicitly.
- Egalitarianism should emerge as a surprise

Exemplified by a version of John Rawls' argument

- A version of John Rawls' attempt with a very thick veil of ignorance
- Individuals do not know what would be their traits (preferences, resources internal or external).
- All souls are placed identically in this original position. There is no difference among them.
- The problem of a social contract among identical souls degenerates into an optimization problem for the representative soul.
- If the representative soul does not know the probability distribution of traits in actual society, it faces a decision problem under ignorance.
- Maskin (1979) axiomatization of the solution which maximizes the minimum possible utility.

Are the constructions under risk so successful ?

- In risk, you know the probability distributions of traits.
- The Rawls' construction under risk would assume an infinite aversion toward risk
- As Harsanyi puts forward, it is dubious to defend the maximin solution on the basis of an extreme risk attitude
- *" If you took the maximin principle seriously then you could not even cross a street ( after all, you might be hit by a car); you could never drive over a bridge (after all, it might collapse), you could never get married (after all, it might end in a disaster). If anybody really acted this way he would soon end up in a mental institution."*

A general objection to the exercise, whatever the result.

- The procedure implies that the allocation and more generally social decisions will depend on the individual attitude towards risk.
- Barry (1989) and Kolm (1995a) think that using risk preferences to compare social decisions may be misleading
- Example : Individuals accept to take small risks for instance to go to a beach where there is possibility of a tsunami. It does not mean that they accept the social consequences of a tsunami, even if the proportion of affected people in the world population and the probability are very small.
- The risk aversion may be much larger for the second exercise than for the first one.

## Outline

- The impartial observer of John Harsanyi (1953) and Prioritarianism
  - Moreno-Ternero Roemer (2005)
- The insurance mechanism of Dworkin and egalitarianism
  - Trannoy (2006), Fleurbaey (2002)
  - Use of Impartial observer Theorem
- A general remark: we are more interested in the properties of the allocation of resources obtained by using the mechanism of the veil of ignorance than in the properties itself of the SWF, if any, obtained through the veil of ignorance.

## Part I: The Harsanyi's impartial observer and prioritism

## 1. The Harsanyi Story

- The observer should "*not know in advance what his own social position would be in each social situation*"
- The impartial observer assesses a wealth distribution  $y = (y_1, \dots, y_n)$  through the utility of a *birth*-lottery giving him an equal chance to be anybody in  $y$ .
- Already in Vickrey 1945 "*The observer has an equal chance of landing in the shoes of each member of the society*"
- Not only he imagines that he gets the allocation of person  $i$  with  $1/n$  (impartiality) but he becomes person  $i$  by adopting his utility level  $v_i(y_i)$  (empathy).

- The observer follows VNM axioms in assessing the birth-lottery so described. The utility of this lottery is thus given by  $\frac{1}{n} \sum_{i=1}^n v_i(y_i)$ .
- The "just" income distribution is the distribution which maximises the above expression which sounds utilitarianist.

–1. The Harsanyi Model ( for more details, see Weymark 1991)

- In the background, there is a wealth amount  $Y$  to be distributed among  $n$  individuals. It does not play any role in deriving the SWF used by the IO.
- The IO may become any of this individuals. He does not know who he will become, which identity he will endorse.
- Each identity  $i$  is endowed with VNM preferences (they satisfy quasi-ordering, continuity and independence) over the set of wealth lotteries. In this context, the space of wealth lotteries = the set of all distribution functions over the positive real line.
- Let  $v = (v_1, \dots, v_n)$  the vector of VNM utility functions, the vector of identities. They are supposed to be increasing in wealth  $y$ .

- It is important to remind that the VNM preferences can be represented by any increasing transformation of VNM utility. Why this representant and why not another? This point of departure has to been admitted.
- We pick a representant among others, the VNM utility, which is defined up to increasing affine transformation.
- An *extended prospect* to becoming person  $i$  with wealth  $y_i = (i, y_i)$
- $L =$  The set of birth-lotteries defined on  $N \times \mathfrak{R}$
- An "impartial" birth-lottery belongs to  $L$

$$l = (1/n \circ (1, y_1), 1/n \circ (2, y_2), \dots, 1/n \circ (n, y_n))$$

- The impartial observer (IO) obeys VNM axioms in evaluating the birth lottery. He receives a utility

$$U(l) = \frac{1}{n} \sum_{i=1}^n U(i, y_i) \quad (1)$$

- What kind the function  $U$  is ?

## The principle of acceptance

- *The Principle of Acceptance* : For each  $i$ , the function  $U(i, \cdot)$  represents the same VNM preferences on wealth lotteries as  $v_i$  represents.
- Now, the VNM theorem says that any two VNM utility functions that represent the same preferences must be positive affine transformations of each other.
- Therefore, for all  $y$  and  $i$  there exist  $a_i > 0$  and  $b_i$  such that

$$U(i, y) = a_i v_i(y_i) + b_i \quad (2)$$

- Substituting formula (2) into (1) gives

$$U(l) = \frac{1}{n} \sum_{i=1}^n U(i, y_i) = \frac{1}{n} \sum_{i=1}^n a_i v_i(y_i) + \frac{1}{n} \sum_{i=1}^n b_i \quad (3)$$

## Weights are undetermined

- Maximising the RHS of (3) is equivalent to maximise 
$$\sum_{i=1}^n a_i v_i (y_i),$$
- Harsanyi concludes that the IO maximises some positive weighted sum of VNM utilities of the persons of the society.
- How are determined the  $a_i$  ? Without these values, the VNM preferences of the IO are undetermined.
- Without these values, the IO cannot decide whether person  $i$  with 1000\$ is better off than person  $j$  with 3000\$. Therefore he can't allocate resources.
- The Harsanyi's model is incomplete

## The reason of undetermination

- The only information the IO has, consists in the profile of risk preferences of the individuals.
- There is absolutely no information permitting interpersonal comparisons
- Indeed, in the parlance of Social Choice, the information basis of the collective choice are cardinally measurable and non comparable (CNC). The individual VNM utility functions are defined up to any positive affine transformation. There is no restriction on the positive slopes and the intercepts.
- By supposing that the IO has complete preferences on the lotteries on the space of extended prospects, it is assumed that the IO is able to say that he prefers  $(i, x_i)$  to  $(j, y_j)$ , i-e to perform a *judgement of extended sympathy* (Suppes 1966).

- That is to say, judgements of the following sort: Wealth  $x$  is better (or worse) for individual  $i$  than wealth  $y$  for individual  $j$ .
- His VNM utility on extended lotteries  $U(i, y_i)$  is able to do so.
- We have assumed with the principle of acceptance that on the intrapersonal dimension,  $U(i, y_i)$  coincides with the personal VNM utility of person  $i$ .
- However, there is no constraint when comparing  $(i, x_i)$  to  $(j, y_j)$ .
- The indeterminateness of the  $a_i$  reflects this lack of constraint.

## The Moreno-Ternero Roemer solution to complete Harsanyi's model

- The simplest solution
- For example, Karni (1998) and Mongin (2001) assume that the observer's preferences conform to the subjective expected utility theory. Provided that one can identify the observer's subjective probabilities, the weights would be determined.
- To assume that there exists from the outset an *extended sympathy* quasi-order and that the IO has to conform with this information.
- Where does the quasi-order come from ?
- An index of primary goods as in Rawls ? an index of human functioning as in Sen ?

- At this stage, it does not matter. Let suppose that it exists.

## The principle of objectivity

- There exists a complete extended quasi-order of preferences on  $N \times \mathbb{R}_+$  denoted  $R$ .
- *The principle of objectivity* : The IO's preferences on extended lotteries must induce a quasi-order identical to  $R$
- $U(i, x_i) \geq U(i, y_i) \Leftrightarrow (i, x_i) R (j, y_j)$
- *Wealthability Assumption*: For all  $i, j, y$ , there exists  $x$  such that  $(i, y) I (j, x)$  where  $I$  denotes the symmetric part of  $R$
- The data available to the IO are  $v, R$  and two principles, the principle of acceptance and objectivity which relate the IO's preferences on extended lotteries with  $R$  and  $v$ .
- Are these data consistent in all circumstances ?

## Risk isomorphism in well-being space

- Due to the *Wealthability Assumption*, there exists equal-well-being wealth distributions, that is a vector  $y$  such that

$$(i, y_i) I (j, y_j) \text{ for every pair } i, j$$

Now consider two more equal-well-being wealth distributions,  $x, z$  such that  $v_i(x_i) > v_i(y_i) > v_i(z_i)$  which implies  $x_i > y_i > z_i$  for all  $i$  since the  $v_i$  are increasing.

- By the principle of acceptance and objectivity,

$$\begin{aligned} x_i I x_j \\ y_i I y_j \\ z_i I z_j \end{aligned} \tag{4}$$

- By continuity property of the VNM quasi-orderings, there exists, for all  $i$ ,  $p_i \in (0, 1)$  such that

$$y_i \sim_i p_i z_i + (1 - p_i) x_i \tag{5}$$

- (4) and principle of acceptance implies that the IO must declare that  $p_i z_i + (1 - p_i) x_i$  and  $p_j z_j + (1 - p_j) x_j$  are indifferent. Since by the principle of objectivity  $z_i$  is indifferent to  $z_j$  and  $x_i$  is indifferent to  $x_j$ , it must be that  $p_i = p_j$ . Otherwise, the data transmitted by the  $v_i$  through the principle of acceptance are inconsistent with the data conveyed transmitted by the extended sympathy quasi-order  $R$ .

- *Definition* : The individuals are isomorphic if for any three equal-well-being wealth distributions,  $x, y, z$ , the  $p_i$  defined by (5) are identical for all  $i$ .
- Individuals have the same risk preferences when prizes are well-being. It does not mean immediately that individuals have the same risk preferences when prizes are just wealth.
- *Proposition 1*: Suppose that IO satisfies the principle of acceptance and objectivity. If the individual VNM utility functions are increasing and if *Wealthability* holds, then the IO cannot form well defined VNM preferences on the lotteries on the space of extended prospects.

Positive or negative result ?

- A matter of interpretation
- Moreno-Ternerero and Roemer offers a negative interpretation:
- "*Risk isomorphism is a singular case, which we rarely if ever hold in 'real world'*"
- A more positive interpretation. It is not a matter of real world.
- If  $R$  is meaningful for a theory of justice, it means that it conveys an objective view about individual well-beings.
- Objective: all individuals agree on the extended sympathy ranking.

- Suppose that all but one single individual are isomorphic. It means that this individual disagrees with the other individuals on the extended sympathy ranking. Then this ranking is no more objective and it cannot be a valid source of information for an impartial theory of distributive justice.
- In my view, risk isomorphism is just another way to say that the "objective extended sympathy ranking  $R$ " exists.
- If it does not exist, then we have to be more inventive
- See the MTR's paper for a proposal, each individual forms a subjective extended sympathy ranking and one takes the average. But then we are back to an aggregation problem and we are leaving the IO framework.

## A remark about the informational basis of collective choice

- Suppose now that risk isomorphism prevails
- The data given by the extended sympathy quasi-ordering  $R$  means that the informational basis are ordinal and fully comparable (OFC)
- Comparing utility levels are meaningful
- That is the information transmitted and therefore the IO's choice is invariant to the same increasing transformation applied to all individual utility functions here VNM utilities.
- Since they are defined up to positive affine transformation, the information basis of collective choice

are *cardinal measurable and fully comparable*, (CFC), that is the transformations of utility functions which leave the social choice invariant are the same positive affine transformation applied to all VNM utility functions.

- CFC is more demanding than *Cardinally measurable and unit comparability* (CUC) required for utilitarianism

## The Positive representation theorem in case of risk isomorphism

- The problem we face with the Harsanyi's result is that we don't know the values of the  $a_i$
- Now we can solve for the  $a_i$ .
- Let us take any two equal-well-being wealth distributions,  $x, z$ .
- By the principle of acceptance, there exists two constants  $k_x, k_z$  such that for all  $i$

$$k_x = a_i v_i(x_i) + b_i$$

$$k_z = a_i v_i(z_i) + b_i$$

- Thus

$$a_i = \frac{k_x - k_z}{v_i(x_i) - v_i(z_i)}; b_i = k_z - a_i v_i(z_i)$$

- Then

$$U(i, y_i) = \frac{k_x - k_z}{v_i(x_i) - v_i(z_i)} v_i(y_i) + k_z - a_i v_i(z_i)$$

- But  $U(i, y_i)$  is defined at a positive affine transformation. Then by setting  $k_x = 1$  and  $k_z = 0$ , one obtains

$$U(i, y_i) = \frac{v_i(y_i) - v_i(z_i)}{v_i(x_i) - v_i(z_i)} \quad (6)$$

- *Proposition 2* Suppose that IO satisfies the principle of acceptance and objectivity. If the individual VNM utility functions are increasing, if *Wealthability* holds and if individuals are risk isomorphic, there exists a unique quasi-order on extended lotteries for IO represented by the VNM utility function defined by (6) where  $x$  and  $z$  are any two equal-well-being wealth distributions.

- (6) is a form of relative utilitarianism which has been also axiomatized by Karni (1998) and Dhillon and Mertens (1999). Suppose that the well-beings of individuals for zero wealth are equal as those when they receive the entire wealth of the society  $Y$ .

$$U(i, y_i) = \frac{v_i(y_i) - v_i(0)}{v_i(Y) - v_i(0)} \quad (7)$$

The possible anti-prioritarianism of the Harsanyi  
IO as appended by Moreno-Ternero and Roemer

- *Definition of disability.* Given  $R$ , individual  $j$  is weakly disabled with respect to individual  $i$  iff for any  $(y_i, y_j)$ ,  $(i, y_i) I (j, y_j) \Rightarrow y_j \geq y_i$ .
- *Definition of Priority* An allocation of wealth  $y$  is prioritarian iff  $j$  is weakly disabled with respect to  $i \Rightarrow y_j \geq y_i$

- *Example*
- Two individuals, Annie ( $i$ ) and Louis ( $j$ ) who are risk neutral  $v(y) = y$ .
- Louis is disabled with respect to Annie.
- $(1, y_1)I(2, y_2) \Rightarrow y_2 = 2y_1$
- Louis needs twice more wealth to achieve the same level of well-being
- $x = (1, 2), z = (0, 0)$ .
- Then by applying (6)  $U(1, y) = y$  and  $U(2, y) = \frac{y}{2}$
- The weight of Louis is half the weight of Annie.

- $Y = 1; (y, 1 - y), y$  Annie's share.

- The IO maximises

$$\frac{1}{2}y + \frac{1}{2}\frac{1-y}{2} = \frac{1}{4} + \frac{y}{2} \quad (8)$$

- Annie receives the lion's share  $y = 1$ .
- Why ? The social marginal utility of Annie is always twice higher than Louis's one. Louis is twice less efficient in transforming wealth into well-being.
- Since IO maximises the sum of well-beings it is efficient to give all the cake to Annie.
- For small degree of risk aversion, Louis receives some share of the cake but less than Annie
- For large degrees of risk aversion, the allocation becomes prioritarian

1. The insurance mechanism of Dworkin and egalitarianism

## Equality Of Opportunity Dworkin, Arneson, Barry, Cohen

- Equality of outcomes is too strong
- Limit the domain of equality requirement
- Critics of the welfarism (consequentialism)
- What matters is not only the result but the process generating inequality
- Ruling out inequality due to factors for which a person should not be held responsible
- Respect of the disparity of allocation among individuals which is due to the exercise of their responsibility

## The Dworkin cut

- Distinction between tastes and ambitions for which a person should be held responsible
- Internal resources like earnings capacity, handicap, which are not transferable from one individual to another.
- External resources like transfers
- Extended resources is the sum of external and internal resources
- The goal of distributive justice is to equalize extended resources

## The Thin veil of ignorance of Dworkin

- Behind the veil of ignorance, individuals know the responsibility factors and ignore the non-responsibility factors
- The individuals know their preferences, in particular their risk preference and their skills
- The individuals don't know their handicaps and their earnings capacity.

## Distinction between option luck and brute luck

- Option luck: you take a bet on (to ski off piste). You should be taken accountable for your choices (responsible)
- Brute luck: luck that you undergo (not responsible)
- Many authors argue against the distinction and that option luck and brute luck will be difficult to disentangle in real world.
- Under the veil of ignorance setting, the distinction is clearer.

## An hypothetical insurance market

- In the absence of a market of contingent claims, your earnings capacity is a result of brute luck.
- But suppose that you can insure against a bad occurrence of earnings by entering into a market of contingent claims.
- If you don't insure because you are risk neutral, it is your own responsibility. If you experience low earnings ex post it is a matter of option luck. You lose your claim to an ex-post redistribution since you could have avoided the bad occurrence of low earnings.
- The outcome of the transactions on this hypothetical insurance market is considered as just by Dworkin because it becomes the result of option luck, a factor of responsibility.

- There is a Nozickian flavour in Dworkin's inspiration. For Nozick (1974), an allocation is just if it results from the exchanges on markets where individuals exert their property rights.
- Individuals under the veil of ignorance have property rights on contingent claims. The market of contingent claims in the original position produces a just allocation in the real world.
- The role of the EOP policy is to mimic the outcome of this hypothetical insurance mechanism.
- The question at stake, what are the distributive properties of the allocation of this hypothetical insurance market ?

- *“That problem seems amenable, at least in principle, to the various types of analysis that economists devote to problems of decision making under uncertainty, and there is no reason to doubt that the computer could furnish an answer”*
- Dworkin’s reasoning is rather fuzzy. It has to be completed somehow. Depending on how you complete it, the answer of the computer is very different.

## The model (First Version)

- No handicap. Only the problem of earnings capacity
- The choice consumption-leisure is made ex post. It is assumed that the individuals work as much as they can. Income or earnings capacity is a full Beckerian income.
- The size of the population is normalized to 1.
- Let us  $\theta$  denote the talent distributed according to  $H(\cdot)$
- Let us  $\omega$  denote the state of the world distributed according to  $F(\cdot)$ , public knowledge.
- The earnings capacity  $x(\omega, \theta)$
- Let  $G(x/\theta) = F(x^{-1}(\cdot/\theta))$

## Assumptions

- Agents must be put on an equal footing behind the veil of ignorance
- *Assumption 1*: They don't have initial resources behind the veil of ignorance
- "*Premiums will be paid from future earnings*"
- Of course this assumption is not enough. If an individual were twice richer in expectation than another one, he might buy twice more insurance premiums.
- *Assumption 2*: The expectation of  $x$  under  $G(x/\theta)$  is independent from  $\theta$
- Indeed, Dworkin requires much more:  $G(x/\theta)$  is independent from  $\theta$ . It is useless.

- *"The individual supposes, in fact, that he has the same chance as anyone else of occupying any particular level of income in the economy".*
- *Remark:* Assumption 2 does not mean that the mechanism of hypothetical contingent claim market is a simple extension of the competitive market with equal incomes.
- For the equality of expectations of gross income does not guarantee the equality of budgets which depend on the prices of contingent claims.
- An individual would be as much rich as his primary income is negatively correlated to the aggregated wealth.

A come back to some kind of utilitarian formula

- Despite the effort made by Dworkin to move away from utilitarianism, the hypothetical market of contingent claim leads to a kind of utilitarian formula
- Let us assume that individuals under the thin veil of ignorance try to maximize their expected utility.
- $U_\theta$  is the VNM utility function for type  $\theta$ , the argument of which is  $y$  which is the level of disposable income which may differ from gross income by the value of net trades or by the indemnity net of the insurance premium.
- The expected utility is

$$\int U_\theta(y(\omega, \theta))dF(\omega)$$

- Thanks to the first theorem of welfare economics, the walrasian market of contingent claims achieves a Pareto optimal allocation ex-ante.
- A necessary condition (at this stage we don't assume concavity of VNM utility) for  $y$  to be a Paretian allocation, is that there exists non negative weights  $\lambda_\theta$  such that the weighted sum of expected utilities admits  $y$  as a critical point (maximum, minimum, saddle point) in the feasible set.

$$\int_{\theta} \lambda_{\theta} \int_{\omega} U_{\theta}(y(\omega, \theta)) dF(\omega) dH(\theta) = \quad (9)$$

$$\int_{\omega} \int_{\theta} \lambda_{\theta} U_{\theta}(y(\omega, \theta)) dH(\theta) dF(\omega) \quad (10)$$

- The redistribution of wealth is state by state, since no redistribution across states is possible

- The constraints which specify the feasible set

$$\int_{\theta} y(\omega, \theta) dH(\theta) = \int_{\theta} x(\omega, \theta) dH(\theta) \text{ for all } \omega \quad (11)$$

- When the paretian  $y(\omega, \theta)$  maximizes or minimizes locally (10) under the constraint (11), the optimization is realized state by state.
- Since (10) is a positive linear expression,  $y(\omega, \theta)$  maximizes or minimizes locally (10) iff it maximizes or minimizes for every state the weighted sum of VNM utilities functions, the weights beign independent from the state, that is

$$\int_{\theta} \lambda_{\theta} U_{\theta}(y(\omega, \theta)) dH(\theta) \text{ under (11)}$$

What about the weights  $\lambda_\theta$  ?

- Can they be equal to  $\frac{1}{n}$  as in Harsanyi's IO theorem ?
- $\int_\theta \lambda_\theta dH(\theta) = 1$
- $\lambda_\theta$  must be equal to the inverse of the expected marginal utility of income for a budget assessed at the equilibrium prices.
- $p(\omega)$  is the walrasian contingent price of one unit of wealth in state  $\omega$
- At the equilibrium, the individual maximizes

$\int U_\theta(y(\omega, \theta)) dF(\omega)$  under

$$\int_\omega p(\omega)(y(\omega, \theta) - x(\omega, \theta)) dF\omega = 0$$

- Let  $\mu_\theta$  the lagrange multiplier

$$\lambda_\theta = \frac{1}{\mu_\theta}$$

- The equality of the weights is far from being obvious, the marginal utilities are not set equal and the incomes are not equal at an utilitarian solution, but special cases.
- What about the consequences of assumption 2 on the weights?
- It is possible to answer to this question if we add an additionnal assumption.
- *Assumption 3* The aggregated wealth is independent from the state of the world

$$z(\omega) = \int_{\theta} x(\omega, \theta) dH(\theta) = K \text{ for all } \omega$$

- Individual risks
- Not formulated by Dworkin but it is necessary for the market of contingent claims to be mimicked by an hypothetical insurance markets with fair actuarial policies by insurance companies

And the solution given by the computer is...

- *Proposition 1* Under assumptions 1, 2 and 3, and provided that all individuals are strictly risk adverse, the hypothetical insurance market leads to a perfect equality in every state of the world, whatever the risk aversion of individuals.
- The reason lies in the mutuality principle (Borch). If all agents are strictly risk adverse, the agent's consumption at an efficient allocation does depend of the state of the world only through the aggregated wealth in this state.
- $y(\omega, \theta) = y(z(\omega), \theta) = y(K, \theta)$
- But since all agents have the same expectation of income, it must be that agents receive the same income in each state.

- Then according to the computer, the EOP solution following the hypothetical insurance solution should be to organize full equality among individuals.

## The model (Version II)

- *Assumption 4* The aggregated wealth depends on the state of the world
- Probably more remote from the thought of Dworkin, since the market of contingent claim cannot be resumed by an insurance market.
- *Well known result.* The efficient allocation of an aggregated risk is such that the sensitivity of each agent's consumption to changes in aggregated wealth equals to the ratio of the corresponding agent's absolute risk tolerance to the average risk tolerance in the economy

$$T(y(z, \theta)) = -\frac{U'(y(z, \theta), \theta)}{U''(y(z, \theta), \theta)}$$
$$\frac{\partial y}{\partial z}(z, \theta) = \frac{T(y(z, \theta))}{\int_{\theta} T(y(z, \theta)) dH(\theta)} \quad (12)$$

- When there is an agent which is risk-neutral, the others being risk-adverse, we deduce from (12) that:
- The allocation is constant for all risk-adverse individuals  $y(z, \theta) = c(\theta)$
- The allocation for the risk-neutral individual depends linearly on  $z$ .  $y(z, \theta) = z + c(\theta)$
- Then it is easy to build examples when the allocation of the risk-neutral individual is equal to 0 in the states where the aggregated wealth is low.

- Benign neglect according to Dworkin !
- *“Some people enjoy, while others hate risks ; but this particular difference in personality is comprehended in a more general differences between the kinds of lives that different people wish to lead. The life chosen by someone who gambles contains, as an element, the factor of risk; someone who chooses not to gamble has decides that he prefers a safer life. We have already decided that people should pay the price of the life they have decided to lead”*
- *Proposition 2* Under assumptions 1, 2 and 4, and on the domain of preferences with strict risk aversion, the infimum of the smallest income over all states of the world generated by the hypothetical market of contingent claims is 0.
- In this second version of the hypothetical market of contingent claim, the EOP recommends to give up any basic income institution.

## Conclusion

- Even if this thought experiments of veil of ignorance are attractive from an intellectual point of view, the appearances can be deceptive. Up to now they cannot be useful to inspire real world redistributive policies.
- Maybe part of the problem comes from the fact that we look at *ex post* inequality generated by fairness *ex ante*. (Diamond (1967)) 'See Fleurbaey (2006) on this.
- The veil of ignorance: A risk trap for political philosophers ?
- Risk theory is complex, the connection between egalitarianism and veil of ignorance still to come...

- Two recent papers about the IO theorem
- Gajdos and Kandil (2006) "*The ignorant observer*".
- As the title suggests they consider uncertainty and not risk. Ignorance modeled as the set of all possible probability distributions over individuals.
- Characterization of a convex combination of Rawls and Harsanyi.
- The anti-prioritarianism still prevails.
- Grant, Kajii, Polak and Safra "*Generalized Utilitarianism and Harsanyi's Impartial Observer Theorem*"
- Contrary to Harsanyi Assumption, Lotteries over identity and lotteries over outcomes are independent.

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