New results about old topics:

the case of uni and multi dimensional utility functions

by

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Goal of the paper

To justify in as simple and intuitive terms as possible the assumption that successive direct or cross derivatives of a utility function U alternate in sign.

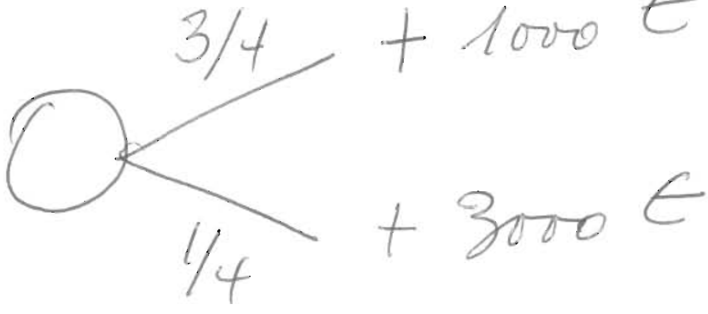
or, in other words.

Can we explain intuitively the notions of direct or cross prudence and temperance?

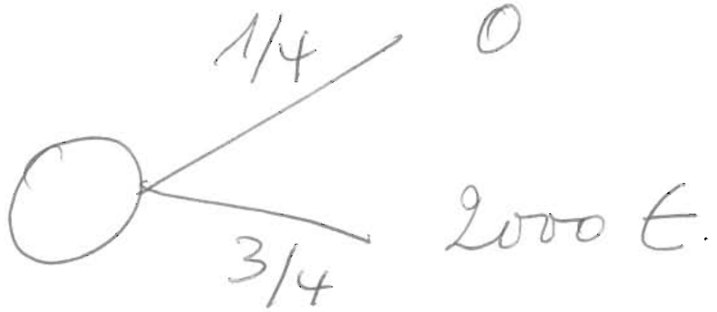
Plan of the talk

1. A (very) partial review of the related literature.
2. The uni-dimensional case (prudence and temperance)
3. The multi-dimensional case (cross prudence and temperance)
4. Extensions.
5. Further questions.

B



A



free access.

you can only play one

no good or bad answer.

I Related literature

1. The period 70-75

1.1. Economics.

- Uni-dimensional models (Mossin, Sandmo)

  assumption of decreasing A_a with

  $$A_a = -\frac{u''}{u'}$$

- multi-dimensional models: Sandmo, Leland and Drèze-Modigliani

  Savings under future income risk.

1.2. O.R. and Mngt Science

Only the multi-dimensional case

- Keeney and Keeney-Raiffa's book
  "Utility independence"
- S. Richard (MngtSc, 1975): an illuminating but also confusing paper on "Multivariate risk aversion"

2. Epstein-Tanny: (1980); Menezes-Geiss-Tressler.

"Increasing generalized correlation"

Clarifies Richard's contribution.
"Increasing Downside risk"

3. The "properness-standardness-vulnerability" period (1990....)

- Unidimensional utility
- Kimball, Pratt-Zeckhauser, Gollier-Pratt
- Caballé-Pomansky.

Further assumptions about $-\frac{u''}{u'}$

II The unidimensional case.

An attempt to unify the concepts of.

risk aversion, prudence, temperance, edginess...
($u''<0$) ($u'''>0$) ($u^{(4)}<0$) ($u^{(5)}>0$)

This attempt is not useless

[see page 5].

Basically

- we start from a principle outside the E-U model ("model free")
- we use a tool: the utility premium (instead of the risk premium) to translate this principle into E-U.

II.1. The principle ("model free").

"People like to disaggregate pains"
"Combine good with bad"

D. Makers like to spread pains on many states of nature instead of concentrating them on a single one

Pains: - a sure loss: $-k$
- a zero mean risk $\tilde{\varepsilon}$

ILLUSTRATIONS →.

This search is not useless.....

- see your answers to the questionnaire?
- see the literature. • first year undergr.

  - risk aversion

    a) several characterizations, some of them being outside E-U.

    b) doesn't refer to an optimizing behavior.

  - prudence

    a) within E-U

    b). based on an optimal choice (precautionary savings)

  - temperance.

    a) no clue ("aversion to alcohol").

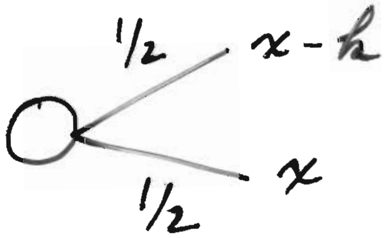
    b). within E.U

    Kimball (1992): an agent is temperant "if an unavoidable risk leads him to reduce his exposure to another indep. risk"

Hence a need for UNIFICATION.

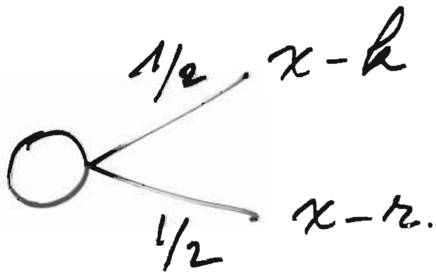
The "pain apportionment" rule

1) Pain apport^t of order 2 (= risk aversion

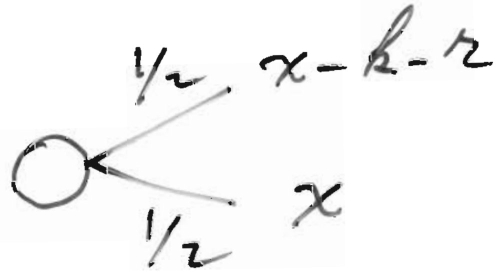


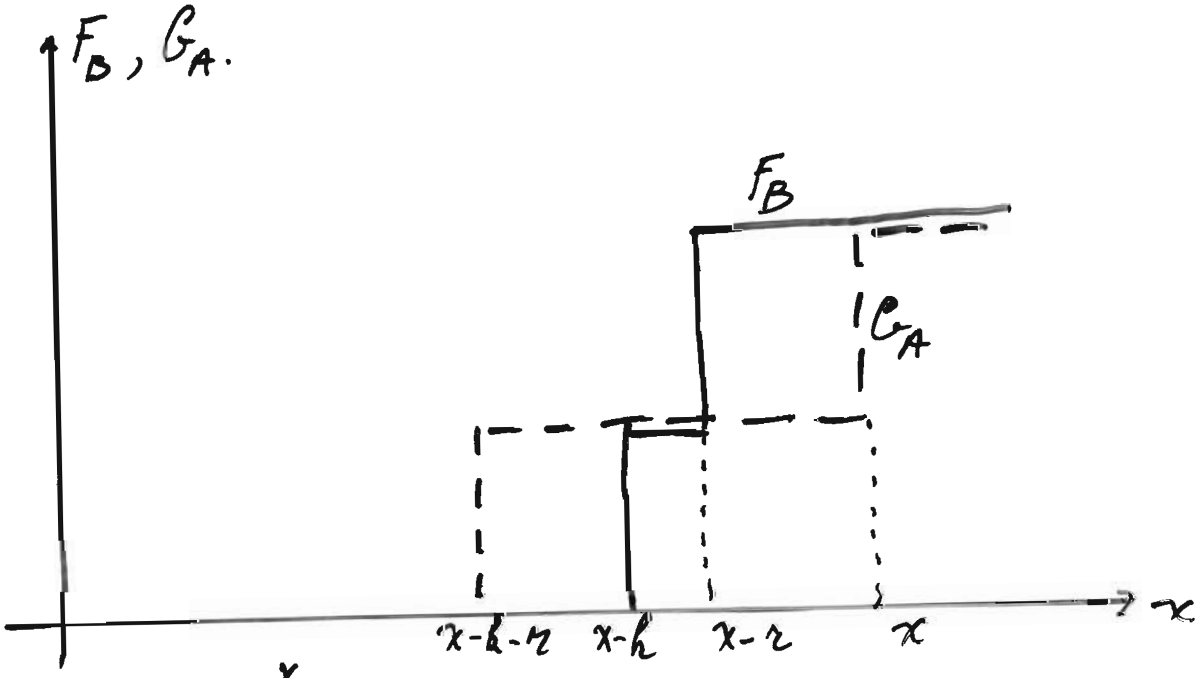
$-r$.

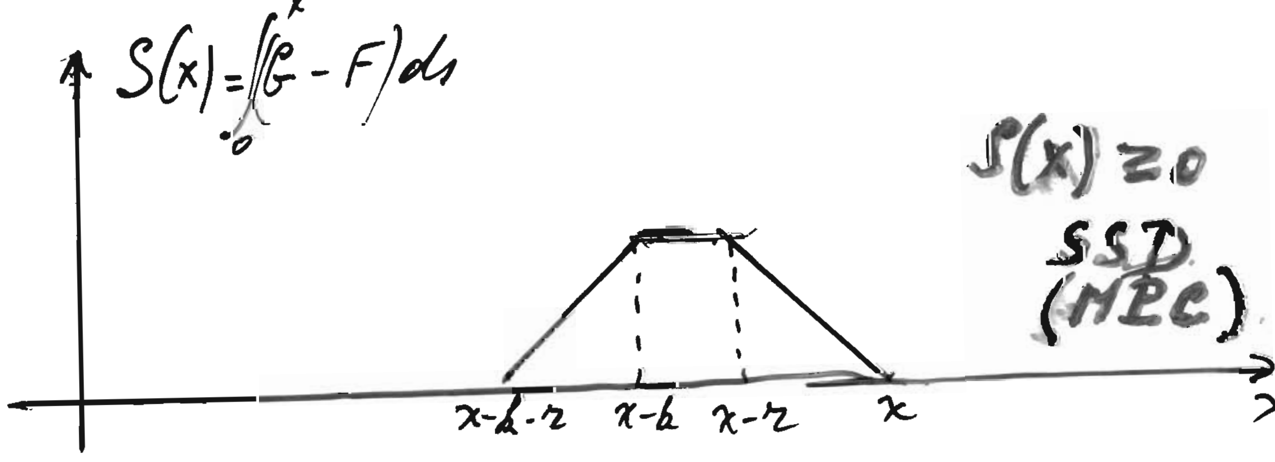
B_2



A_2







$S(x) \geq 0$

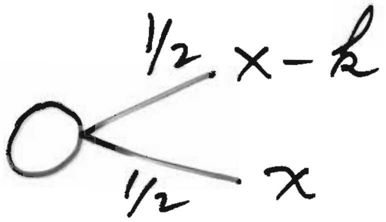
SSD

(MEC)

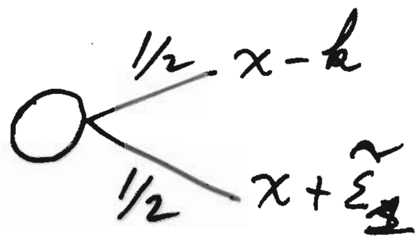
2) Pain apportionment of order 3

(= prudence)

$\tilde{\varepsilon}_1$



B_3



A_3

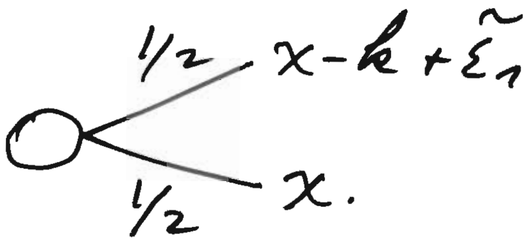
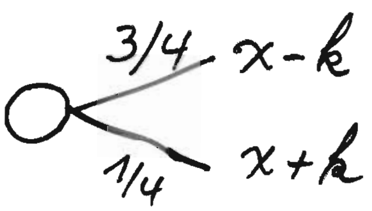
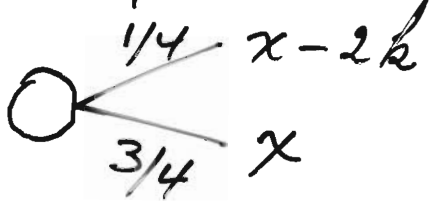


Illustration: $\tilde{\varepsilon}_1$ is binary with $\varepsilon_1 = k$.

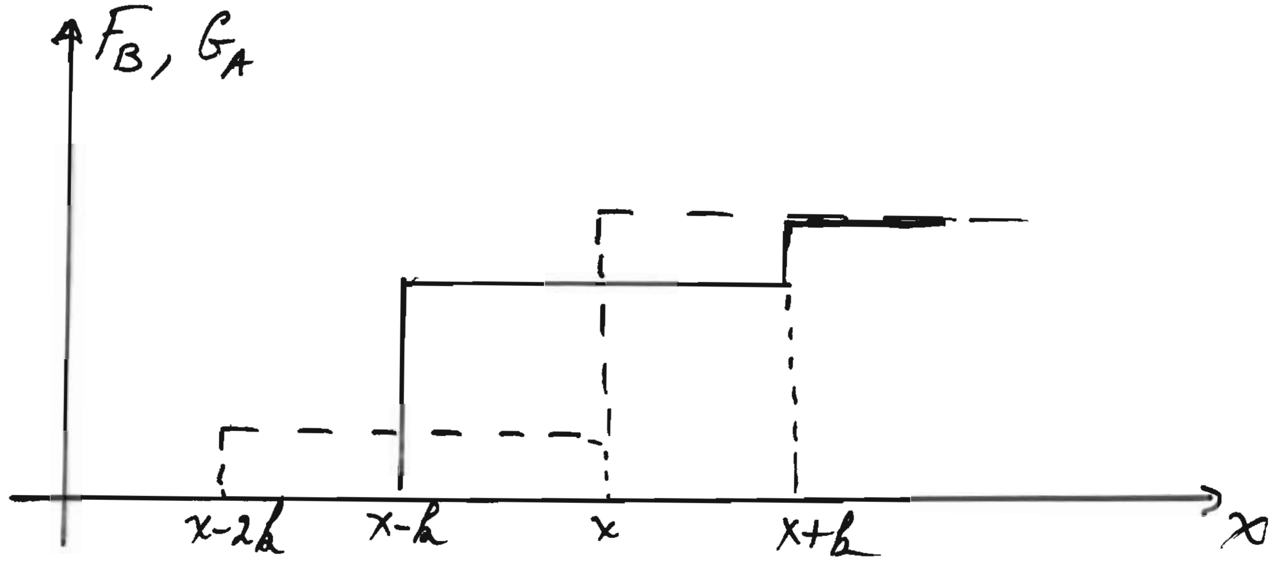
B_3

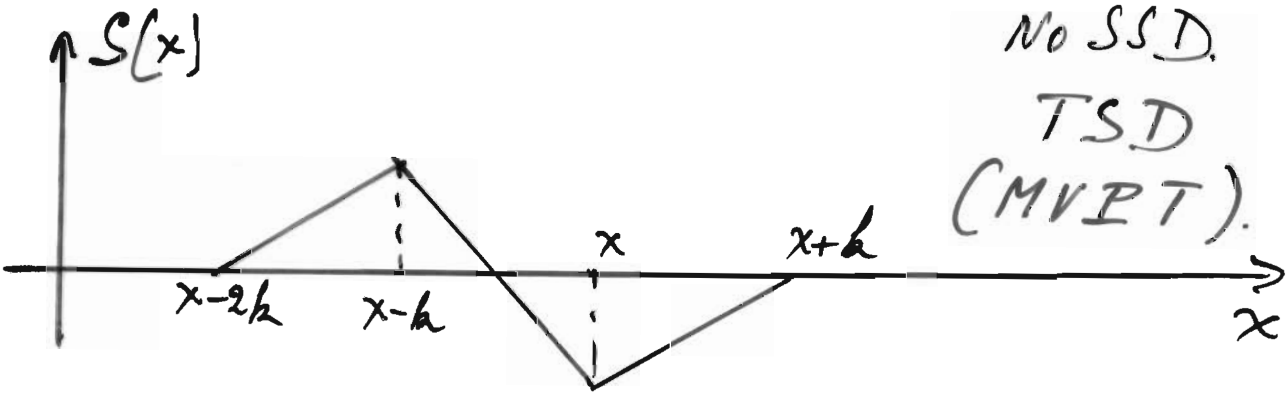


A_3



(= Menezes, Geiss, Tressler 1980)





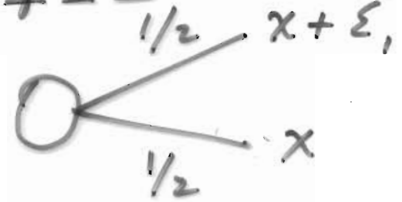
No SSD.

TSD

(MVPT).

3) ~~Prin~~ Pain apportionment of order 4 (Temperance).

Start from:

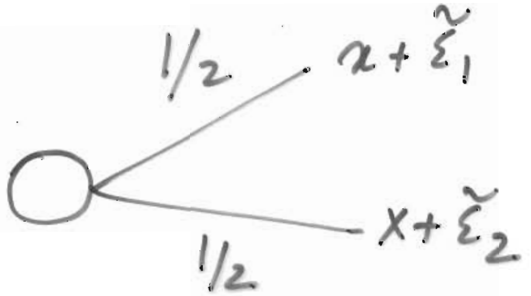


$\tilde{\varepsilon}_2$ with

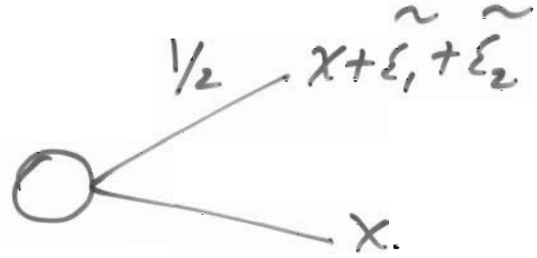
$E(\tilde{\varepsilon}_2) = 0$

$\tilde{\varepsilon}_2 \perp \tilde{\varepsilon}_1$

B_4



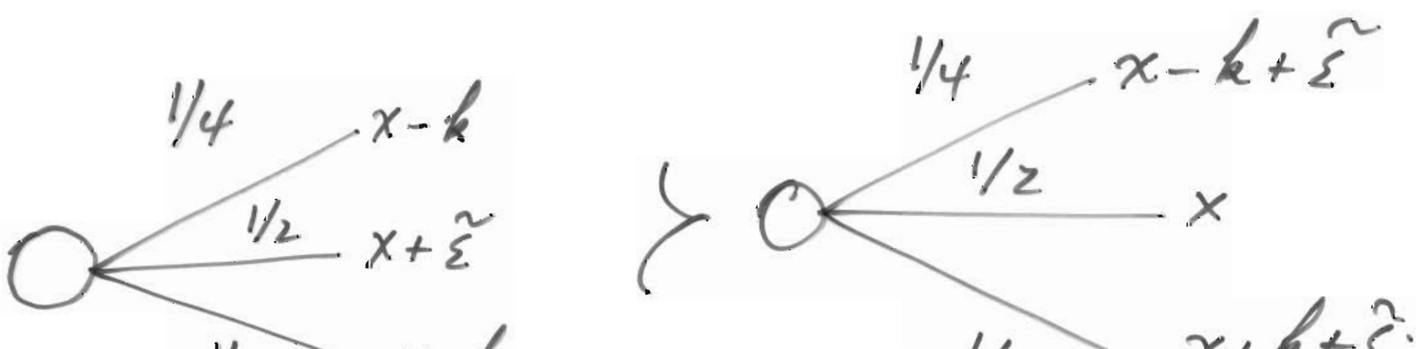
A_4



$B_4 \succ A_4 \Rightarrow$ temperance.

Related work: Menezes and Wang (J^al of Math. Ec. 2005).

"Outer risk aversion"

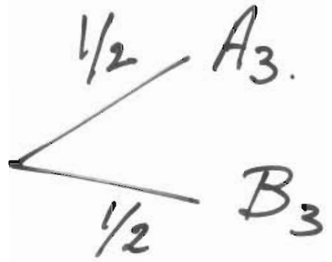


See E. G. S. (1995)

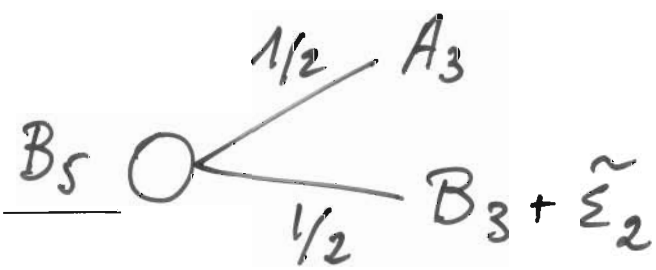
"Do not gamble with the location of the risk"

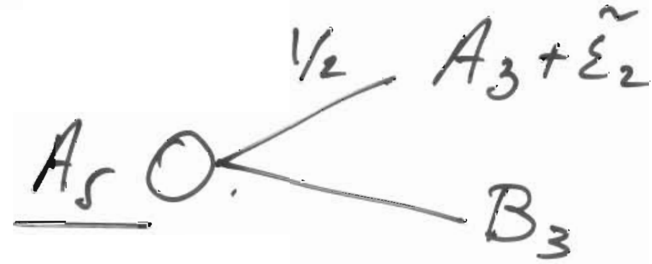
4~~3~~) And going further....

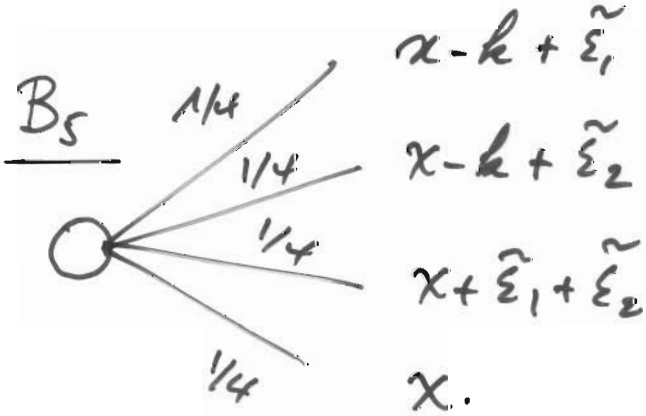
R.A. of order 5

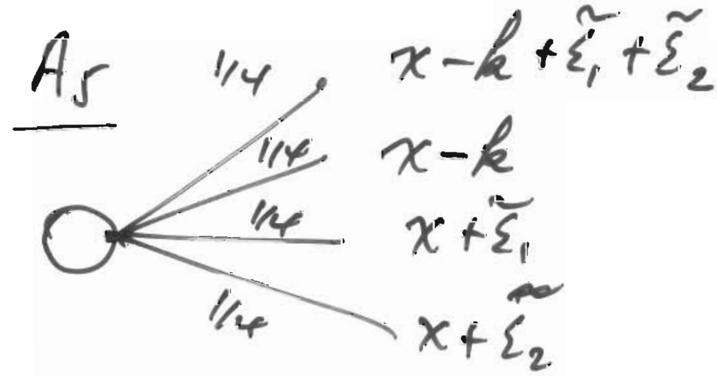


$\tilde{\varepsilon}_2$









$B_5 \succ A_5$: I like to disaggregate pains: they are better apportioned in B_5.

$[2, 2, 2, 0]$ $\qquad$ $[3, 1, 1, 1]$.

R.A. of order n:

see formula p. 284.

II 2. The TRANSLATION into E.U.

II 2.a. The tool we are going to use.

The cost of risk can be measured

Interpersonal comparisons are POSSIBLE

•) by the RISK PREMIUM. (99%)

$$E\left[u(x+\tilde{\varepsilon})\right] = u(x-\pi)$$

π = MONEY.

•) by the PROBABILITY premium (.7%)

$$\begin{array}{ccc} & \xrightarrow{1/2} & x-h \\ \bullet & & \\ & \xrightarrow{1/2} & x+h \end{array} \qquad \begin{array}{ccc} & \xrightarrow{1/2-h} & x-k \\ \bullet & & \\ & \xrightarrow{1/2+h} & x+h \end{array}$$

h = risk aversion = PROBABILITY

no interp. comparison

•) by the UTILITY PREMIUM.

$$W_1(x) = E\left[u(x+\tilde{\varepsilon})\right] - u(x)$$

See
- Friedman Savage (1948)
- Hanson Menezes (1971).
- Psychology literature
- O.R. papers + actuarial sc.

Properties of the utility premium.

$$W_1(x) = E\left[u(x+\tilde{\varepsilon})\right] - u(x) \approx \frac{\sigma^2}{2} u''(x)$$

$$\pi = -\frac{\sigma_\varepsilon^2}{2} \cdot \frac{u''(x)}{u'(x)} = \frac{-\text{utility premium}}{u'} = \frac{\text{pain}}{u'}$$

it BUNDLES two elements.

Illustration

u quadratic:

ut. premium is c^t when $x \uparrow$

u' decreases when $x \uparrow$

π increases " " "

u exponential.

ut. premium is $\updownarrow$ when $x \uparrow$

u' decreases " $x \uparrow$

π is c^t when $x \uparrow$

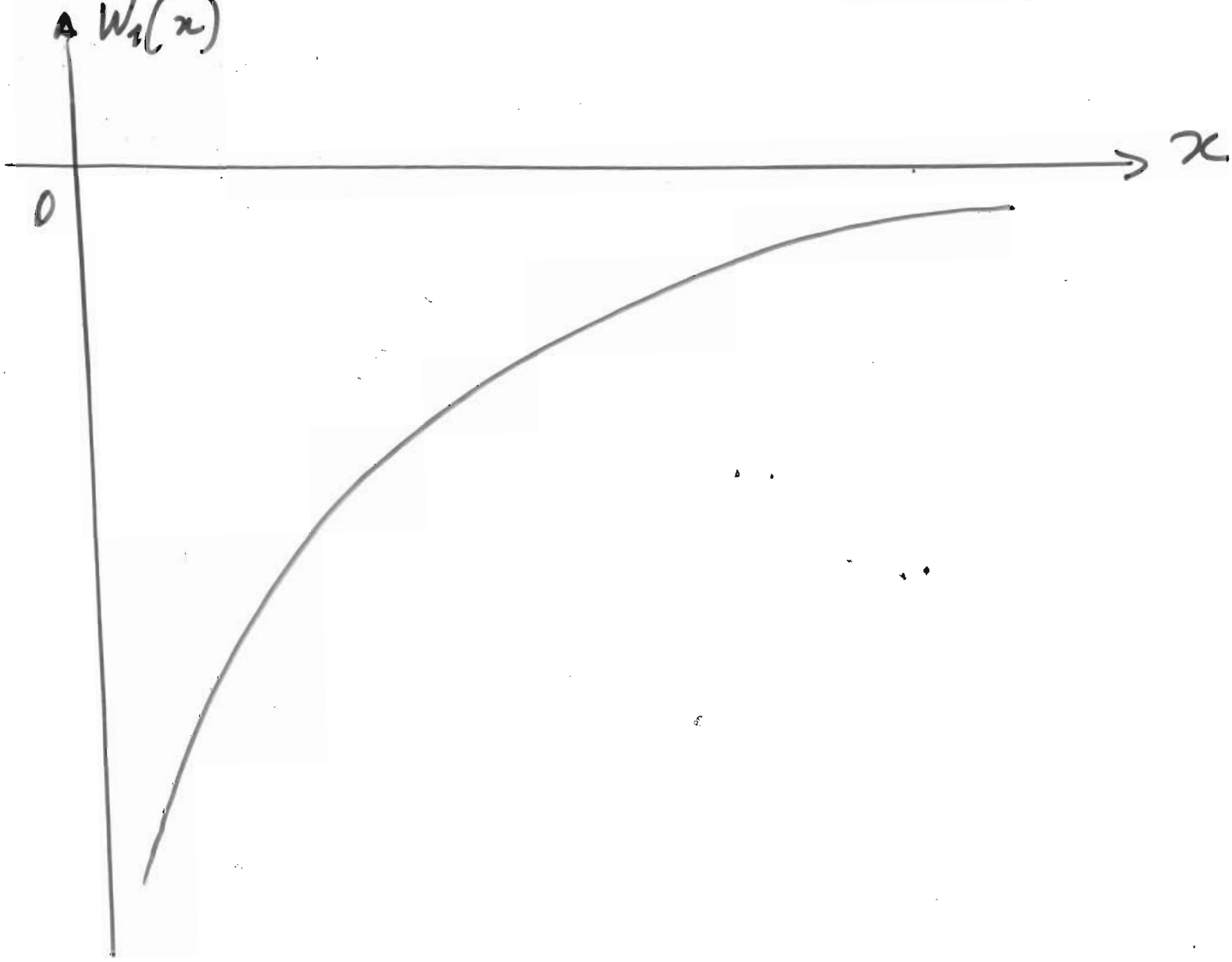
Notice also: by Jensen's inequ.

$$W_1(x) = E\left[u(x+\tilde{\varepsilon})\right] - u(x) < 0 \iff u'' < 0$$

$$W_1'(x) = E\left[u'(x+\tilde{\varepsilon})\right] - u'(x) > 0 \iff u''' > 0$$

$$W_1''(x) = E\left[u''(x+\tilde{\varepsilon})\right] - u''(x) < 0 \iff u^{(4)} < 0$$

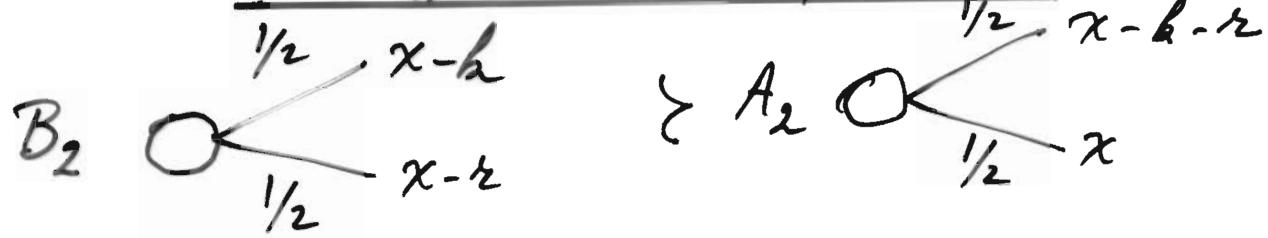
The $W_1(x)$ function with $u''<0$, $u'''>0$, $u^{(iv)}<0$.



$U''' > 0$ and $U^{(4)} < 0 \Longrightarrow$ $W_1(x)$ is an increasing and concave utility function of its own.

II 2 b The utility equivalence: our main results under E.U.

II 2 b. 1. Pain apportionment of order 2



In E. utility terms:

$$\frac{1}{2}\,U(x-k)+\frac{1}{2}\,U(x-r) > \frac{1}{2}\,U(x-k-r)+\frac{1}{2}\,U(x)$$

or.

$$U(x-k)-U(x-k-r) > U(x)-U(x-r)$$

It is more painful to lose r when one starts from $x-k$ than when one starts from x

In Kimball's terminology, pains are mutually aggravating.

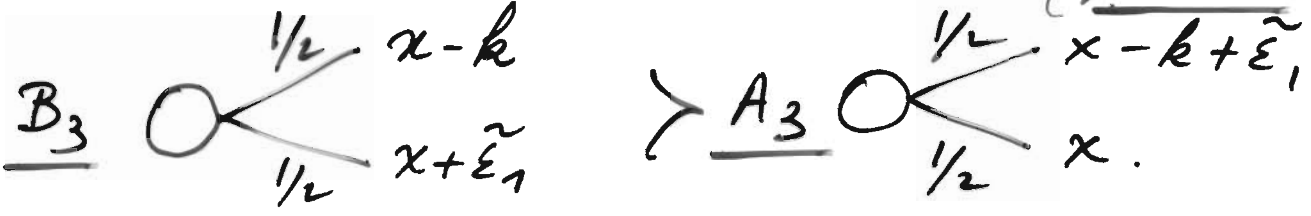
$$U'(x-k) > U'(x)$$

U' is decreasing in x.

$$U''(x) < 0.$$

risk aversion under E-U.

II 2 b. 2. Pain apportionment of order 3 (prudence)



In utility terms under E-U

$$\frac{1}{2}\,u(x-k)+\frac{1}{2}\,E\left[u(x+\tilde{\varepsilon}_1)\right] > \frac{1}{2}\,E\left[u(x-k+\tilde{\varepsilon}_1)\right] + \frac{1}{2}\,u(x)$$

or.

$$u(x-k) - E\left[u(x-k+\tilde{\varepsilon}_1)\right] > u(x) - E\left[u(x+\tilde{\varepsilon}_1)\right].$$

The pain attached to $\tilde{\varepsilon}_1$ is higher when one starts from $(x-k)$ than when one starts from x.

This is also equivalent to

$$E\left[u(x+\tilde{\varepsilon}_1)\right] - E\left[u(x+\tilde{\varepsilon}_1-k)\right] > u(x)-u(x-k).$$

Loosing k is more painful when the initial endowment is $x+\tilde{\varepsilon}_1$ rather than x.

$$E\left[u'(x+\tilde{\varepsilon}_1)\right] > u'(x).$$

u' is convex in x

$$u''' > 0.$$

$\doteq$ prudence under E-U.

II 2 b 3. Pain apportionment of order 4 (temperance)

$$B_4 \quad \bigcirc \begin{cases} \xrightarrow{1/2} x+\tilde{\varepsilon}_1 \\ \xrightarrow{1/2} x+\tilde{\varepsilon}_2 \end{cases} \quad \succ \quad A_4 \quad \bigcirc \begin{cases} \xrightarrow{1/2} x+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2 \\ \xrightarrow{1/2} x \end{cases}$$

In an E-U environment:

$$\frac{1}{2}E\left[u(x+\tilde{\varepsilon}_1)\right]+\frac{1}{2}E\left[u(x+\tilde{\varepsilon}_2)\right] > \frac{1}{2}E\left[u(x+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2)\right]+\frac{1}{2}u(x).$$

or

$$\underbrace{\left(E\left[u(x+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2)\right]-E\left[u(x+\tilde{\varepsilon}_2)\right]\right)}_{=E\left[W_1(x+\tilde{\varepsilon}_2)\right]} - \underbrace{\left(E\left[u(x+\tilde{\varepsilon}_1)\right]-u(x)\right)}_{=W_1(x)} < 0$$

$$E\left[W_1(x+\tilde{\varepsilon}_2)\right] - W_1(x) < 0.$$

Hence

$$B_4 \succ A_4 \Rightarrow E\left[W_1(x+\tilde{\varepsilon}_2)\right] < W_1(x)$$

i.e. W_1 concave

i.e. $u^{(4)} < 0$.

Pain apport^t of order 4 is temperance in the E-U model

... and so on.

Conclusion.

If an undergraduate student asks: "Professor, why do you assume that $u^{(7)}$ is positive?"

Now the answer is easy:

It is because decision makers do not like to aggregate pains!

Remark

Before, the answer was

"Because it leads to results that make sense".

III The multidimensional case

III 1. Early literature

1.a. Economics: savings under future income risk.

The concept of "decreasing temporal risk aversion"

$$E\left[u(c_1, \tilde{c}_2)\right] \Rightarrow \quad A_a = -\frac{u_{22}}{u_2}$$

Assumptions: $\frac{\partial}{\partial c_2}\left(-\frac{u_{22}}{u_2}\right) < 0$

and $\frac{\partial}{\partial c_1}\left(-\frac{u_{22}}{u_2}\right) \geq 0.$

Justification:

a. "It leads to sensible results"

b. If $c_1 \uparrow$, D. Makers are less. willing to take risks about c_2 because they are used to a high consumption level.

Notice that

$$\frac{\partial}{\partial c_1}\left(-\frac{u_{22}}{u_2}\right) = -\frac{u_2 u_{221} - u_{22} u_{12}}{u_2^2}$$

and the signs of u_{12} and u_{221} are not discussed. (except for the sign of u_{12} linked to the income effect).

1. b. The notion of "utility independ."

- Keeney (73) Econometrica
- " (74) Operations Research
- Keeney - Raiffa (76): "Decision with multiple objectives"

page 226 : central assumption

$U(x,y)$: local risk aversion towards x $\left(-\frac{U_{xx}}{U_x}\right)$ does not depend upon y.

$$\frac{\partial}{\partial y}\left(-\frac{U_{xx}}{U_x}\right) = 0.$$

Mutual ut. indep. $\Rightarrow$

$$U(x,y) = k_1 U_X(x) + k_2 U_Y(y) + k_3 U_X \cdot U_Y$$

Justification : make possible (and easy) the elicitation of a D.M's. utility function.

Notice that $\frac{\partial}{\partial y}\left(-\frac{U_{xx}}{U_x}\right) \Rightarrow$ that U_{12} and U_{221} alternate in sign.

1. c. Scott Richard's paper (Mngt Sc, 1975)

"Multivariate risk aversion, utility indep. and separable utility functions"

<u>Main result</u>: risk aversion is defined

---- by the sign of U_{12}!

$$U_{12} \begin{array}{l} <0 \\ =0 \\ >0 \end{array} \Rightarrow \begin{array}{l} \text{multiv. risk aversion.} \\ \text{" " neutrality} \\ \text{" " loving.} \end{array}$$

+ a definition of "<u>more multivariate risk averse</u>".

<u>Epstein - Tanny (1980)</u>: Richard should have said that the sign of U_{12} determines an attitude to correlation.

<u>Goal of this section:</u> interpret the sign of successive cross derivatives of $U(x, y)$

<u>Remark</u>: the impact of comorbidities in health decision making.

III 2 Three concepts: correlation aversion, cross prudence, cross temperance

$U(x,y)$ $\quad x =$ wealth
$\quad y =$ health.

4 pains: 2 sure pains: $-k$ and $-c$
2 random " : $\tilde{\varepsilon}$, $\tilde{\delta}$ with
$E(\tilde{\varepsilon}) = E(\tilde{\delta}) = 0$ and $\tilde{\varepsilon} \perp \tilde{\delta}$

$\boxed{\text{Outside E.U. ("model free")}}$ Assumptions: if I do not like to aggregate pains (i.e. "I prefer to mitigate pains"), then.

(a.1) Correlation aversion

$$\left\{\tfrac{1}{2}(x-k,y), \tfrac{1}{2}(x,y-c)\right\} \succ \left\{\tfrac{1}{2}(x-k,y-c), \tfrac{1}{2}(x,y)\right\}.$$

(a.2) Cross prudence

- in health.

$$\left\{\tfrac{1}{2}(x+\tilde{\varepsilon},y), \tfrac{1}{2}(x,y-c)\right\} \succ \left\{\tfrac{1}{2}(x+\tilde{\varepsilon},y-c), \tfrac{1}{2}(x,y)\right\}.$$

- in wealth.

$$\left\{\tfrac{1}{2}(x,y+\tilde{\delta}), \tfrac{1}{2}(x-k,y)\right\} \succ \left\{\tfrac{1}{2}(x-k,y+\tilde{\delta}), \tfrac{1}{2}(x,y)\right\}.$$

(a.3) Cross temperance

$$\left\{\tfrac{1}{2}(x+\tilde{\varepsilon},y), \tfrac{1}{2}(x,y+\tilde{\delta})\right\} \succ \left\{\tfrac{1}{2}(x+\tilde{\varepsilon},y+\tilde{\delta}), \tfrac{1}{2}(x,y)\right\}.$$

If I like to aggreg. pains $\succ$ becomes $\prec$ (b.1).

III 3. Translation into the E-U model

1. To introduce the topics let's consider

2 Special cases

- Perfect substitutes

$$U(x,y) = U(x+y)$$

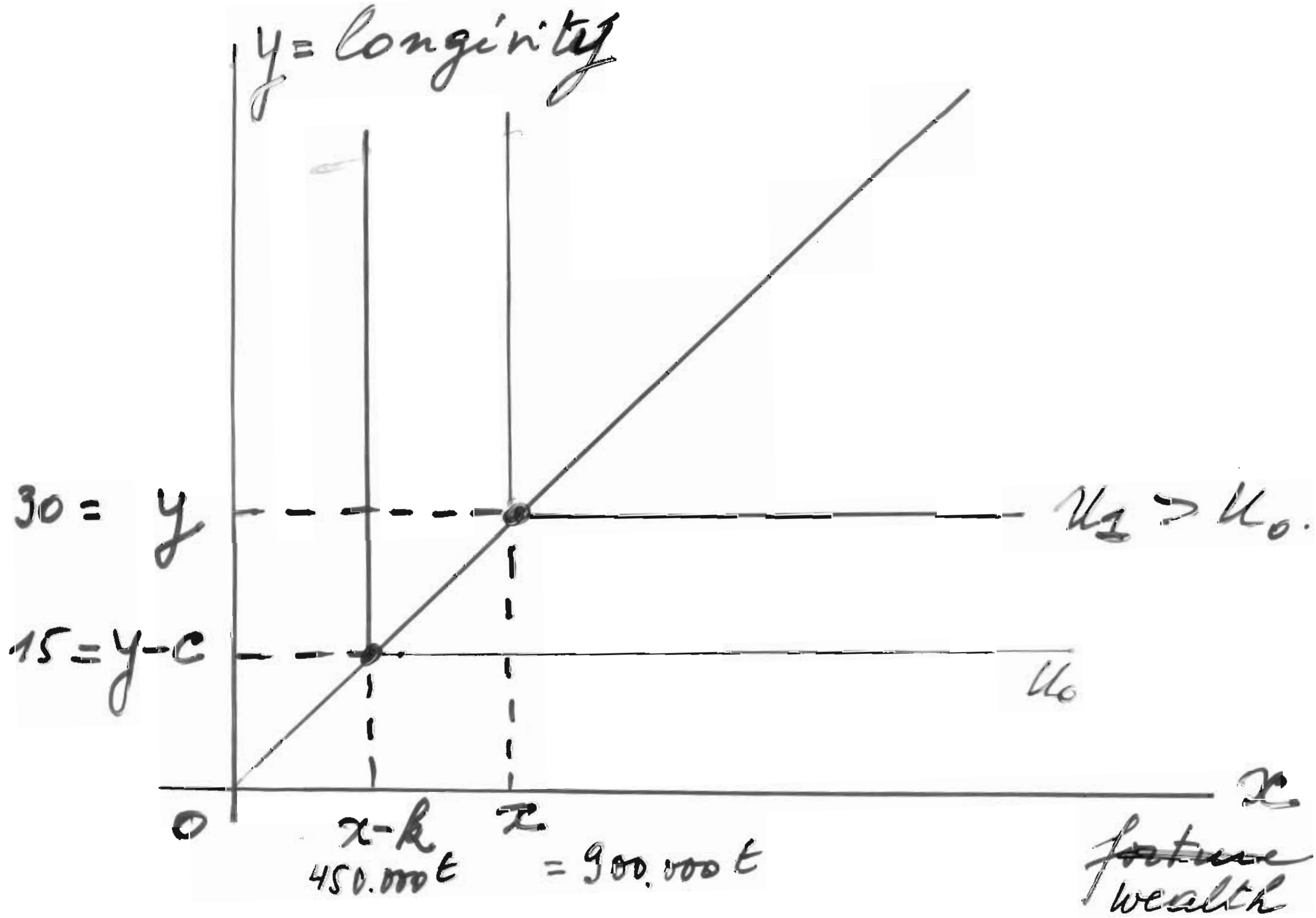
e.g. Vaitianatham, J.H.E, November 2006, pp. 1193-1202

$$U(c,l) = U(c+g(l)).$$

- Perfect complements:

$$U(x,y) = U(\min(x,y)).$$

Correlation prone.



$$\left\{ \frac{1}{2}(x-k, y), \frac{1}{2}(x, y-c) \right\} = \frac{1}{2} U_0 + \frac{1}{2} U_0 = U_0.$$

$$\left\{ \frac{1}{2}(x-k, y-c), \frac{1}{2}(x, y) \right\} = \frac{1}{2} U_0 + \frac{1}{2} U_1.$$

Comme $U_1 > U_0$.

Je préfère l'agrégation des peines

Data.

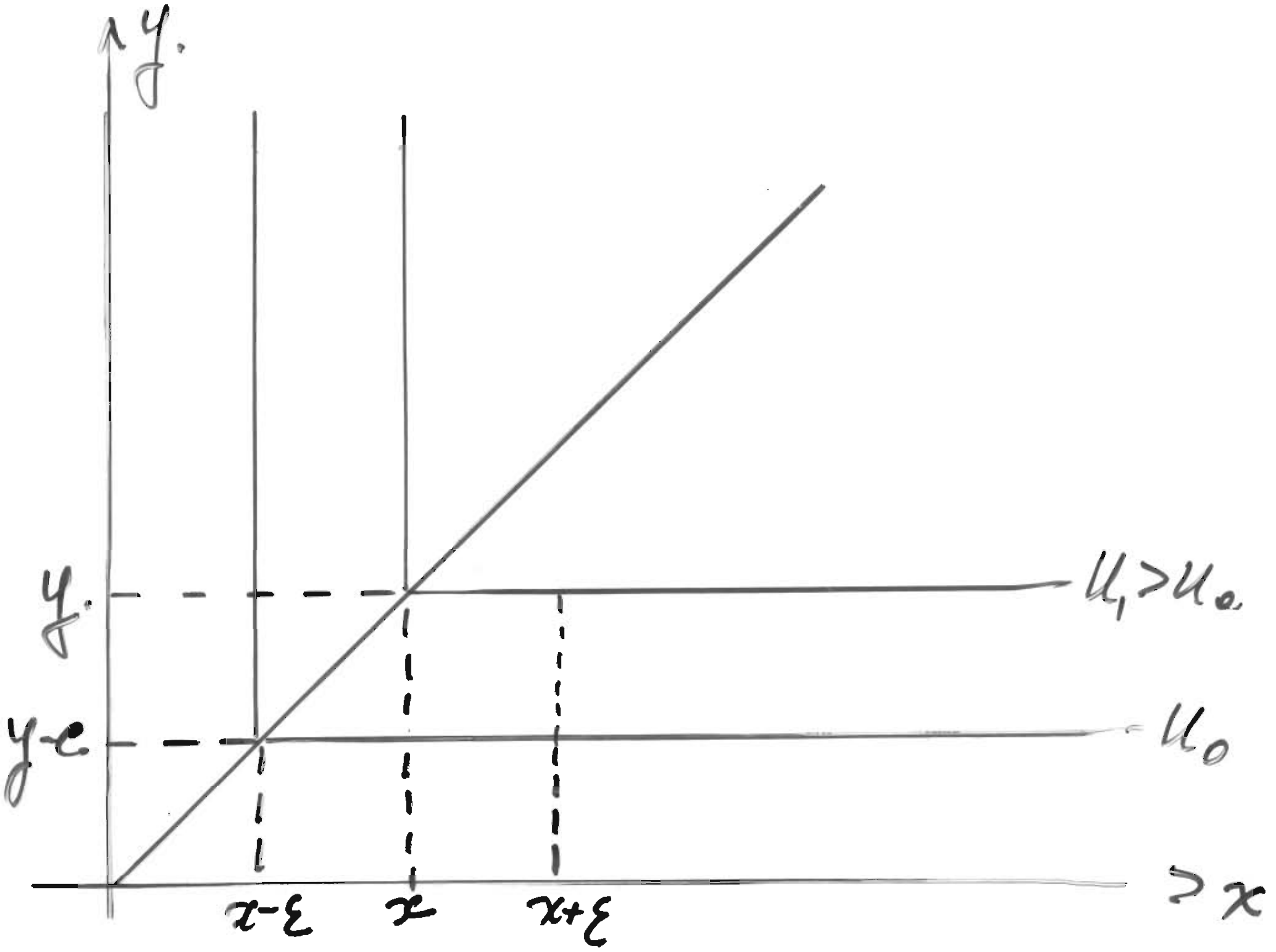
Données: 60 ~~ans~~ years

célibataire sans enfants ("no bequest motive")

30.000 £ ~~par an~~ year

INTUITION: fixed proportions.

Cross imprudence



$$\left\{\tfrac{1}{2}(x+\tilde{\varepsilon}, y), \tfrac{1}{2}(x, y-c)\right\} = \tfrac{1}{2}\left(\tfrac{1}{2}u_0 + \tfrac{1}{2}u_1\right) + \tfrac{1}{2}u_0$$

$$= \tfrac{3}{4}u_0 + \tfrac{1}{4}u_1.$$

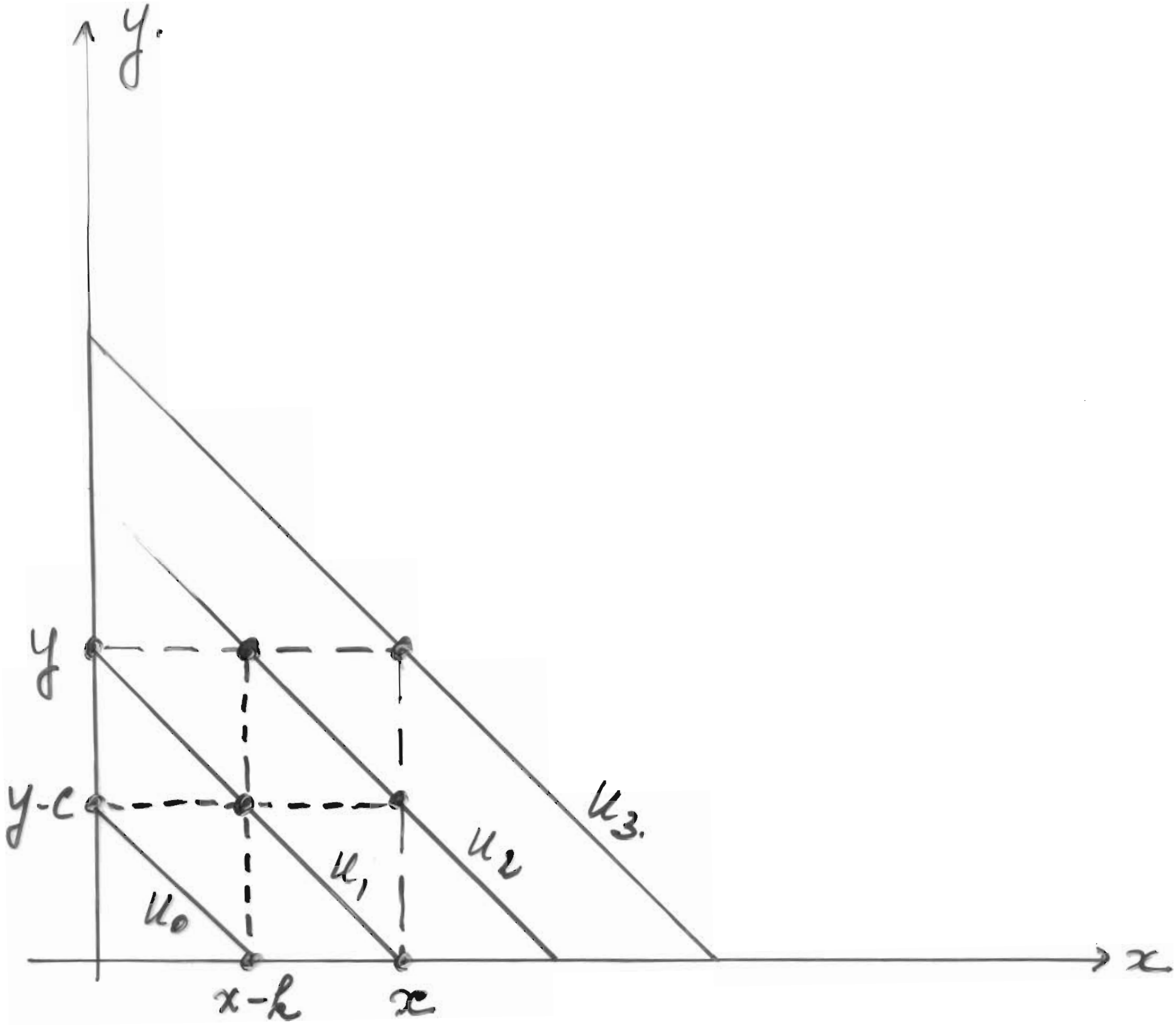
$$\left\{\tfrac{1}{2}(x+\tilde{\varepsilon}, y-c), \tfrac{1}{2}(x, y)\right\} = \tfrac{1}{2}\left(\tfrac{1}{2}u_0 + \tfrac{1}{2}u_0\right) + \tfrac{1}{2}u_1.$$

$$= \tfrac{1}{2}u_0 + \tfrac{1}{2}u_1$$

$u_1 > u_0 \Rightarrow$ cross-imprudence.

J'aime l'agrégat des lines

Corrélation aversion.



$$\left\{\tfrac{1}{2}(x-k, y), \tfrac{1}{2}(x, y-c)\right\} = \tfrac{1}{2} u_2 + \tfrac{1}{2} u_2 = u_2$$

$$\left\{\tfrac{1}{2}(x-k, y-c), \tfrac{1}{2}(x, y)\right\} = \tfrac{1}{2} u_1 + \tfrac{1}{2} u_3$$

<u>Si U est concave</u>, $u_2 > \tfrac{1}{2} u_1 + \tfrac{1}{2} u_3$

<u>Je préfère désagr. les peines.</u>

U concave et différent $\Rightarrow u_{11} = u_{22} = u_{12} < 0$!

2. The sign of successive cross derivatives.

Proposition: in the E-U model

$$(a.1) \Longleftrightarrow U_{12} < 0$$

$$(a.2) \Longleftrightarrow U_{122} > 0 \text{ and } U_{112} > 0$$

$$(a.3) \Longleftrightarrow U_{1122} < 0.$$

Proofs.

(a.1) $\Rightarrow$ $\frac{1}{2} U(x-k, y) + \frac{1}{2} U(x, y-c)$

$$> \frac{1}{2} U(x-k, y-c) + \frac{1}{2} U(x, y)$$

or.

$$U(x-k, y) - U(x-k, y-c) > U(x,y) - U(x, y-c)$$

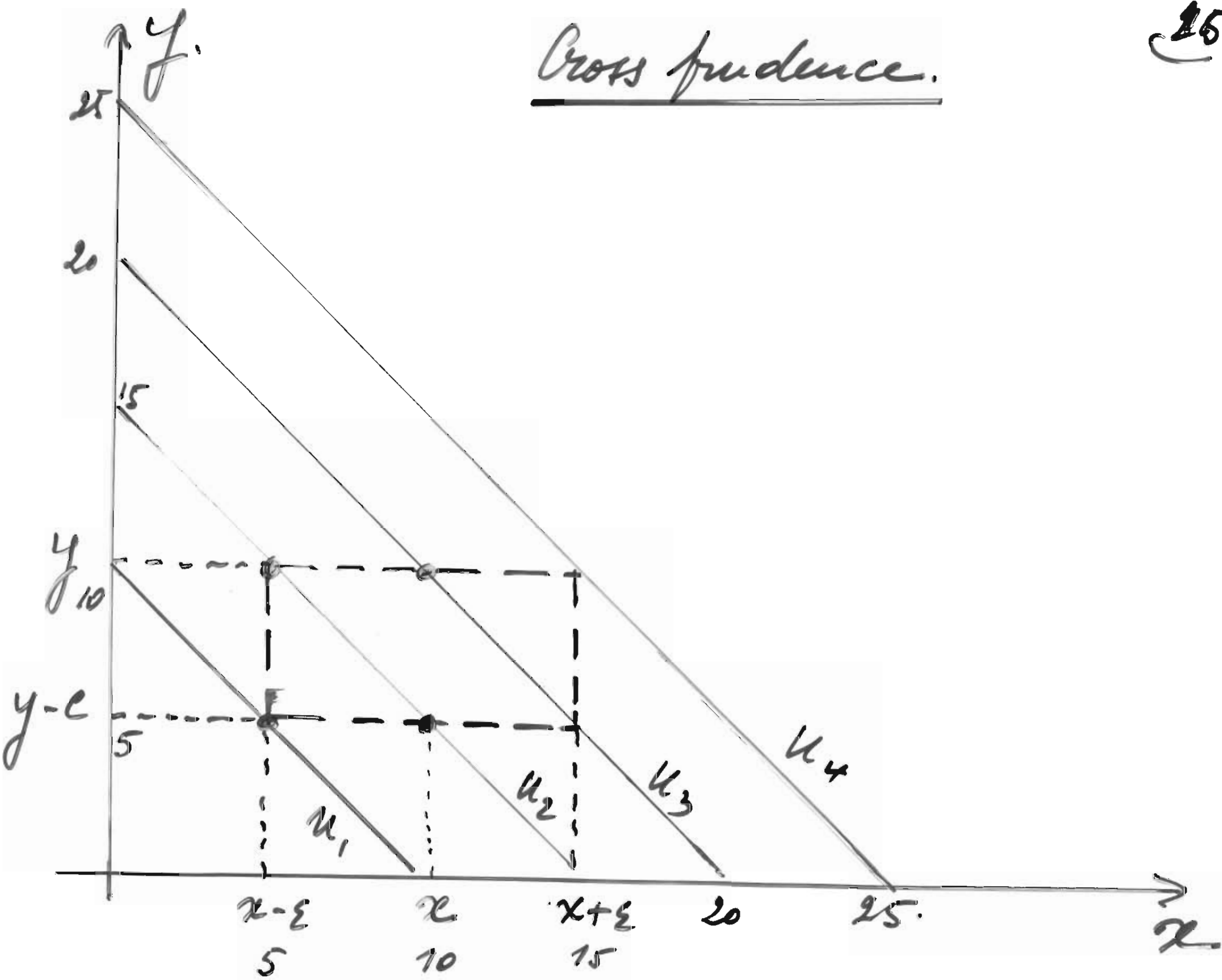
The pain of loosing c units of health when I am poor is larger than when I am rich.

$$U_2(x-k, y) > U_2(x,y)$$

$$U_{21} < 0.$$

(When I am rich, money compensates for the reduced health: health and wealth are subst.)

Cross prudence.

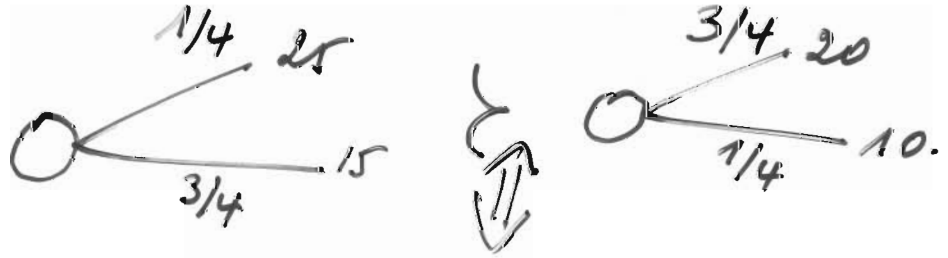


$$\left\{\frac{1}{2}(x+\tilde{\varepsilon}, y), \frac{1}{2}(x, y-c)\right\} = \frac{1}{2}\left(\frac{1}{2}u_2 + \frac{1}{2}u_4\right) + \frac{1}{2}u_2$$

$$= \frac{1}{4}u_4 + \frac{3}{4}u_2$$

$$\left\{\frac{1}{2}(x+\tilde{\varepsilon}, y-c), \frac{1}{2}(x, y)\right\} = \frac{1}{2}\left(\frac{1}{2}u_1 + \frac{1}{2}u_3\right) + \frac{1}{2}u_3$$

$$= \frac{1}{4}u_1 + \frac{3}{4}u_3.$$



Si $\underline{u'''} = u_{111} = u_{222} = \underline{u_{122}} = \underline{u_{211}} > 0.$

Qd j'aime dissocier les peines il y a cross-prudence (a.2): One considers again a utility premium.

$$V(x,y) = U(x,y) - E\left[U(x+\tilde{\varepsilon}, y)\right].$$

so that $V > 0 \iff U_{11} < 0$.

If an increase in y mitigates the pain due to risky wealth one has

$$V_2 < 0$$

But $V_2 = U_2(x,y) - E\left[U_2(x+\tilde{\varepsilon}, y)\right]$.

Hence $V_2 < 0 \iff U_{211} > 0$

~~which correspond~~

Now $V_2 < 0$ at any point means:

$$\Big(U(x,y) - U(x,y-c)\Big) - \Big(E\left[U(x+\tilde{\varepsilon},y)\right] - E\left[U(x+\tilde{\varepsilon},y-c)\right]\Big) < 0.$$

or.

$$\frac{1}{2}U(x,y) + \frac{1}{2}E\left[U(x+\tilde{\varepsilon}, y-c)\right] < \frac{1}{2}U(x,y-c) + \frac{1}{2}E\left[U(x+\tilde{\varepsilon},y)\right]$$

which is the E.U. translation of cross-prudence.

Note: a few utility functions

$$u(x,y) = \underline{-}\ \underline{x}^{-\alpha}\ \underline{y^{-\beta}}$$

$\Longrightarrow$ (a.1), (a.2), (a.3)

$$u(x,y) = \underline{x}^{\alpha}\ \underline{y^{\beta}} \qquad (0<\alpha<1,\ 0<\beta<1)$$

$\Longrightarrow$ (b.1), (b.2), (b.3)

$$u(x,y) = \underline{xy} - \underline{\tfrac{1}{2}}\ \underline{x^2}\ \underline{y^2}.$$

(a.1) : sometimes yes, sometimes not.

(a.2) : no

(a.3) : yes.

III 4 Applications.

$$\underline{\text{Max } U(s)} = u(x-s, y) + u(x+s, y)$$

$$s^* = 0$$

4.1.

$$\max_s \; u(x-s, y) + u(x+s, y-c)$$

If $u_{12} < 0$ $\quad s^* > 0$: I accumulate wealth to mitigate the pain due to $-c$.
(This has nothing to do with mvt in health)

4.2. $$\max_s \; u(x-s, y) + E\left[u(x+s, y+\tilde{S})\right]$$

$$\left.\frac{dU}{ds}\right|_{s=0} = -u_1(x,y) + E\left[u_1(x, y+\tilde{S})\right]$$

This is positive if $u_{122} > 0$

4.3. Exogenous income risk ($\tilde{\varepsilon}$) to be spread on the 2 period.

$$\max_\alpha \; E\left[u(x+\alpha\tilde{\varepsilon}, y)\right] + E\left[u(x+(1-\alpha)\tilde{\varepsilon}, y)\right]$$

Sol: $\alpha^* = 1/2$

$$\max_\alpha \; E\left[u(x+\alpha\tilde{\varepsilon}, y)\right] + E\left[u(x+(1-\alpha)\tilde{\varepsilon}, y+\tilde{S})\right]$$

$$u_{1122} < 0 \implies \alpha^* > 1/2.$$

IV Extensions.

IV 1. TSETLIN's theorem.

Let $\tilde{X}_1 \succsim_n \tilde{Y}_1$.

$\tilde{X}_2 \succsim_m \tilde{Y}_2$.

Then, if I prefer to "combine good with bad":

$$\left\{\frac{1}{2}(\tilde{X}_1+\tilde{Y}_2), \frac{1}{2}(\tilde{X}_2+\tilde{Y}_1)\right\} \succsim_{?} \left\{\frac{1}{2}(\tilde{X}_1+\tilde{X}_2), \frac{1}{2}(\tilde{Y}_1+\tilde{Y}_2)\right\}$$

Answer: $n+m$.

Illustrations.

•) $1 \succsim_1 0$ $\quad \left\{\frac{1}{2}(0), \frac{1}{2}(0)\right\} \succ \left\{\frac{1}{2}(1), \frac{1}{2}(-1)\right\}$

$0 \succsim_1 -1$

$$0 \succsim_{?} \left\{\frac{1}{2}(1), \frac{1}{2}(-1)\right\}$$

yes under SSD

•) $k \succsim_1 0$

$0 \succsim_2 \tilde{\varepsilon}$ $\quad \left\{\frac{1}{2}(k+\tilde{\varepsilon}), \frac{1}{2}(0)\right\} \succsim_3 \left\{\frac{1}{2}(k), \frac{1}{2}(\tilde{\varepsilon})\right\}$

Prudence: I prefer to face the "bad" $\tilde{\varepsilon}$ when I have the "good" k.

[To fully coincide with III, write instead $0 \succsim_1 -k$.]

IV 2. B. REY's suggestion

To use the idea of "correlation aversion" (coming from the multidim. problem) into the unidimensional case.

Hypothesis: I do not like correlation of the pairs.

	$-k$	0	
$-r$	$p_1\theta$	$1-p_1\theta$	p_1
0	$p_2 - p_1\theta$	$1-p_2-p_1+p_1\theta$	$1-p_1$
	p_2	$1-p_2$	

$(p_1 \leq p_2)$

$0 \leq \theta \leq 1.$

$\theta = p_2$: indep.

$\theta = 0$: negat. correlat.

$\theta = 1$: positive "

$$\frac{dE[u]}{d\theta} = p_1\left[u(x-k-r) + u(x) - \left(u(x-k) + u(x-r)\right)\right]$$

and $\frac{dE[u]}{d\theta} < 0 \implies u'' < 0.$

Then replace $-r$ by $\tilde{\varepsilon}$ (or by $-r+\tilde{\varepsilon}$) and you generate prudence.

V. Further questions.

1. From the "sign" to the intensity.

2. What about non E-U models?
   i.e. how to translate "fair apportionment" into prospect theory?