

# Intergenerational Fairness

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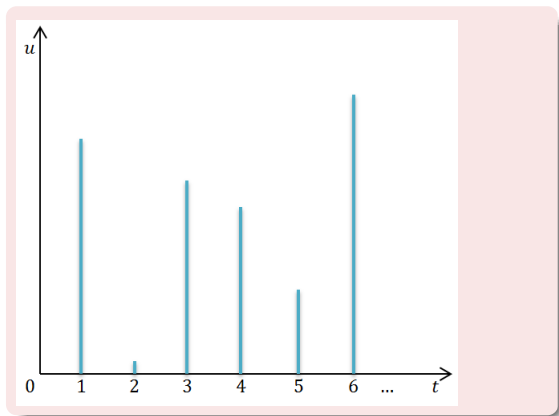
Canazei, 14th January 2015

- What can be understood as a *fairness approach* to *intergenerational justice*?
- What are the advantages/difficulties compared to other approaches?
- Some results and open challenges.

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- Infinite utility streams
- Seminal contributions
- Properties



- Infinite utility streams
  - **Seminal contributions**
  - Properties
- The axiomatic approach dates back to Koopmans (1960) and Diamond (1965).
  - A large part of the literature deals with operationalizing the discounted utilitarian approach. A smaller part studies how to avoid Diamond's impossibility.
  - For a review see Asheim (2010).

- Infinite utility streams
- Seminal contributions
- **Properties**
  - Infinite time horizon
  - Ordinal level comparability (and cardinal measurability)
  - One size fits all

## Asheim (1991)

This is the first article that highlights the importance of the specific economic situation for intergenerational justice.

After ruling out "unjust intergenerational allocations" and for a specific class of technologies, the previous difficulties of the literature are (mostly) avoided.

## Asheim et al.(2001), Asheim et al. (2010)

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The domain restriction introduced is "eventual productivity".

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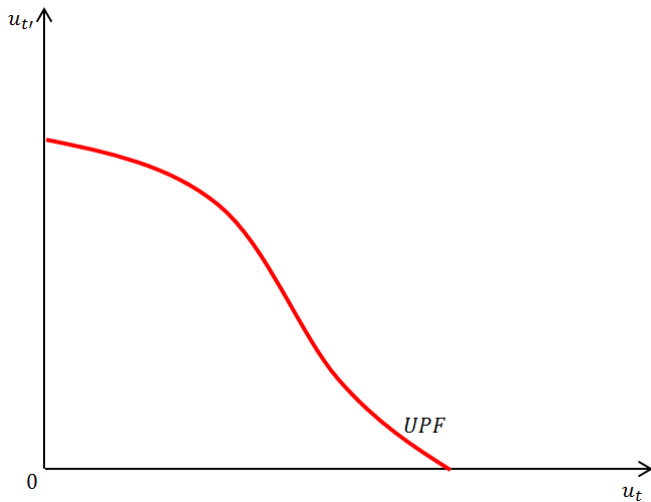
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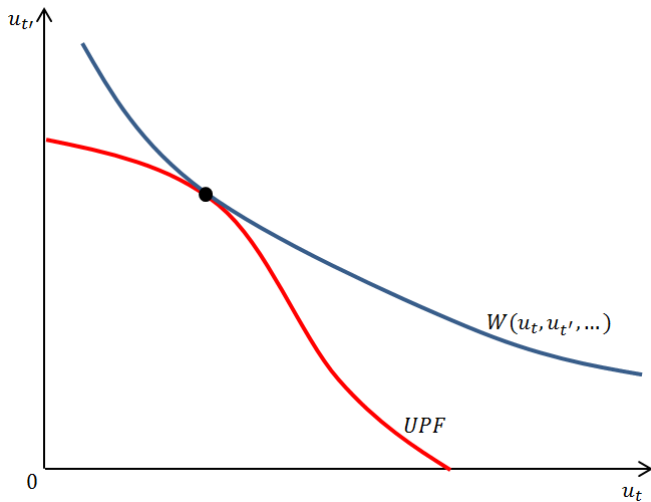
Asheim et al. (2010) chooses equitable and efficient paths of consumptions. Productivity of capital needs to be larger than 1.

- The social desirability of alternatives is judged by how resources are distributed to the agents.
- Ethical considerations:
  - take into account economic circumstances (production technology, timing and resolution of risk, extinction possibility, population dynamics, ...);
  - respect each agent's preferences.
- The social viewpoint can be expressed as an allocation rule (see Thomson, 2011) or as a social ordering function (see Fleurbaey and Maniquet, 2011).

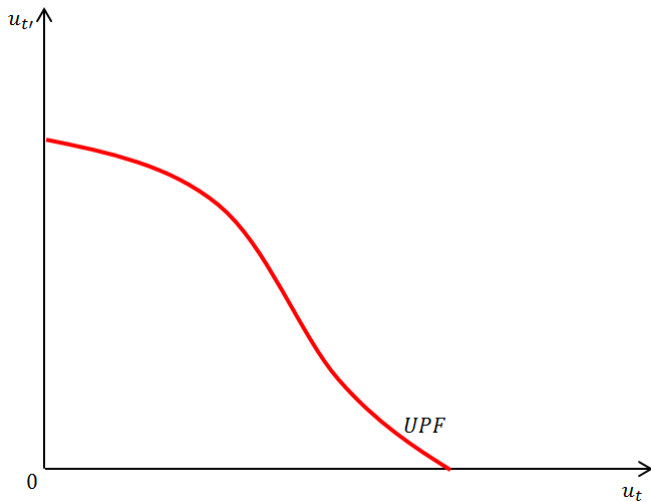
# The utility possibility frontier



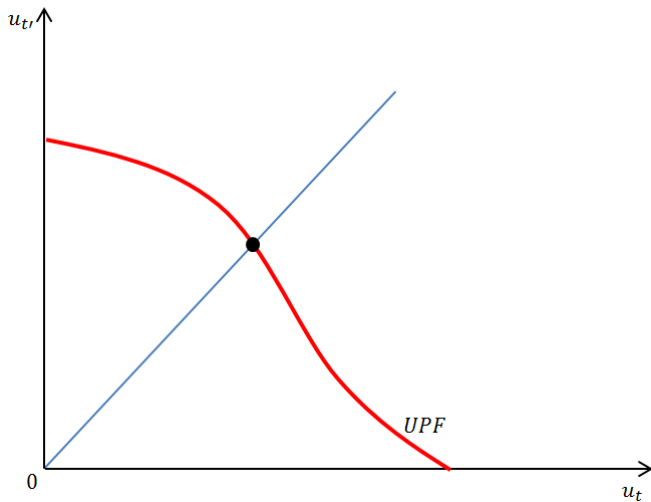
# First best choice for $W$



# A more basic problem



When is it that  $u_t = u_{t'}$ ?



- When  $u_t = u_{t'}$ , generations  $t$  and  $t'$  are viewed as equally well-off.
- Then it is natural to associate to equal utilities an allocation of resources that is egalitarian and (under minor domain restrictions) efficient.
- The first result I will discuss is the emergence of an “*Equity gap*” (based on “*Intergenerational Egalitarianism*,” 2014a).
- Efficiency and equity are extremely difficult to combine:
  - impossibility results arise already with very weak principles of justice;
  - thus, equalizing utilities does not correspond to very egalitarian distributions of goods.

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# The model (1)

- Time is discrete and finite, i.e.  $T \equiv \{0, 1, \dots, \bar{t}\}$ .
- In each period  $t$ , a stock of capital  $k_t \in \mathbb{R}_+^2$  is used for production.
- Technology is time-invariant, linear, and separable across goods:  $y_t \leq F(k_t) = \begin{pmatrix} \rho^1 & 0 \\ 0 & \rho^2 \end{pmatrix} k_t$ .
- Production is shared between consumption of the currently living generation, i.e.  $x_t \in \mathbb{R}_+^2$ , and savings for future generations, i.e.  $k_{t+1}$ .
- For each  $t \in T$ , generation  $t$  has a preference relation  $R_t$  defined over  $\mathbb{R}_+^2$ , which can be represented by a Cobb-Douglas utility function.

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## The model (2)

- An **economy** is a list  $E \equiv (\underline{k}, F, R)$ .
- Let  $\mathcal{E}$  be the **domain** of economies  $E$  with at least one feasible allocation.
- For each  $E \in \mathcal{E}$ , let  $A(E)$  be the set of **feasible allocations of  $E$** .
- An **allocation rule** is a correspondence that selects a non-empty subset of feasible allocations for each economy in the domain.

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## (Pareto) efficiency

For each  $E \in \mathcal{E}$ ,  $a \in A(E)$  is (Pareto) **efficient** if there is no  $a' \in A(E)$  such that for each  $t \in T$ ,  $x'_t R_t x_t$ , and for some  $t \in T$ ,  $x'_t P_t x_t$ .

## $\varepsilon$ -no-domination

Let  $\varepsilon \in [0, 1]$ . For each  $E \in \mathcal{E}$ ,  $a \in A(E)$  satisfies  **$\varepsilon$ -no-domination** if for each pair  $t, t' \in T$ ,  $x_t \not\ll \varepsilon x_{t'}$ .

## $\varepsilon'$ -equal treatment of equals

Let  $\varepsilon' \in [0, 1]$ . For each  $E \in \mathcal{E}$ ,  $a \in A(E)$  satisfies  **$\varepsilon'$ -equal treatment of equals** if for each pair  $t, t' \in T$  such that  $R_t = R_{t'}$ ,  $x_t R_t \varepsilon' x_{t'}$ .

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## Theorem

*On the domain  $\mathcal{E}$  and for each pair  $\varepsilon, \varepsilon' \in (0, 1]$ , no rule satisfies efficiency,  $\varepsilon$ -no-domination, and  $\varepsilon'$ -equal treatment of equals.*

- *Efficiency* forces taking into account the different conditions at different times (relative scarcity of goods); whereas
- equity impedes placing much importance to the time generations live in. Moreover:
  - *$\varepsilon$ -no-domination* is based on comparing physical amounts;
  - *$\varepsilon'$ -equal treatment of equals* is based on preferences;
- Thus: there is no common ground evaluation that allows combining the axioms, no matter how small (but positive)  $\varepsilon$  and  $\varepsilon'$  are.

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## $n$ -period $\varepsilon$ -no-domination

Let  $n \geq 2$  and  $\varepsilon \in [0, 1]$ . For each  $E \in \mathcal{E}$ ,  $a \in A(E)$  satisfies  **$n$ -period  $\varepsilon$ -no-domination** if for each pair  $t, t' \in T$  with  $|t - t'| < n$ ,  $x_t \not\prec \varepsilon x_{t'}$ .

## $n'$ -period $\varepsilon'$ -equal treatment of equals

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Let  $\varepsilon, \varepsilon' \in [0, 1]$  and  $n, n' \geq 2$ . On the domain  $\mathcal{E}$ :

- If  $n$  and  $\varepsilon$  are such that  $(n - 2)\varepsilon \neq 0$ , then there exists a rule satisfying efficiency and  $n$ -period  $\varepsilon$ -no-domination. Furthermore, no such rule satisfies  $n'$ -period  $\varepsilon'$ -equal treatment of equals, unless  $\varepsilon' = 0$ .

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- *If  $\varepsilon$  and  $\varepsilon'$  are such that  $\varepsilon\varepsilon' \neq 0$ , then there exists a rule satisfying efficiency, 2-period  $\varepsilon$ -no-domination, and 2-period  $\varepsilon'$ -equal treatment of equals. Furthermore, when  $nn' > 4$ , there exists no rule no rule satisfying efficiency,  $n$ -period  $\varepsilon$ -no-domination and  $n'$ -period  $\varepsilon'$ -equal treatment of equals.*

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# An ethical dilemma

## Time independent rules

- Efficiency + no-domination; *or*
- Efficiency + equal treatment of equals.

## Sequential rules

- Efficiency + 2-period no-domination + 2-period equal treatment of equals.

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  - a strong conflict between equity and efficiency arises;
  - and unveils an “equity gap.”
- Overcoming such tension, leads to a new ethical dilemma for intergenerational justice:
  - short-term/long-term inequality trade-off.

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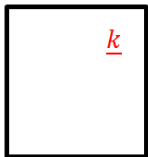
- Assume we made up our mind about the meaning of intergenerational egalitarianism.
- How to use such reference (identified by the allocation rule) to construct “acceptable” social preferences?
- I will try to give you some intuition for a different, but related, model dealing with risk (“Fair intergenerational utilitarianism,” 2014b).

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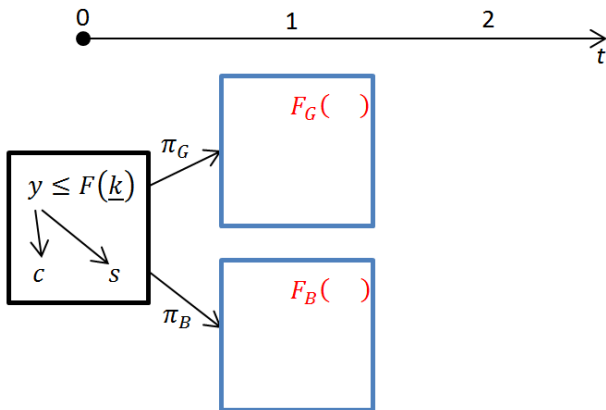
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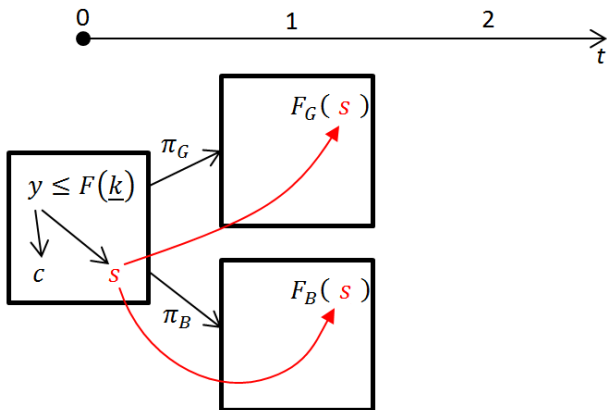


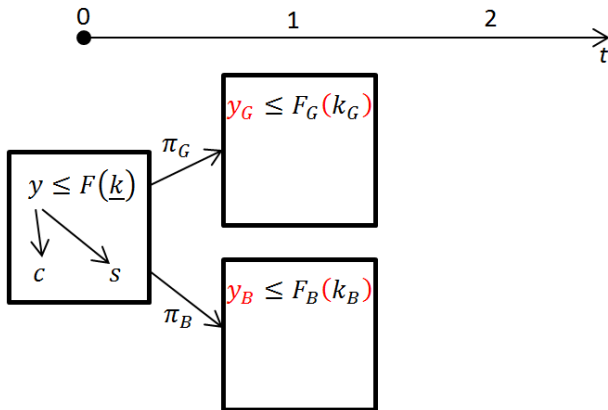
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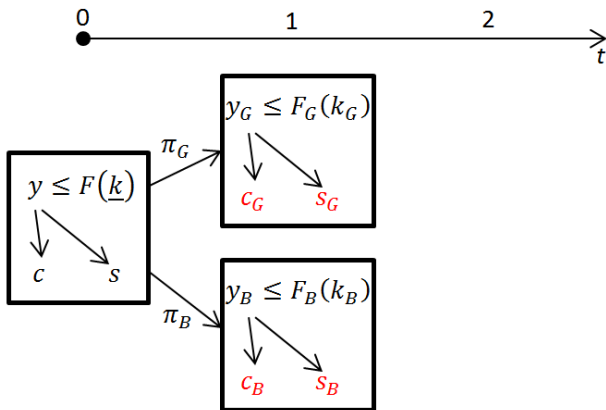
Two red arrows originate from the expression  $y \leq F(\underline{k})$ . One arrow points down to the letter  $c$ , and the other points down and to the right to the letter  $s$ .

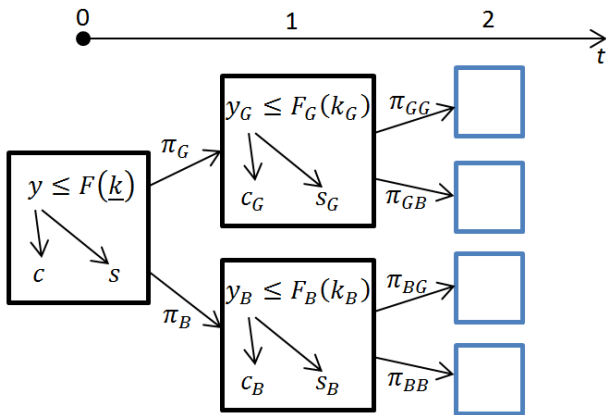
# Period 1











$$D \equiv \langle \langle \pi_n, F_n \rangle_{n \in N}, \underline{k} \rangle$$

where:

- $N$  is the event tree;
- $\underline{k}$  is the initial capital stock.

For each  $t \in T$  and  $n \in N_t$ :

- $\pi_n \in (0, 1]$  is the probability that node  $n$  is reached at  $t$ ;
- $F_n$  is the production function that transforms input  $k_n$  into output  $y_n \in \mathbb{R}_+$ ;  $F_n$  is continuous, strictly increasing, and satisfies no-free lunch;
- output  $y_n$  can be consumed,  $c_n$ , or saved  $s_n$ ;
- $s_n$  determines the capital stock of the immediate successor nodes:  $k_{n'} = s_n$  for each  $n' \in N_{+1}(n)$ .



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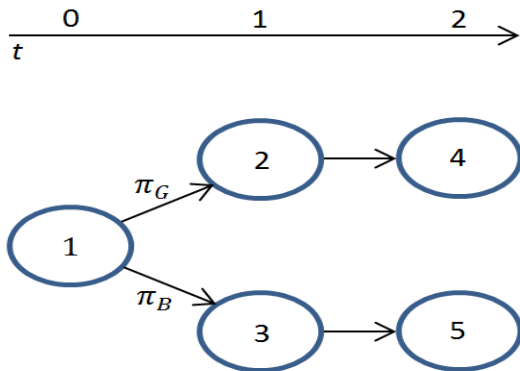
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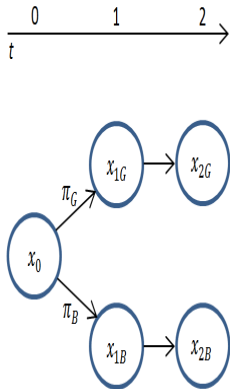
# A risky intergenerational prospect



$$c \equiv (\{c_n\}_{n \in N}) \in C(\mathbf{D})$$

- The **fair prospect**  $x \equiv (\{x_n\}_{n \in N})$  is uniquely identified by the allocation rule  $\phi : \mathcal{D} \rightarrow 2^{C(\mathcal{D})} \setminus \{\emptyset\}$ .
- The fair rule  $\phi$  satisfies:
  - *Maximality*; and
  - *“interim egalitarianism.”*

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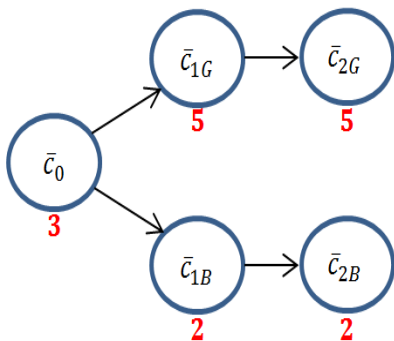
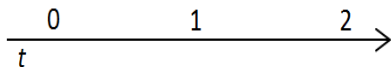
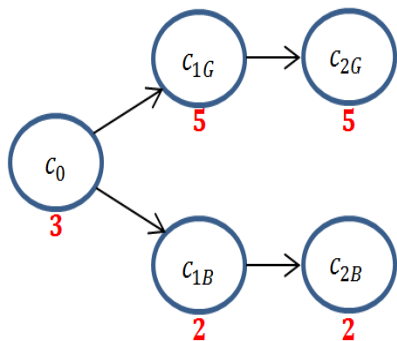
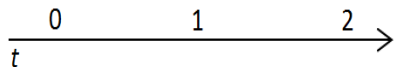
## Interim egalitarianism

Let  $\mu : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  be strictly increasing and concave. For each  $D \in \mathcal{D}$ , each  $x \in \phi(E)$ , each  $n \in N$ , and each  $t > t_n$ :

$$x_n = \mu^{-1} \left( \frac{\sum_{n' \in N_t(n)} \pi_{n'} \mu(x_{n'})}{\sum_{\bar{n} \in N_t(n)} \pi_{\bar{n}}} \right)$$

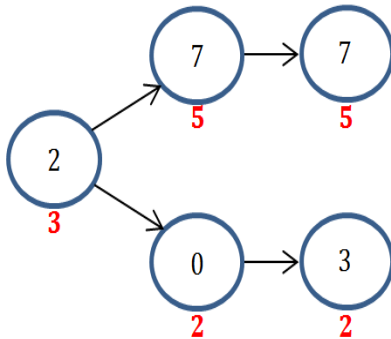
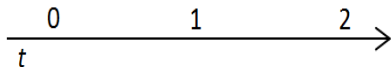
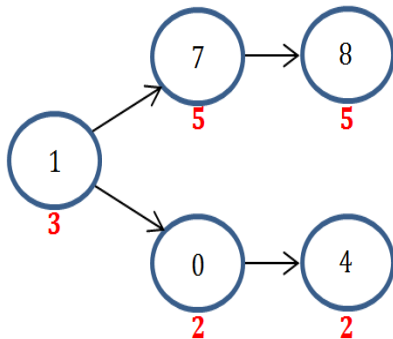
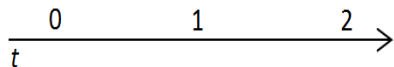
- For each  $D \in \mathcal{D}$ , a **social ordering** of  $D$  is a complete and transitive binary relation defined over the prospects  $C(D)$ .
- A **social ordering function**  $\succsim$  assigns to each decision tree  $D \in \mathcal{D}$  a social ordering of  $D$  denoted  $\succsim_D$ .
- Thus,  $c \succsim_D \bar{c}$  means that the prospect  $c$  is socially at least as desirable as  $\bar{c}$  for decision tree  $D$ .
  - The symmetric and asymmetric counterparts of  $\succsim_D$  are  $\sim_D$  and  $\succ_D$ .
- $V(c; D)$  is a welfare representation of  $\succsim_D$ .

# Comparing alternatives

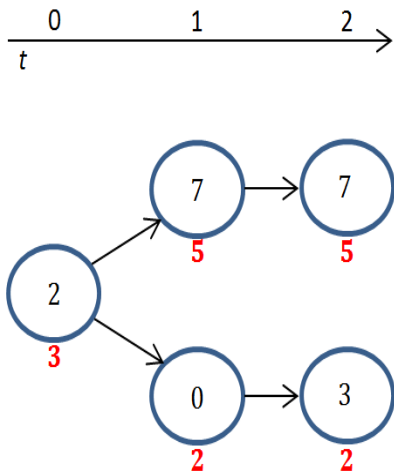
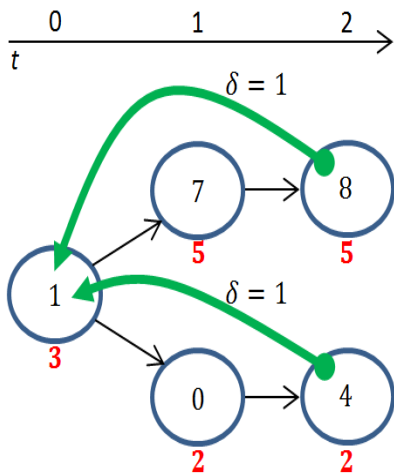




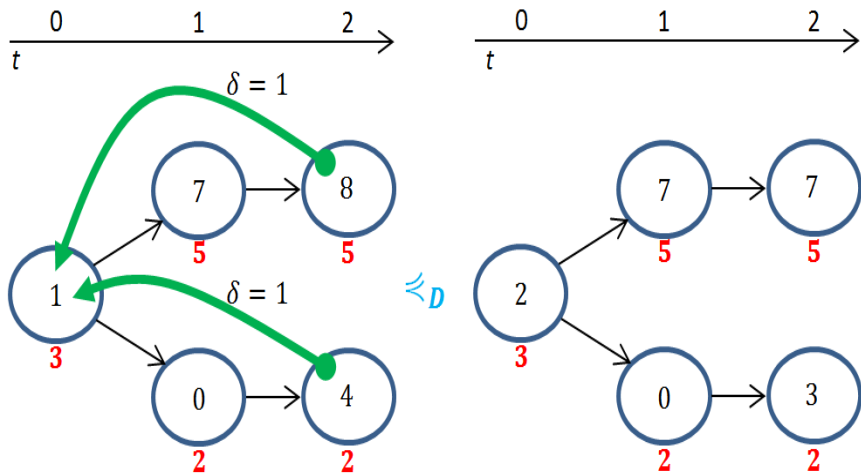
# AXIOM: Intergenerational equity



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# AXIOM: Intergenerational equity



## Intergenerational equity

For each  $D \in \mathcal{D}$ , for each pair  $c, \bar{c} \in C(D)$ , for each pair  $t, t' \in T$  and each  $\delta \in \mathbb{R}_+$  such that:

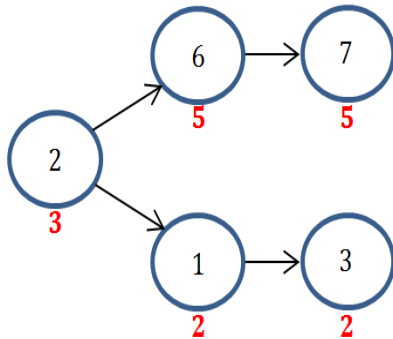
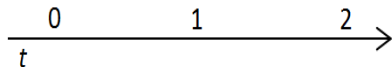
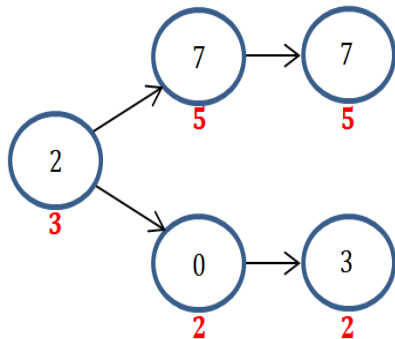
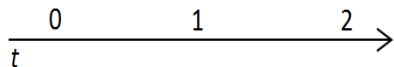
i) **[donor]**  $c_n = \bar{c}_n - \frac{\delta}{\beta^t} \geq x_n$  for each  $n \in N_t$ ;

ii) **[recipient]**  $c_{n'} = \bar{c}_{n'} + \frac{\delta}{\beta^{t'}} \leq x_{n'}$  for each  $n' \in N_{t'}$ ;

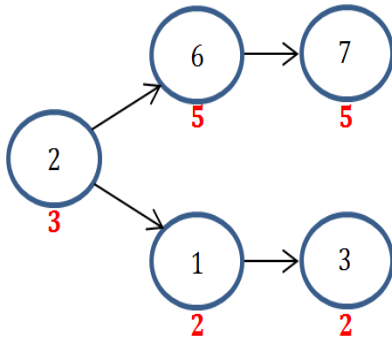
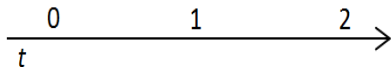
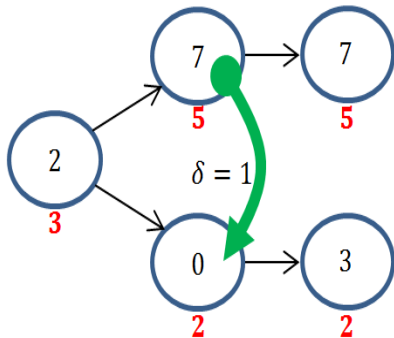
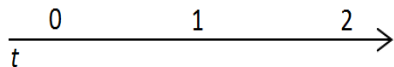
iii) **[ceteris paribus]**  $c_{n''} = \bar{c}_{n''}$  for each  $n'' \in N \setminus \{N_t \cup N_{t'}\}$ ,

then  $c \succsim_D \bar{c}$ .

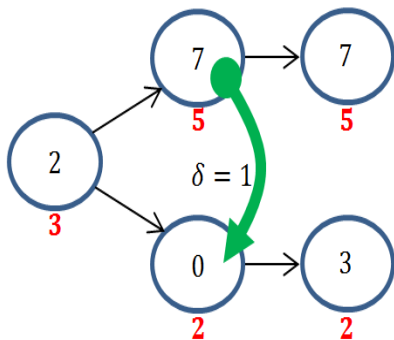
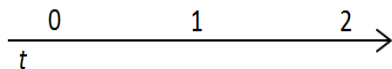
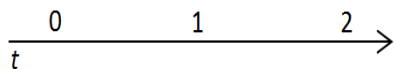
# AXIOM: Risk-reducing transfer



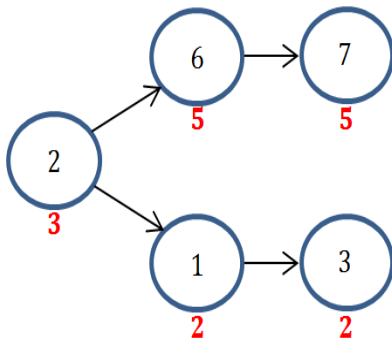
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$< D$



## Risk-reducing transfer

For each  $D \in \mathcal{D}$ , for each pair  $c, \bar{c} \in C(D)$ , for each  $t \in T$ , each pair  $n, n' \in N_t$ , and each  $\delta \in \mathbb{R}_+$  such that:

i) **[donor]**  $c_n = \bar{c}_n - \frac{\delta}{\pi^n} \geq x_n$ ;

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## Monotonicity

For each  $D \in \mathcal{D}$  and each pair  $c, \bar{c} \in C(D)$ ,  $c > \bar{c}$  implies  $c \succ_D \bar{c}$ .

Generalized utilitarianism ( $\equiv$  separability + continuity)

For each  $D \in \mathcal{D}$ ,  $\succsim_D$  can be represented by  
 $V(c; D) = \sum_{t \in T} v_t \left( \sum_{n \in N_t} u_n(c_n) \right)$ , with  $v_t, u_n$  continuous functions.

Proportionality

For each  $D, D' \in \mathcal{D}$ , if the set of feasible alternatives of  $D$  is proportional to that of  $D'$  then  $\succsim_D = \succsim_{D'}$ .

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## Theorem

*The following statements are equivalent:*

- 1 a SOF  $\succsim$  satisfies:
  - *intergenerational equity;*
  - *risk-reducing transfer;*
  - *monotonicity;*
  - *generalized utilitarianism;*
  - *proportionality;*
- 2 each  $\succsim_D$  can be represented by the *FIU* criterion.

# The Fair Intergenerational Utilitarian criterion

**FIU**

$$V(\mathbf{c}; D) =$$

## FIU

$$V(c; D) = \sum_t \frac{v_t(c_t; D)^{1-\rho}}{1-\rho}$$

$$v_t(c_t; D) =$$

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$$V(c; D) = \sum_t \tilde{\beta}_t \frac{v_t(c_t; D)^{1-\rho}}{1-\rho}$$

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$$v_t(c_t; D) = \left[ \sum_{n \in N_t} (c_n)^{\gamma} \right]^{\frac{1}{\gamma}}$$

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$$\tilde{\beta}_t = \beta^t \left( \frac{\sum_{\bar{n} \in N_t} \pi_{\bar{n}} x_{\bar{n}}}{x_0} \right)$$

# What does the FIU family do? (1)

- It allows to disentangle:
  - intergenerational inequality aversion (captured by the parameter  $\rho$ );
  - aversion to *intrinsic risk* (captured by the concavity of the function  $\mu$  in the definition of the fair prospect);
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## What does the FIU family do? (2)

- It introduces a role for the time disclosure of risk and, as a consequence, different discounting formulas are characterized:
  - no technological difference across histories  $\Rightarrow$  exponential discounting;
  - indifference to intrinsic risk (linear  $\mu$ )  $\Rightarrow$  exponential discounting;
  - 1 step resolution of risk  $\Rightarrow$  quasi-hyperbolic discounting.
- In general, discounting depends on risk, its resolution over time, and the planner's risk attitude.

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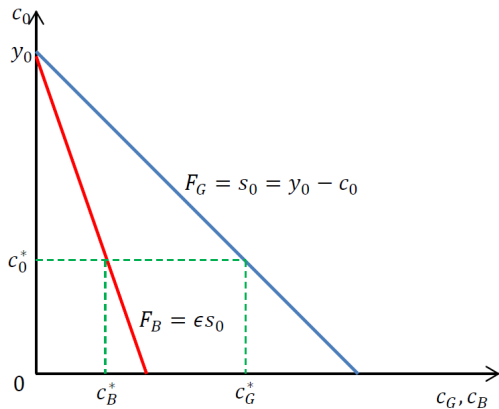
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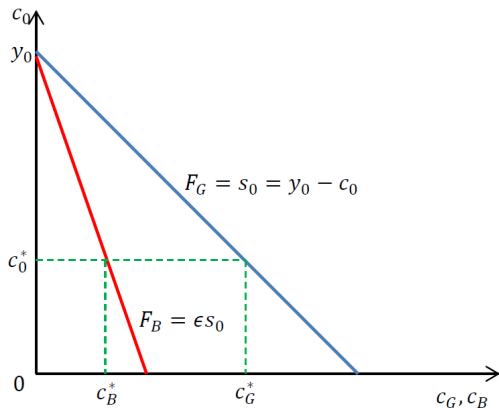
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- the FOC for the utilitarian criterion (with CRRA utility) is:

$$(c_0^*)^{-\lambda} = \beta \left[ \pi_G (1 - c_0^*)^{-\lambda} + \pi_B \varepsilon (\varepsilon (1 - c_0^*))^{-\lambda} \right]$$

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- Intergenerational fairness seems to be a powerful tool to investigate intergenerational distributive justice.
- A two step approach can be adopted:
  - first, identify the meaning of equity by means of an allocation rule;
  - second, evaluate the social trade-off between equity and the quantity of resources distributed.
- More work is needed:
  - Sequential rules (axiomatic justification, growth/development consequences, ...);
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Thank you!