MD transfer trouble

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UD versus *MD*

MD transfer trouble

Personal view

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UD approach

- Evaluation of *UD* distributions via $E(y)$, with $y = (y_1, y_2, \ldots, y_n)$

  - Impartiality (for welfare, poverty & inequality) ↔ symmetry
  - Efficiency (for welfare & poverty) ↔ monotonicity
  - Equity (for welfare, poverty & inequality) ↔ P-D transfers

MD approach

- Evaluation of *MD* distributions via $\mathcal{E}(X)$, with $X = (x_1, x_2, \ldots, x_n)$

- Different interpretations for x_i :
  - Income and a description of needs $x_i = (y_i, z_i)$
  - A bundle of attributes $x_i = (x_{i1}, x_{i2}, \ldots, x_{im})$
  - A bundle and a preference relation $x_i = (x_{i1}, x_{i2}, \ldots, x_{im}, R_i)$

- Why not simply use $W(U(X))$, $P(U(X))$, and $I(U(X))$ for $\mathcal{E}$, with
  - $U(X) \equiv \left(U(x_1), U(x_2), \ldots, U(x_n)\right)$
  - (standard) *UD* properties for *W*, *P*, and *I*?

MD transfer trouble

- Works fine for impartiality and efficiency, but trouble with transfers in the needs, attributes, and fairness literature …

Contrary to *UD*, where transfers can only affect equity, transfers in *MD* may change both equity and efficiency

- Overview of *MD* transfer trouble (in a welfare setting)
  - In the needs literature
  - In the attributes and fairness literature (fairness, with same R_i; see Fleurbaey & Trannoy, 2003)

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Needs literature

Attributes/fairness literature

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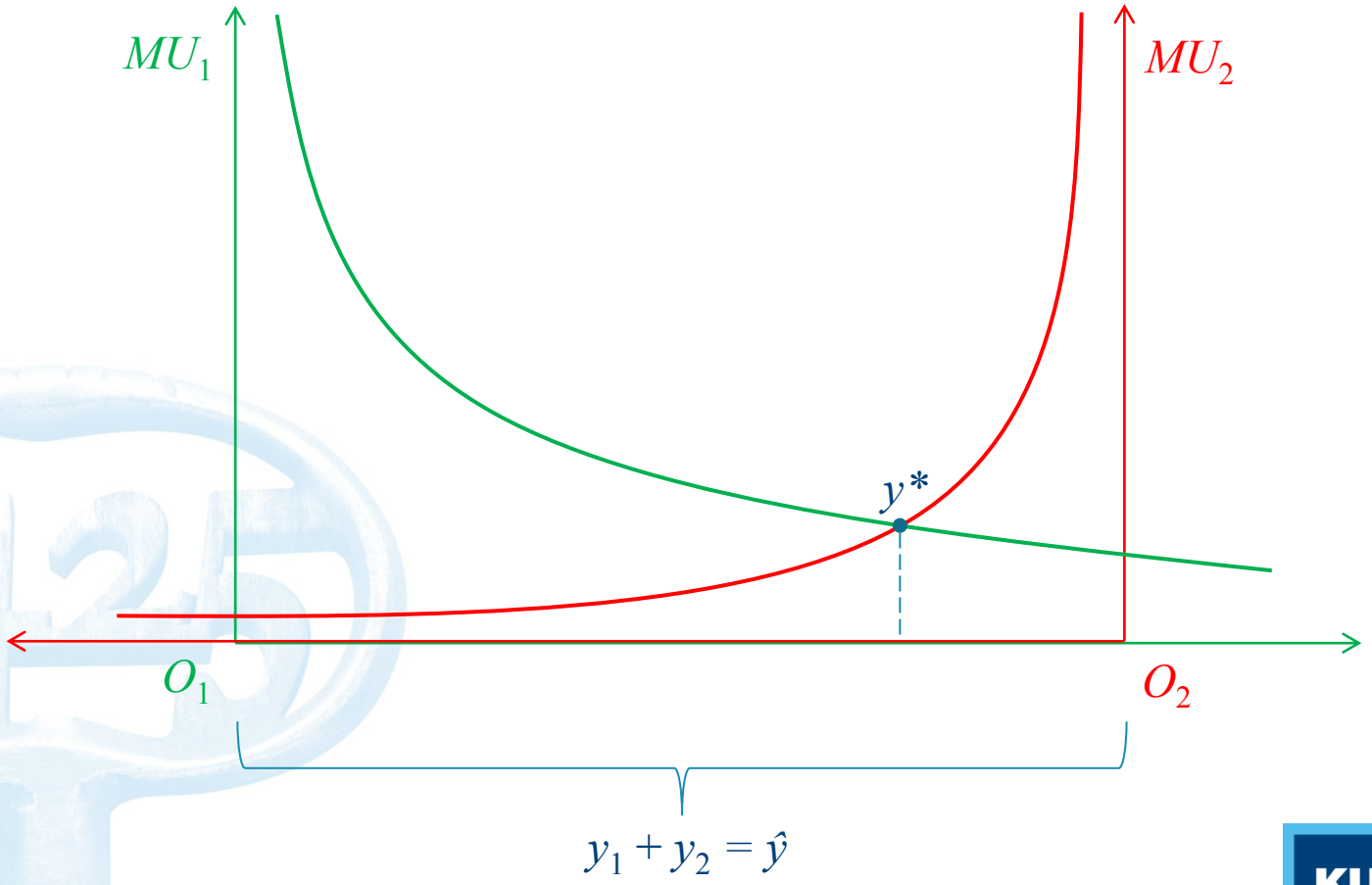
Needs - Sen (1973)

- Sen's (1973) critique on utilitarianism
- $E(X) = U(x_1) + U(x_2) = U(y_1,1) + U(y_2,2) \equiv U_1(y_1) + U_2(y_2)$
- Suppose "one person A derives exactly twice as much utility as person B from any given level of income, say, because B has some handicap, e.g., being cripple."
- So, we have
  - $U_1(y) = 2 \times U_2(y)$ for all y, and thus also
  - $MU_1(y) = 2 \times MU_2(y) > MU_2(y)$ for all y
- For the same income level, individual 1 is always
  - better off in utility levels, but also
  - a more efficient pleasure machine …

Needs - Sen (1973)

- Division of a fixed amount of income $\hat{y}$ ...



The weak equity axiom - Sen (1973)

- "Let person *i* have a lower level of welfare than person *j* for each level of individual income. Then in distributing a given total of income among *n* individuals, including *i* and *j*, the optimal solution must give *i* a higher level of income than *j*."

- Utilitarianism
  - does not satisfy the weak equity axiom
  - is therefore "a blunt approach" and "a non-starter"

- Sen also shows that a strictly concave welfare function over utilities is necessary, but not sufficient to satisfy weak equity → a sufficient degree of concavity is required …

Needs – Glewwe (1991)

- Glewwe (1991) observes that "The use of equivalence scales when measuring income inequality can lead to the paradoxical result that transferring money from poor to wealthy households may reduce measured inequality."

- Glewwe's numerical example uses Theil's inequality measure

- But, as noted by Glewwe "One can extend this paradox to social welfare […] regressive transfers may increase social welfare even if the social welfare function is *S*-concave."

Needs – Glewwe (1991)

- In the 'equivalence scale' approach, each individual in a couple with household income sy is equally well-off as a single with y
  - Needs and scale economies imply $1 < s < 2$
  - Living standards measured by 'equivalent income' y/s

- Two-stage (welfarist) approach:
  - First, replace a couple with y by two singles with y/s
  - Second, aggregate, as if UD, s.t., $E(X) = W(y_1, y_2/s, y_2/s)$

- In Sen's words, a couple is a more efficient 'pleasure-machine'

Needs – Glewwe (1991)

- Start from $\mathcal{E}(X) = W(y_1, y_2/s, y_2/s)$ and assume
  - equal equivalent incomes, so, $y_2 = sy_1$
  - W symmetric & differentiable $\rightarrow MW_i(u,u,u) \equiv MW(u) > 0$
- To confirm Glewwe's claim, a small regressive transfer from the single to the couple increases social welfare, because indeed

$$\Delta\mathcal{E} = MW(y_1)[-1 + (2/s)] > 0$$

- Note that concavity of W is not needed, so, Sen' solution—make W sufficiently concave—cannot avoid the paradox …
- Other solutions? Recall W symmetric and differentiable …

Needs – Shorrocks (2004)

- Use $E(X) = W(y_1, y_2/s, y_2/s)$ as before, but drop differentiability …
- Sen's trick—'sufficient concavity'—may work again

- For example, use a Gini-type W, e.g.,
  $W(u_1, u_2, u_3) = w_1 \times u_{(1)} + u_{(2)} + u_{(3)}$, with $w_1 \geq 2/s$

- Not "for practical purposes," according to Shorrocks …
- (and, in a more general setting, close to leximin)

Needs - Ebert (1997) and Shorrocks (2004)

- Use $\mathcal{E}(X) = W(y_1, y_2/s, y_2/s)$ as before, but now drop symmetry

- For example, choose $W(u_1, u_2, u_3) = \varphi(u_1) + \frac{s}{2}\varphi(u_2) + \frac{s}{2}\varphi(u_3)$, leading to $\mathcal{E}(X) = \varphi(y_1) + s\varphi(y_2/s)$
  - "weighting by the equivalence scale"
  - guarantees that increasing household income has the same social welfare effect whenever living standards are the same

- Besides violating impartiality, also violating needs monotonicity

Summary needs literature

- Sen (1973): progressive transfers have equity as well as efficiency implications (that utilitarianism cannot handle appropriately)
- Concave W necessary, but not sufficient to satisfy weak equity

- Glewwe (1991): paradox that regressive transfers can improve social welfare in the equivalence scale approach

- Possible to circumvent the paradox (Ebert, 1997, Shorrocks, 2004)
  - either, by dropping differentiability and imposing again a sufficiently strong (typically extreme) concavity
  - or, by dropping symmetry and "weight by equivalence scale"

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Needs literature

Attributes/fairness literature

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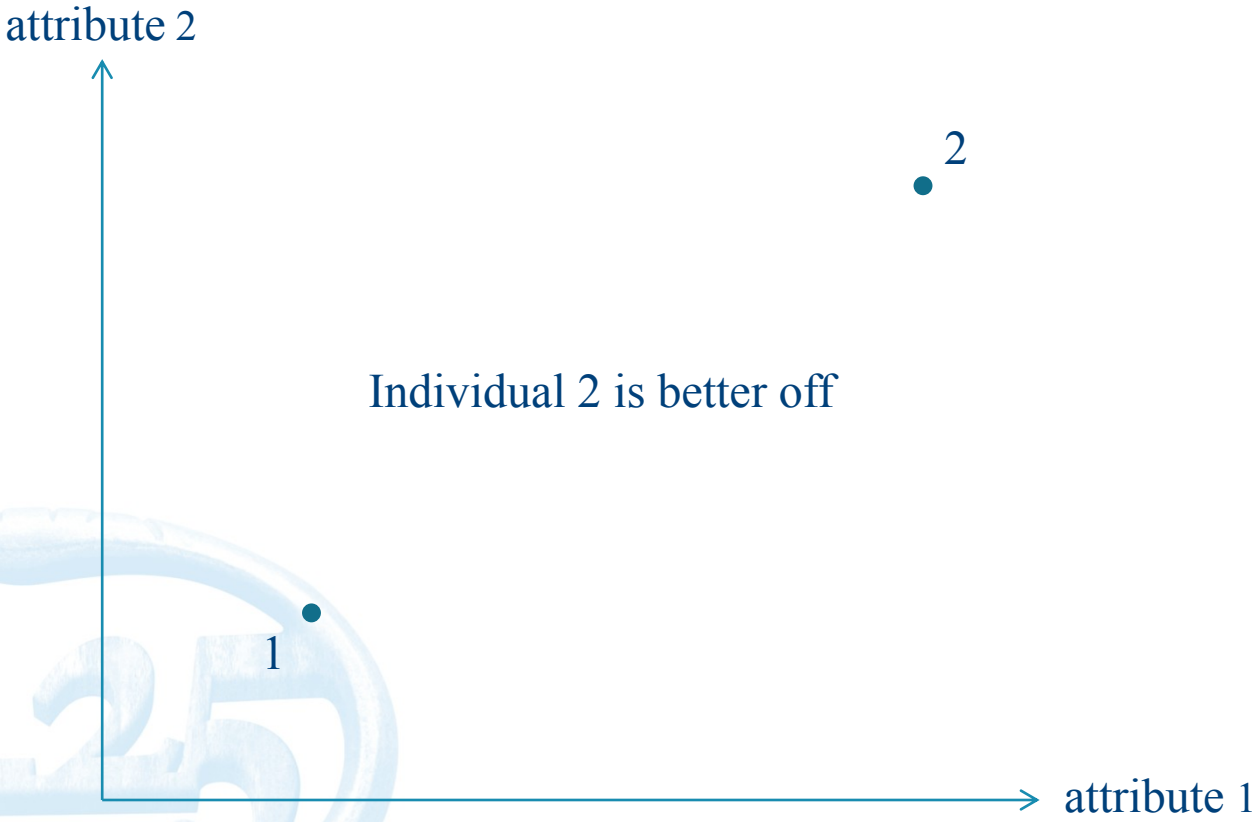
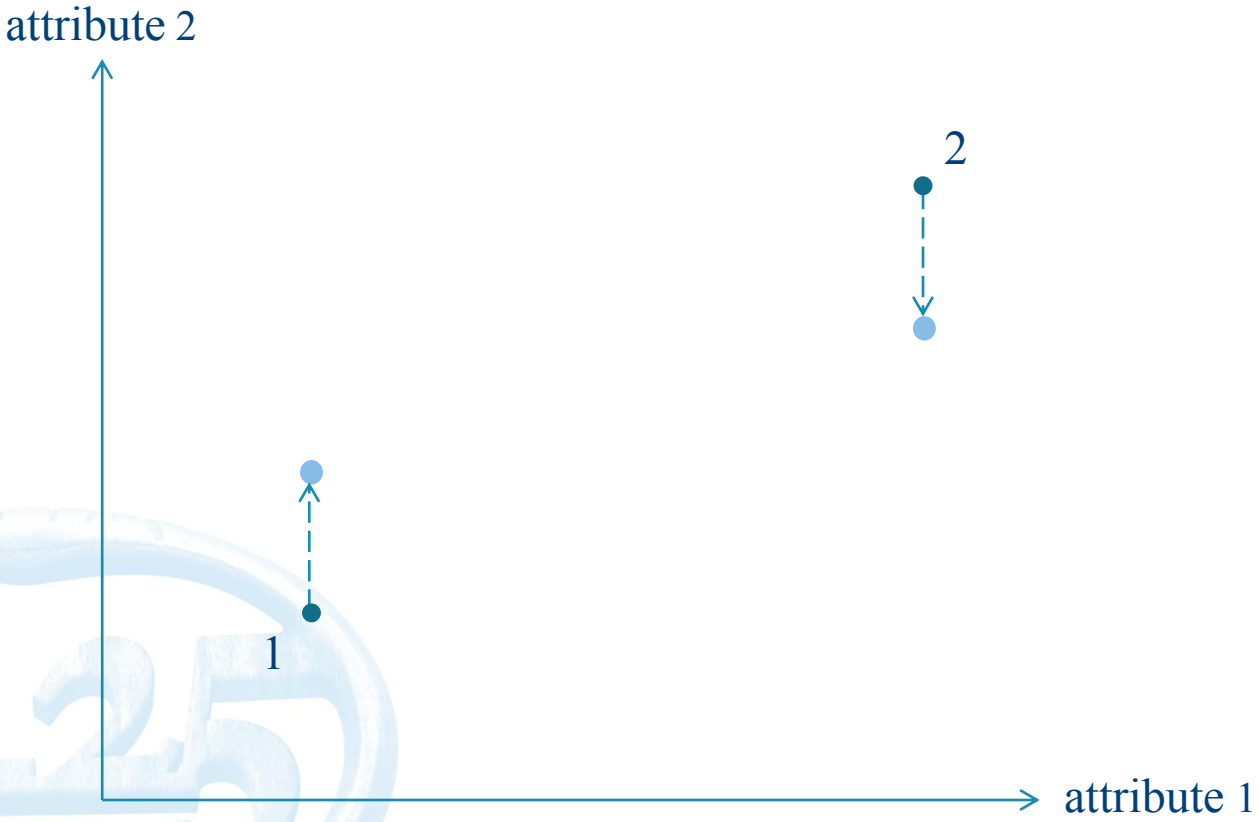
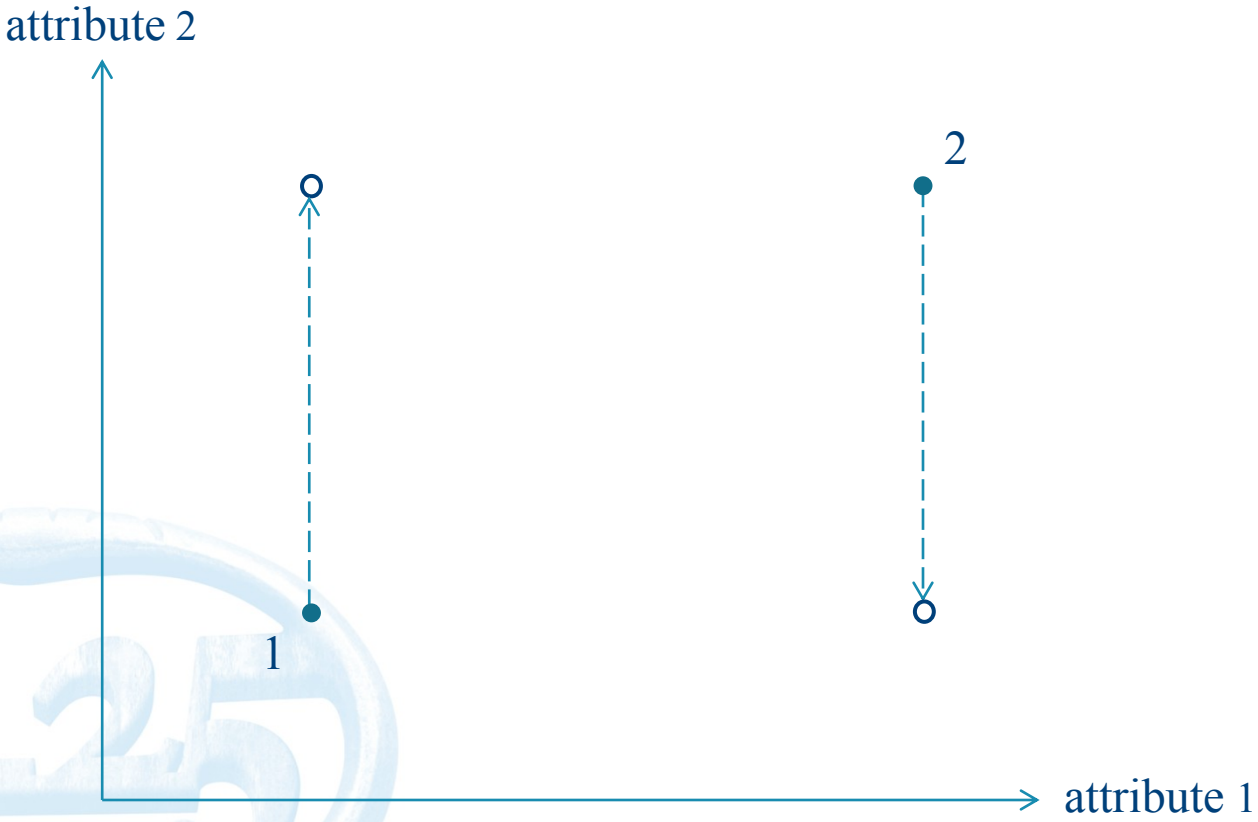
Attributes – an ‘extreme’ example



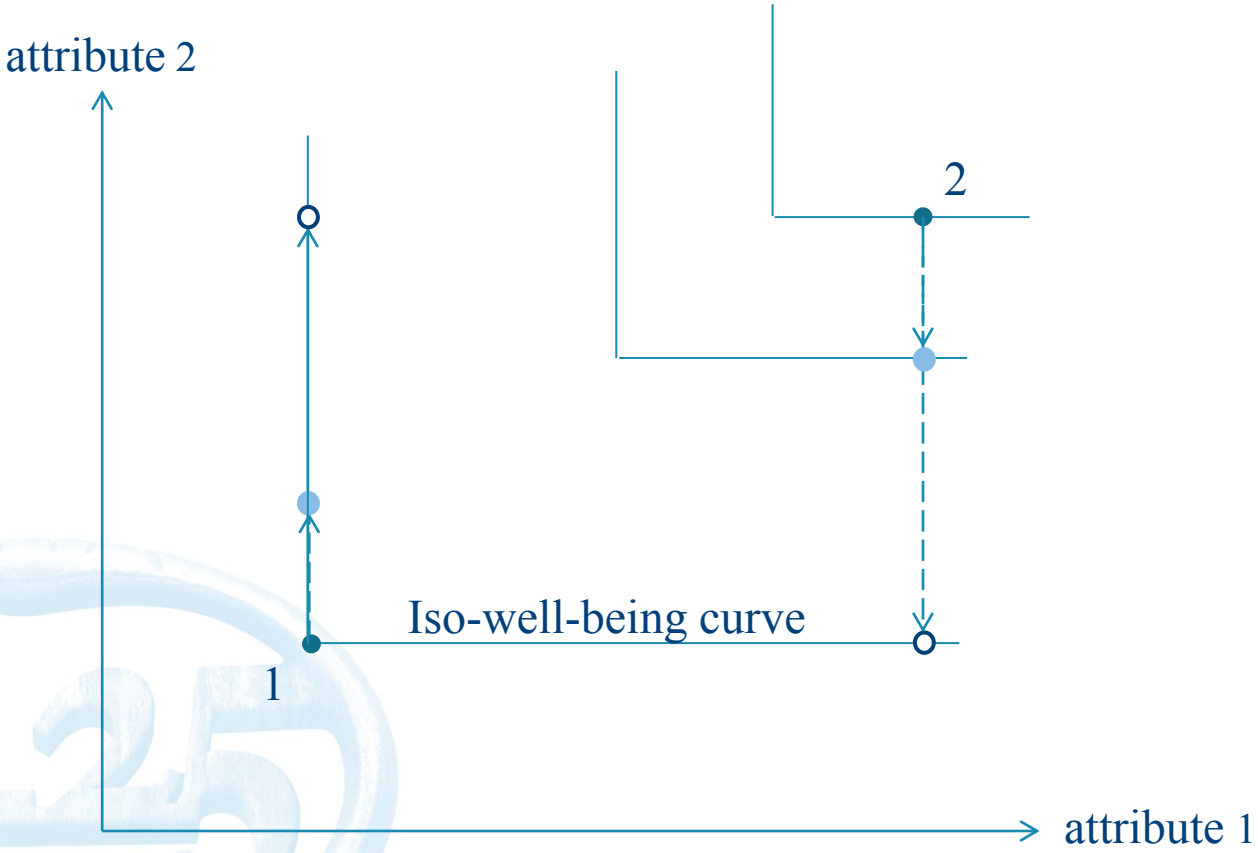
Attributes & P-D transfer in one attribute



Attributes & C-D switch (if ordinal)



Attributes & conflict with efficiency*



* Only if the social welfare specification is individualistic

PD transfers & solutions (more generally)

- Suppose P-D transfers in all attributes and/or less extreme well-being functions, then no direct conflict, but 'leaks' may occur …

- Blackorby and Donaldson (1988):
  - 'standard' procedure "may result in social judgements that contradict normal distributional judgements"
  - "social welfare analysis based on money metrics is flawed"

- Fleurbaey and Maniquet (2011): Adding some mild conditions, 'absolute priority' is required to satisfy the P-D transfer principle

CD switches & solutions (more generally)

- Concerning C-D switches, suppose $\mathcal{E}(X) = W(U(X))$, with
  - $U(x_i) = (\sum_{j=1}^{m} r_j (x_{ij})^{1-\beta})^{\frac{1}{1-\beta}}$
  - $W(u) = (\sum_{i=1}^{n} (u_i)^{1-\alpha})^{\frac{1}{1-\alpha}}$
  - (α called 'inequality aversion' and β 'complementarity')

- Atkinson (2003): C-D switch if and only if $\alpha \geq \beta$

Summary attributes/fairness literature

- *MD* P-D transfers or CD switches always improve equity, but may also have an adverse efficiency impact

- Blackorby and Donaldson (1988): concavity of the social welfare function not sufficient
- Fleurbaey and Maniquet (2011): (under some mild conditions) progressive transfers approved of if and only if leximin

- Atkinson (2003): CD switches approved of if and only if the degree of inequality aversion is larger than the degree of complementarity

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Personal view for *MD* welfare and poverty

- *MD* transfers mix equity and efficiency considerations (mix can be avoided, but that's not my main point here)

- In case of *MD* welfare and poverty comparisons:
  - be aware and understand the causes of paradoxical results, but do not try to circumvent these paradoxes …
  - these paradoxes do not contradict anything, they only tell us that an equity-efficiency trade-off may occur for *MD* transfers

Personal view for *MD* normative inequality

- The normative approach measures inequality as
  - the social welfare gain that could be obtained
  - by optimally redistributing the available goods
- In a *UD* setting:
  - welfare can be increased only by redistribution
  - inequality therefore coincides with inequity
- In a *MD* setting:
  - also efficiency improvements become possible
  - by non-realized mutually beneficial exchanges of goods
- *MD* inequality mixes inequity and (allocative) inefficiency; it might be interesting to disentangle both …

A decomposition – Bosmans et al. (2013)

- Start from $E(X) = W(U(X))$, with some properties on W and U
- Measure normative *MD* inequality
- Decompose *MD* inequality into its inefficiency and inequity part
  - (in)efficiency ≈ (Debreu) coefficient of resource utilization
  - (in)equity ≈ the remainder
- To obtain *MD* normative equality is $d(X) \times e(X)$, with

$$1 - e(X) = 1 - \frac{EDE\ well \cdot being}{average\ well \cdot being}$$

- Inequity is a *UD* inequality index applied to well-being ≈ 2-stage
- (normative justification for using 2-stage measures)

Conclusion

- Several transfer paradoxes in the *MD* setting reflect a classical equity-efficiency trade-off

- In my view, important to understand, but not troublesome
  - *except* probably for normative *MD* inequality measurement
  - decomposition shows that the inequity part is 'two-stage'

- So, simply use $W(U(X))$, $P(U(X))$, and $I(U(X))$, with standard *UD* properties for *W, P,* and *I*, after all?

"And the end of all our exploring
Will be to arrive where we started
And know the place for the first time."

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