Income mobility and welfare

Markus Jäntti[1]

[1] Swedish Institute for Social Research, Stockholm University

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Acknowledgements

This lecture builds on joint work with Stephen P Jenkins

Markus Jäntti and Stephen Jenkins (2015). *"Income mobility"*. *In: Handbook of Income Distribution. Ed. by Anthony B Atkinson and François Bourguignon. Vol. 2. Elsevier. Chap. 10, pp. 807–935.* DOI: doi:10.1016/B978-0-444-59428-0.00011-4. URL: http://www.sciencedirect.com/science/article/pii/B9780444594280000114

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Outline

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- we know and understand much less about whether or not more or less mobility is *socially desirable*
- we will consider both *intra-* and *inter-generational* mobility

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- Gary S Fields and Efe A Ok (1999). "The Measurement of Income Mobility: An Introduction to the Literature". In: *Handbook of Income Inequality Measurement*. Ed. by Jacques Silber. Recent Economic Thought. Boston: Kluwer Academic Publishers. Chap. 19, pp. 557–598

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- let $M(F)$ be a (statistical) measure of mobility; our concern is *not* to assess if

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- instead, we are concerned with assessing if, given a welfare function(al) W,

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- we shall mostly look at discrete distributions for analytical tractability

Outline

Mobility concepts

. . . the mobility literature does not provide a unified discourse of analysis. This might be because the very notion of income mobility is not well-defined; different studies concentrate on different aspects of this multi-faceted concept. At any rate, it seems safe to say that a considerable degree of confusion confronts a newcomer to the field (Fields and Ok, 1999, p. 557).

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- mobility can be thought of as transformation linking the marginal distribution f_1 with marginal distribution f_2
- sometimes, study of a single (longitudinal) population can be informative...
- but as a rule, mobility is about *comparing* two populations A and B (two countries, two different periods, etc)

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  - relationship to equality of opportunity

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  - rank reversal ($p_{ij} > 0 \quad i = 1, \ldots, n, j = n, \ldots, 1$; all entries in transition matrix on the anti-diagonal)

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- measures: directional growth (gains vs. losses) as opposed to non-directional growth

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- inequality reduction from longitudinal averaging now re-interpreted as a measure of income risk (and has different normative implications)

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- in two-period case, the utility-of-income function is $U(Y_1, Y_2)$, and weighted by the joint probability density $f(y_1, y_2)$:

$$W = \int_0^{a_2} \int_0^{a_1} U(y_1, y_2) f(y_1, y_2) dy_1 dy_2 \qquad (1)$$

where $U(Y_1, Y_2)$ is differentiable and a_1 and a_2 are the maximum incomes in periods 1 and 2.

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  where $U(Y_1, Y_2)$ is differentiable and a_1 and a_2 are the maximum incomes in periods 1 and 2.
- increases in income in either period assumed desirable (so positive income growth raises utility): $U_1 \geq 0$ and $U_2 \geq 0$.

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- if U additively separable (so $U_{12} = 0$), mobility is irrelevant and only marginal distributions matter
- if $U(Y_1, Y_2)$ is a concave transformation of the sum of the per-period utilities, then $U_{12} < 0$

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  - $\mathbf{f}_1'\mathbf{P} = \mathbf{f}_2$
- consider the problem of choosing the transition matrix **P** that maximizes welfare, given the fixed marginal distribution and a social evaluation function U:

$$\max_{\mathbf{P}} W = \sum_i \sum_j U(Y_{1i}, Y_{2i}) p_{ij} f_{1i}$$

subject to

$$\sum_i f_{1i} p_{ij} = f_{2j} = f_{1j}, \quad j = 1, \ldots, n \qquad (2)$$

$$\sum_j p_{ij} = 1, \quad i = 1, \ldots, n$$

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- note that “origin independence” plays no role here

Exchange and structural mobility

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- an alternative is to rely on the *social evaluation U* to decompose mobility

Exchange and structural mobility – welfare-based

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- note that $\tilde{f}^k, k = A, B$ is a hypothetical steady-state distribution, not the actual

Exchange and structural mobility – an example

- to examine this more closely, consider $n = 2$ and focus on the case of identical marginal distributions in the two time periods:

$$\mathbf{P} = \begin{bmatrix} p_1 & 1 - p_1 \\ 1 - p_2 & p_2 \end{bmatrix} \qquad (5)$$
$$1 > p_i > 0, i = 1, 2; \quad \mathbf{f} = (f_1, f_2)' = (f_1, 1 - f_1)'$$

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- the welfare function (expected/average utility) for this economy is

$$\begin{aligned} W =& U(Y_1, Y_2)p_1 f_1 + U(Y_1, Y_2)(1 - p_1)f_1 + \\ & U(Y_2, Y_1)(1 - p_2)(1 - f_1) + U(Y_2, Y_2)p_2(1 - f_1) \end{aligned} \tag{6}$$

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- this can re-written as

$$W = [\{U(Y_2, Y_2) - U(Y_2, Y_1)\} - \{U(Y_1, Y_2) - U(Y_1, Y_1)\}]p_1 f_1 + C \tag{7}$$

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  (C does not depend on p_1 or p_2)
- to maximize welfare wrt. p_1 we choose a low value when [] is negative (and high when it is positive); the sign of [] equals the sign of the cross-partial derivative (as $Y_1 < Y_2$)

Exchange and structural mobility – an example

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  - $p_1 = p_2 = 0$ has here been ruled out on feasibility grounds so complete rank reversal is not a solution

Exchange and structural mobility – graphical representation

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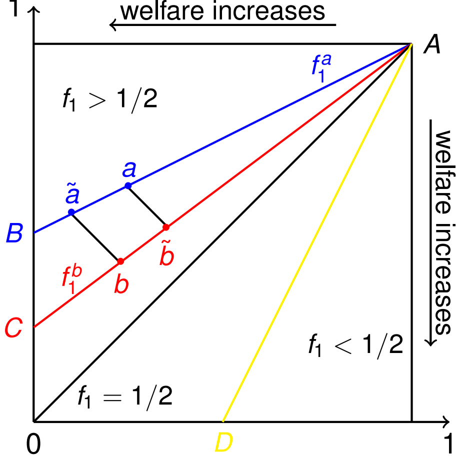

$p_2\left(=\frac{1-2f_1}{\ldots}\right)$

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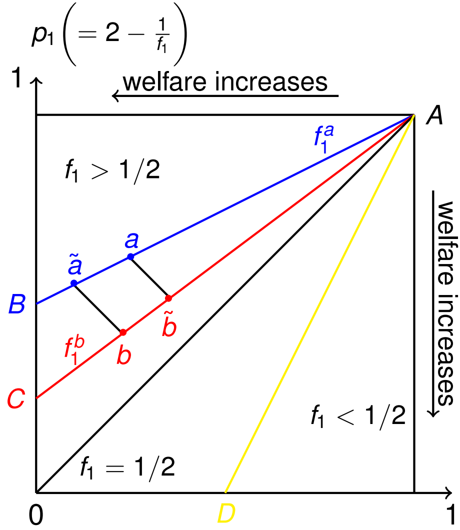

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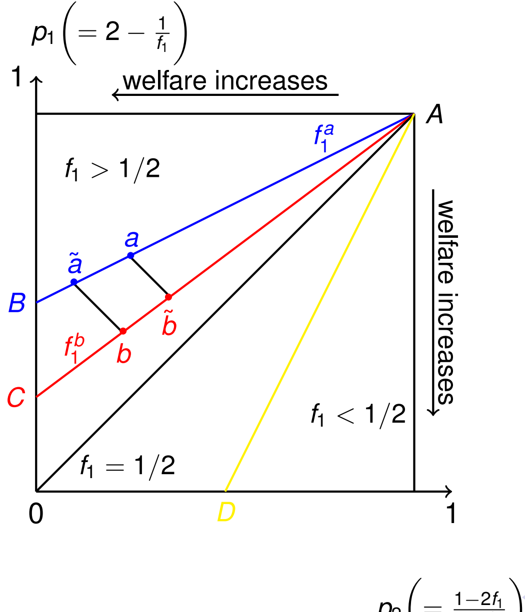

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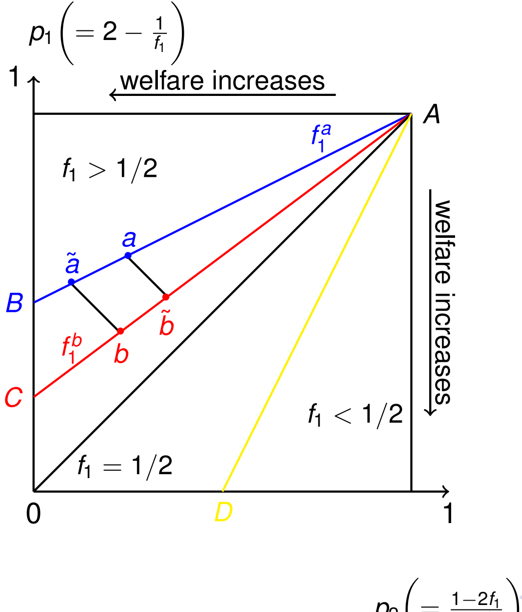

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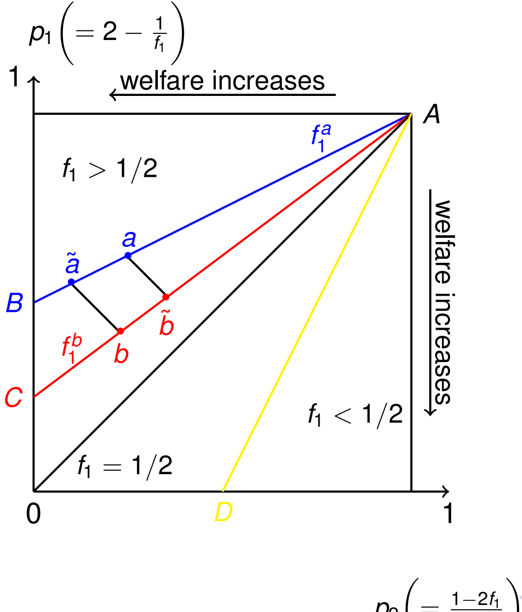

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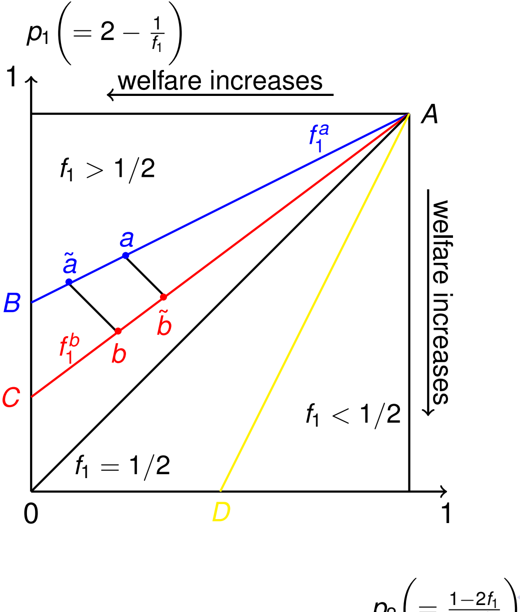

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- move from $\tilde{a}$ to b preserves welfare but represents structural mobility $f_1^a \to f_2^b$

Exchange and structural mobility – decomposition

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Decomposition I

$$\begin{aligned} \text{Total change in welfare} &= W^b - W^a \\ \text{Exchange mobility} &= W^{\tilde{a}} - W^a \\ \text{Structural mobility} &= W^b - W^{\tilde{a}} \end{aligned}$$

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Remarks

- one might also take point A (perfect immobility) as the reference for for decomposing, but that would make no use of welfare information.

Welfare dominance in more general bivariate distributions

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- the problem is still to compare two distributions, f^A and f^B with

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- Atkinson and Bourguignon (1982) consider other classes of U and derive higher-order dominance conditions

A closer look at U

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- when $\rho = 0$ and perfect substitution of income between periods, one is only interested in the reduction of multi-period inequality

Mobility dominance

- an example that would generate a welfare improvement is a ‘correlation-reducing transformation’ which leaves the marginal distributions unchanged but reduces the correlation between Y_1 and Y_2 (for $\eta, h, k > 0$):

$$\left\{ \begin{array}{ccc} & y_1 & y_1 + h \\ y_2 & \text{density reduced by } \eta & \text{density increased by } \eta \\ y_2 + k & \text{density increased by } \eta & \text{density reduced by } \eta \end{array} \right\}$$

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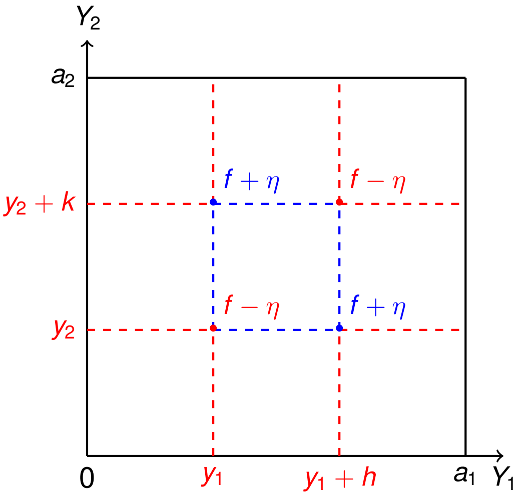
- mobility dominance powerful in theory but not used much in practice – results apply to simplified situations (identical margins, homothetic preferences, positional mobility)
- Dardanoni (1993) provides an alternative approach to dominance (stochastic dominance results for mobility processes summarised by transition matrices with the same steady-state income distribution)

Mobility dominance – graphical illustration



Mobility dominance – examples

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- the welfare function, using the expectations operator, is then

$$\hat{W} = \{\mathrm{E}_0[Y_1^{1-\rho} + (\mathrm{E}_1[Y_2^{1-\gamma}])^{1/(1-\gamma)})^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)} \tag{13}$$

Welfare dominance with origin independence

- Gottschalk and Spolaore (2002) prove that time independence is value if and only if

  $$\epsilon \geq \gamma \text{ and } \rho \geq \gamma$$

  i.e., origin independence only matters in the *ex ante* sense that individuals, looking forward, value a sure thing relative to a lottery and that valuation is high enough to dominate aversion to both multiperiod utility (ϵ) and intertemporal variation in income (ρ)

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- Gottschalk and Spolaore (2002) prove that time independence is value if and only if

$$\epsilon \geq \gamma \text{ and } \rho \geq \gamma$$

  i.e., origin independence only matters in the *ex ante* sense that individuals, looking forward, value a sure thing relative to a lottery and that valuation is high enough to dominate aversion to both multiperiod utility (ϵ) and intertemporal variation in income (ρ)
- moreover, in the 2×2 example, setting $p_1 = p_2 = p$, they show that the welfare-maximizing p depends on the relationship between ϵ and ρ

$$p \lesseqgtr 1/2 \text{ if } \rho \lesseqgtr \epsilon$$

Measurement of welfare loss

Welfare measures and extended Atkinson indices

Welfare	Index
No mobility preference: $W_s = \{E_0[Y_1^{1-\rho} + Y_{12}^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_s = 1 - \frac{W_s}{\bar{Y}}$
Reversals improve welfare: $W_r = \{E_0[Y_1^{1-\rho} + Y_2^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_r = 1 - \frac{W_r}{\bar{Y}}$
Origin independence improves welfare: $W_o = \{E_0[Y_1^{1-\rho} + (E_1[Y_2^{1-\gamma}])^{1/(1-\gamma)})^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_o = 1 - \frac{W_o}{\bar{Y}}$

Note: Y_{12} is income in period 2 under the assumption of no mobility, i.e., $Y_{12} = F_2^{-1}[F_1(Y_1)]$.

Measurement of welfare loss – empirical illustration

Decomposition of welfare gains from mobility

	$\underbrace{A_o - A_s}_{\text{Overall diff}}$	$=$	$\underbrace{A_o - A_r}_{\text{diff from origin independence}}$	$+$	$\underbrace{A_r - A_o}_{\text{diff from reversals}}$
Germany	$-.096$		$-.041$		$-.055$
US	$-.090$		$-.044$		$-.046$

Source: Gottschalk and Spolaore (2002), Table 1, p 202

Intra- or inter-generational mobility

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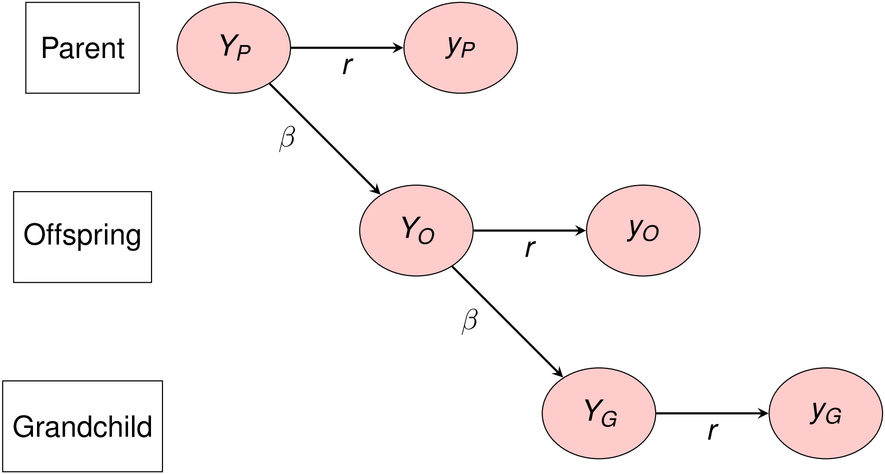
- the "plasticity" of the framework hides the fact that in intergenerational analysis, individuals experience (welfare-reducing) income fluctuations within generations
- next, we'll look at a simple way of integrating intra- and inter-generational mobility based on Atkinson (2008)

Intra- and inter-generational mobility



Inter- and intragenerational mobility

- focus for now on the 2-generation case, but allow each generation to have annual income that fluctuates around the long-run average such that

$$Y_j = \prod_{t_1}^{T} \tilde{y}_{jt}^{1/T} \text{ and } \ln Y_j = \frac{1}{T} \sum_{t=1}^{T} y_{jt} \quad j = F, S \tag{14}$$

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$$U(Y_P, Y_O) = [\ln Y_P + \delta \ln Y_O]/\Delta,\ \Delta = 1 + \delta \tag{15}$$

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- we'll measure social welfare by $-$Var[], so we need

$$\begin{aligned} \text{Var}[U(Y_P, Y_O)] = &\text{Var}[\ln Y_P] + \delta^2 \text{Var}[\ln Y_O] + \\ &\delta 2\beta \text{Var}[\ln Y_P]^{1/2} \text{Var}[\ln Y_O]^{1/2} \end{aligned} \tag{16}$$

(β is the intergenerational income *correlation*; δ is the discount rate)

Inter- and intragenerational mobility

- assuming a within-person correlation r_j and stationary transitory error variance $\sigma^2_{v_j}$, the welfare function is

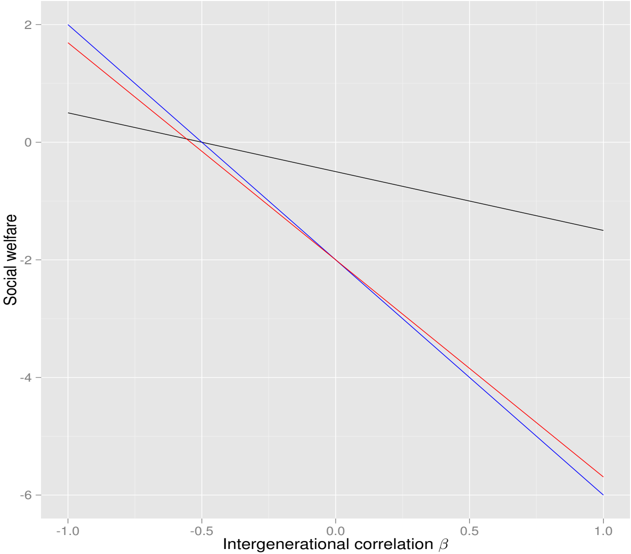
$$
\begin{aligned}
W = -\mathrm{Var}[U(Y_P, Y_O)] = -\Bigg\{ & \sigma_P^2\left(\frac{1}{T} + \frac{T-1}{T} r_P\right) + \frac{\sigma^2_{v_P}}{T} + \\
& \delta^2\left[\sigma_O^2\left(\frac{1}{T} + \frac{T-1}{T} r_O\right) + \frac{\sigma^2_{v_O}}{T}\right] + \\
& \delta 2\beta \sqrt{\sigma_P^2\left(\frac{1}{T} + \frac{T-1}{T} r_P\right) + \frac{\sigma^2_{v_P}}{T}} \times \\
& \sqrt{\sigma_O^2\left(\frac{1}{T} + \frac{T-1}{T} r_O\right) + \frac{\sigma^2_{v_O}}{T}} \Bigg\} / \Delta^2
\end{aligned}
\tag{17}
$$

Inter- and intragenerational mobility

- assume T large and impose stationarity ($\sigma_P = \sigma_O = \sigma; r_P = r_O = r$):

$$W = -\text{Var}[U(Y_P, Y_O)] = -\sigma^2[r(1+\delta^2) + \delta 2\beta]/\Delta^2 \quad (18)$$

Welfare and intergenerational correlation (2-gen)



(black=$\sigma^2 = 1; \delta = 1$; blue=$\sigma^2 = 2; \delta = 1$; red=$\sigma^2 = 2; \delta = 1.5$)

Welfare and intergenerational correlation (3-gen)

- taking a 3-generation perspective changes this only a little

Welfare and intergenerational correlation (3-gen)

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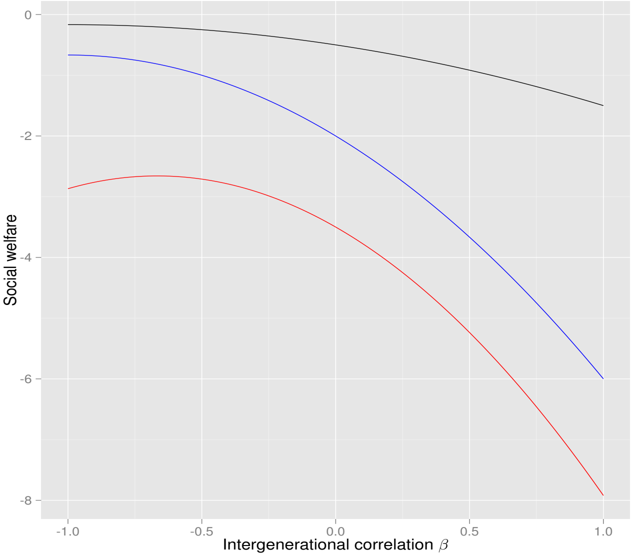
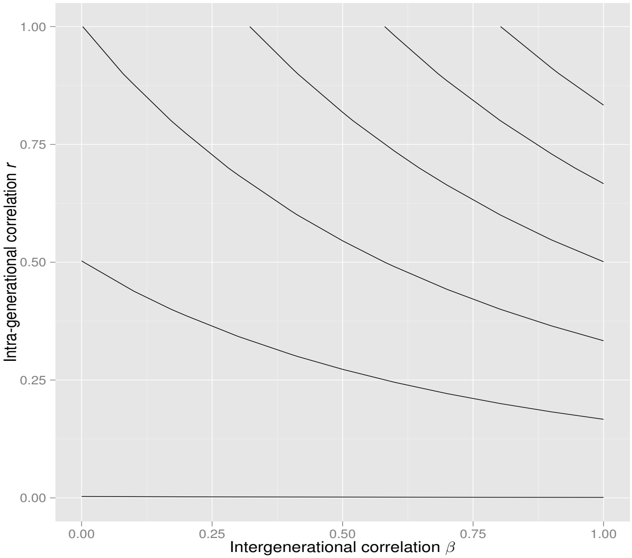
- welfare is now non-linear (in fact, quadratic) in the intergenerational correlation so it is more sensitive to generational variance and discount factor

Welfare and intergenerational correlation (3-gen)



(black=$\sigma^2 = 1; \delta = 1$; blue=$\sigma^2 = 2; \delta = 1$; red=$\sigma^2 = 2; \delta = 1.5$)

Intra- and intergenerational correlation – trade-off



Outline

Concluding comments

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  - it is not clear why *society* should value a sure thing for the offspring generation (“period 2”) relative to the uncertain lottery
  - it is more clear that such valuations make sense within the same individual
- integration of intra- within intergenerational analysis promising, but more complex processes likely useful
  Homoscedastic transitory variances? (Bingley and Cappellari, 2012)

Outline

Decile transition matrices: USA, (a) 1979–1988

Note: Income refers to equivalized real annual family disposable income, distributed among all individuals (adults and children). The decile groups are ordered from poorest (1) to richest (10). Source: Hungerford (2011, Tables 2 and 3), based on PSID data.

Go back

					Destination					
Origin	1	2	3	4	5	6	7	8	9	10
1979					**1988**					
1	44.3	18.3	12.4	9.2	7.1	3.0	1.8	2.0	0.7	1.3
2	18.1	25.3	21.0	11.7	7.5	5.4	4.7	3.2	1.9	1.1
3	10.6	18.2	15.3	16.8	11.6	9.0	8.8	4.9	3.1	1.7
4	7.2	8.9	14.0	14.0	14.7	15.7	12.0	5.6	6.0	2.1
5	6.1	9.2	10.9	12.8	13.3	16.9	12.3	7.5	7.7	3.4
6	4.1	5.2	8.8	10.3	11.8	10.0	14.2	16.9	12.6	6.2
7	3.5	6.5	6.9	8.6	10.4	13.4	13.3	16.8	13.4	7.2
8	3.1	4.6	3.2	7.7	12.3	9.5	12.6	15.7	17.7	13.6
9	1.2	2.2	4.8	6.3	6.9	10.2	12.2	14.7	18.0	23.5
10	2.1	1.5	2.8	2.5	4.2	7.0	8.5	12.8	18.6	40.0

Decile transition matrices: USA, (b) 1989–1998

Note: Income refers to equivalized real annual family disposable income, distributed among all individuals (adults and children). The decile groups are ordered from poorest (1) to richest (10). Source: Hungerford (2011, Tables 2 and 3), based on PSID data.

Go back

					Destination					
Origin	1	2	3	4	5	6	7	8	9	10
1989					**1998**					
1	41.9	21.6	13.7	7.0	4.6	3.7	2.7	2.2	1.9	0.7
2	20.4	22.5	15.4	11.6	11.0	8.1	4.0	4.0	1.7	1.2
3	12.5	20.8	17.1	16.4	10.9	10.3	5.2	3.2	1.7	1.9
4	6.9	11.6	15.5	16.9	14.5	11.4	10.1	7.7	2.3	3.1
5	4.8	6.2	12.2	13.8	16.0	14.2	12.4	7.1	7.5	5.8
6	3.2	3.7	9.1	11.6	16.0	14.4	15.7	11.7	7.7	6.9
7	3.2	4.5	7.6	9.3	8.7	12.2	16.3	15.6	16.8	5.8
8	3.0	4.7	5.2	5.4	7.9	12.1	17.2	17.0	19.3	8.3
9	2.5	3.1	4.0	4.9	7.5	7.1	10.7	18.2	21.8	20.3
10	1.7	1.0	0.4	3.2	3.0	6.3	6.0	13.1	19.3	46.1

Differences in cumulative density: USA, 1979–1988 versus 1989–1998

Source: Authors' calculations from (Hungerford, 2011, Tables 2 and 3), based on PSID data.

Go back

	Destination group								
Origin group	1	2	3	4	5	6	7	8	9
1	0.2	–0.1	–0.2	0.0	0.3	0.2	0.1	0.1	–0.1
2	0.0	0.0	0.4	0.6	0.5	0.2	0.2	0.1	0.0
3	–0.2	–0.5	–0.2	0.0	0.0	–0.5	–0.1	–0.1	0.0
4	–0.2	–0.7	–0.6	–0.6	–0.7	–0.7	–0.2	–0.3	0.1
5	0.0	–0.3	–0.3	–0.5	–0.7	–0.5	0.0	–0.1	0.4
6	0.1	–0.1	–0.1	–0.4	–1.1	–1.3	–0.9	–0.5	0.4
7	0.1	0.2	0.0	–0.3	–0.8	–0.9	–0.8	–0.3	0.3
8	0.1	0.2	–0.2	–0.2	–0.3	–0.7	–1.1	–0.7	–0.3
9	0.0	–0.1	–0.3	–0.2	–0.4	–0.4	–0.7	–0.6	–0.6
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Intergenerational transition matrices in disposable income among all persons for Germany, the UK and the USA

Source: Authors' calculations from Eberharter (2013, Table 3). ‣ Go back

A. Germany

	Offspring				
	1	2	3	4	5
Father					
1	34	29	14	17	7
2	15	23	32	15	16
3	12	16	22	26	24
4	9	11	18	29	33
5	7	11	19	25	39

B. UK

	Offspring				
	1	2	3	4	5
Father					
1	48	22	14	12	5
2	22	26	21	22	10
3	11	18	25	25	21
4	6	16	25	26	25
5	4	16	16	27	36

C. USA

	Offspring				
	1	2	3	4	5
Father					
1	37	31	13	13	5
2	21	23	24	17	15
3	12	23	18	24	24
4	9	11	21	33	26
5	2	10	15	26	46

Cumulated differences in intergenerational transition matrices in disposable income among all persons for Germany, the UK and the USA

Source: Authors' calculations from Eberharter (2013, Table 3). ▸ Go back

A. USA – Germany

	Offspring				
	1	2	3	4	5
Father					
1	3	5	5	1	0
2	9	11	4	2	0
3	9	18	6	2	0
4	9	18	9	9	0
5	4	13	1	2	0

B. USA – UK

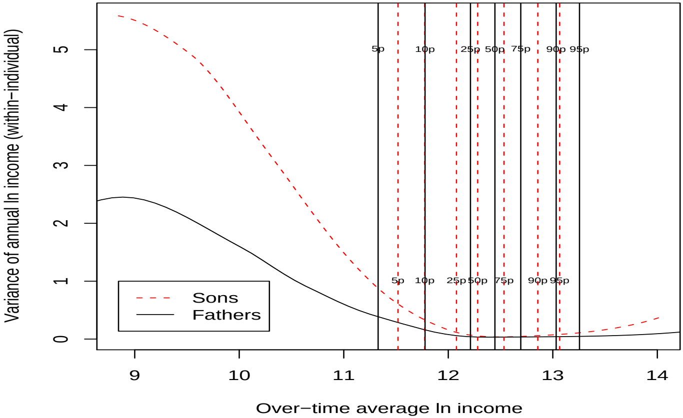
	Offspring				
	1	2	3	4	5
Father					
1	−10	−1	−1	0	0
2	−11	−5	−2	−6	0
3	−11	1	−4	−9	0
4	−8	−3	−12	−10	−1
5	−10	−11	−21	−20	−1

C. UK – Germany

	Offspring				
	1	2	3	4	5
Father					
1	14	6	7	2	0
2	20	16	6	8	0
3	20	18	11	11	0
4	17	20	21	19	1
5	15	24	22	23	1

Transitory errors and long-run income

The variation of annual ln income across over-time mean of ln income – Swedish fathers and sons



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Atkinson, A. B. and F. Bourguignon (1982). “The Comparison of Multi-Dimensioned Distributions of Economic Status”. In: *Review of Economic Studies* 49.2, pp. 183–201.

Atkinson, Anthony B (1981). “The Measurement of Economic Mobility”. In: *Essays in Honor of Jan Pen*. Ed. by P. J. Eigjelshoven and L. J. van Gemerden. Utrecht: Het Spectrum.

– (1983). “The measurement of economic mobility”. In: *Social Justice and Public Policy*. Ed. by A. B. Atkinson. Cambridge, MA: MIT Press. Chap. 3, pp. 61–76.

– (2008). “Mobility, Meritocracy and Markets”. Unpublished lecture at Russell Sage Foundation, New York.

Bingley, Paul and Lorenzo Cappellari (2012). *Alike in many ways: Intergenerational and sibling correlations of brothers' earnings*. CESIFO Working Paper 3994. http://www.cesifo-group.de/DocDL/cesifo1_wp3994.pdf. Munich: CESifo.

Dardanoni, V. (1993). “Measuring Social Mobility”. In: *Journal of Economic Theory* 61, pp. 372–394.

Eberharter, Veronika V (2013). “The Intergenerational Dynamics of Social Inequality – Empirical Evidence from Europe and the United States”. Paper presented at the ECINEQ Conference, Bari, July 2013, http://www.ecineq.org/ecineq_bari13/FILESxBari13/CR2/p118.pdf.

Fields, Gary S and Efe A Ok (1999). “The Measurement of Income Mobility: An Introduction to the Literature”. In: *Handbook of Income Inequality Measurement*. Ed. by Jacques Silber. Recent Economic Thought. Boston: Kluwer Academic Publishers. Chap. 19, pp. 557–598.

Gottschalk, P. and E. Spolaore (2002). “On the Evaluation of Economic Mobility”. In: *Review of Economic Studies* 69, pp. 191–208.

Hungerford, T. L. (2011). “How Income Mobility Affects Income Inequality: US Evidence in the 1980s and 1990s”. In: *Journal of Income Distribution* 20.1, pp. 83–103.

Jäntti, Markus and Stephen Jenkins (2015). “Income mobility”. In: *Handbook of Income Distribution*. Ed. by Anthony B Atkinson and François Bourguignon. Vol. 2. Elsevier. Chap. 10, pp. 807–935. DOI: doi:10.1016/B978-0-444-59428-0.00011-4. URL: http://www.sciencedirect.com/science/article/pii/B9780444594280000114.

Markandya, A. (1984). “The Welfare Measurement of Changes in Economic Mobility”. In: *Economica* 51, pp. 457–471.