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Abstract

We model voters' gender bias as a prejudice on women's competence coming from a distorted prior. We analyse the effect of this bias in a two-period two-party election in which voters care about both ideology and competence. We find that female politicians are less likely to win office but, when elected, have higher competence on average. As a consequence, they choose to seek re-election more often. We also show that if parties endogenously select candidates, the effect of gender bias is stronger, in that we observe fewer female candidates and elected politicians, and of higher competence. This holds even when parties are not biased.

JEL-Codes: D72, D91, J16.
Keywords: gender bias, elections, female politicians.

1 Introduction

Under-representation of women in parliaments worldwide is an issue that has been puzzling political scientists and political economists for a long time. While gender gaps in many areas have been reduced or have completely disappeared, women are still a minority in elected

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legislatures. They account for 24,9% of the members of parliaments worldwide, in spite of being roughly 50% of the population.

This is clearly a failure of descriptive representation but this deficit is of even greater concern because women's policy preferences receive more attention when a larger percentage of women sit in elected legislatures. It is therefore a question of substantive, not just formal, representation, as recent studies have shown that female politicians are more effective at addressing women's policy needs (Chattopadhyay and Duflo 2004, Clots-Figueras 2011, Funk and Gathmann 2015, Braga and Scervini 2016).

The puzzle among scholars of various fields comes from the fact that elected women tend to be more qualified than men and while in office they seem to be better representatives for their district (Volden *et al.* 2012). If, given this superiority, women are still such a minority among legislators, it must be that voters hold some kind of gender bias when evaluating candidates (Pearson and McGhee 2013). Studies have shown that voters hold women to a higher standard, and vote them only if they are both capable and likeable (Anzia and Berry, 2011). Also, there is evidence of the belief that women are more suitable for dealing with some issues like healthcare and education and less with others like homeland security (Lawless 2004, Falk and Kenski 2006).

In this work we model the presence of a gender bias without assuming explicit voters' preference for male politicians and we study its implications on politicians' quality and behaviour, and on their probability of winning an election.

Explanations of female under-representation. Female under-representation in politics has been shown to be due both to a shortage of candidates (supply side) and to a lower appeal of women politicians in elections (demand side). As for the supply story, according to the Inter Parliamentary Union, the top five motivations that globally keep women from entering politics are: domestic responsibilities, prevailing cultural attitudes regarding the role of women in society, lack of support from family, lack of confidence and lack of finance. Females are typically the primary caregivers both for children and the elderly and therefore tend to have a higher opportunity cost of running for elections (Fox, Lawless and Feely 2001). On top of that they tend to be less numerous in the professions which constitute the pipeline of the political career (Welch 1977, Clark 1994). But other explanations have been proposed for the low level of female entry in politics: women tend to be under-confident and do not think they possess the characteristics for being successful candidates (Fox and Lawless 2011), women are willing to represent their group but are election averse (Kanthak and Woon 2015), women have lower political ambition due to reduced political socialization within the family (Fox and Lawless 2014), and there is a scarcity of political role models for women (Campbell and Wolbrecht 2006).

On the demand side, instead, a large survey evidence gathered by psychologists and political scientists suggests that voters largely think that "men are better suited emotionally", "men make better leaders"and that there are circumstances in which being "tough" is really important (Huddy and Terkildsen 1993, Dolan 2004). In fact, when elections focus on terrorism, defence and homeland security the willingness to vote for female politicians is lower (Lawless 2004, Falk and Kenski 2006).

Yet, information (or the lack of it) may play a relevant role in determining gender bias. Sanbonmatsu (2002a) shows how voters may use gender as a low-information short-cut to make decisions at the poll station, as a way of simplifying the assessment of complex probability judgements (as in Kahneman and Tverski 1972). As a matter of fact, voters who pay little attention to politics and do not gather enough information may vote according to heuristics, the first being party affiliation, and the second one being the candidate's gender. This suggests that gender stereotypes affect voting behaviour because they influence, more or less consciously, the way in which candidates are evaluated. The influence is stronger the lower the information level, as the effect of gender attitudes can be attenuated by providing more information on the candidates' qualifications and past experiences (see Mo 2015).

Recently Bordalo *et al.* (2016) have formalized the formation of stereotypes building on the work of Tverski and Kahneman (1983). In their work, a sterotype is a probability distribution that over-(under-)estimates the likelihood of an event but it builds on a *kernel of truth* that is the first thing that "comes to mind" when making decisions. When dealing with gender stereotypes in politics though, this element of truth cannot be found. The evidence is indeed consistent with the fact that women in office are on average better than the men in the same elected body. More precisely, there is evidence that women tend to have greater prior political experience (Pearson and McGhee 2013), that they deliver more federal funds for their district (Anzia and Berry 2011), that they put more bills through the legislative process (Wolde *et al.* 2012) and that they deliver more speeches on the house floor (Pearson and Dancey 2011). Intuitively in fact, if voters hold a bias unfavourable to female candidates, only the most talented and hard working women will succeed in the electoral process.

Bohren *et al.* (2019) show how the dynamics of discrimination may be different depending on whether discrimination itself is preferences-based or belief-based (with either correct of incorrect beliefs). In the second case, over time, we might even observe reverse discrimination. Our source is similar but our "evaluation" is much coarser since it's the result of a political election. To the best of our knowledge ours is the first voting model that generates belief-based gender discrimination.

If information is the key explanation, being exposed to very talented and qualified female politicians may actually reduce the sex bias with which voters evaluate candidates when making their voting decision. In this respect, quotas and other affirmative action policies may speed up the process as institutional changes are useful to modify cultural and social norms that otherwise evolve extremely slowly. Regarding this, it is worth noting that these policies may work even when they are temporary (De Paola *et al.* 2010), they also seem to improve the general quality of politicians (Baltrunaite *et al.* 2014) and may even cause an increase in the quality of elected males through some sort of competition effect (Besley *et al.* 2017).

Modelling gender bias as an information bias. In our model we will study the implications of a voters' gender bias on the probability of females being elected, on politician's quality, implemented policies and chances of re-election without assuming explicit preferences for male politicians. In order to achieve this, we build a multi-period model where political candidates have private information on two of their own characteristics: ideology and valence (see Bernhardt *et al.* 2011). Voters have heterogeneous ideological bliss points, which characterise their policy preferences, while they are homogeneous in their preference for higher valence, which is a measure of the politician's competence. Candidates instead are heterogeneous both in their ideology and in their level of competence (valence). In our model, the valence of male and female politicians is drawn from the same distribution but voters believe that women are drawn from a distribution where lower valences have higher probability than men. Voters observe a signal on the candidates' valences before they run for the first time, while valences are observed during the first period in office (following Bernhardt *et al.* 2011).

Note that we model the bias against females as due to a misperception of the characteristics of the group. This prejudice, however, implies that if the voters observe the same signal on candidates' valences they are going to expect that female candidates are of a worse type than male ones. The fact that valences are observed when in office, instead, implies that the bias against women can be reduced since voters update their prior on female valences.

The assumption that voters have a distorted prior may be grounded in history. Indeed, in the not so distant past, it was true that women had a different distribution of political competence due to gaps in education and labour market participation. Nowadays, the gap in education has disappeared in most countries, and the gap in labour market participation is steadily shrinking. On the opposite, the gap in political performance and participation suffers from this additional information aspect. Women may be elected only if they manage to convince voters of their competence. However, voters, due to their distorted belief, elect them less often than men. Hence, the bias in politics is more persistent than elsewhere.

Our results on the implication of such an informative gender bias are consistent with the evidence discussed above. We find that women win elections less often, but elected female politicians have higher competence (on average) than male ones. This implies also that female incumbents are re-elected more often than male ones, a result which is also consistent with empirical evidence (Ferreira and Gyourko 2014, Bhalotra *et al.* 2018).

Finally, the fact that more observations of female elected politicians allow voters to update their prior and reduce their bias is consistent with the evidence that temporary measures in support of female candidacies have a persistent effect on the presence of women in politics (Beaman *et al.* 2008, Bahvnani 2009, De Paola *et al.* 2010).

The structure of the paper is as follows: Section 2 introduces the model, Section 3 characterises the equilibrium and discusses its dynamics and implications, Section 4 extends the model to address endogenous candidacy, Section 5 concludes. An Appendix contains the proofs.

2 The model

We consider a two-period model: in each period there is an electoral competition, in which two candidates, one from party L and one from party R, face each other, and a policy period, in which the elected politician implements a policy. Politicians are characterised by their ideology x^k, and their valence v^k, $k = L, R$, which is essentially a measure of their competence. Voters are heterogeneous in their policy preference, while they all prefer higher valence. We model the potential trade-off between competence and ideology, as in Bernhardt *et al.* (2011), and we adopt their utility function.

Politicians. Politicians are characterised by ideology and valence and they are policy oriented. In every period they receive utility from the implemented policy, $y_t \in \mathbb{R}$ and the valence of the elected politician, v_t^P, where $P = L, R$ is the identity of the elected politician in period t. The utility of a politician from party k in period t is:

$$u_t^k\left(y_t, v_t^P\right) = -\left(x^k - y_t\right)^2 + v_t^P.$$

A politician from party L has ideology $x^L \sim U[-1, 0]$, and a politician from party R has ideology $x^R \sim U[0, 1]$. The valence of a candidate is essentially a measure of his competence. It is private information of each candidate before election, and $v^k \sim U[0, 1]$ for $k = L, R$. Note that while ideology and valence of a politician are constant across periods, the identity of each party's candidate may vary. Therefore we let x_t^k and v_t^k denote the ideology and valence of party k's candidate in period t. When a politician is elected his/her valence and ideology are observed.[1]

Voters. Each voter i has ideological preferences characterised by a bliss point x^i. Bliss points $x^i \sim U[-1, +1]$, so that the median voter has bliss point $x^m = 0$. Period t utility of each voter depends on the implemented policy y_t and on the valence of the elected politician v_t^P as follows:

$$u_t^i\left(y_t, v_t^P\right) = -\left(x^i - y_t\right)^2 + v_t^P.$$

Note that voters and politicians have the same utility function.

Gender and gender bias. The first time in which they run for office, candidates are randomly selected from a gender-balanced population (male/female with equal probability). The gender of the candidate matters, in that there is a distortion in the voters' perception of female candidates.[2] Even though the valence of the candidates is $v_t^k \sim U[0, 1]$ regardless of the candidate's gender, voters' prior belief on female candidates is that there is a probability ϕ_t that they come from a worse distribution, specifically $v_t^k|F \sim U[0, V]$ where $V \in \left(\frac{1}{2}, 1\right)$,[3] and from the true one $v_t^k|F \sim U[0, 1]$ with the complementary probability $(1 - \phi_t)$.

[1] We depart from the standard assumption of Bernhardt *et al.* (2011) by assuming that also ideology is observed.

[2] We assume that politicians are instead not biased. The analysis is robust to the alternative assumption that politicians are gender biased as well. A discussion of the consequences of this alternative assumption on the re-candidacy decision can be found in the proof of Proposition 1 (Appendix A).

[3] A discussion of the case $V < \frac{1}{2}$ can be found in Section 3.2.

Therefore, the higher is ϕ_t the higher is the gender bias. Period 1 belief ϕ_1 is taken as given, while period 2 belief ϕ_2 may be updated if a female politician is elected in period 1 and her valence is observed.

Signal. When a candidate runs for the first time, voters observe a signal $\sigma_t^k \in \{\underline{v}, \overline{v}\}$ on his/her valence. The signal reveals whether the valence of the candidate is below ($\underline{v}$) or above ($\overline{v}$) the median of its group. Hence the perceived expected valence given the signal σ differs for male and female candidates as follows:

$$\mathbb{E}\left[v_t^k|\sigma_t^k, M\right] = \begin{cases} \frac{3}{4} & \text{if } \sigma_t^k = \overline{v} \\ \frac{1}{4} & \text{if } \sigma_t^k = \underline{v} \end{cases} ; \quad \mathbb{E}\left[v_t^k|\sigma_t^k, F\right] = \begin{cases} \frac{3}{4}\left(1 - \phi_t\left(1 - V\right)\right) & \text{if } \sigma_t^k = \overline{v} \\ \frac{1}{4}\left(1 - \phi_t\left(1 - V\right)\right) & \text{if } \sigma_t^k = \underline{v} \end{cases} .$$

As a consequence, the expected valence of a female candidate is lower than the one of a male candidate for any possible signal. Moreover $\mathbb{E}\left[v_t^k|\sigma_t^k, F\right]$ is decreasing in the bias ϕ_t. Given the assumption $V > \frac{1}{2}$, it is always the case that a woman with a high signal has a higher expected valence than a man with a low signal for any possible ϕ_t.

Timing. The sequence of events at any period t is:

- Two candidates (one per party) run for election. In period 1 they are two randomly drawn untried candidates, while in period 2 one of them may be the incumbent.
- Signals on candidates are observed.
- Given the information about candidates (party affiliation, gender and σ for the untried candidates; party affiliation, gender, x, v and past policy choice for the incumbent) citizens vote for their preferred candidate.
- The winning politician, P, with ideology x_t^P and valence v_t^P, implements the policy choice $y_t = p\left(x_t^P, v_t^P\right)$.
- At the end of period 1 only, the incumbent optimally chooses whether to run for re-election or not.

3 Equilibrium analysis

An equilibrium of this political game is composed by policy choices and voting decisions that may involve incumbents, for which ideology x^I and valence v^I have already been observed, or untried candidates for which only party affiliation is known. Proposition 1 characterises a Perfect Bayesian Equilibrium equilibrium with weakly undominated voting strategies.[4]

Proposition 1 *The following pure strategies constitute a political equilibrium in which the electoral outcome is decided by the median voter where* $x^m = 0$*.*
Policy choice. *In every period* $t = 1, 2$ *the elected politician,* P*, implements* $y_t = x_t^P$*.*
Voting on untried candidates. *When candidates are both untried the median voter votes according to the following ranking of gender-signal pairs*

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}),$$

randomizing with equal probability when indifferent.
Voting for the incumbent. *The median voter votes for the incumbent:*

1. *for any type of challenger when* $v^I \geq \left(x^I\right)^2 + \frac{5}{12}$*;*
2. *if the challenger gender-signal is in* $\{(F, \overline{v}), (M, \underline{v}), (F, \underline{v})\}$ *when* $v^I \geq \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi_2(1 - V)$*;*
3. *if the challenger gender-signal is in* $\{(M, \underline{v}), (F, \underline{v})\}$ *when* $v^I \geq \left(x^I\right)^2 - \frac{1}{12}$*;*
4. *if the challenger gender-signal is* $(F, \underline{v})$ *when* $v^I \geq \left(x^I\right)^2 - \frac{1}{12} - \frac{1}{4}\phi_2(1 - V)$

Re-candidacy decision. *The incumbent of* $t = 1$ *runs for re-election in* $t = 2$ *if one of the following conditions holds:*

1. $v^I \geq \max\left\{-\left(x^I\right)^2 + \frac{1}{3}x^I + \frac{1}{4}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi(1 - V)\right\}$*;*
2. $v^I \in \left[\max\left\{\left(x^I\right)^2 - \frac{1}{12}, -\left(x^I\right)^2 + x^I + \frac{1}{6}\right\}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi(1 - V)\right]$.

[4] We consider a variant of the PBE which incorporates the independence assumption that is characteristic of sequential equilibria, the "no-signaling-what-you-don't-know" condition, as in Fudenberg and Tirole (1991). In our framework, the assumption notably implies that observed deviations from the incumbent do not signal anything on the challenger's type.

In the last period of the game every elected politician implements his/her bliss point as policy choice since there are no incentives to do otherwise to gain votes for re-election. Moreover, recall that in this model we have assumed that both ideology and valence are observed when a politician is elected. As a consequence, also in period one the elected politician optimally implements a policy equal to his/her bliss point, as there is no incentive to mimic a different ideological position. Given that politicians' ideologies are uniformly distributed and parties are symmetric around the median voter's position, the median voter's expected utility from the policy component is the same for any untried candidate. Therefore, in elections involving an open seat (*i.e.*, with two untried candidates) the median voter only focuses on the politician's expected valence. In this respect, politicians are instead different even ex-ante, and information on the expected valence can be inferred from the gender-signal pairs. The median voter votes for the candidate with the highest expected valence as described by the ranking in Proposition 1, randomizing with equal probability when choosing between candidates with the same gender-signal pair. The case of elections with an incumbent is different. When an incumbent runs for re-election both his/her ideology and valence matter for the median voter's decision as they are known, and for this reason the incumbent's gender does not affect the voting decision of the electorate. The incumbent is more likely to be re-elected for higher valences and lower ideological biases. Re-election is also more likely the lower the expected valence of the challenger. Finally the incumbent anticipates this electoral outcome and chooses to run for re-election only when his/her valence is sufficiently high.

3.1 Effects of the gender bias

The presence of a bias in voters' appraisal of candidates has naturally many consequences for the career of our politicians conditional on their gender. In what follows we show in detail how women have a lower chance of being elected but, when they do, they are characterised by higher valences on average and will therefore be stronger incumbents so that they will typically run for re-election more often.

Female probability of winning. The first effect of gender bias is a distortion in the winning probability of women, in that female candidates are less likely to win for any given signal. Women are thought to be drawn from a distribution that gives higher probability to lower valences. This has two implications. First, when running for an open seat, they lose the electoral competition against males with the same signal. The winning probabilities given σ in an open seat election are as follows:

$$\Pr_t\left[\text{win}|\sigma_t^k, M\right] = \begin{cases} \frac{7}{8} & \text{if } \sigma_t^k = \overline{v} \\ \frac{3}{8} & \text{if } \sigma_t^k = \underline{v} \end{cases} \qquad \Pr_t\left[\text{win}|\sigma_t^k, F\right] = \begin{cases} \frac{5}{8} & \text{if } \sigma_t^k = \overline{v} \\ \frac{1}{8} & \text{if } \sigma_t^k = \underline{v} \end{cases},$$

so that the winning candidate is a woman with probability $\frac{3}{8}$ which is lower than the fraction of female candidates $\left(\frac{1}{2}\right)$.

Second, the bias also affects the probability of winning when running against an incumbent. Note that challengers with low signals, regardless of their gender, always lose competitions against incumbents who optimally seek re-election. Thus, the difference is driven by challengers with high signals. In this group, male challengers win against incumbents with $v^I < \left(x^I\right)^2 + \frac{5}{12}$, while female challengers only win against a subset of these incumbents, i.e., those with $v^I < \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi_2\left(1-V\right)$. Hence, incumbents with $v^I \in \left[\left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi_2\left(1-V\right), \left(x^I\right)^2 + \frac{5}{12}\right)$ defeat female candidates of the same (true) expected valence as the one of the male candidates who defeat them.

Expected valence of elected politicians. A second effect of the bias is that the expected valence of an elected politician depends on his/her gender. Indeed the expected valence of elected politicians is higher for women, and this is driven by the electoral competitions for open seats. Male politicians who are elected in an open seat competition are above the median $(\sigma_t^P|M = \overline{v})$ with a lower probability than elected females. As the true median is the same for both groups the result follows:

$$\begin{aligned}
\mathbb{E}\left[v_t^P|M\right] &= \Pr\left(\sigma_t^P = \overline{v}|M\right)\mathbb{E}\left[v_t^P|\overline{v}, M\right] + \Pr\left(\sigma_t^P = \underline{v}|M\right)\mathbb{E}\left[v_t^P|\underline{v}, M\right] = \frac{3}{5} \\
\mathbb{E}\left[v_t^P|F\right] &= \Pr\left(\sigma_t^P = \overline{v}|F\right)\mathbb{E}\left[v_t^P|\overline{v}, F\right] + \Pr\left(\sigma_t^P = \underline{v}|F\right)\mathbb{E}\left[v_t^P|\underline{v}, F\right] = \frac{2}{3}.
\end{aligned}$$

Voting for the incumbent. In our model the gender bias has no effect on voters' perception of the incumbent, since both his/her valence and ideology are observed once in office. What affects the incumbent's chances of re-election, instead, is the challenger's gender-signal pair. Note that in an election between an incumbent of type $\left(x^I, v^I\right)$ and an untried challenger, as the quality of the challenger as predicted by the gender-signal pair falls, the set of incumbent's characteristics $\left(x^I, v^I\right)$ that allows him/her to gain re-election grows larger.

Running for re-election. As discussed above, voters' evaluation of an incumbent does not depend on his/her gender.[5] Therefore, re-election incentives are the same for male and female politicians in office. Specifically, incumbents that are so good that they win against any type of challenger always run for re-election. Candidates who win at most against a female challenger with a low signal, instead, never choose to do so, as running implies that the opposing party wins too often, thus implementing a policy that belongs to the opposite side of the ideological spectrum. Candidates with intermediate types, who win against candidates with low signals, and possibly also against female candidates with high signals, only run for re-election if their valence is high enough given their ideological type. These results hold whatever the gender of the incumbent, nonetheless there is an indirect effect of gender on the re-candidacy decision. As a matter of fact, as discussed before, elected women have higher valences and will be harder-to-defeat incumbents (Fulton 2012, Ferreira and Gyourko 2014). Hence, elected women are more likely to be in those parametric regions in which re-candidacy is optimal, so that on average they run for re-election more often (Bhalotra *et al.* 2018).

3.2 Dynamics of the gender bias

We have assumed that voters hold the prejudice that female politicians are on average less competent than men, that is they are drawn from a distribution which gives more weight to lower valences. In the Introduction, we argued that this prejudice may come from the past. As it used to be the case that women had lower or no access to education and to political experience, if we interpret competence as a trait that does not only depend on intrinsic qualities but also on learning, it is reasonable to think that their distribution of competence may have been different from the male one until the not so distant past. We are thus interested in understanding how a prejudice which has its origin in the past may evolve over time.

To understand the dynamics of the bias, we ask first what estimator can voters use to estimate the parameter of the distribution of valences. The true distribution is indeed a uniform on $[0, 1]$. However, as discussed in Section 2, voters believe that the distribution of female valences is the mixture of two uniforms, one over $[0, V]$ and one over $[0, 1]$, with weight ϕ.

[5] Shair-Rosenthal and Hinojosa (2014) find evidence in Chilean data that incumbency eliminates voters' bias.

In order to use the results by Craigmile and Tirrerington (1997) we express the problem in a different, but equivalent, formulation. We can represent all densities in our model as a mixture density of the following form:

$$f(x|p, V) = pU[0, V] + (1 - p)U[V, 1],$$

i.e., as a mixture of a uniform over $[0, V]$ and a uniform over $[V, 1]$, with weight p. In this formulation, the true distribution of female competences has density $VU[0, V] + (1 - V)U[V, 1]$, while voters' belief in the presence of a prejudice is characterised by a higher weight on the uniform over $[0, V]$, i.e., $p > V$. Note that $p = \phi + (1 - \phi)V$, so that $p = V$ is equivalent to $\phi = 0$, i.e., to the true unbiased distribution.

When V is known, a consistent and unbiased estimator of p is given by (Gupta and Miyawaki 1978)

$$\tilde{p} = 1 + V - 2M,$$

where M is the sample average. The existence of a consistent (and unbiased) estimator of p implies that if voters disregarded their original prejudice and estimate p on the basis of observed competences only, their estimates are immediately unbiased and reflect the actual distribution of female competences. However, prejudices die hard. We therefore assume that there is some degree of inertia in the beliefs. Modelling the specific functional form of this assumed inertia is not the aim of this paper, yet, some relevant properties of the dynamics can be derived under general assumptions on the update process itself.

Let us call $\hat{p}_t$ the estimator of p at time $t \geq 1$.[6] We assume that at time $t = 1$, that is, before the election of period 1, the estimator has a known (non random) value p_1.[7] To account for the inertia of the prejudice we assume that $\{p_t\}$ is an autocorrelated process, so that the convergence of the estimator to the (true) value V happens gradually and not instantaneously.

In this case, we can measure how much voters reduce their prejudice by looking at the

[6] To be consistent with our original model, we call $\hat{p}_t$ the estimator of p at the beginning of time t, before the election takes place. Therefore $\hat{p}_t$ does not include the information on the female politicians that will be elected at time t.

[7] Note that $\phi_1 = \frac{p_1 - V}{1 - V}$.

expected square variation of $\hat{p}_t$ between period 1 and period 2:[8]

$$\mathbb{E}\left[(\hat{p}_2 - p_1)^2\right] = Var[\hat{p}_2] + (\mathbb{E}[\hat{p}_2] - p_1)^2 .$$

Note that the expected variation is greater the larger the difference between the original belief p_1 and the expected value of the estimator $\mathbb{E}[\hat{p}_2]$.[9] This difference depends both on the strength of the prejudice and on the inertia of the estimator.[10] To understand the effects of these two elements, let us consider, as an example, an easy autoregressive process such as:

$$\hat{p}_2 = \alpha p_1 + (1-\alpha)\tilde{p} = \alpha p_1 + (1-\alpha)(1 + V - 2M).$$

In this case $\mathbb{E}[\hat{p}_2] = \alpha p_1 + (1-\alpha)V$, so that $(\mathbb{E}[\hat{p}_2] - p_1)^2 = (1-\alpha)^2(V-p_1)^2$. Moreover, $Var[\hat{p}_2] = (1-\alpha)^2 Var[\tilde{p}] = \frac{(1-\alpha)^2}{3n}$, which does not depend on V.[11] Overall, the expected square variation can be written as

$$\mathbb{E}\left[(\hat{p}_2 - p_1)^2\right] = \frac{(1-\alpha)^2}{3n} + (1-\alpha)^2(V-p_1)^2 = (1-\alpha)^2\left(\frac{1}{3n} + (V-p_1)^2\right).$$

From this we can easily note that a higher persistence of the prejudice (higher α) reduces the expected variation, while a stronger bias (higher distance between V and p_1) increases it, thus favouring a larger reduction of the prejudice. This result holds regardless of the specific functional form of the process, provided it is positively autocorrelated.

Strength of the bias and the effects of V. As discussed a few lines above, a stronger bias will be reduced more easily than a weaker one. This persistence of weak biases is consistent with the empirical evidence in the literature that suggests that an increased

[8] We consider the expected square variation because we are not interested in the sign of the expected variation itself.

[9] Even though our theoretical model has only two periods, we can derive the expected square variation between two generic periods t and $t+1$, which is

$$\mathbb{E}\left[(\hat{p}_{t+1} - \hat{p}_t)^2\right] = Var[\hat{p}_{t+1}] + Var[\hat{p}_t] + (\mathbb{E}[\hat{p}_{t+1}] - \mathbb{E}[\hat{p}_t])^2 - 2Cov[\hat{p}_{t+1}, \hat{p}_t].$$

We can note that the expected variation between two generic periods t and $t+1$ displays the same comparative statics discussed for the variation between periods 1 and 2. This formulation highlights the effect of autocorrelation on the process, through the covariance term. Higher covariance, which corresponds to greater inertia, induces lower expected square variation.

[10] To understand the two effects more clearly, we can decompose the difference $\mathbb{E}[\hat{p}_2] - p_1 = (\mathbb{E}[\hat{p}_2] - V) - (p_1 - V)$, where $(p_1 - V)$ is a measure the strength of the prejudice.

[11] On the computation of $Var[\tilde{p}]$ see Craigmile and Titterington (1997).

exposure to female politicians has a stronger positive effect on the number of elected women in countries where the bias can be thought to be stronger (Beaman *et al.* 2009), or when women make their début in the political arena (Gilardi 2015), than in mature western democracies where open discrimination should be a thing of the past (Broockman 2014). As the prejudice becomes weaker it is harder to remove it.

In our paper we take V as constant and exogenously given, and we model the intensity of the bias through the parameter ϕ (and through the parameter p in the above discussion of the dynamics of the bias). However, V can also be considered a measure of the strength of the gender bias, and it is likely to be heterogeneous across countries, due to the influence of specific cultural characteristics or historical background. Specifically, if V is low, voters allow for the possibility that female politicians are characterised by valences that are much lower than the males' ones.

In the paper, we assume $V > \frac{1}{2}$, as it is the most interesting case. If instead the bias is stronger in terms of V, i.e., $V < \frac{1}{2}$, two different situations may arise. At first, when $V \in \left(\frac{1}{3}, \frac{1}{2}\right)$, women are elected with the same probability as in our model. However, when a female candidate with high signal is elected, her valence is always greater than V, so that convergence to the undistorted distribution is extremely fast. When the bias is even stronger, i.e., $V < \frac{1}{3}$, female candidates with high signals lose the electoral competition also against male candidates with low signals. Because of the lack of observations of female competences, the valence of women is observed very rarely and the bias results extremely persistent.

Possible effects of gender quotas. Affirmative action policies, such as gender quotas, can be interpreted in our model as an exogenous variation in the frequency of observations coming from female politicians. In other words any policy that increases exogenously the number of candidate or elected women will allow voters to acquire information on a larger set of women. In our environment this implies a faster reduction in the gender bias, as increasing the number of observations from the true distribution helps estimating the parameters faster.

It is worth noting that this effect would persist even in case these affirmative policies are then removed and that this is consistent with the empirical evidence (see Beaman *et al.* 2008, Bahvnani 2009 and De Paola *et al.* 2010).

4 Selection of candidates: the role of parties

In the baseline version of the model we worked under the assumption that candidates are randomly drawn from the population. This may not be a realistic assumption as parties choose their candidates to maximise the probability of being in power. We now modify the model by adding an initial stage in which parties select their candidates from a pool that is smaller than the whole population (as in Le Barbanchon and Suvagnat 2018).

We extend the model by introducing parties L and R as separate agents whose aim is to maximise the total probability (across periods) of having one of their members elected. They do so by optimally selecting which politician is going to run for them.

We make the simplest possible assumption: each party, when facing an open seat election (either in period one, or in period two if the incumbent chooses not to run), has to select a candidate. Specifically, the party has to choose between two politicians that are randomly drawn from the whole population.

Proposition 2 characterises the political equilibrium of the model with endogenous selection of candidates. Note that policy choices and voters' behaviour are the same as in the baseline model (see Proposition 1).

Proposition 2 *The following pure strategies, together with the policy choice and voting behaviour characterised in Proposition 1, constitute a political equilibrium in which the electoral outcome is decided by the median voter where $x^m = 0$.*
Selection of candidates. *In every period and type of election, parties choose their candidate according to the following ranking of gender-signal pairs*

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}),$$

randomizing with equal probability when indifferent.
Re-candidacy decision. *The incumbent of $t = 1$ runs for re-election in $t = 2$ if one of the following two conditions holds:*

1. $v^I \geq \max\left\{-\left(x^I\right)^2 + \frac{14}{27}x^I + \frac{39}{108}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi(1-V)\right\}$;
2. $v^I \in \left[\max\left\{\left(x^I\right)^2 - \frac{1}{12}, -\left(x^I\right)^2 + 2x^I + \frac{7}{24}\right\}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi(1-V)\right]$.

Proposition 2 highlights how the equilibrium behaviour is modified by the presence of parties in two aspects.

On the one hand, the thresholds for the incumbent's re-candidacy decision change. This is due to the fact that the candidates' selection process increases both the expected quality of the potential challenger and of the alternative candidate for the same party if the incumbent chooses not to run. These two effects make re-candidacy less appealing.

On the other hand, it is also necessary to describe the optimal process for selecting the candidates.

In the second period parties maximise the probability that their current candidate is elected. Therefore, when facing an open seat election, they choose their candidate according to the same ranking that describes the median voter's preferences:

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}) \, .$$

If instead the second period election involves an incumbent, indifferences may arise. For example, the party is indifferent among each type of politician if $v^I \geq \left(x^I\right)^2 + \frac{5}{12}$, as every type of challenger is worse than the incumbent and loses the elections. Similarly, if $v^I \in \left[\left(x^I\right)^2 - \frac{1}{12}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi_2(1-V)\right]$ the party is indifferent between a female and a male candidate with a high signal as they both win against the incumbent, and between a female and a male candidate with a low signal as they both lose. We assume that, even when indifferent, the party ranks politicians according to the same preference ordering as the median voter, *i.e.*,

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}) \, .$$

We justify this assumption by noticing that this is strictly optimal if there exists the possibility of an unforeseen event that induces party k to replace the incumbent with another candidate (*e.g.*, the death of the incumbent).

Finally, parties choose according to the same ranking also in the first period as they do not know the true valences of the candidates but only their gender-signal pair. This is true even if parties are not gender biased, as they anticipate the presence of a bias in the electorate.[12] If parties maximise the probability of winning elections, it is indeed optimal to acknowledge that voters are biased when they evaluate candidates. Moreover, if, traditionally, the voters of a party have a lower bias we should observe more female candidates running for that party, which is in fact what happens in the US with the

[12] Bagues and Esteve-Volart (2012) find evidence of party bias for Spain, where women were strategically put on less favourable positions on the ballot with the result of nearly nullifying the newly adopted affirmative action party policies.

Democratic and Republican parties (Sanbonmatsu 2002b). Additionally, if we combine the dynamics of the bias with the role of parties, we can further explain the gap in female participation as due to parties who may undersupply women candidates if they haven't acknowledged the reduction in the bias of the voters (Murray 2008).

The introduction of endogenous selection of candidates by parties changes the effects of gender bias on electoral outcomes. In what follows we discuss the main implications of the introduction of political parties.

Female probability of candidacy and election. Each party faces two politicians that are randomly drawn from the population, so that each politician is characterised by any specific gender-signal pair with equal probability.[13] Given that parties select the candidates according to the ranking described above, the probability of having a candidate of a specific gender-signal pair is no longer the same across pairs. As a consequence, the candidate is a woman with probability $\frac{3}{8}$, which is lower than the fraction of female politicians (and therefore lower that the fraction of female candidates in the baseline model). The two selected candidates, one for each party, compete against each other in the election. This further unbalances the probability of observing the election of a woman. Females now account only for roughly 25.7% of elected politicians, which is less than what happens in the baseline model (around 37.5%). This is due to fact that the gender bias distorts the selection process twice: first at the party candidacy decision level and then at the voters level in the election itself.

Expected valence of female candidates and elected politicians. Female politicians who have been selected as candidates are of higher expected valence than before. As a matter of fact, the expected valence of a female candidate in the baseline model is $\frac{1}{2}$ (as they are randomly drawn from the population) while after party selection it is $\frac{2}{3}$. Also, the expected valence of an elected female politician is now higher than her expected valence when candidacy is exogenous.

Endogenous selection of candidates highlights the effect of the gender bias since it operates twice. First of all, as discussed above, it reduces the overall probability of observing

[13] As discussed in the Introduction, we observe that women are under-represented also in the pool of possible candidates. If we made the more realistic assumption that the pool of politicians is unbalanced, our results would be strengthened.

female politicians at every level at which it is at play (here both candidates and elected politicians). Second this reduction in probability is greater for women with low types, so that the expected valence of female politicians active in the political arena increases as the bias operates at more levels.

Note that the endogenous selection of candidates improves also the expected quality of male candidates and male elected politicians, as it decreases the probability of observing a male politician with a low signal at every level of the process. Male candidates and elected politicians have still a lower expected valence than their female counterparts although the difference between the expected valence of female and male elected politicians is smaller than in the baseline model. This is due to the fact that the selection process improves the quality of male politicians more than the quality of female politicians, as female politicians with low quality were already very unlikely to be elected.

The incumbent's candidacy decision. When parties select candidates, incumbents run for re-election weakly less often than in the baseline model. This is an effect of the increased quality of challengers, which has a twofold consequence. First, it increases the incumbent's expected utility from not running, and second it decreases his/her chances of winning. This implies that, differently from the baseline model, not all the incumbents who would be defeated only by a male challenger with a high signal now run for re-election. This is due both to the fact that competing against a male challenger with a high signal is more likely than before and to the higher expected quality of the alternative candidate of the incumbent's own party if he steps down.

Female conditional probability of winning Women's probability of winning conditional on being a candidate is lower than the equivalent in the baseline model. This is due to the fact that parties' selection of candidates increases the likelihood of competing against a male candidate with a high signal. We acknowledge that this low conditional probability of winning is in contrast with empirical evidence that shows that when women run they win at the same rate as men (Burrell 1994, Darcy and Schramm 1977, Seltzer 1997). In our model, this is due to the discrete nature of the signal which does not allow parties to select women with such an high signal that lead them to be perceived as equally qualified as men. We conjecture that increasing the number of possible signals on competence, having a finer classification of abilities, should have the twofold effect of reducing the number of female candidates and increasing their conditional winning probabilities, thus reducing the conditional differences between men and women.

5 Concluding remarks

The results of US midterm elections in November 2018 have granted to 2018 the name of "The Year of the Woman" for the unprecedented number of female candidates. In the American political system, in order to run for one of these elections, the candidate must have won the corresponding primary race. The national elections, however, took this phenomenon a step forward, by bringing a record number of women to serve as legislators. Only future elections will allow us to understand whether this result is part of a trend or a one-off event. In either case, female under-representation in politics is far from being a problem of the past.

This paper has proposed an explanation of the presence of gender bias in politics as a demand effect driven by a prejudice of voters on the distribution of competence in the female population.

In a simple two-period model we have shown how modelling gender bias as an informational bias of voters in the evaluation of candidates generates results that are consistent with the empirical evidence of the existing political science and political economy literature. For example, we found that women win less often than men but those who are elected are on average more competent than the elected male politicians.

We also discussed how prejudices may evolve over time. We have highlighted how being exposed to qualified women politicians contributes to its reduction, and we have argued that this should be considered a theoretical ground for those policies of affirmative action that favour women political participation. Our model predicts that they may have a positive effect even when temporary. Moreover, we found that stronger prejudices are easier to discard, while the more subtle ones tend to be more persistent.

We also extended the model to consider the strategic candidacy choice made by parties. When we allow parties to choose their candidates strategically, we observe that they choose women candidates only when they are sufficiently strong, even if parties are not gender biased, because they anticipate the gender bias of the electorate. Therefore, the bias operates twice, first at the candidate selection stage, and then at the election one. As a consequence, the probability of having a female candidate decreases, and the probability of electing a woman falls even further, while on the other hand the expected valence of elected women increases more than the expected valence of the elected men.

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A Proof of Proposition 1

Policy choice. In the last period of the game the policy choice of the elected politician affects only his/her second period utility, as the game ends afterwards. Therefore he/she implements his/her most preferred policy to maximise his/her utility. In the first period of the game, the elected politician P in period t knows that his/her ideology x_t^P and his/her valence v_t^P are both observed by voters when he/she is in office. Moreover, voters know that in the last period every elected politician implements a policy equal to his/her ideology. As a consequence, the policy choice of the politician who is in office in period $t = 1$ does not affect his/her probability of winning the election in $t = 2$ (nor his/her re-candidacy choice). Therefore, also in period 1 $y_1 = x_1^P$.

Voting on untried candidates. Voters anticipate the politicians' behaviour. At the time of the election the median voter cannot distinguish two untried candidates according to their ideological position, as candidates are ex-ante symmetric in this dimension. Therefore the election is decided on the basis of the information available on candidates' valence. The median voter's utility is linear in v_t^P therefore he will prefer the candidate with the highest expected valence. The expected valence given the candidate's gender and his/her σ_t^k is:

$$\mathbb{E}\left[v_t^k|\sigma_t^k, M\right] = \begin{cases} \frac{3}{4} & \text{if } \sigma_t^k = \overline{v} \\ \frac{1}{4} & \text{if } \sigma_t^k = \underline{v} \end{cases} \quad ; \quad \mathbb{E}\left[v_t^k|\sigma_t^k, F\right] = \begin{cases} \frac{3}{4}\left(1 - \phi_t\left(1 - V\right)\right) & \text{if } \sigma_t^k = \overline{v} \\ \frac{1}{4}\left(1 - \phi_t\left(1 - V\right)\right) & \text{if } \sigma_t^k = \underline{v} \end{cases},$$

therefore the median voter ranks the candidates according to their gender-signal pair as follows $(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v})$.

Voting for the incumbent. Consider now a period 2 election in which the incumbent from period 1 runs for re-election. In period 2 voters know both valence v^I and ideology x^I of the incumbent. Given that the valence is observed, the incumbent's gender does not affect his/her probability of re-election. Moreover, voters know that any politician winning in period 2 will implement a policy equal to his/her bliss point.

Consider therefore the voting incentives of the median voter when comparing an incumbent (x^I, v^I) with an untried challenger characterised by his/her gender-signal pair. As the median voter is located at $x^m = 0$, it is indifferent whether the incumbent is from party R and the challenger from party L or viceversa.

The median voter's expected policy disutility in period 2 from an untried challenger from party k, given the equilibrium policy choice, is:

$$\begin{aligned} -\mathbb{E}(0-y_2)^2 &= -\mathbb{E}(x_2^k)^2 = -Var[x_2^k] - (\mathbb{E}[x_2^k])^2 \\ &= -\frac{1}{12} - \frac{1}{4} = -\frac{1}{3}. \end{aligned}$$

Therefore, the median voter's expected utility in period 2 when facing a challenger, depending on his/her gender-signal pair, is:

- $\mathbb{E}u_2^m(M,\overline{v}) = \mathbb{E}(0-y_2)^2 + \mathbb{E}\left[v_2^k|\overline{v},M\right] = -\frac{1}{3}+\frac{3}{4} = \frac{5}{12}$;
- $\mathbb{E}u_2^m(F,\overline{v}) = -\frac{1}{3}+\frac{3}{4}(1-\phi_2(1-V)) = \frac{5}{12}-\frac{3}{4}\phi_2(1-V)$;
- $\mathbb{E}u_2^m(M,\underline{v}) = -\frac{1}{3}+\frac{1}{4} = -\frac{1}{12}$;
- $\mathbb{E}u_2^m(F,\underline{v}) = -\frac{1}{3}+\frac{1}{4}(1-\phi_2(1-V)) = -\frac{1}{12}-\frac{1}{4}\phi_2(1-V)$.

The median voter's expected utility in period 2 from an incumbent characterised by (x^I, v^I) is instead: $\mathbb{E}u_2^m\left(x^I, v^I\right) = -(x^I)^2 + v^I$. Hence, the median voter votes for the incumbent given the challenger's gender-signal pair, in the following parametric regions:

- if the challenger's gender-signal pair is $(M,\overline{v})$, when $v^I \geq (x^I)^2 + \frac{5}{12}$, that is, when $x^I \in \left[-\sqrt{v^I - \frac{5}{12}}, \sqrt{v^I - \frac{5}{12}}\right]$;
- if the challenger's gender-signal pair is $(F,\overline{v})$, when $v^I \geq (x^I)^2 + \frac{5}{12} - \frac{3}{4}\phi_2(1-V)$, that is, when $x^I \in \left[-\sqrt{v^I - \frac{5}{12} + \frac{3}{4}\phi_2(1-V)}, \sqrt{v^I - \frac{5}{12} + \frac{3}{4}\phi_2(1-V)}\right]$;
- if the challenger's gender-signal pair is $(M,\underline{v})$, when $v^I \geq (x^I)^2 - \frac{1}{12}$, that is, when if $x^I \in \left[-\sqrt{v^I + \frac{1}{12}}, \sqrt{v^I + \frac{1}{12}}\right]$;
- if the challenger's gender-signal pair is $(F,\underline{v})$, when $v^I \geq (x^I)^2 - \frac{1}{12} - \frac{1}{4}\phi_2(1-V)$, that is, when $x^I \in \left[-\sqrt{v^I + \frac{1}{12} + \frac{1}{4}\phi_2(1-V)}, \sqrt{v^I + \frac{1}{12} + \frac{1}{4}\phi_2(1-V)}\right]$.

Remark 3 *Note that, in an election between an incumbent $\left(x^I, v^I\right)$ and an untried challenger, as the quality of the challenger falls, the set of x^I by the incumbent that allows him/her to gain re-election grows larger, and also the threshold for v^I decreases.*

The incumbent's candidacy decision. Consider an incumbent from party R, characterised by $x^I \geq 0$ and v^I. If he/she does not run, voters have the same ex-ante probability of electing a politician from party L or R. Therefore the incumbent, in period 2, obtains expected utility:

$$\begin{aligned}
\mathbb{E}u_2^I &= -\frac{1}{2}\mathbb{E}[(x^I - x_2^L)^2] - \frac{1}{2}\mathbb{E}[(x^I - x_2^R)^2] + \mathbb{E}\left(v_2^P\right) \\
&= -\left(x^I\right)^2 - \frac{1}{2}\mathbb{E}[\left(x_2^L\right)^2] + x^I\mathbb{E}\left[x_t^L\right] - \frac{1}{2}\mathbb{E}[\left(x_2^L\right)^2] + x^I\mathbb{E}\left[x_t^L\right] + \mathbb{E}\left(v_2^P\right) \\
&= -\left(x^I\right)^2 - \frac{1}{3} + \frac{5}{8} \\
&= -\left(x^I\right)^2 + \frac{7}{24}.
\end{aligned}$$

Note that in the model we assume that politicians are not biased and therefore their expected utility is not affected by the gender of the challenger. If we assumed instead that the incumbent suffered from the same gender bias as the voters, his/her utility from not running would have been lower, and therefore he/she would have run for re-election (sub-optimally) more often.

Given the incumbent's expected utility if he/she does not run in the second period, we discuss the optimal re-candidacy choice, which depends on the chances of re-election.

- If elected in period 2, the incumbent implements his/her bliss point and his/her utility is equal to v. Therefore, an incumbent who wins against any type of challenger, i.e., an incumbent with $v^I \geq \left(x^I\right)^2 + \frac{5}{12}$, runs for re-election in period 2 for $v^I \geq \frac{7}{24} - \left(x^I\right)^2$. The first condition implies the second one, therefore an incumbent who wins against any type of challenger always runs for re-election.

- An incumbent who wins against everybody but $(M, \overline{v})$, *i.e.*, an incumbent such that $v \in \left[\left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi_2(1-V), \left(x^I\right) + \frac{5}{12}\right)$, has an expected utility from running for re-election which is the weighted average of the utility of winning (with probability 3/4) and of the utility of losing against a challenger of the opposite party with a gender-signal pair equal to $(M,\overline{v})$, that is:

$$\begin{aligned}
& \frac{3}{4}v^I+\frac{1}{4}\left(-\left(x^I\right)^2-\frac{1}{3}-x^I+\mathbb{E}\left(v_2^L|M,\overline{v}\right)\right) \\
= & \frac{3}{4}v^I+\frac{1}{4}\left(-\left(x^I\right)^2-\frac{1}{3}-x^I+\frac{3}{4}\right) \\
= & \frac{3}{4}v^I-\frac{1}{4}\left(x^I\right)^2-\frac{1}{4}x^I+\frac{5}{48}.
\end{aligned}$$

Therefore this incumbent will run for re-election if the following holds:

$$\begin{aligned}
\frac{3}{4}v^I-\frac{1}{4}\left(x^I\right)^2-\frac{1}{4}x^I+\frac{5}{48} & \geq -\left(x^I\right)^2+\frac{7}{24}, \\
v^I & \geq \frac{4}{3}\left(-\frac{3}{4}\left(x^I\right)^2+\frac{1}{4}x^I+\frac{7}{24}-\frac{5}{48}\right), \\
v^I & \geq -\left(x^I\right)^2+\frac{1}{3}x^I+\frac{1}{4}.
\end{aligned}$$

Therefore, an incumbent in this region seeks re-election for:

$$v^I\in\left[\max\left\{\left(x^I\right)^2+\frac{5}{12}-\frac{3}{4}\phi(1-V),-\left(x^I\right)^2+\frac{1}{3}x^I+\frac{1}{4}\right\},\left(x^I\right)^2+\frac{5}{12}\right].$$

- An incumbent who wins only against challengers with low signals (of any gender), *i.e.*, an incumbent such that $v\in\left[\left(x^I\right)^2-\frac{1}{12},\left(x^I\right)^2+\frac{5}{12}-\frac{3}{4}\phi_2(1-V)\right)$, has an expected utility from running which is the weighted average of the utility of winning (with weight 1/2) and of the utility of losing against a challenger of the opposite party with a high signal. Recall that politicians are not gender biased, so that the expected valence of a candidate with a high signal is $\frac{3}{4}$ regardless of his/her gender. Hence, in this parametric region, the incumbent's utility from running in period 2 is:

$$\begin{aligned}
& \frac{1}{2}v^I+\frac{1}{2}\left(-\left(x^I\right)^2-\frac{1}{3}-x^I+\frac{3}{4}\right) \\
= & \frac{1}{2}v^I-\frac{1}{2}\left(x^I\right)^2-\frac{1}{2}x^I+\frac{5}{24},
\end{aligned}$$

which is higher than the expected utility of the incumbent who chooses not to run in period 2 if:

$$\begin{aligned}\frac{1}{2}v^I - \frac{1}{2}\left(x^I\right)^2 - \frac{1}{2}x^I + \frac{5}{24} &\geq -\left(x^I\right)^2 + \frac{7}{24}, \\ v^I &\geq 2\left(-\frac{1}{2}\left(x^I\right)^2 + \frac{1}{2}x^I + \frac{1}{12}\right), \\ v^I &\geq -\left(x^I\right)^2 + x^I + \frac{1}{6}.\end{aligned}$$

Therefore, an incumbent in this region seeks re-election for:

$$v^I \in \left[\max\left\{\left(x^I\right)^2 - \frac{1}{12}, -\left(x^I\right)^2 + x^I + \frac{1}{6}\right\}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi(1-V)\right].$$

- An incumbent that gains re-election only against $(F, \underline{v})$, *i.e.*, an incumbent such that $v \in \left[\left(x^I\right)^2 - \frac{1}{12} - \frac{1}{4}\phi_2(1-V), \left(x^I\right)^2 - \frac{1}{12}\right)$, has an expected utility from running which is the weighted average of the utility of winning (with weight 1/4), of the utility of losing against a challenger of the opposite party with a high signal (with weight 1/2), and of the utility of losing against a challenger of the opposite party with a low signal. Hence, in this parametric region, the incumbent's utility from running in period 2 is:

$$\begin{aligned}&\frac{1}{4}v^I + \frac{1}{2}\left(-\left(x^I\right)^2 - \frac{1}{3} - x^I + \frac{3}{4}\right) + \frac{1}{4}\left(-\left(x^I\right)^2 - \frac{1}{3} - x^I + \frac{1}{4}\right) \\ = \;&\frac{1}{4}v^I - \frac{3}{4}\left(x^I\right)^2 - \frac{3}{4}x^I + \frac{3}{16},\end{aligned}$$

which is higher than the expected utility of the incumbent who chooses not to run in period 2 if:

$$\begin{aligned}\frac{1}{4}v^I - \frac{3}{4}\left(x^I\right)^2 - \frac{3}{4}x^I + \frac{3}{16} &\geq -\left(x^I\right)^2 + \frac{7}{24} \\ v^I &\geq 4\left(-\frac{1}{4}\left(x^I\right)^2 + \frac{3}{4}x^I - \frac{3}{16} + \frac{7}{24}\right) \\ v^I &\geq -\left(x^I\right)^2 + 3x^I + \frac{5}{12}.\end{aligned}$$

However, $-\left(x^I\right)^2 + 3x^I + \frac{5}{12} > \left(x^I\right)^2 - \frac{1}{12}$. Hence, there is no pair (x^I, v^I) in this parametric region such that the incumbent finds it optimal to run for re-election.

- Finally, an incumbent who wins against no type of challenger does never find it optimal to seek re-election as by running he ensures that the challenger from the opposite party (which can have a high or low signal with equal probability) wins. His/her expected utility from running is:

$$\begin{aligned} & \frac{1}{2}\left(-\left(x^I\right)^2 - \frac{1}{3} - x^I + \frac{3}{4}\right) + \frac{1}{2}\left(-\left(x^I\right)^2 - \frac{1}{3} - x^I + \frac{1}{4}\right) \\ = & -\left(x^I\right)^2 - x^I + \frac{1}{6} < -\left(x^I\right)^2 \leq -\left(x^I\right)^2 + \frac{7}{24}. \end{aligned}$$

Combining the conditions above, we conclude that the incumbent characterised by a pair (x^I, v^I) chooses to run for re-election if one of the following conditions hold:

1. $v^I \geq \max\left\{-\left(x^I\right)^2 + \frac{1}{3}x^I + \frac{1}{4}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi(1-V)\right\}$;
2. $v^I \in \left[\max\left\{\left(x^I\right)^2 - \frac{1}{12}, -\left(x^I\right)^2 + x^I + \frac{1}{6}\right\}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi(1-V)\right]$.

Results for an incumbent from party L are derived symmetrically.

■

B Proof of Proposition 2

The elected politician's policy choice, and the voting behavior of voters are the ones characterised in Proposition 1. Please refer to the proof of Proposition 1 for their optimality.

Selection of candidates I: open seats elections in the second period. In the second period, parties maximise the probability that their current candidate is elected. Therefore when facing an open seat election, they choose their candidate according to the same ranking that describes the median voter's preferences:

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}).$$

Each party faces two politicians that are randomly drawn from the population, so that each politician is characterised by any specific gender-signal pair with equal probability. As the two politicians are independently drawn, the conditional candidacy probabilities are as follow:

$$\Pr_2\left[\text{cand.}|\sigma_2^k, M\right] = \begin{cases} \frac{7}{8} & \text{if } \sigma_2^k = \overline{v} \\ \frac{3}{8} & \text{if } \sigma_2^k = \underline{v} \end{cases} ; \quad \Pr_2\left[\text{cand.}|\sigma_2^k, F\right] = \begin{cases} \frac{5}{8} & \text{if } \sigma_2^k = \overline{v} \\ \frac{1}{8} & \text{if } \sigma_2^k = \underline{v} \end{cases},$$

so that the candidate is a woman with probability $\frac{3}{8}$. The probability of having a candidate of a specific gender-signal pair is:

$$\Pr_2\left[\left(\sigma_2^k, M\right)|\text{cand.}\right] = \begin{cases} \frac{7}{16} & \text{if } \sigma_2^k = \overline{v} \\ \frac{3}{16} & \text{if } \sigma_2^k = \underline{v} \end{cases} ; \quad \Pr_2\left[\left(\sigma_2^k, F\right)|\text{cand.}\right] = \begin{cases} \frac{5}{16} & \text{if } \sigma_2^k = \overline{v} \\ \frac{1}{16} & \text{if } \sigma_2^k = \underline{v} \end{cases}.$$

The two selected candidates, one for each party, compete against each other in the election. Their probability of winning, conditional on the gender-signal pair and on having been selected as candidates, is:

$$\Pr_2\left[\text{win}|\left(\sigma_2^k, M\right), \text{cand.}\right] = \begin{cases} \frac{25}{32} & \text{if } \sigma_2^k = \overline{v} \\ \frac{5}{32} & \text{if } \sigma_2^k = \underline{v} \end{cases} ; \quad \Pr_2\left[\text{win}|\left(\sigma_2^k, F\right), \text{cand.}\right] = \begin{cases} \frac{13}{32} & \text{if } \sigma_2^k = \overline{v} \\ \frac{1}{32} & \text{if } \sigma_2^k = \underline{v} \end{cases}.$$

As a consequence, the probability of electing a politician with a specific gender-signal pair is:

$$\Pr_2\left[\left(\sigma_2^k, M\right)|\text{win}\right] = \begin{cases} \frac{175}{256} & \text{if } \sigma_2^k = \overline{v} \\ \frac{15}{256} & \text{if } \sigma_2^k = \underline{v} \end{cases} ; \quad \Pr_2\left[\left(\sigma_2^k, F\right)|\text{win}\right] = \begin{cases} \frac{65}{256} & \text{if } \sigma_2^k = \overline{v} \\ \frac{1}{256} & \text{if } \sigma_2^k = \underline{v} \end{cases}.$$

Selection of candidates II: second period election with an incumbent. Consider a second period election in which an incumbent from party k is running for re-election. The opposing party knows (x^I, v^I) and has to choose among two possible candidates. In this case, indifferences may arise. For example, the party is indifferent among each type of politician if $v^I \geq \left(x^I\right)^2 + \frac{5}{12}$, as every type of challenger is worse than the incumbent and loses the elections. Similarly, if $v^I \in \left[\left(x^I\right)^2 - \frac{1}{12}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi_2(1-V)\right]$ the party is indifferent between a female and a male candidate with a high signal as they both win against the incumbent, and between a female and a male candidate with a low signal as they both lose. We assume that, even when indifferent, the party ranks politicians according to the same preference ordering as the median voter, *i.e.*,

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}).$$

We justify this assumption by noticing that this is strictly optimal if there exists the possibility of an unforeseen event that induces party k to replace the incumbent with another candidate (*e.g.*, the death of the incumbent).

Selection of candidates III: first period elections Parties do not observe the true valences of politicians but only the public signal on them. Therefore they cannot anticipate who will have higher chances of running for re-election in the second period, when they compare two politicians with the same signal (but possibly different gender). Therefore, they select candidates by maximising the probability that their candidate wins the first period election, and the problem is equivalent to the selection of candidates for an open seat election in the second period. So, they choose the candidate according to the order

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}) \, .$$

The incumbent's candidacy decision. Consider an incumbent from party R, characterised by $x^I \geq 0$ and v^I. If he/she does not run, voters have the same ex-ante probability of electing a politician from party L or R. Note that, in this case, the expected valence of the elected politician is

$$\begin{aligned}
\mathbb{E}[v_2^P] &= (\Pr_2[(M,\bar{v})|\text{win}] + \Pr_2[(F,\bar{v})|\text{win}])\frac{3}{4} + (\Pr_2[(M,\underline{v})|\text{win}] + \Pr_2[(F,\underline{v})|\text{win}])\frac{1}{4} \\
&= \left(\frac{175}{256} + \frac{65}{256}\right)\frac{3}{4} + \left(\frac{15}{256} + \frac{1}{256}\right)\frac{1}{4} = \frac{23}{32}.
\end{aligned}$$

Therefore the incumbent, when choosing not to run, has a period 2 expected utility as follows:

$$\begin{aligned}
\mathbb{E}[u_2^I] &= -\frac{1}{2}\mathbb{E}[(x^I - x_2^L)^2] - \frac{1}{2}\mathbb{E}[(x^I - x_2^R)^2] + \mathbb{E}\left[v_2^P\right] \\
&= -\left(x^I\right)^2 - \frac{1}{2}\mathbb{E}[\left(x_2^L\right)^2] + x^I\mathbb{E}\left[x_t^L\right] - \frac{1}{2}\mathbb{E}[\left(x_2^L\right)^2] + x^I\mathbb{E}\left[x_t^L\right] + \mathbb{E}\left[v_2^P\right] \\
&= -\left(x^I\right)^2 - \frac{1}{3} + \frac{23}{32} \\
&= -\left(x^I\right)^2 + \frac{37}{96}.
\end{aligned}$$

Recall that, as in the baseline model, politicians are not biased and therefore their expected utility is not affected by the gender of the challenger.

Given the incumbent's expected utility if he/she does not run in the second period, we discuss the optimal re-candidacy choice, which depends on the chances of re-election.

- If elected, the incumbent in period 2 implements his/her bliss point and his/her utility is equal to v^I. Therefore, an incumbent who wins against any type of challenger (*i.e.*, such that $v^I \geq \left(x^I\right)^2 + \frac{5}{12}$) runs for re-election in period 2 if $v^I \geq -\left(x^I\right) + \frac{37}{96}$. As the first condition implies the second one, this happens for every value of $\left(x^I, v^I\right)$.

- An incumbent who wins against everybody but $(M, \overline{v})$, *i.e.*, an incumbent such that $v^I \in \left[\left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi_2(1-V), \left(x^I\right)^2 + \frac{5}{12}\right)$, has an expected utility from running for re-election which is the weighted average of the utility of winning (with probability 9/16) and of the utility of losing against a challenger of the opposite party with a gender-signal pair equal to $(M, \overline{v})$, that is:

$$\begin{aligned} & \frac{9}{16}v^I + \frac{7}{16}\left(-\left(x^I\right)^2 - \frac{1}{3} - \frac{2}{3}x^I + \mathbb{E}\left[v_2^L|M,\overline{v}\right]\right) \\ = \; & \frac{9}{16}v^I + \frac{7}{16}\left(-\left(x^I\right)^2 - \frac{1}{3} - \frac{2}{3}x^I + \frac{3}{4}\right) \\ = \; & \frac{9}{16}v^I - \frac{7}{16}\left(x^I\right)^2 - \frac{14}{48}x^I + \frac{35}{192}. \end{aligned}$$

  Note that the incumbent finds it optimal to run for re-candidacy when

$$\frac{9}{16}v^I - \frac{7}{16}\left(x^I\right)^2 - \frac{14}{48}x^I + \frac{35}{192} \geq -\left(x^I\right)^2 + \frac{37}{96},$$

  that is, when $v^I \geq -\left(x^I\right)^2 + \frac{14}{27}x^I + \frac{39}{108}$.

- An incumbent who wins only against challengers with a low signal (of any gender), *i.e.*, an incumbent such that $v^I \in \left[\left(x^I\right)^2 - \frac{1}{12}, \left(x^I\right)^2 + \frac{5}{12} - \frac{3}{4}\phi_2(1-V)\right)$, has an expected utility from running which is the weighted average of the utility of winning (with weight 1/4) and of the utility of losing against a challenger of the opposite party with a high signal. Recall that politicians are not gender biased, so that the expected valence of a candidate with a high signal is $\frac{3}{4}$ regardless of his/her gender.

Hence, in this parametric region, the incumbent's utility from running in period 2 is:

$$\begin{aligned} & \frac{1}{4}v^I + \frac{3}{4}\left(-\left(x^I\right)^2 - \frac{1}{3} - \frac{2}{3}x^I + \frac{3}{4}\right) \\ = & \frac{1}{4}v^I - \frac{3}{4}\left(x^I\right)^2 - \frac{1}{2}x + \frac{5}{16}, \end{aligned}$$

which is higher than the expected utility of the incumbent who chooses not to run in period 2 if:

$$v^I \geq -\left(x^I\right)^2 + 2x^I + \frac{7}{24}.$$

- An incumbent that gains re-election only against $(F, \underline{v})$, *i.e.*, an incumbent such that $v^I \in \left[\left(x^I\right)^2 - \frac{1}{12} - \frac{1}{4}\phi_2(1-V), \left(x^I\right)^2 - \frac{1}{12}\right)$, has an expected utility from running which is the weighted average of the utility of winning (with weight 1/16), of the utility of losing against a challenger of the opposite party with a high signal (with weight 3/4), and of the utility of losing against a challenger of the opposite party with a low signal. Hence, in this parametric region, the incumbent's utility from running in period 2 is:

$$\begin{aligned} & \frac{1}{16}v^I + \frac{3}{4}\left(-\left(x^I\right)^2 - \frac{1}{3} - \frac{2}{3}x^I + \frac{3}{4}\right) + \frac{3}{16}\left(-\left(x^I\right)^2 - \frac{1}{3} - \frac{2}{3}x^I + \frac{1}{4}\right) \\ = & \frac{1}{16}v^I - \frac{15}{16}\left(x^I\right)^2 - \frac{5}{8}x^I + \frac{21}{64}, \end{aligned}$$

which is higher than the expected utility of the incumbent who chooses not to run in period 2 if:

$$v^I \geq -\left(x^I\right)^2 + 10x^I + \frac{11}{12}.$$

Given that $-\left(x^I\right)^2 + 10x^I + \frac{11}{12} > -\left(x^I\right)^2 - \frac{1}{12}$ for every $x^I \in [-1, 1]$ the above inequality is never satisfied, so that such an incumbent never chooses to run for re-election.

- Finally, an incumbent who wins against no type of challenger does never find it optimal to seek re-election as by running he ensures that the challenger from the opposite party (which can have a high or low signal with equal probability) wins.

His/her expected utility from running is:

$$\begin{aligned} & \frac{3}{4}\left(-\left(x^I\right)^2-\frac{1}{3}-\frac{2}{3}x^I+\frac{3}{4}\right)+\frac{1}{4}\left(-\left(x^I\right)^2-\frac{1}{3}-\frac{2}{3}x^I+\frac{1}{4}\right) \\ = & -\left(x^I\right)^2-\frac{2}{3}x^I+\frac{1}{3}<-\left(x^I\right)^2+\frac{37}{96}. \end{aligned}$$

Combining the conditions above, we conclude that the incumbent characterised by a pair (x^I, v^I) chooses to run for re-election if one of the following conditions hold:

1. $v^I \geq \max\left\{-\left(x^I\right)^2+\frac{14}{27}x^I+\frac{39}{108}, \left(x^I\right)^2+\frac{5}{12}-\frac{3}{4}\phi(1-V)\right\}$;
2. $v^I \in \left[\max\left\{\left(x^I\right)^2-\frac{1}{12}, -\left(x^I\right)^2+2x^I+\frac{7}{24}\right\}, \left(x^I\right)^2+\frac{5}{12}-\frac{3}{4}\phi(1-V)\right]$.

Results for an incumbent from party L are derived symmetrically.

■