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# **Bootstrapping the Gini Index of the Network Degree: An Application for Italian Corporate Governance**

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## **Abstract**

We propose a new approach based on bootstrapping to compare complex networks. This is an important task when we wish to compare the effect of a (policy) shock on the structure of a network. The bootstrap test compares two values of the Gini index, and the test is performed on the difference between them. The application is based on the interlocking directorship network. At the director level, Italian corporate governance is characterized by the widespread occurrence of interlocking directorates. Article 36 of Law 214/2011 prohibited interlocking directorates in the financial sector. We compare the interlocking directorship networks in 2009 (before the reform) with 2012 (after the reform) and find evidence of an asymmetric effect of the reform on the network centrality of the different companies but no significant effects on Gini indices.

**Keywords:** Complex Networks, Gini Index, Bootstrap Method

**JEL codes:** C13, G34

## 1 Introduction

During the past decades, many scholars have come up with theories to explain the presence of interlocking directorates (board members that simultaneously sit on more than one board: for a review of the literature see [10] and [14]). From an economic standpoint, interlocking directorates are important because they can increase collusion among different companies whose directors sit on their respective boards, reducing consumer welfare. The effectiveness of ‘busy’ board members sitting on several boards may also diminish, with less ability to check the chief executive officer’s decisions, exposing companies to high risks [16]. Network analysis has been applied several times to the analysis of Italian corporate governance and ownership ([1], [2], [8], [3], [22], [26]).

The chapter is organized as follows. In section 2 we introduce our approach with simulations performed in section 3. Section 4 presents our application, analyzing the effect of a reform banning interlocking directorates in banks and insurance companies in Italy, to see whether this led to significant changes in the structure of the network after the reform. Section 5 concludes.

## 2 The Approach

Given a network  $G = (V, E)$  where  $V$  indicates the vertices of the network and  $E$  the edges, for each node we calculate the degree [5]:

$$CD(v) = deg(v) \quad (1)$$

The degree is the number of nodes that are their neighbors. In this way, each node is characterized by its local measure of centrality [25]. Then for each network, we compute the Gini inequality index [17]:

$$G_k = \frac{1}{n-1} \left( n + 1 - 2 \left( \frac{\sum_{j=1}^n (n+1-i)x_j}{\sum_{j=1}^n y_j} \right) \right) \quad (2)$$

Where  $n$  is the number of nodes,  $x$  is the degree that characterizes the  $j^{\text{th}}$  node. We have perfect equality if the Gini index is equal to 0, which means that all the degrees show the same value. For each network  $k$  we obtain different values for the Gini index.

The bootstrap test compares two Gini indices,  $G_1$  and  $G_2$ , and tests the statistical significance of the difference  $D = G_1 - G_2$  [21, 7]). It is important to note that we can bootstrap the distribution of  $G_1$  or  $G_2$  similarly to bootstrapping the distribution of  $D$  [21]. Following [21], it is important to note that the bootstrap distribution  $\hat{F}(D)$  allows us to obtain the values for the hypothesis testing on  $D$ . With the bootstrap hypothesis testing method [12] we draw samples from the original data with replacement from data to obtain the bootstrap sample. Then we approximate the distribution of  $D$  by the bootstrap distribution of  $D^*$ . Finally, from the sampling distribution of  $D$  by obtaining their bootstrap estimate hypothesis testing [21, 13] can be carried out. Finally, the results are used to test whether the change in inequality in degree can be considered statistically significant. The code is written in Stata [4, 20] and in R [9].

### 3 Simulations

To try out the methodology, we simulated some networks on which we performed the algorithm to observe the changes that occur when impacted by structural changes.<sup>1</sup> We consider different networks and scenarios to test whether significant changes over time can be identified. The simulation is performed by randomly generating six networks and then executing the bootstrap test. In each test, we use two network topologies with the same number of nodes. In the first set, we start by considering two extreme cases: the first with a highly centralized network structure (a star) versus a structure evolved in an Erdős Renyi model [15] and the second a typically centralized (the Barabasi Albert model [6]) and a non-centralized network (the Erdős Renyi model).

We then provide further experiments by increasing the number of nodes/edges to construct more complex structures that may react differently to a shock. In this way, a more challenging environment is created for the null hypothesis. The second batch of simulations is based on the evolution of a network over time in which the deletion of some edges is

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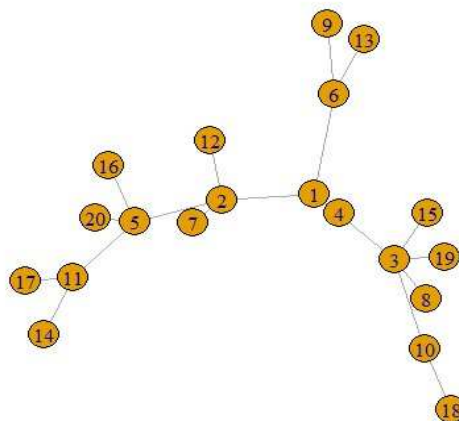
<sup>1</sup> In the simulations we have considered very general networks in which there are no effects caused by preferential attachment, triadic closure, clustering tendencies, constraints on the degree distribution due to transaction costs, which may characterize economic networks. The different additions/destructions of the links occur randomly. In the future, we will consider more complex cases that consider these features. These more complex structures may lead to complex reactions to the addition/destruction of the different nodes. In particular, the process of addition/destruction may be quicker or slower with the implication that the test may be more or less likely to reject the null hypothesis.

simulated, followed by the addition of other edges in the second round. More precisely, we start from a Barabasi Albert Model and then randomly delete 4 edges in the first round and add 12 edges in the second. The initial network for both simulations is represented in Figure 1. In the second experiment, we consider a higher additional number of edges by randomly removing and adding edges of the first network evolution simulation, i.e. 4 edges are deleted and 20 added.

We consider all possible connections on the nodes. The adjacency matrix is related to all possible connections which can occur in the network. Addition can occur randomly at each theoretically plausible connection and if the edge already exists, it is kept, otherwise, a new link is added. Therefore, the edge is added only where the connection is non-existent.

The final networks are shown in figure 2 (simulation 1) and figure 3 for simulation 2. It is interesting to note that very different network structures can be obtained quickly with growth in the mechanism of addition or deletion of the single edges.

To simulate the networks, and for the other estimates, the package igraph on R [9, 19] was used. We simulate the network and compute the degree for each node, then calculate the Gini index for both networks.



**Fig.1.** Evolutive network simulations 1 and 2: initial network

Finally, we consider the third batch of simulations. We simulate the destruction (first case) and creation (second case) of a random number of edges between 100 and 1. We plot the initial networks (figures 4 and 6) and their final state after the destruction (figure 5) and the construction (figure 7) of new edges.

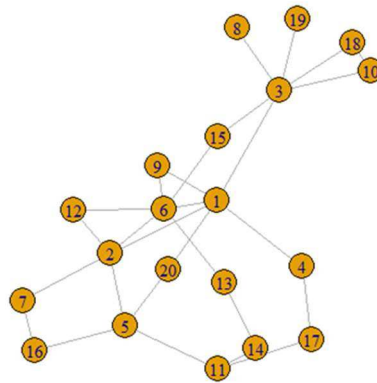
In the simulation considered (Table 1), the null hypothesis of a similar structure can be rejected in the first and simplest case but not in the second. In other words, in the first case

we found a significant effect on the Gini index of the degree of the network, but not in the second case. In the two more complex simulations, the null hypothesis in Network evolution 1 cannot be rejected, whereas it may be slightly rejected in Network evolution 2. Finally, we are unable to reject the null hypothesis for network edge deletion, but we can reject the null for network edge creation.

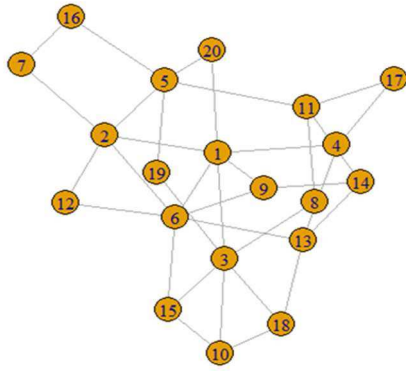
**Table 1.** Simulation results

Coef.	Std. Err.	Z	P> z
Centralized Structure (Star Model) versus Erdős-Renyi Model			
-0.19101	0.04908	-3.89	0.000
Model with similar structures (Barabasi-Albert Models)			
-0.02894	0.08562	-0.34	0.735
Network evolution (1)			
-0.06268	0.05898	1.06	0.288
Network evolution (2)			
-0.11300	0.04575	-2.47	0.014
Network edge deletion			
0.07047	0.10186	0.69	0.489
Network edge creation			
-0.23972	0.07925	-3.02	0.002

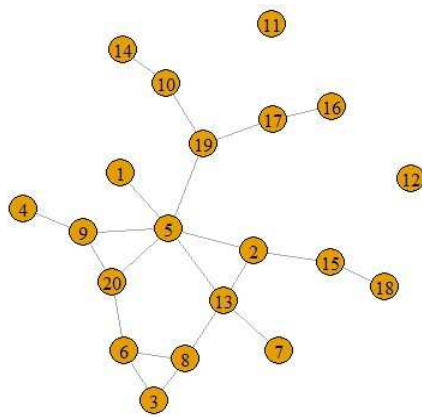
Number of observations: 400, replications 2000.



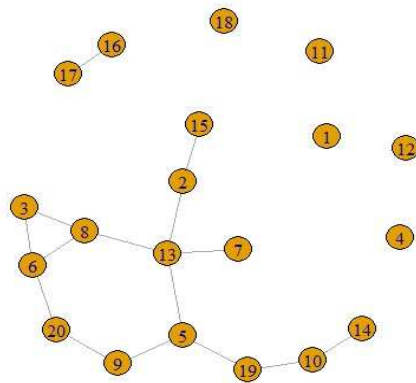
**Fig. 2.** Evolutive network 1 end of the simulation



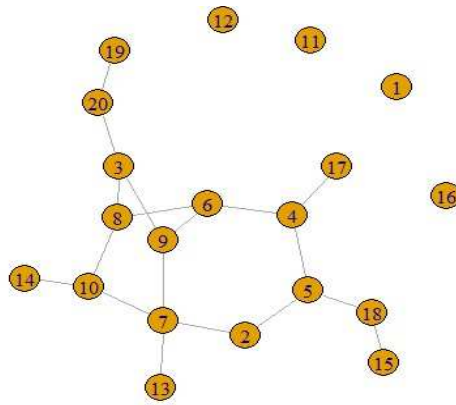
**Fig. 3.** Evolutive network 2 end of the simulation



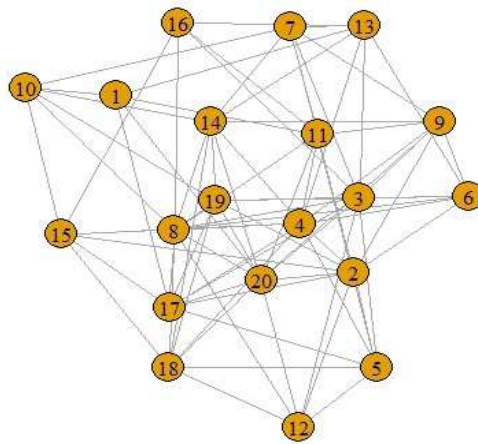
**Fig. 4.** Network edge deletion, the initial state



**Fig. 5.** Network edge deletion after a simulated shock



**Fig. 6.** Network edge creation, the initial state



**Fig. 7.** Network edge creation after a simulated shock

#### 4 Application

The application is based on the interlocking directorates network. Italian corporate governance features a high concentration of ownership and the presence of control-enhancing mechanisms that are conducive to controlling shareholders' dominance and exploitation of minorities. At the director level, corporate governance is characterized by the widespread recourse to interlocking directorates (directors sitting on more than one board at the same time, hereafter referred to as ID). Through cross-ownerships, circular ownerships and interlocking directorates, the Italian system has been characterized by pyramidal groups headed by a small number of families that permanently control the firms. A few reforms have been implemented over the last 15 years to open up the market for corporate control, reduce the scope for collusion, and to protect minorities from exploitation by controlling shareholders able to extract private benefits at the expense of the minority. The latest addition to this wave of reforms was a new measure introduced in 2011, article 36 banning interlocking, part of the Save Italy Decree which started life as

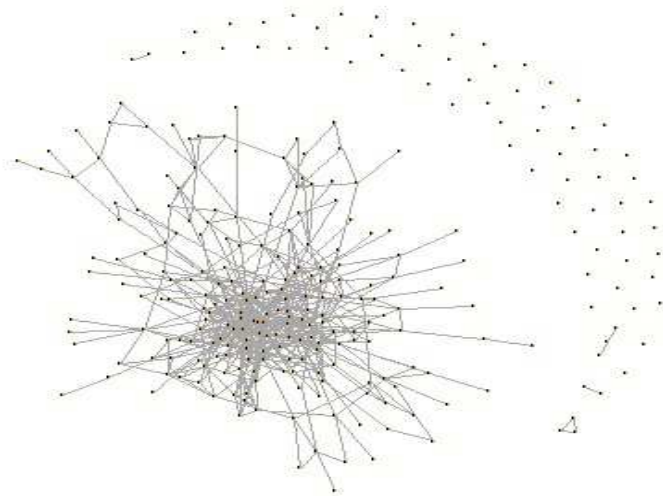


Legal Decree 201/2011, published in Official Gazette on December 6, 2011. This decree was converted into law 214 with amendments in 2011 and published in the Official Gazette on December 27, 2011. Under article 36 paragraph 2b the requirement of 120 days to comply with the law ran from December 27, 2011. Therefore, the director of a bank or insurance company with incompatible appointments was required to choose one of the two (or more) positions by April 27, 2012, and failing this, would lose all the positions. The effects of the Law were in place when the data for our study were collected (December 31, 2012), hence it is legitimate to compare 2012 with 2009 to see if the provision was effective in reducing ID in the financial sector. The reform aimed to break the ties between the sectors, increasing competition between financial companies. If this were true, we would observe a sparser network after the reform, with a significantly lower concentration in the Gini Index and more communities. Data were collected from listed companies in light of the Board of Directors for each firm on 31/12. Only the management board is considered for the few companies with the two-tier system. We used publicly available data from Consob (the Italian stock market regulator) and to collect the network data, we considered individual names and the related company and then created the two-way matrix, from which we were able to perform the one mode projection to obtain the adjacency matrices both for the network of directors and for the network of companies. A weighting represents the number of directors shared by connected companies.<sup>2</sup> Nodes represent companies. The isolates are companies that do not share any director with other companies.

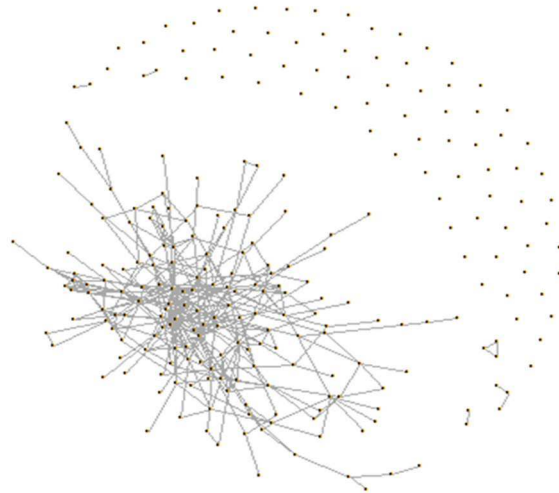
Figures 8 and 9 show the networks in the two years of interest. Visually comparing the structures of the networks, we find a group of nodes connected with each other on a first component and several isolates. Table 2 reports the descriptive statistics for the two networks.

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<sup>2</sup> Working with the projected two-mode would make it possible to analyze the inequality in degree both on the company and the director side.



**Fig. 8.** The network in 2009



**Fig. 9.** The network in 2012

**Table 2.** Descriptive statistics

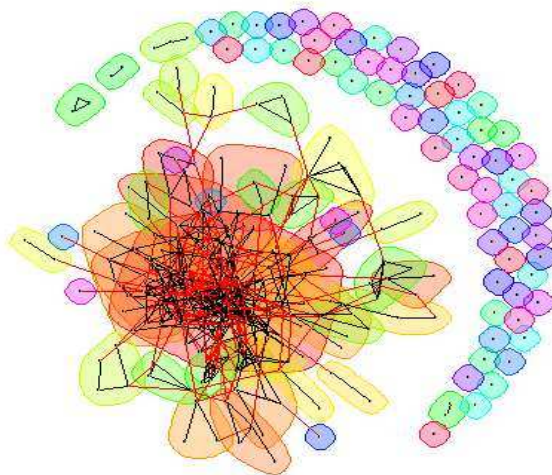
	Network 2009	Network 2012
Nodes	278	251
Edges	576	387
Density	0.01496	0.012335
Islands\clusters	66	80
Global cluster coefficient	0.252179	0.260765
Diameter	10	15
Betweenness (mean)	225.4018	162.8853
Degree (mean)	4.143885	3.083665

We then perform a community detection analysis to decompose the networks in communities. We use the walktrap community detection approach [23] because it can

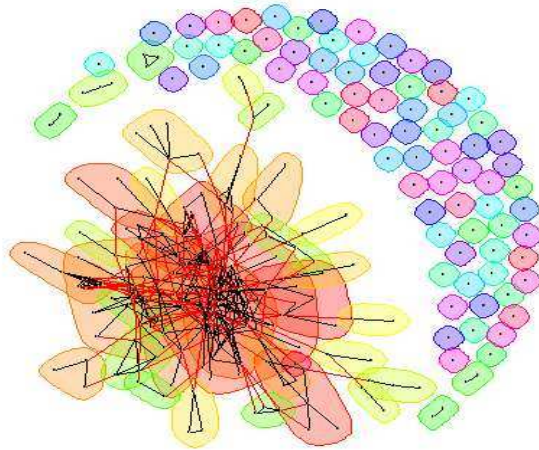
detect and separate the different communities in this context. Walktrap is an appealing approach in our context because the random walks can be related to the dynamics of the information in the network.

The first community detection relating to the year 2009 (figure 10) shows 105 communities. Communities vary in size from 49 for the largest group to 1 for the isolates. The result shows that there are different groups of nodes strongly connected to each other. The communities tend to connect weakly compared to dense intra-community networks, and the network is utterly dissimilar to a random graph. Hence the expectation that the nodes have a different number of connections and different centrality in the same network. This result is particularly important because it confirms the need to consider the Gini index analysis to investigate the structure of the distribution of the degree over time.

Regarding the community structure of the network in 2012 (figure 11), the number of communities increases to 111. There is greater fragmentation of the communities, with the largest community including 32 nodes against 1 for the isolates.

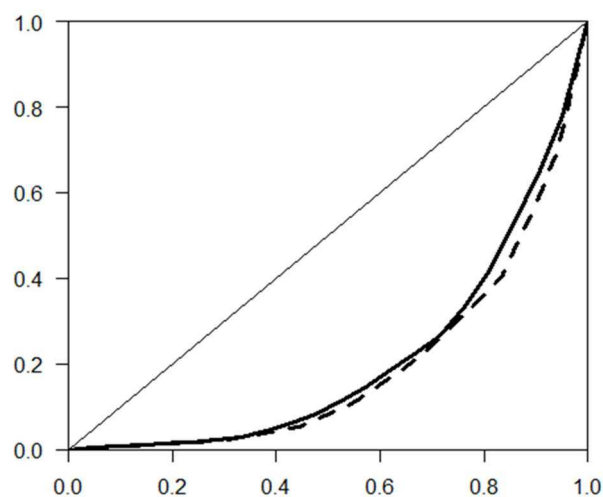


**Fig. 10.** Network in 2009: community structure



**Fig. 11.** Network in 2012: community structure

The heterogeneity in the distribution of the links between the nodes tends to decrease from 2009 to 2012 regarding the degree, for the first network it is in a range of 1 to 34, whereas in 2012 the range is from 1 to 21. In addition, the median and the mean are higher in 2009 (4.00 and 5.144, respectively) than in 2012 (3.00 and 4.084, respectively). Before the reform, the average value of the degree for each node is higher, possibly related to a different structure in the two networks. Finally, the variance of the degree for 2009 and 2012 moves from 21.71 to 13.43. Given the observed reduction in the heterogeneity of the degree, this may have an impact on its distribution, and this calls for an analysis of the Gini index of the degree in the two years.



**Fig. 12.** Lorenz Curves (black line for 2009, dotted line for 2012)

The coefficient observed is 0.02, the bootstrap standard error is 0.02575, with 2000 replications in the bootstrap process. Therefore, we detect no statistical significance, since we cannot reject the null hypothesis at 5%. In this case, we have included only the nodes which show a positive degree because we considered only nodes with at least a linkage, neglecting isolates. If we consider all the nodes, isolates and non-isolates, and repeat the analysis we obtain a Gini index for the year 2009 of 0.55 and for 2012 of 0.60. These results are interesting because the deletion of some links due to the reforms has created an increase in the Gini index. The computed coefficient is 0.04151 with the bootstrap standard error equal to 0.02881. In this case too, we cannot reject the null hypothesis at the 5% significance level. We also plot the results obtained for the Lorenz Curves (figure 12) comparing the results for the year 2009 (black line) and for 2012 (dotted line). Overall, no strong effect of the reform on the network can be found (as expected; see [8, 9]) whereas an asymmetric effect on the distribution of the edges (by considering both connected nodes and isolated) is evident. The asymmetric effect is due to the presence of some nodes that became more central in a local sense, because of edge deletion. Interestingly, this potentially shows some unintended effects of the Law: by deleting some edges, some nodes became even more important than before (see [24] showing this effect in the period 1998-2006 for S&P MIB financial companies).

This result is similar to [11], where community detection techniques for the analysis of the networks in 2009 and 2012 ascertained the effect of the reform on the network of Italian directorates. They find that, although the number of interlocking directorates decreases in 2012, the reduction takes place mainly at the periphery of the network. The result is due to the fact the creation/deletion process fails to activate the “structural change threshold”. The activation is probably due when the structure of the network is designed to be structurally changed so the inequality tends to grow significantly or not (about the network flow optimization problem see [18]).

## **5 Conclusions**

This chapter presents a new method for the detection of statistically significant changes in the network structure by a bootstrap test of the degree. The approach appears to be very useful in analyzing the impact of exogenous shocks on a network and in detecting impacts on the network structure. In our case, the shock was a statutory amendment aimed at cutting the links between banking and insurance companies in terms of directors sitting on more than one board. The methodology can be extended to other node-level statistics, such as

closeness, betweenness and so on. It is also possible to think about the centrality not only considering single nodes but also groups of nodes (entire communities). The advantage of this approach is the ability to observe the relative significance of shocks which can occur in a network system over time and space. The approach is also promising because it can be applied to other network structural measures such as betweenness. Possible problems can be identified in the fact that bootstrapping is not robust to outliers.

Our results show the limited effects of this legislative measure on the network of companies, possibly suggesting that a more far-reaching intervention was needed to achieve the desired outcome. Another possible reason could be that interlocking directorates are a symptom of cross-shareholding and therefore regulation aimed at breaking these networks should address the former rather than the latter. Future research should try to understand the ultimate causes of the intertwining of listed Italian companies.

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