UNIVERSITÀ di VERONA | Department of ECONOMICS

Working Paper Series
Department of Economics
University of Verona

Electoral competition with strategic voters

Claudia Meroni

WP Number: 7 May 2017

ISSN: 2036-2919 (paper), 2036-4679 (online)

ELECTORAL COMPETITION WITH STRATEGIC VOTERS

CLAUDIA MERONI[†]

ABSTRACT. A recent literature has found a positive relationship between the disproportionality of the electoral system and the convergence of parties' positions. Such a relationship depends crucially on the assumption that voting is sincere. We show that, when voters are players in the game and not simply automatons that vote for their favorite party, two policy-motivated parties always take extreme positions in equilibrium.

KEY WORDS. Voting theory, strategic voting, electoral competition, power sharing.

JEL CLASSIFICATION. C72, D72.

1. INTRODUCTION

The policies announced before an election is called are crucial to determine the electoral outcome and the resulting policy outcome. How voters are expected to behave and how the electoral outcome translates into actual political power can influence parties' positioning choice. In particular, the mapping from the distribution of votes to the distribution of power varies according to the electoral system. For instance, under majoritarian rule the winning party takes all the political power, while under proportional representation each party's power share coincides with her share of votes. In this paper we analyze the electoral competition between two policy-motivated parties when they expect voters to vote strategically and under multiple power-sharing rules.

The political process consists of three stages. First, a leftist and a rightist party choose a policy they commit to, then voters vote, and finally the policy outcome is implemented. The outcome function is a weighted average of parties' positions, where weights are given by the corresponding power shares.[1] Specifically, we follow Herrera et al. (2014) and employ the "contest success function" (Tullock, 1980), whose parameter allows parties' weights to span continuously across different electoral systems (see also Saporiti, 2014; Herrera et al., 2015;

[†] DEPARTMENT OF ECONOMICS, UNIVERSITY OF VERONA, VERONA, ITALY.

Email addresses: claudia.meroni@univr.it.

Date: May 5, 2017.

[1] This kind of outcome function is used also in, e.g., Ortuño-Ortín (1997); Lizzeri and Persico (2001); Llavador (2006); De Sinopoli and Iannantuoni (2007); Merrill and Adams (2007).

Matakos et al., 2016). In fact, its two extreme values correspond to the purely proportional and to the majoritarian systems, in which, respectively, the degree of power sharing is maximum and null. The intermediate values correspond to power-sharing rules that are in between those two extremes.

The form of the outcome function implies that policy-motivated parties potentially face a trade-off. The more moderate is the policy that they choose, the higher is their share of votes and their relative political power, but such a weight will be assigned to a less extreme position. Since the power share depends in turn on the electoral system, one may expect this to influence the net effect of the two forces. As the proportionality decreases, indeed, the incentive to obtain more votes than the opponent by proposing a more moderate policy intensifies, because the vote share translates into a higher power share.

Some recent papers have explored such a conjecture and found results in this direction.[2] Saporiti (2014) considers two parties who have mixed motives and shows that power sharing and ideology favor the centrifugal force increasing party polarization. Matakos et al. (2016) assume policy-motivated parties and find that the degree of platform polarization increases in the level of electoral rule proportionality and in the number of competing parties. Both studies assume that voters vote sincerely, that is, they just vote for their favorite party.

This work is the first study of party electoral competition under *strategic voting*; that is, voters are players in the game. As Matakos et al. (2016) we assume that parties are policy-motivated, but we show that for any positive degree of power sharing (i.e., for all the electoral rules except the majoritarian one) the unique equilibrium of the game is characterized by platform polarization. In fact, parties always choose extreme positions in equilibrium. Only in the limit case of the winner-take-all, parties' positions can converge. The disproportionality of the electoral system does not favor policy convergence precisely because the equilibrium outcome is always strictly increasing in each party's proposed platform, that is, the more rightist is a party's platform the more rightist is the policy outcome. A consequence is that the centrifugal force is always dominant for a policy-oriented party when there is some power sharing. This suggests that the aforementioned results in favor of a positive relationship between the electoral rule disproportionality and the convergence of parties' positions depend crucially on the assumption that voting is sincere.

[2] Previous studies considered party positioning in the two extreme cases of majoritarian and purely proportional rule. For the first case, we refer to the pioneering works of Hotelling (1929), Downs (1957), Ledyard (1981, 1984), and to Wittman (1977), Calvert (1985), Roemer (1994). For the second one, see Ortuño-Ortín (1997).

To characterize voters' strategic behavior we borrow the results of De Sinopoli and Iannantuoni (2007), who analyze it under purely proportional rule and multiple parties. They show that, as the number of voters goes to infinity, in equilibrium basically voters split in two and only the two extremist parties take votes. This result guarantees an unambiguous interpretation of strategic voting in a game with a continuum of voters in which, in principle, nobody can affect the outcome. In fact, such a game can be seen as limit of finite games and, therefore, voters' strategic behavior is precisely identified by the limit of their equilibria.

The paper is organized as follows. The basic spatial model is described in the next section. In Section 3 we analyze the voting subgame, while in Section 4 we study the electoral stage.

2. Preliminaries

Let the one-dimensional *policy space* be represented by the closed interval $\mathbb{X} = [0,1]$. Parties L and R announce simultaneously their platforms $x_L, x_R \in \mathbb{X}$, to which they are committed. Knowing these positions, every voter votes for a party, and given the electoral outcome a policy is implemented. Each party j is policy-motivated, that is, she has single-peaked preferences characterized by an ideal policy $\theta_j \in \mathbb{X}$, with $\theta_L < \theta_R$.

The implemented policy, or *policy outcome*, is a function of parties' platforms and vote shares, and of the degree of power sharing. This is captured by the parameter of a standard contest success function (Tullock, 1980), which allows to embed different electoral systems ranging from proportional representation to majoritarian rule. Let v be the vote share of party L. The policy outcome $\hat{x}$ is given by

$$\hat{x}_\gamma(x_L, x_R) = \frac{v^\gamma}{v^\gamma + (1-v)^\gamma} x_L + \frac{(1-v)^\gamma}{v^\gamma + (1-v)^\gamma} x_R, \tag{2.1}$$

where $\gamma \geq 1$. In a purely proportional system the weight of each party's policy is the corresponding share of votes, i.e. $\gamma = 1$. As γ increases, the weight of the party who gets the majority of the votes increases, up to the limit case ($\gamma \to \infty$) in which she takes all the power and her proposed policy is implemented. Thus, higher values of γ correspond to lower degrees of power sharing. Figure 1 represents a party's power share as a function of her share of votes for the values $\gamma = 1$ (thick line), $\gamma = 2$ (thin line), $\gamma = 10$ (dashed line), and $\gamma \to \infty$ (dotted line).

The electorate consists of a continuum of voters of unit mass, whose bliss points are distributed over $[0,1]$ according to the distribution function F (i.e.,

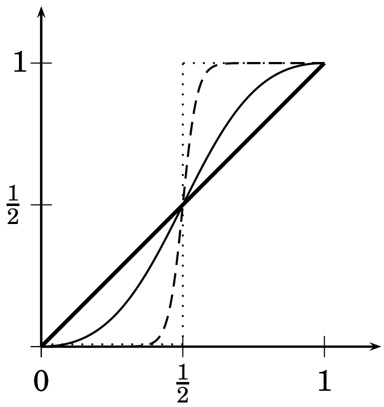

FIGURE 1. A party's power share as a function of her vote share for different values of γ.

$F(x)$ is the fraction of voters whose ideal policy is at most $x \in \mathbb{X}$).[3] We assume that F is continuously differentiable and strictly increasing. Each voter i has single-peaked preferences with bliss point $\theta_i \in \mathbb{X}$, which can be represented by a real-valued utility function $u(\hat{x}, \theta_i)$ that is continuously differentiable in $\hat{x}$.[4]

The whole game is a three-stage process and we solve it by backward induction. The last stage consists simply in the implementation of the policy outcome according to 2.1.

3. THE VOTING SUBGAME

Let us analyze the second stage in which voters, having observed the announced platforms x_L and x_R, vote strategically for one of the two parties. To this end, we can extend to our framework the results in De Sinopoli and Iannantuoni (2007), who study this kind of game under proportional rule and with multiple parties assuming that their positions are exogenous.

Consider two announced policies $x_A < x_B$ and a finite value of γ. Let the *cutpoint outcome* x^* be the unique policy implemented when all the voters to its left vote for party A and all the voters to its right vote for party B, i.e., the unique solution of

$$x^* = \frac{F(x^*)^\gamma}{F(x^*)^\gamma + (1-F(x^*))^\gamma} x_A + \frac{(1-F(x^*))^\gamma}{F(x^*)^\gamma + (1-F(x^*))^\gamma} x_B.$$

De Sinopoli and Iannantuoni (2007) prove that, for $\gamma = 1$, x^* is the unique equilibrium outcome of the game with a continuum of voters when this is seen as limit of games with a finite electorate, where voters are strategic.[5] In any mixed-strategy equilibrium of finite games, indeed, all the voters to the left of

[3] A continuum of voters is assumed also in Saporiti (2014) and Matakos et al. (2016).

[4] These assumptions are those needed in De Sinopoli and Iannantuoni (2007).

[5] We talk about the *unique* equilibrium of a game with a continuum of players with slight abuse of terminology.

the cutpoint outcome vote for party A and all the voters to its right vote for party B, except for a neighborhood that shrinks as the number of voters increases. They also remark that this result remains true for any continuous and monotonic transformation of the purely-proportional outcome function. In fact, the function needs just to be continuous in the parties' vote shares and satisfy an ordinal assumption that, in the two-party case, reduces to strict increasingness in the rightist party's vote share. It is easy to see that our outcome function satisfies these assumptions.

Thus, take a sequence of finite games as the number of voters goes to infinity, whose associated sequence of bliss point distributions converges to the function F. Each game in this sequence has two particular voters, the rightmost one such that, in any equilibrium, she and all the voters to her left vote for party A, and the leftmost one such that, in any equilibrium, she and all the voters to her right vote for party B. The assumptions on the distribution function imply that the two corresponding sequences of these voters' bliss points converge to the same limit point, which is precisely the cutpoint outcome associated to F. This allows to fully characterize strategic voting in the game with a continuum of voters. Therefore, we can conclude that x^* is the unique equilibrium outcome of the voting subgame whenever parties' announced policies differ. Obviously, when the two policies are equal, every strategy profile is a Nash equilibrium also for a finite electorate, but the equilibrium outcome is unique and coincides with them.

4. THE ELECTORAL STAGE

We study now the two-party electoral competition. An immediate consequence of the assumption that $\theta_L < \theta_R$ and of the form of the outcome function is that, in every equilibrium, $x_L < x_R$. Otherwise, at least one party has the incentive to deviate from her proposed platform, in order to induce an outcome closer to her favorite one.

We can characterize the equilibrium for any fixed (and finite) value of γ. First, we show that the equilibrium outcome is strictly increasing in both parties' proposed platforms, independently of the (non-null) degree of power sharing. That is, by offering a less extreme policy, the higher power share that a party gets does never balance the loss from assigning this weight to a more moderate position, and the implemented policy is always moved in the direction of the change. The implication for the equilibrium of the game is that parties' positions will always be extreme.

Given the degree of power sharing γ, let $x^*_\gamma(0,1)$ be the equilibrium outcome of the voting subgame when party L's proposed platform is 0 and party R's proposed platform is 1.

Proposition 1. *If* $\theta_L < x^*_\gamma(0,1) < \theta_R$*, then* $(x_L, x_R) = (0,1)$ *is the unique Nash equilibrium. If* $x^*_\gamma(0,1) \le \theta_L < \theta_R$*, then the unique equilibrium is* $(x_L, x_R) = (\tilde{x}_L, 1)$ *with* $\tilde{x}_L$ *such that* $x^*_\gamma(\tilde{x}_L, 1) = \theta_L$*. If* $\theta_L < \theta_R \le x^*_\gamma(0,1)$*, then the unique equilibrium is* $(x_L, x_R) = (0, \tilde{x}_R)$ *with* $\tilde{x}_R$ *such that* $x^*_\gamma(0, \tilde{x}_R) = \theta_R$*.*

Proof. We first prove that, for every γ and $x_L < x_R$, x^* is strictly increasing in x_L and x_R. Let $P(x) = \frac{F(x)^\gamma}{F(x)^\gamma + (1-F(x))^\gamma}$. We have $x^* = P(x^*)x_L + (1 - P(x^*))x_R$. Taking the partial derivatives with respect to x_L and x_R of both sides of this expression and rearranging, we have

$$\frac{\partial x^*}{\partial x_L} = \frac{P(x^*)}{1 + P'(x^*)(x_R - x_L)},$$

and

$$\frac{\partial x^*}{\partial x_R} = \frac{1 - P(x^*)}{1 + P'(x^*)(x_R - x_L)}.$$

When $x^* \notin \{0,1\}$ and for every γ, both expressions are strictly positive, since $P(\cdot)$ is strictly increasing and $x_R - x_L > 0$.

Now, let (x_L, x_R) be an equilibrium such that $\theta_L < x^*_\gamma(x_L, x_R) < \theta_R$. Obviously, party L would prefer the outcome to be more to the left, while party R would prefer it to be more to the right. Given the previous result, the only equilibrium of this kind is necessarily $(0,1)$.

Then, let $x^*_\gamma(0,1) \le \theta_L < \theta_R$ (an analogous and symmetric argument applies to prove the last case). Clearly, $(\tilde{x}_L, 1)$ such that $x^*_\gamma(\tilde{x}_L, 1) = \theta_L$ is a Nash equilibrium, as party L gets her ideal policy implemented and party R cannot induce a more rightward outcome.[6] To see that this equilibrium is unique, note first that if party R's position is 1 then party L has a unique best reply. Thus, suppose that there exists another equilibrium $(\bar{x}_L, \bar{x}_R)$ with $\bar{x}_R \neq 1$. By the above result, this can be the case only if $x^*_\gamma(\bar{x}_L, \bar{x}_R) \ge \theta_R$. But then party L has a unique best reply, viz. $\bar{x}_L = 0$. Hence, we would have $x^*_\gamma(0, \bar{x}_R) \ge \theta_R$ and, by assumption and given that $\partial x^*/\partial x_R > 0$ for every γ, also $x^*_\gamma(0, \bar{x}_R) < x^*_\gamma(0,1) \le \theta_L < \theta_R$, a contradiction. □

Thus, when there is some power sharing, parties' positions always diverge in equilibrium. According to how voters' bliss points are distributed, either the two parties choose extreme platforms and the equilibrium outcome is between θ_L and θ_R, or one party's choice is extreme and the other party can induce her

[6] Note that there exists a unique x_L such that $x^*_\gamma(x_L, 1) = \theta_L$, since $\partial x^*/\partial x_L > 0$ for every γ.

favorite policy by choosing, however, a more extreme one. The reason why, differently from the case of sincere voting, the "centrifugal" force is always dominant is that voters' strategic behavior counterbalances parties' incentive to converge to obtain a higher power share. Precisely, some strategic voters will vote for their less favorite party in order to bring the policy outcome closer to their bliss point. Hence, when voters are strategic, a party who proposes a more moderate policy gains less votes (and so less power) than when voters are sincere, and these votes are not enough to induce policy convergence.

The above results hold for finite values of γ. Let us finally consider the limit case $\gamma \to \infty$ of the majoritarian electoral system, in which there is no power sharing.[7] Since the policy outcome is the winner's ideal policy and there are just two parties, voting strategically coincides with voting sincerely. In this case the basic insight of Hotelling (1929) model survives, even if parties care only about the enacted policy rather than about winning the election. In fact, each party has the incentive to move towards the other, since now the change in the power share is very abrupt. It follows that in equilibrium parties' positions converge.

Let m be the *median* ideal policy, i.e., the unique position such that $F(m) = \frac{1}{2}$. For the proof of the next result, we refer to Osborne (1995, Proposition 2).[8]

Proposition 2. *Under majoritarian rule,* $(x_L, x_R) = (m, m)$ *is always a Nash equilibrium. If* $\theta_L < m < \theta_R$*, then it is unique. If* $m \leq \theta_L < \theta_R$*, then also the pairs* (x_L, x_R) *such that either* $x_L = x_R \in (m, \theta_L]$*, or* $x_L = \theta_L$ *and* $x_R \in [\theta_L, 1]$ *are Nash equilibria. If* $\theta_L < \theta_R \leq m$*, then also the pairs* (x_L, x_R) *such that either* $x_L = x_R \in [\theta_R, m)$*, or* $x_L \in [0, \theta_R]$ *and* $x_R = \theta_R$ *are Nash equilibria.*

References

Calvert, R. L. Robustness of the multidimensional voting model: candidate motivations, uncertainty, and convergence. *American Journal of Political Science*, 29:69–95, 1985.

De Sinopoli, F. and Iannantuoni, G. A spatial voting model where proportional rule leads to two-party equilibria. *International Journal of Game Theory*, 35:267–86, 2007.

Downs, A. *An economic theory of democracy.* Harper Collins, New York, 1957.

Herrera, H., Morelli, M., and Palfrey, T. Turnout and power sharing. *The Economic Journal*, 124(574):131–62, 2014.

Herrera, H., Morelli, M., and Nunnari, S. Turnout across democracies. *American Journal of Political Science*, 60(3):607–24, 2015.

Hotelling, H. Stability in competition. *The Economic Journal*, 39:41–57, 1929.

[7] Assume that each party has preferences that can be represented by a continuous utility function and evaluates lotteries over winning outcomes according to its expected value.

[8] See also Wittman (1977, 1990), Calvert (1985), Roemer (1994).

Ledyard, J. O. The paradox of voting and candidate competition: a general equilibrium analysis. In *Essays in Contemporary Fields of Economics*, ed. George Horwich and James P. Quirk. Purdue University Press, West Lafayette, 1981.

Ledyard, J. O. The pure theory of large two-candidate elections. *Public Choice*, 44:7–41, 1984.

Lizzeri, A. and Persico, N. The provision of public goods under alternative electoral incentives. *American Economic Review*, 91:225–39, 2001.

Llavador, H. Electoral platforms, implemented policies, and abstention. *Social Choice and Welfare*, 27:55–81, 2006.

Matakos, K., Troumpounis, O., and Xefteris, D. Electoral rule disproportionality and platform polarization. *American Journal of Political Science*, 60(4):1026–43, 2016.

Merrill, S. and Adams, J. The effects of alternative power-sharing arrangements: do "moderating" institutions moderate party strategies and government policy outputs? *Public Choice*, 131;413–34, 2007.

Ortuño-Ortín, I. A spatial model of political competition and proportional representation. *Social Choice and Welfare*, 14(3):427–38, 1997.

Osborne, M. J. Spatial models of political competition under plurality rule: a survey of some explanations of the number of candidates and the positions they take. *The Canadian Journal of Economics*, 28(2):261–301, 1995.

Roemer, J. E. A theory of policy differentiation in single issue electoral politics. *Social Choice and Welfare*, 11:355–80, 1994.

Saporiti, A. Power sharing and electoral equilibrium. *Economic Theory*, 55(3):705–29, 2014.

Tullock, G. Efficient rent seeking. In *Toward a Theory of the Rent-Seeking Society*, ed. J. M. Buchanan, R. D. Tollison, and G. Tullock, pp. 97–112. Texas A&M University Press, College Station, 1980.

Wittman, D. Candidates with policy preferences: a dynamic model. *Journal of Economic Theory*, 14:180–9, 1977.

Wittman, D. Spatial strategies when candidates have policy preferences. In *Advances in the Spatial Theory of Voting*, ed. James M. Enelow and Melvin J. Hinich. Cambridge University Press, Cambridge, 1990.