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Adult Mortality and Modern Growth*

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Abstract

In this paper we analyze the relationship between (adult) mortality and the long-run development of countries from an empirical and theoretical perspective. A quantitative exploration of the model shows that improvements in adult survival rates alone bring an economy towards a Malthusian regime in the long run, while a transition from a Malthusian to a modern regime requires substantial advances in technological progress. Limited gains in technological progress associated with a strong decline in adult mortality can produce a sort of "false" take-off, i.e. an economy passed from a Malthusian to a pre-modern regime can be pushed back by the increasing demographic pressures.

Keywords: Unified Growth Theory, Human Capital, Adult mortality, Non-linear Dynamics, Endogenous Fertility, Industrial Revolution.

JEL code: O10, O40, I20.

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Contents

1 Introduction

The aim of this paper is to discuss the relationship between (adult) mortality and the long-run development of countries from an empirical and theoretical perspective.

Empirical evidence points to a general upward trend in life expectancy at birth and adult survival rate (an inverse proxy of mortality) of countries, and to a positive correlation between these two variables and income, but with some notable exceptions: i) the null (negative) correlation for most western countries in recent years; ii) a negative correlation for the same countries at the end of the nineteenth century; and iii) always a negative correlation for some very poor countries after the second world war. Another remarkable fact is the strong convergence in life expectancy at birth and adult survival rate among countries after 1950/1960 (see Livi-Bacci (2001)).

From the theoretical point of view, literature on the relationship between mortality and income is very broad and increasing all the time. A first strand of literature investigates the causes of mortality decline in western countries; scholars can be divided into two main groups: the first attributes the observed mortality decline mainly to income growth via better nutrition (McKeown, 1976); the second emphasizes the role of public health and sanitary intervention (Easterlin, 2004). A second strand of literature explores various channels through which mortality decline affects income: i) increasing labor productivity due to better health (Fogel, 2004); ii) providing higher incentives to the accumulation of physical and human capital (Bloom and Canning, 2003 and Cervellati and Sunde, 2005); but also iii) decreasing the resources devoted to factor accumulation due to higher consumption by older (unproductive) generations (Zhang et al., 2003); and iv) imposing a higher demographic pressure on resources (Acemoglu and Johnson (2007)).

To clarify the mechanics linking mortality and income we propose a theoretical model in the spirit of the Unified Growth Theory proposed by Galor (2005), augmented by the presence of adult mortality, physical capital accumulation, and with two technologies: a traditional one, using unskilled labour and land, and a modern one, using physical and human capital.

The focus on adult mortality is justified by the intuition that many of the channels discussed above mainly refer to this variable. The introduction of physical capital accumulation and of two technologies instead aims to match the structural changes and the related demographic phenomena (i.e. decline in fertility rate and the increase in population size) displayed along the transition to modern economic growth of many countries. In the theoretical model an increase in adult survival has two opposing effects on transfer to offspring: a negative effect because it raises the lifetime consumption of the parents and a positive effect because it increases parental income (via an increase in their labor income). The dynamic is characterized by three different regimes: (i) a *Malthusian regime*, where output is produced only with traditional technology and income is checked by fertility rate; (ii) a *pre-modern regime*, where an increasing share of aggregate output is produced in an industrial sector with only physical capital; and, finally, (iii) a *modern regime* where both physical and human capital are used in the industrial sector.

A quantitative exploration of the model highlights that improvements in adult survival rates alone bring an economy towards a Malthusian regime in the long run, while a transition from a Malthusian to a modern regime requires substantial advances in technological progress. Limited gains in technological progress associated with a strong decline in adult mortality can produce a sort of "false" take-off, i.e. an economy passed from a Malthusian to a pre-modern regime can be pushed back by the increasing demographic pressures (which are not contrasted by sufficiently high improvements in technological progress).

The paper is organized as follows: Section 2 discusses some stylized facts on mortality and income of countries; Section 3 reviews the related literature; Section 4 introduces the theoretical model; Section 5 presents a quantitative exploration of the model; and Section 6 concludes.

2 Stylized Facts on Mortality and Income

This section is devoted to discuss some remarkable historical consistencies revealed by empirical investigations. In particular, as regards mortality and income, the following "stylized facts" emerge:

1. Since 1870 for most western countries there exists an upward, continuous and sustained trend in life expectancy (at birth) and adult survival rate (see, e.g., Easterlin, 2004, p. 84).[1]

2. Since 1870 for most western countries fluctuations in life expectancy and adult survival have been strongly declining over time, with a clear break after the second world war (see, e.g., Livi-Bacci, 2001, p.119).

3. Since 1870 differences in life expectancy and adult survival among most of western countries have been strongly declining (see, e.g., Livi-Bacci, 2001, p. 125).

[1] World Bank adopts as definition of adult survival rate the probability of surviving from 15 to 60 years; here we will use this expression in a broader sense, i.e. the probability that an individual actually in her childhood survives to the end of her adulthood.

4. Around 1870 western countries showed a very low positive correlation between life expectancy and GDP per capita; over time this positive correlation has been gaining strength, reaching a peak just before the second world war; but by 1980 this correlation became very low (and negative) again. From 1950 to 1980 low and lower-middle income countries have shown a pattern of increasing positive correlation between life expectancy and GDP per capita, which remains very high until 2008.

5. Around 1870 western countries displayed a negative correlation between adult survival rate and GDP per capita; over time correlation has been changing its sign, to become almost null in 2008. From 1950 to 1980 in low and lower-middle income countries the positive correlation between adult survival rate and GDP per capital increased, to successively decline from 1980 to 2008, but still remaining strictly positive.

Below, we motivate in details these stylized facts.

2.1 Long-run Trends in Life Expectancy and Adult Survival Rate

Fig. 1 shows the time series of life expectancy for a sample of 17 countries and of a proxy of adult survival rate for a more restricted sample of 11 countries[2] (the restriction of sample size derives from the lack of data on infant mortality).[3]

The time paths of life expectancy and adult survival rate look very similar (see Fig. 1). The maximum level of life expectancy and adult survival rate follow an upward trend for the whole period, albeit with large fluctuations at least until the 1920s; from 1920 to 1940 both grew at increasing rates, while from 1940 to 1960 at decreasing rates, to finally follow a linear trend.

The dynamics of the minimum level of life expectancy and adult survival rate, in particular of their swings, signals the change in mortality regimes. Before 1940 it was still possible to observe sudden drops in life expectancy followed by marked increments; after 1940 the emergence in

[2]See Appendix A for the country list.

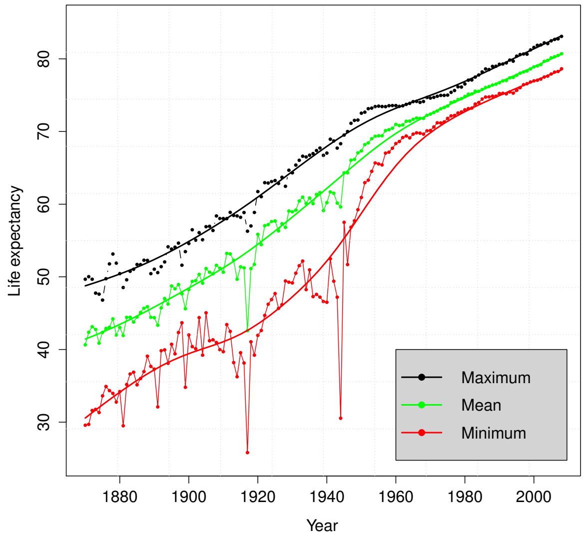
[3]The infant mortality rate is the number of deaths of infants under one year old per 1,000 live births. The proxy of adult survival rate is calculated by "filtering out" the impact of infant mortality from life expectancy at birth. In particular, assuming that the maximum length of life is 100 years, $1-\pi$ is the infant mortality rate, p the child and adult survival probability, then life expectancy at birth is given by:

$$LE = \pi + \pi p 99, \tag{1}$$

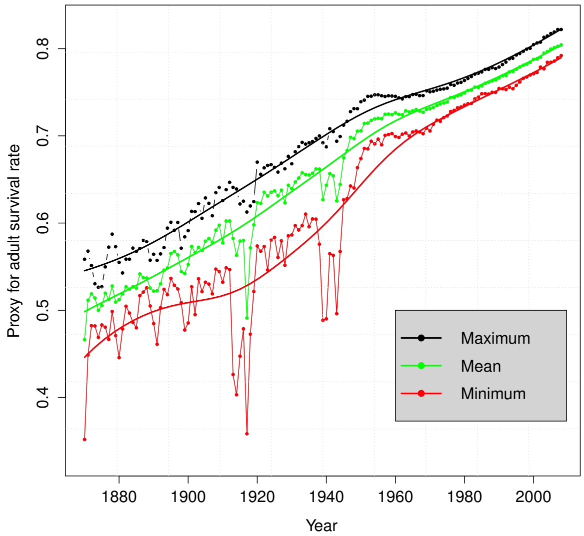
from which the child and adult survival rate:

$$p = \frac{LE - \pi}{\pi 99}. \tag{2}$$

We use p as a proxy for the adult survival rate.



(a) Trends in maximum, average, and minimum life expectancy in 1870-2008 for a sample of 17 western countries. Source: `https://www.clio-infra.eu/`



(b) Trends in maximum, average and minimum (for a proxy of) adult survival rate in 1870-2008 for a sample of 11 western countries. Source: `https://www.clio-infra.eu/`

Figure 1: Trends in life expectancy at birth and adult survival rate in western countries in the period 1870-2008.

all countries of the sample of the "new knowledge of disease and new technologies of disease control"(Easterlin, 2004, p.101) allows a progressive attenuation of mortality fluctuations. In the words of Livi-Bacci (2001, p.119) "Out of the disorder of earlier times, owing to random and unpredictable mortality, the processes of life became orderly times. Two connected factors essentially explain the earlier capricious nature of death. The first was the frequent and irregular occurrence of mortality crises...The second factor was the risk that the natural age-linked and chronological succession of death would be overturned".

Finally, since 1870 the range between the maximum and minimum life expectancy and adult survival rate has been strongly decreasing, reaching the smallest range in 1970, and then remaining stable until the present day.

From 1870 to 1910 the countries contributing to maximum life expectancy are mostly Sweden and Norway, from 1910 to 1978 Denmark and Netherlands, and from 1978 to 2008 Japan. From 1870 to 1940 countries that contribute more frequently to the minimum life expectancy are Italy and Spain, from 1946 to 2000 Japan, Italy, Spain, Finland, Austria and Denmark succeed one another; finally, from 2000 to 2008 The United States shows the lowest life expectancy. In general, the ranking of countries in term of life expectancy and adult survival rate shows a notable persistence in the last 30 years.

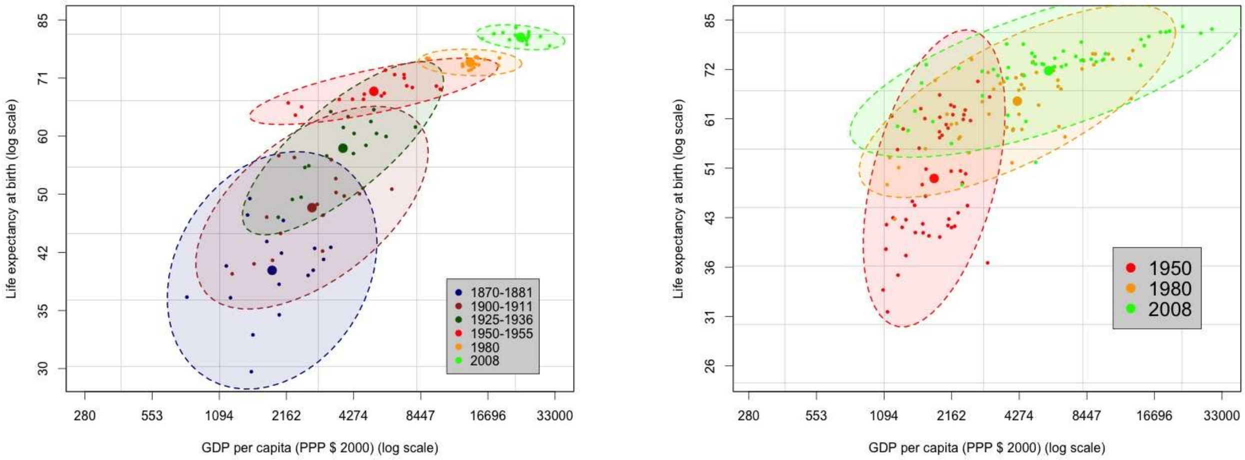
2.2 Mortality and Income in the Long Run

Fig. 2(a) reports the relationship between GDP per capita and life expectancy at birth for a sample of 17 western countries and for a sample of 52 lower-middle income countries in 1950, i.e. countries with a level of GDP per capita in 1950 in the range defined by the minimum and the maximum level of the GDP per capita in 1870 of 17 western countries[4].

Around 1870 there is a very weak correlation between life expectancy and GDP per capita (see Fig. 2(a) and Table 1); from 1900 to 1955 life expectancy and GDP per capita instead display a positive and strong correlation; but after 1980 correlation reversed. At the same time levels of life expectancy across countries converged, while inequality of GDP per capita firstly increased between 1870 and 1950 and then declined after 1950 (see Table 1). In 2008 the inequality in life expectancy appears very limited with respect to the one of GDP per capita. As we will discuss in Section 3 this evidence supports the idea that western countries, starting from 1870, experienced a Mortality Revolution, similar to the celebrated Industrial Revolution, whose effects are still present today (see Easterlin, 2004).

As regards the sample of 52 lower-middle-income countries, from 1950 to 2008 they display

[4]See Table 6 in Appendix A for the list of these lower-middle countries)



(a) The long-run relationship between life expectancy and GDP per capita for 17 western countries. Source: `https://www.clio-infra.eu/`

(b) The relationship between life expectancy and GDP per capita for a sample of 52 lower-middle income countries in 1950.

Figure 2: The relationship between life expectancy and GDP per capita in western and lower-middle-income countries.

Period	S.d. of LE	S.d. of Y	Corr. between LE and Y	R^2
1870 – 1881	0.13	0.39	0.23	0.05
1900 – 1911	0.11	0.44	0.55	0.30
1925 – 1936	0.10	0.38	0.78	0.60
1950 – 1955	0.04	0.47	0.75	0.56
1980	0.01	0.19	-0.10	0.01
2008	0.01	0.17	-0.32	0.10

Table 1: Descriptive statistics for a sample of 17 western countries.*Source*: `https://www.clio-infra.eu/`. LE: log of life expectancy; Y: log of GDP per capita; and S.d.: standard error. R^2 is the share of the total variance of life expectancy explained by GDP per capita calculated by the estimate of model $log(LE_t) = \alpha + \beta log(Y_t)$.

a positive and increasing correlation between life expectancy and GDP per capita, with a decreasing sample variance in life expectancy but strongly increasing in GDP per capita.

Period	S.d. of ASR (LE)	S.d. of Y	Corr. between ASR (LE) and Y	R^2
1960(1950)	0.10 (0.20)	0.37 (0.28)	0.50 (0.45)	0.25 (0.20)
1980	0.08 (0.13)	0.63	0.67 (0.72)	0.45 (0.51)
2008	0.10 (0.12)	0.80	0.58 (0.70)	0.34 (0.49)

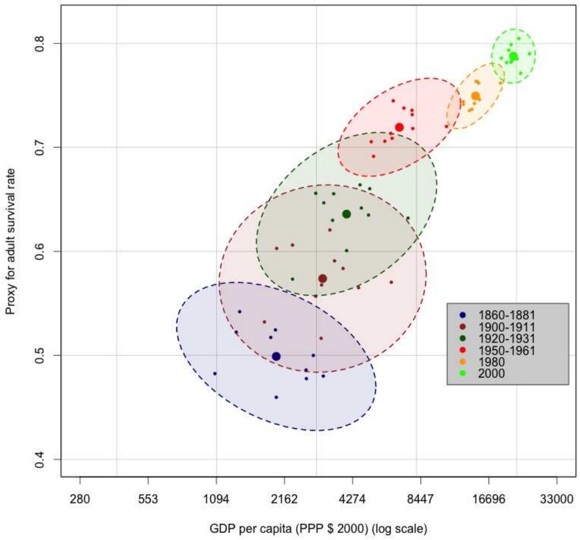
Table 2: Descriptive statistics for a sample of 52 lower-middle-income countries. *Source*:`https://www.clio-infra.eu/` and World Development Indicators, 2015 `https://http://databank.worldbank.org/`. R^2 is the share of the total variance of adult survival rate explained by GDP per capita calculated by the estimate of model $ASR_t = \alpha + \beta log(Y_t)$. The same statistics for life expectancy are reported in parenthesis.

In the period 1870−1881 adult survival rate and GDP per capita show a negative correlation, which turned positive from 1920 − 1980, to finally become negligible in 2000 (see Fig. 3(a) and Table 3). For the sample of lower-middle-income countries in the period 1960-2008 adult survival rate and GDP per capita always show a first an increasing and then a decreasing positive correlation.

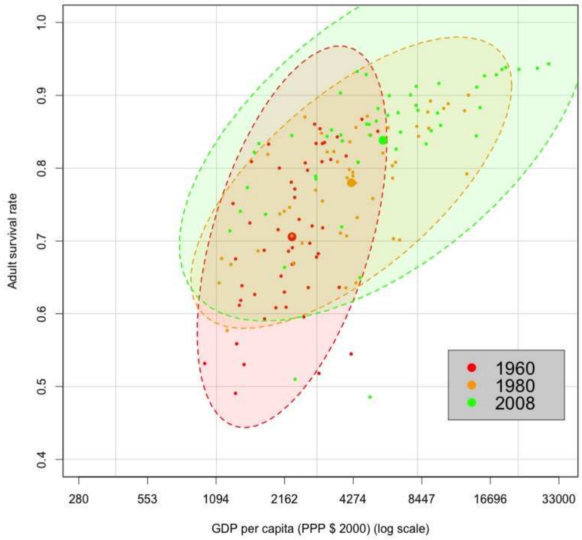
Period	S.d. of ASR (LE)	S.d. of Y	Corr. between ASR (LE) and Y	R^2
1860 – 1881	0.02 (0.10)	0.35	-0.40 (-0.24)	0.16 (0.06)
1900 – 1911	0.03 (0.09)	0.36	0.05 (-0.06)	0.00 (0.00)
1920 – 1931	0.03 (0.06)	0.32	0.36 (0.39)	0.13 (0.15)
1950 – 1961	0.02 (0.03)	0.21	0.50 (0.53)	0.25 (0.29)
1980	0.01 (0.02)	0.10	0.61 (0.59)	0.37 (0.35)
2000	0.01 (0.01)	0.08	0.04 (0.02)	0.00 (0.00)

Table 3: Descriptive statistics for a sample of 11 countries. *Source*: authors' calculations from `https://www.clio-infra.eu/`. ASR: a proxy for adult survival rate; Y: log of GDP per capita. R^2 is the share of the total variance of adult survival probability explained by GDP per capita calculated by the estimate of model $ASR_t = \alpha + \beta log(Y_t)$.The same statistics with life expectancy are reported in parenthesis.

In 1870, for the western countries the negative correlation between adult survival rate and per capita GDP is much more stronger than the correlation between life expectancy at birth and per capita GDP (see Tables 3). This is in agreement with the idea that although infant and adult mortality are strongly correlated, adult mortality should have the highest impact on



(a) The long-run relationship between a proxy for adult survival and GDP per capita for 11 western countries. Source: `https://www.clio-infra.eu/`.



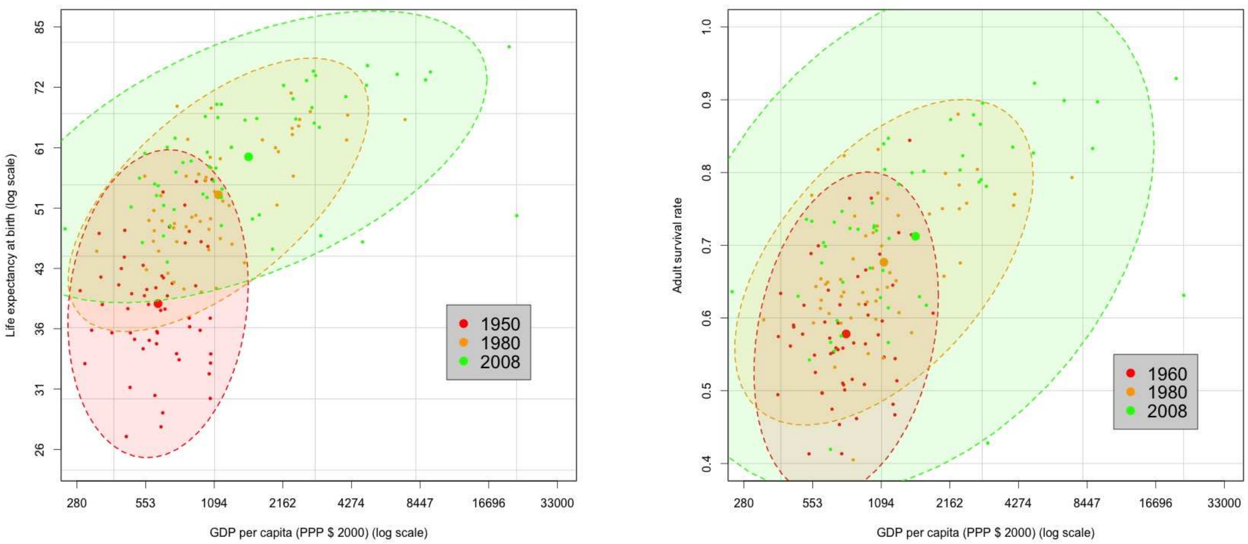
(b) The relationship between adult survival rate and GDP per capita for 52 lower-middle income countries. Sources: `https://www.clio-infra.eu/` and World Development Indicators, 2015, `https://http://databank.worldbank.org/`.

Figure 3: Long-run relationship between life expectancy and GDP per capita

growth through the channels of the accumulation of human and physical capital (see Lorentzen et al., 2008), while infant mortality should be more related to the "demographic waste" of resources (see Livi-Bacci, 2001, p.114).

2.3 Mortality and Income in Poor Countries

Fig. 4 shows the positive, and first increasing and then decreasing correlation between life expectancy, adult survival rate and GDP per capita in a sample of low-income countries selected on the basis of their GDP per capita in 1950, which is below the minimum GDP per capita in 1870 of the sample of 17 western countries (see Table 4).



(a) The long-run relationship between life expectancy and GDP per capita for 57 low-income countries. Source: `https://www.clio-infra.eu/`.

(b) Correlation between adult survival rate and GDP per capita for 57 low-income countries. Source: `https://www.clio-infra.eu/` and World Development Indicators, 2015 `https://http://databank.worldbank.org/`

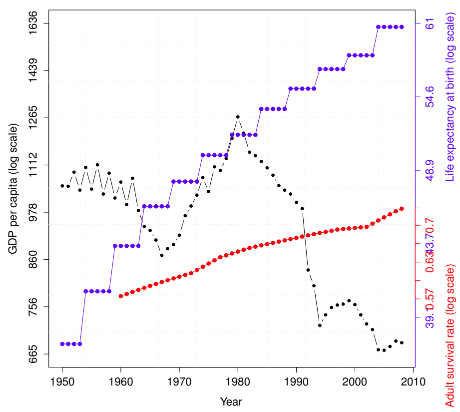
Figure 4: Correlation between life expectancy, adult survival rate and GDP per capita for a sample of 57 low-income countries

From 1960 to 2008 within-sample variance is fairly constant for life expectancy, while it is increasing for adult survival rate.

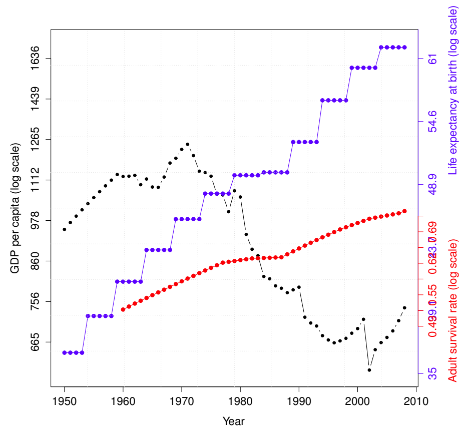
However, as shown in Fig. 5, within this sample some countries display a negative correlation between life expectancy, adult survival rate, and GDP per capita. The overall impression is that the dynamics of life expectancy and adult survival is independent of GDP per capita dynamics.

Period	S.d. of ASR (LE)	S.d. of Y	Corr. between ASR (LE) and Y	R^2
1960 (1950)	0.09 (0.17)	0.36	0.24 (0.12)	0.06 (0.01)
1980	0.09 (0.15)	0.59	0.53 (0.62)	0.28 (0.39)
2008	0.14 (0.16)	0.94	0.36 (0.53)	0.13 (0.28)

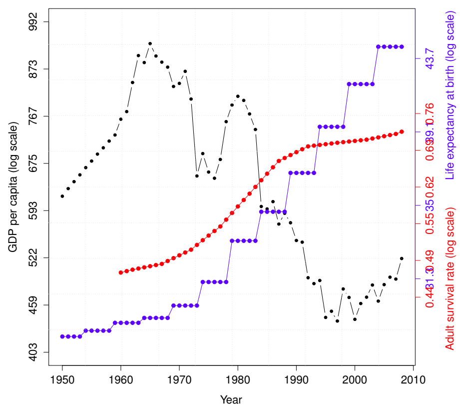
Table 4: Descriptive statistics for 57 low-income countries. *Source*:`https://www.clio-infra.eu/` and World Development Indicators, 2015 `https://http://databank.worldbank.org/`. R^2 is the share of the total variance of adult survival rate explained by GDP per capita calculated by the estimate of model $ASR_t = \alpha + \beta log(Y_t)$. The same statistics for life expectancy are reported in parenthesis.



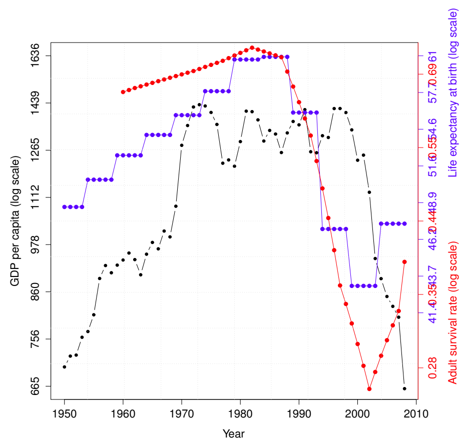
(a) Haiti



(b) Madagascar



(c) Niger



(d) Zimbabwe

Figure 5: Countries with a negative relationship between GDP per capita and life expectancy/adult survival rate, and a country experiencing a dramatic fall in life expectancy/adult survival rate.

The case of Zimbabwe reported in Fig. 5(d) also suggests that the sudden drop in life expectancy due to an exogenous shock, the HIV epidemics, did not have an immediate effect on the dynamics of GDP per capita, which experienced a dramatic fall only after 1998 as result of an aggressive program of forced land redistribution.

3 Related Literature

Literature on the relationship between mortality and income is very wide and increasing all the time. As a line of exposition we choose to divide the analysis between the contributions which focus on the causes of mortality decline in western countries and those addressing the economic effects of changes in mortality. These two strands of literature are clearly interconnected, for example, in models where human capital accumulation produces a decline in mortality and this reduction, in turn, stimulates the accumulation of human capital.

3.1 What Caused Mortality Decline in Western Countries: Exogenous versus Endogenous Explanations

The sources of mortality decline have been extensively debated by historical demographers, historians of medicine and economic historians (see Schofield et al., 1991).

The literature on the factors causing mortality decline can be grouped into two main strands: one attributes a crucial role to income growth, and, by implication, to better nutrition; the other one emphasizes the role of public health and sanitary intervention.

The former is associated with the pioneering contribution of McKeown (1976) who attributed the principal cause of mortality decline in England from 1838 to the present day to the modern economic growth which, by increasing living standards, and particularly the nutritional level of the population, inevitably increased resistance of the population to infectious diseases.

According to McKeown's thesis Floud, Fogel, Harris and Hong argue "As we have already seen, those data suggested that the average height of the population and average levels of life expectancy rose between circa 1750 and 1820, followed by a period of stagnation or decline, and then further improvement from the 1850s onwards. These trends are broadly consistent with our new estimates of food availability (certainly for the periods 1750 1800 and 1850-1914), and help to reinforce the link between nutrition and mortality which McKeown could only infer when he attempted to account for the modern rise of population in the 1970s "(see Floud et al. (2011), p. 162). However they also point out "However, one of the main themes of our argument has been to emphasize the synergistic nature of the relationship between nutrition and infection, and it would certainly not be correct to conclude that dietary change was the only reason for improvements in either height or life expectancy."(see Floud et al. (2011), p. 162).

Robert Fogel while emphasizing the strict relationship between better nutrition and mortality reduction, finds evidence of a very limited or even opposing relationship between economic growth and improvement in nutritional status and health during the eighteenth and nineteenth centuries both in Europe and America[5]. In particular, Fogel points out that "The overall improvement in health and longevity during this period is less than might be expected from the rapid increases in per capita income indicated by national income accounts for most of the countries in question. More puzzling are the decades of sharp decline in height and life expectancy, some of which occurred during eras of undeniably vigorous economic growth."(see Fogel, 2004 p. 18).

The McKeown's view that improvements in nutrition are the main determinant of decreasing mortality has been widely criticized by many other scholars. As Easterlin (1996, p.81) asserts "Some economic historians have taken the trend in stature simply as indicating improved nutrition due to modern economic growth. But this confuses nutrition with nutritional status. Although stature is commonly taken as substantially influenced by nutritional status, nutritional status itself depends not only on nutrition (i.e., nutritional intake), but on the retention of nutrients. The retention of nutrients in turn depends especially on the incidence of disease among infants and children, particularly gastrointestinal disease. Thus, an improvement in health of infants and children-in particular, a reduced incidence of gastrointestinal disease-would improve nutritional status and make for increased stature, even with nutritional intake unchanged. Hence, theory suggests that trends in stature may reflect changes in nutritional intake (connected with economic conditions), better retention of nutrients (due to the Mortality Revolution and associated improvement in health), or both.".

Again Livi Bacci (2007) asserts "This theory is countered by a number of considerations which make us look to other causes. In the first place, the link between nutrition and resistance to infection holds primarily in cases of severe malnutrition; and while these were frequent during periods of want, in normal years the diet of European populations seems to have been adequate. Second the latter half of the eighteenth century and the first decades of the nineteenth, the period during which this mortality transition began, do not appear to have been such a fortunate epoch."(see Livi Bacci, 2007, p.71). Moreover, Livi Bacci (2007) argues that the increase in

[5]See also Deaton (2006, p. 111) "In Britain, the United States, and much of Europe, there were periods in the nineteenth century when urbanization ran ahead of the rate of public health provision and population health deteriorated during periods of rapid economic growth."

longevity was caused principally by a reduction in youth and infant mortality which occurred "not because of better nutrition, but because of improved child-rearing methods and better protection from the surrounding environment."(see Livi Bacci, 2007 p.71). In general Livi Bacci (2007, p. 118) writes: "The mortality decline which began in the second half of the eighteenth century is generally ascribed partly to exogenous factors including the reduced frequency of epidemic cycles and the disappearance of plague; partly the reduction of famine due to better economic organization; and to sociocultural practices which helped to reduce the spread of infectious diseases and improve survival, especially infants ".

With respect to the period 1900-1960, Preston (1975) finds that economic growth explains only $10-25$ percent of the increase in life expectancy whereas the remaining $75-90$ per cent of the growth in life expectancy is attributable to factors exogenous to countries' income. In particular, he emphasizes the crucial role of the widespread diffusion of medical innovation: "It seems to have been predominantly broad-gauged public health programmes of insect control, environmental sanitation, health education and maternal and child health services that transformed the mortality picture in less developed areas, while it was primarily specific vaccines, antibiotics and sulphonamides in more developed areas. But the technologies were not, for the most part, indigenously developed by countries in either group. Universal values assured that health breakthroughs in any country would spread rapidly to all others where the means for implementation existed. The importance of exogenous, largely imported, health technology in the now-developed countries may have been underestimated for earlier periods as well."(Preston, 1975, p. 243). Preston (1980) shows that in the period 1940-1970 life expectancy in low-income countries increased even though income and average daily calories remained constant. Preston's conclusion finds an empirical support in Fig.2(a), where our sample of western countries after 1980 GDP per capita appears not positively correlated with life expectancy.

According to Easterlin (2004, p.84) the rise in life expectancy principally depends on the emergence and increasing importance of medical and technological innovations: "Since 1870, life expectancy at birth in many areas of the world has soared from values around 40 years or less to 70 years or more. The reduction in mortality has been accompanied by an associated improvement in health as the incidence of contagious disease has dramatically lessened. This lengthening of life and associated reduction in morbidity brought about by the Mortality Revolution has meant at least as much for human welfare as the improvement in living levels due to modern economic growth. Certainly the Mortality Revolution has substantially affected a much wider segment of the world's population."

In particular, he identifies three major breakthroughs which led to mortality reduction: 1) new methods of preventing disease transmission, including clean water supply and education in personal hygiene; 2) new vaccines to prevent certain diseases which started in the 1890's; and 3) new drugs to cure infectious diseases (antimicrobials) which started in the late 1930's (see Easterlin, 2004, p. 104 and also Deaton, 2006, p. 110).

Finally, the recent paper by Harris (2004) provides a balanced discussion on the empirical evidence of the relationship between mortality and income in England and Wales in the period 1750-1914.

3.2 The Economic Effects of Mortality Decline

From the second half of the 1990s an increasing literature developed focusing on the relationship between mortality and economic growth, but with mixed results. In particular, the argument whether the increase in longevity has a positive effect on economic growth has been the subject of lively discussion in recent years.

From the empirical point of view, in a recent article Acemoglu and Johnson (2007, p. 975) concludes: "There is no evidence that the increase in life expectancy led to faster growth of income per capita or output per worker. This evidence casts doubt on the view that health has a first-order impact on economic growth."The results of Acemoglu and Johnson (2007) are called into question by Bloom et al. (2014), arguing that the empirical results in Acemoglu and Johnson (2007) are not robust towards omitted variables. They point out that "In a more general empirical framework which includes initial life expectancy, the Acemoglu and Johnson results are reversed, with both levels and changes in health displaying positive associations with economic growth."In turn, Acemoglu and Johnson (2014) extend their previous analysis to take into consideration Bloom et al. (2014)'s critique, controlling for the impact of lagged life expectancy. They claim that their analysis provides additional empirical support to their previous results, concluding "All these approaches confirm our main results are robust: there is no evidence that increases in life expectancy after 1940 had a positive effect on GDP per capita growth."

From a theoretical point of view the main contributions dealing with the effect of longevity on economic growth can be divided into two main groups: i) those that suggest a positive effect and ii) those that point to a negative or null effect.

Contributions that argue for a positive effect of longevity on growth identify at least three channels in which health improvements affect economic growth.

The first channel is labor productivity, i.e. healthier workers are both physically and mentally more efficient. In his 1994 seminal contributions Fogel asserts: "Changes in health, in the composition of diet, and in clothing and shelter can significantly affect the efficiency with which ingested energy is converted into work output. Reductions in the incidence of infectious diseases increase the proportion of ingested energy. [...] I believe, on the huge social investments made between 1870 and 1930, whose payoffs were not counted as a part of national income during the 1920's and 1930's even though they produced a large stream of benefits during these decades.... (Fogel, 1994, p.386) ". From another point of view Bar and Leukhina (2010) suggest that the increase in longevity positively affects total factor productivity improving the process of knowledge transmission across generations and encouraging innovation. Weil, 2007 claims that there is also an indirect way by which higher longevity, increasing labor productivity and therefore the marginal product of capital, positively affects income: higher incentives to physical capital accumulation.

The second channel is the investment in physical capital, that is improvements in health have a positive effect on savings because it creates the need to finance a longer period of retirement (see, e.g. Bloom et al., 2003, Kageyama, 2003, Zhang et al., 2003). Within this framework Lorentzen et al. (2008) highlight the key role of adult mortality (as opposed to child mortality).

The third channel is the accumulation of human capital. The argument, dating back to Ben-Porath (1967), is that improvements in longevity promote human capital accumulation "because they increase the horizon over which benefits from investments in human capital can be enjoyed, therefore increasing the returns to education. This eventually raises educational attainment and increases the productivity of individuals in the labor market and in the household sector "(Soares, 2005, p.581). In other words, higher longevity reduces the rate of depreciation of investment in education and increases its return (see, for example, De la Croix and Licandro, 1999, Boucekkine et al., 2002, Boucekkine et al., 2003, Cervellati and Sunde, 2005, 2014, Lagerlof, 2003, Weisdorf, 2004 among others).

The view that health improvements play a positive role on economic growth has been however countered by a strand of the literature that has emerged in recent years. A common theme in most of these papers is that while the increase in longevity, on the one hand, increases human capital and labor productivity, on the other, it may also boost population size. Thus, if the positive effects are not sufficiently high, and/or if there exist some fixed factors of production, mortality reduction can depress economic growth (Acemoglu and Johnson, 2007, 2014). In Acemoglu and Johnson (2014, p.929)'s words "Increased life expectancy raises population, which initially reduces capital-to-labor and land-to-labor ratios, thus depressing income per capita. This initial decline is later compensated by higher output as more people enter the labor force and as more capital is accumulated. This compensation can be complete and may even exceed the initial level of income per capita if there are significant productivity benefits from longer life expectancy. Yet, the compensation may also be incomplete if the benefits from higher life expectancy are limited and if some factors of production, for example, land, are supplied inelastically". In line with this reasoning Young (2005) and Voigtlander and Voth (2013) argue for a positive effect on long-run growth of negative *shocks*, such as the Black Death or the HIV/AIDS epidemic.

Hazan (2009) provides a critique to the idea that longer time horizon positively affects schooling and shows that the increase in longevity positively affects human capital accumulation only when it also increases lifetime labor supply, which has not been the case for US in the last 150 years.

Another argument in favor of a negative relationship between health and income growth is urbanization. A well documented fact in the literature is that the increasing concentration of population in urban areas and in manufacturing factories following the Industrial Revolution, negatively affected life expectancy by increasing the exposure of population to contagious disease. As Easterlin (2004, p.89) argues "Thus, although modern economic growth may have increased resistance to disease, it also increased exposure, and the net balance of these two factors is not clear". Moreover, Schofield et al. (1991, p.14) point out: "The rapid process of industrialization and urbanization in nineteenth-century European society created new obstacles to improved health. Towns had always been characterized by higher mortality rates due mainly to greater population densities which facilitated infection and filth; and during the nineteenth century increased proportions of the population were living in these urban centers. The poor living conditions of the age were probably one of the principal reasons why mortality ceased to improve during most of the central decades of the century". However, it is difficult to separate the negative impact on mortality of conditions of work from the increase in population density in urban city. Schofield et al. (1991, p.182), for example write "The expectation is that specific occupational hazards would lead to higher mortality among the working population. It is difficult to separate this from the effects of urban environments. In an early study, Copenhagen and other towns in Denmark for the period 1865-1974, it was found that death rates for men aged 20 and over increased monotonically from the higher class (capitalists, professionals, wholesale, dealers, higher officers) to middle class (master mechanics, petty officers, teachers, clerks, small shopkeepers) to the poorest class (workmen, servants, almshouse inmates) ".

A final explanation of the negative effect of increasing adult survival on income is related to the decrease in accidental bequests, and therefore to the lower physical capital accumulation (see Zhang et al., 2003).

The recent paper of Cervellati and Sunde (2011) finds support to the idea that the effect of mortality on income is ambiguous; in particular, they find that the increase in longevity has a negative effect before the demographic transition (not necessarily related to the level of income), and a strong and positive effect in post-transition countries.

4 A Theoretical Model

In this section we briefly introduce a theoretical model capable of capturing most of the features related to the relationship between mortality and income present in literature; we refer to Fiaschi and Fioroni (2014) for more details.

The model is inspired by Galor and Moav (2004). Consider an economy populated by overlapping generations of people who potentially live for two periods: childhood and adulthood. Though certain to live through childhood, they are subject to the risk of dying during adulthood. Let the expected length of adulthood in period t be denoted by $p_t \in (0,1)$ and the total number of adults at the beginning of period t by L_t. Hence the *actual aggregate labour supply* in period t is equal to $p_t L_t$.

4.1 Production

In every period, the economy produces a single material good, whose price is normalized to 1. Production may take place using two different technologies: a traditional technology that employs unskilled labour and land, and an industrial technology that employs physical capital and skilled labour. While the traditional technology is always operating, the industrial technology, as we shall see below, will become available once technology has sufficiently progressed (for production we follow Aghion and Howitt, 2009).
The traditional production function is given by:

$$Y_t^a = A_t^a (p_t L_t^a)^{1-\lambda} (T)^{\lambda}, \tag{3}$$

where $\lambda \in (0,1)$, A^a is a productivity parameter, pL_t^a is the *actual* amount of unskilled labour employed in the traditional sector in period t and T is the quantity of land. The industrial production function is given by:

$$Y_t^m = A_t (p_t h_t L_t^m)^{1-\alpha} K_t^{\alpha}, \tag{4}$$

where $\alpha \in (0,1)$, $A > 0$ is a technological parameter, and $p_t h_t L^m$ is the *actual* amount of skilled labour employed in the industrial sector given by the individual level of human capital and the *actual* workforce $p_t L_t^m$. As discussed below, human capital increases with the resources invested in education, and when these resources are zero $h_t(0) = 1$. Hence the industrial sector employs unskilled labour.

Instead, when production is conducted using only traditional technology the wage rate is given by:

$$w_t^a = (1-\lambda)A_t^a p_t^{1-\lambda}(L_t^a)^{-\lambda}T^{\lambda}. \tag{5}$$

When industrial technology is operating, the rate of return to capital r_t and the wage rate per efficiency unit of labour w_t^m are given by:

$$r_t = \alpha A_t p_t^{1-\alpha}\left(\frac{K_t}{h_t L_t^m}\right)^{\alpha-1}, \text{ and} \tag{6}$$

$$w_t^m = (1-\alpha)\, A_t p_t^{1-\alpha}\left(\frac{K_t}{h_t L_t^m}\right)^{\alpha}. \tag{7}$$

In the early stages of development production is conducted using traditional technology while industrial technology is latent since no physical or human capital have yet been accumulated. The economy will start employing industrial technology together with traditional technology when income is sufficiently high. As discussed below, improvements in technological progress and adult survival will lead parents to leave a positive transfer to their children first in the form of investments in physical capital and subsequently in both physical and human capital; this, in turn, will activate industrial technology.

Total output is therefore given by:

$$Y_t = Y_t^a + Y_t^m. \tag{8}$$

As workers are assumed to be perfectly mobile between the two sectors, wages are equalized across the sectors in every period, i.e. $w_t^a = w_t^m h_t \ \forall t$. This implies that employment in the traditional sector is chosen in order to maximize profits. The amount of labor employed in the industrial sector is therefore:

$$L_t^m = L_t - L_t^a. \tag{9}$$

Assuming, for the sake of simplicity that:

$$\alpha = \lambda,$$

and that productivity in the traditional sector has the same trend as productivity in the industrial sector, that is[6]:

$$A_t^a = \phi A_t,$$

[6] Broadberry et al. (2013) discuss evidence in support of such a hypothesis.

with $\phi < 1$, the aggregate production is given by[7]:

$$Y_t = A_t p_t^{1-\alpha} L_t^{1-\alpha} \left(\tilde{T} + h_t^{\left(\frac{1-\alpha}{\alpha}\right)} K_t \right)^{\alpha}, \tag{13}$$

where $\tilde{T} \equiv \phi^{1/\alpha} T$.

The *actual income per worker* in period t is therefore given by:

$$y_t = A_t p_t^{1-\alpha} \left(\frac{\tilde{T}}{L_t} + h_t^{\left(\frac{1-\alpha}{\alpha}\right)} k_t \right)^{\alpha}, \tag{14}$$

where $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$.

From Eq. (12) income per worker can be written as follows:

$$y_t = A_t p_t \left(\frac{\tilde{T}}{p_t L_t l_t^a} \right)^{\alpha}, \tag{15}$$

where $1/l_t^a$ is a proxy for the accumulation of physical and human capital.

4.2 Consumption and Total Transfer

In childhood individuals acquire education and make no decisions; in adulthood individuals work, have n_t children, and can save in order to accumulate wealth for their offspring, and invest in their education.

To analyse adults' behaviour it is useful to conceptualise adulthood (of length p_t) as divided into small time increments (for example years or months). At each time increment individuals (born in period $t-1$) allocate their income between consumption c_t, a transfer to their offspring b_t and raising children (note from Eq. (14) that parents' income y_t is already expressed in terms of p_t):

---

[7]From the condition $w_t^a = w_t^m h_t$:

$$L_t^a = \left[\frac{A_t^a p_t^{1-\lambda} (1-\lambda)(h_t L_t^m)^{\alpha}}{A_t p_t^{1-\alpha} (1-\alpha) h_t K_t^{\alpha}} \right]^{1/\lambda} T; \tag{10}$$

with $\lambda = \alpha$, it yields:

$$L_t^a = \frac{L_t^m \tilde{T}}{K_t h_t^{\frac{1-\alpha}{\alpha}}}. \tag{11}$$

Thus from Eq. (9) it follows that the labour share in the traditional sector is given by:

$$l_t^a = \frac{L_t^a}{L_t} = \frac{\tilde{T}/L_t}{k_t h_t^{\frac{1-\alpha}{\alpha}} + \tilde{T}/L_t}, \tag{12}$$

with $\partial l_t^a/\partial h_t < 0$, $\partial l_t^a/\partial k_t < 0$ and $\partial l_t^a/\partial T > 0$.

$$y_t = p_t c_t + p_t b_t + \delta n_t y_t. \tag{16}$$

where $p_t c_t$ is the *actual consumption* during adulthood per individual, $p_t b_t$ is the *actual transfer* which each parent gives to their children during their life and δ is the opportunity cost of raising children, that is the fraction of parents time required to raise each child (see Galor, 2005)

The transfer $p_t b_t$, in turn, is allocated between *actual spending on children's education* $p_t e_t$, and *actual saving* $p_t s_t$ for the future wealth of children:

$$pb_t = p_t s_t + p_t e_t. \tag{17}$$

The investment in education is devoted to increasing children's human capital. In particular, each child with a total parental investment in education $p_t e_t$ receives an amount of $\bar{e}_t \equiv p_t e_t / n_t$ and acquires:

$$h_{t+1} = h\left(\bar{e}_t\right) = \left(1 + D\bar{e}_t\right)^{\gamma}, \text{with } \gamma \in (0,1) \text{ and } D > 0, \tag{18}$$

efficiency units of human capital, where $h(0) = 1$, $h'(0) = \gamma D$ and $\lim_{\bar{e}_t \to \infty} h'(\bar{e}_t) = 0$ (see Galor and Moav, 2004, 2006). For $\gamma \geq 1$ human capital accumulation alone could generate positive long-run growth. D is a scale parameter.

Individual preferences are defined for a consumption above a *subsistence level* $c^{\text{MIN}} > 0$, the transfer to their children b_t and the total number of children who survive to adulthood n_t. Easterlin (2004) and Galor (2005) show that the main variable for fertility choice is the number of children surviving to adulthood. The expected utility function of altruistic individuals born in period $t-1$ is therefore:[8]

$$U = p_t \left[(1-\beta)\log(c_t) + \beta \log(b_t + \theta) + \epsilon \log(n_t)\right], \tag{20}$$

where $\theta > 0$ implies that children receive a positive transfer only when the parents' income is sufficiently high (see Eq. (24) below).

The optimal problem of parents is therefore:

$$(c_t^*, b_t^*, n_t^*) = \underset{c_t, b_t, n_t}{\arg\max} \left\{p_t \left[(1-\beta)\log(c_t) + \beta \log(b_t + \theta) + \epsilon \log(n_t)\right]\right\}, \tag{21}$$

[8] Following Rosen (1988) we assume that the expected utility in the second period is given by the utility of the state "life", which is given by the utility from consumption and the transfer to the children, and the utility of the state "death"which is given by M. In the analysis, M, without loss of generality, is assumed to be equal to zero:

$$U = p_t \left[(1-\beta)\log(c_t) + \beta \log(b_t + \theta)\epsilon \log(n_t)\right] + (1-p)M, \tag{19}$$

subject to:

$$\begin{aligned} y_t &= p_t c_t + p_t b_t + \delta n_t y_t; \\ c_t &\geq c^{\text{MIN}}; \text{and} \\ b_t &\geq 0, \end{aligned}$$

Assumption 1 *In order to ensure that for low levels of income optimal consumption is increasing with income per worker, while optimal bequest is zero (i.e. $y^{\text{MIN}} < y^{\text{CAP}}$, see Eqs. (26) and (28) below) we assume that:*

$$c^{\text{MIN}} < \frac{(1-\beta+\epsilon)\theta}{\beta}. \tag{22}$$

The optimal levels of consumption, transfer and number of surviving children are given by:[9]

$$c_t^* = \begin{cases} c^{\text{MIN}} & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{SUB}}] \\ \dfrac{(1-\beta)y_t}{(1-\beta+\epsilon)p_t} & \text{if } y_t \in (y_{\text{SUB}}, y^{\text{CAP}}] \\ \dfrac{(1-\beta)(y_t+p_t\theta)}{(1+\epsilon)p_t} & \text{if } y_t \in (y^{\text{CAP}}, \infty) \end{cases} \tag{23}$$

and:

$$b_t^* = \begin{cases} 0 & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{CAP}}] \\ \dfrac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{p_t(1+\epsilon)} & \text{if } y_t \in (y^{\text{CAP}}, \infty) \end{cases} \tag{24}$$

and:

$$n_t^* = \begin{cases} \dfrac{y_t - p_t c^{\text{MIN}}}{\delta y_t} & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{SUB}}] \\ \dfrac{\epsilon}{\delta(1-\beta+\epsilon)} & \text{if } y_t \in (y^{\text{SUB}}, y^{\text{CAP}}] \\ \dfrac{\epsilon(y_t+\theta p_t)}{\delta(1+\epsilon)y_t} & \text{if } y_t \in (y^{\text{CAP}}, \infty) \end{cases} \tag{25}$$

where:

$$y^{\text{MIN}} = p_t c^{\text{MIN}}; \tag{26}$$

$$y^{\text{SUB}} = \frac{p_t c^{\text{MIN}}(1-\beta+\epsilon)}{1-\beta}, \tag{27}$$

and:

$$y^{\text{CAP}} = \frac{\theta(1-\beta+\epsilon)p_t}{\beta}. \tag{28}$$

[9] See Appendix in Fiaschi and Fioroni (2014).

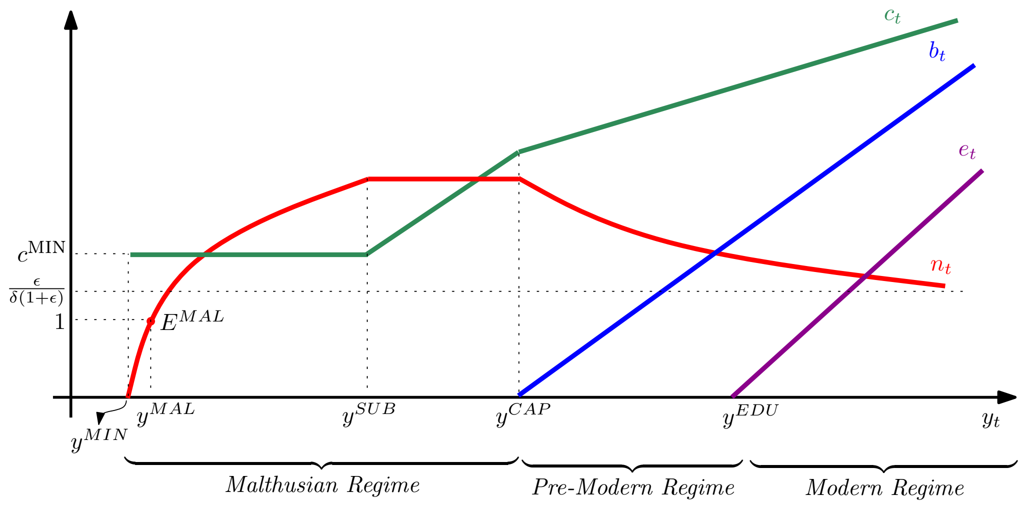


Figure 6: Optimal choices of individuals with endogenous fertility

Fertility is increasing with income in the first range and reaches its maximum level in the range $y_t \in (y^{\text{SUB}}, y^{\text{CAP}}]$, and then declines (see Fig. 6). The *quality-quantity trade-off* for children discussed in Becker et al. (1990) starts operating at the moment parents leave a positive transfer to children.

For very low levels of income per worker, i.e. $y_t < y^{\text{SUB}}$, consumption is at its subsistence level, the optimal transfer is zero, while the optimal number of children increases with respect to y_t (see Fig. 6): thus the economy is a Malthusian regime. In a Malthusian regime any increase in income per worker (due to improvement in technological progress and/or adult survival) results in a surge of population, which, in presence of diminishing returns to labour, leads to a subsequent fall in income per worker; y_t is therefore doomed to stagnate just above its minimum level in the long run. Simple calculations show that in the range $y_t \in (y^{\text{MIN}}, y^{\text{SUB}}]$ there exists a unique stable equilibrium level of y_t for which the population growth is zero (i.e. $n_t = 1$), that is:

$$y^{MAL} = \frac{p_t c^{\text{MIN}}}{1-\delta}, \tag{29}$$

to which the equilibrium level of the observed adult population corresponds [10]:

$$p_t L^{MAL} = \left[\frac{A_t(1-\delta)}{c^{\text{MIN}}}\right]^{1/\alpha} \tilde{T}. \tag{30}$$

When y_t rises (due to strong increases in technological progress and mortality reductions) above the level y^{SUB}, this allows parents to escape from subsistence consumption and the fertility

[10] Eq. (30) follows from:

$$y_t = A_t p_t^{1-\alpha} \left(\frac{\tilde{T}}{L_t}\right)^{\alpha} = \frac{p_t c^{\text{MIN}}}{1-\delta}.$$

rate becomes constant. However, the economy is still in the Malthusian regime since parents do not have a sufficient level of income to leave a positive transfer to their children.

If y_t continues to increase, the constancy of the fertility rate ensures that at a certain period $y_t > y^{\text{CAP}}$: the economy moves into the pre-modern regime, where parents start devoting a fraction of income for the wealth of their children and the relationship between income and population growth becomes negative (the *quantity-quality trade-off* starts operating).

4.3 Adult Survival and the Optimal Choices

Eq. (25) shows that fertility has a non-linear relationship with adult survival; first increasing and then decreasing. In particular, when income per worker is very low, such that consumption is around its subsistence level, fertility decreases with adult survival. The increase in adult survival, indeed, raises the total amount of resources needed to maintain consumption at subsistence level. In other words, at the subsistence level of consumption, the increase in adult survival raises population beyond "the capacity sustainable by the available resources and thus it was reduced by the preventive check"(Galor, 2011, p.67).

On the contrary, when income is sufficiently high, i.e. $y_t > y^{\text{CAP}}$, fertility increases with adult survival. An increase in adult survival, indeed, at high level of income results in an increase in the number of children through a positive income effect (see Cervellati and Sunde, 2014). In other words the increase in adult survival at a high level of income has two opposing effect: a direct positive effect and an indirect negative effect through the increase in income. However since the elasticity of income with respect to adult survival is lower than 1, then the increase in adult survival leads to an increase in fertility.[11] Intuitively, an increase in adult survival increases the time available for the parent and thereby positively affects the number of births.

As shown in Eq.(23), a rise in adult survival in period t, when income is above subsistence level, increases *actual* consumption in adulthood, i.e. $\partial p_t c_t / \partial p_t = (1-\beta)(\partial y_t/\partial p_t + \theta) > 0$.

By contrast, the *actual* transfer pb_t has an inverted U-shaped relationship with respect to adult survival. The decline in adult mortality $1 - p_t$ has in fact two opposing effects on transfer: on the one hand, higher longevity increases parents' consumption, thus reducing the overall transfer to their offspring; on the other, parents with a longer working life, experience

[11] In particular:

$$\frac{\partial n_t}{\partial p_t} = \frac{\epsilon\theta}{\delta(1+\epsilon)y_t}\left(1 - \frac{p_t}{y_t}\frac{\partial y_t}{\partial p_t}\right) > 0, \tag{31}$$

where:

$$\frac{p_t}{y_t}\frac{\partial y_t}{\partial p_t} = 1 - \alpha. \tag{32}$$

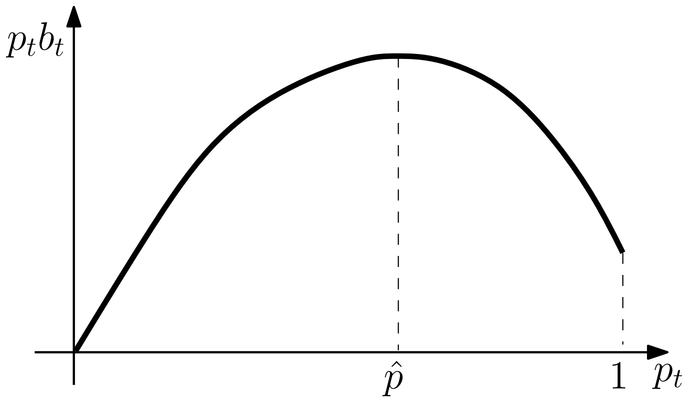


Figure 7: Actual transfer versus adult survival rate

an increase in their income, thus raising the transfer to their offspring. When the initial level of income is sufficiently high, the latter effect always prevails, whereas at low levels of income the opposite is true. In particular, from Eq. (14) there exists a threshold level of adult survival rate denoted by $\hat{p}$, such that for $p_t < (>)\hat{p}$ the rise in adult survival rate positively (negatively) affects the total transfer to the offspring (see Fig. 7):

$$\hat{p} = \left[\frac{\beta(1-\alpha)A_t}{\theta(1-\beta+\epsilon)}\right]^{1/\alpha}\left(\frac{\tilde{T}}{L_t} + h_t^{\frac{1-\alpha}{\alpha}} k_t\right). \tag{33}$$

This threshold increases with the level of development of the country, i.e. with respect to A_t, h_t and k_t. On the other hand, it decreases with respect to the size of the workforce L_t.

4.4 Accumulation of Physical and Human Capital

The transfer is allocated between saving, i.e. accumulation of physical capital, and education, i.e. accumulation of human capital (see Eq. (17)). However, the economy begins to accumulate physical capital only when parents are sufficiently rich (i.e. $y_t > y^{\text{CAP}}$, see Eq. (24)) to leave a positive transfer to their offspring; and to accumulate human capital for a still higher level of income (i.e. $y_t > y^{\text{EDU}}$, see Eq. (40) below).

The total physical capital stock in period $t+1$ is given by the aggregate saving in period t:

$$K_{t+1} = L_t p_t s_t = L_t\left(p_t b_t - p_t e_t\right). \tag{34}$$

Given the fertility rate n_t, adult population L_t evolves according to:

$$L_{t+1} = n(y_t)L_t. \tag{35}$$

The capital/labour ratio is therefore equal to:

$$k_{t+1} = \bar{b}_t - \bar{e}_t, \tag{36}$$

where $\bar{b}_t \equiv p_t b_t/n_t$ is the actual transfer per child, and $\bar{e}_t \equiv p_t e_t/n_t$ is the actual investment in education per child. In particular:

$$\bar{b}_t = \frac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{(1+\epsilon)n(y_t)} \tag{37}$$

Parents choose the amount to invest in their children's education in order to maximise the future income of their offspring i.e. y_{t+1}. In the early stages of development, when productivity in the industrial sector is relatively low with respect to productivity in the traditional sector, individuals do not have the incentive to invest in the human capital of their children. However, improvements in technological progress, the accumulation of physical capital and raising the efficiency of industrial technology, will all lead to a positive demand for human capital. From Eqs. (14) and (18) it follows that:

$$e_t^* = \arg\max_{e_t\in[0,b_t]} \left\{ A_{t+1}p_{t+1}^{1-\alpha}\left[\frac{\tilde{T}}{L_{t+1}} + \left(1+\frac{Dp_t e_t}{n(y_t)}\right)^{\frac{\gamma(1-\alpha)}{\alpha}} k_{t+1}\right]^{\alpha}\right\}, \tag{38}$$

where k_{t+1} is given by Eq. (36). Eq. (24) shows that the optimal level of education is positive only if income is sufficiently high, i.e.:

$$\bar{e}_t^* = \begin{cases} 0 & \text{if } y_t \in [y^{\text{MIN}}, y^{\text{EDU}}]; \\ \dfrac{\beta y_t - \theta(1-\beta+\epsilon)p_t - \tilde{b}n(y_t)(1+\epsilon)}{(1+D\tilde{b})(1+\epsilon)n(y_t)} & \text{if } y_t \in (y^{\text{EDU}}, \infty) \end{cases} \tag{39}$$

where y^{EDU} is the solution of the following implicit function:

$$y_t^{\text{EDU}} = \frac{\tilde{b}\epsilon/\delta + \theta(1-\beta+\epsilon)p_t + \sqrt{[\tilde{b}\epsilon/\delta + \theta(1-\beta+\epsilon)p_t]^2 - \frac{4\beta\tilde{b}\epsilon\theta p_t}{\delta}}}{2\beta}, \tag{40}$$

and:

$$\tilde{b} \equiv \frac{\alpha}{D(1-\alpha)\gamma}. \tag{41}$$

From Eqs. (36) and (39) the capital-labour ratio in period $t+1$ is given by:

$$k_{t+1} = \begin{cases} 0 & \text{if } y_t \in [y^{\text{MIN}}, y^{\text{CAP}}]\,; \\ \dfrac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{n(y_t)(1+\epsilon)} & \text{if } y_t \in (y^{\text{CAP}}, y^{\text{EDU}}]\,; \\ \left(\dfrac{\tilde{b}}{1+D\tilde{b}}\right)\left\{1 + D\left[\dfrac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{n(y_t)}\right]\right\} & \text{if } y_t \in (y^{\text{EDU}}, \infty)\,. \end{cases} \tag{42}$$

4.5 The Stages of Development

Eqs. (24), (39) and (42) allow us to characterise the dynamic of income per worker in period $t+1$ as follows:

$$y_{t+1} = \begin{cases} \frac{A_{t+1}p_{t+1}^{1-\alpha}\tilde{T}^{\alpha}}{L_t^{\alpha}}\left(\frac{\delta y_t}{y_t - p_t c^{\text{MIN}}}\right)^{\alpha} & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{SUB}}]; \\ \frac{A_{t+1}p_{t+1}^{1-\alpha}\tilde{T}^{\alpha}}{L_t^{\alpha}}\left[\frac{\delta(1-\beta+\epsilon)}{\epsilon}\right]^{\alpha} & \text{if} y_t \in (y^{\text{SUB}}, y^{\text{CAP}}]; \\ A_{t+1}p_{t+1}^{1-\alpha}\left[\frac{\delta(1+\epsilon)y_t}{\epsilon(y_t+\theta p_t)}\right]^{\alpha}\left[\frac{\tilde{T}}{L_t} + \frac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{1+\epsilon}\right]^{\alpha} & \text{if } y_t \in (y^{\text{CAP}}, y^{\text{EDU}}]; \\ A_{t+1}p_{t+1}^{1-\alpha}\left\{\frac{\tilde{T}}{L_t}\frac{\delta(1+\epsilon)y_t}{\epsilon(y_t+\theta p_t)} + \tilde{b}\left[\frac{\epsilon(y_t+\theta p_t) + D(\beta y_t - \theta(1-\beta+\epsilon)p_t)\delta y_t}{\epsilon(y_t+\theta p_t)(1+D\tilde{b})}\right]^{\frac{\gamma(1-\alpha)}{\alpha}+1}\right\}^{\alpha} & \text{if } y_t \in [y^{\text{EDU}}, +\infty), \end{cases} \tag{43}$$

The three ranges of y_t correspond to i) the Malthusian regime, where $y_t \in (y^{\text{MIN}}, y^{\text{CAP}})$ and production is conducted using traditional technology; ii) the pre-modern regime where $y_t \in (y^{\text{CAP}}, y^{\text{EDU}})$ and output is the result of using physical capital and unskilled labour; and iii) the modern regime where $y_t > y^{\text{EDU}}$ and both physical and human capital are jointly used in production.

Assumption 2 *Assume that $y_t \geq p_t c^{\text{MIN}}$ $\forall t$, that is $A > A^{\text{MIN}}$, where:*

$$A^{\text{MIN}} \equiv c^{\text{MIN}}\left(\frac{pL_t}{\tilde{T}}\right)^{\alpha} \tag{44}$$

As A increases the economy will pass through all three regimes. The following Proposition states the conditions under which we observe one or more than one equilibrium in the three regimes.

Proposition 1 *Suppose that the level of technological progress and adult survival are constant over time, i.e. $A_{t+1} = A_t = A$ and $p_{t+1} = p_t = p$, under Assumptions 1 and 2 and*

$$\frac{\tilde{T}}{L_t} > \frac{\tilde{b}}{\left[\frac{\tilde{b}(1+\epsilon)+\theta(1-\beta+\epsilon)}{\theta(1-\beta+\epsilon)}\right]^{\frac{1-\alpha}{\alpha}}\left(\frac{\tilde{b}+\theta}{\theta}\right) - 1}$$

- *if $A \in [A^{\text{MIN}}, A^{\text{TRA}})$, then there exists at least one stable equilibrium in the Malthusian regime, where A^{MIN} is defined in Eq.(44) and:*

$$A^{\text{TRA}} = \frac{\theta(1-\beta+\epsilon)^{1-\alpha}}{\beta}\left(\frac{\epsilon p L_t}{\delta\tilde{T}}\right)^{\alpha}. \tag{45}$$

- *If $A \in [A^{\text{TRA}}, A^{\text{PRE-MOD}}]$, then there exists at least one stable equilibrium in the pre-modern regime:*

$$A^{\text{PRE-MOD}} = \frac{\left[\tilde{b}\left(1+\epsilon\right)+\theta\left(1-\beta+\epsilon\right)p\right]^{1-\alpha}\left[\epsilon\left(\tilde{b}+\theta p\right)\right]^{\alpha}}{\beta\delta^{\alpha}\left(\tilde{T}/L_t+\tilde{b}\right)^{\alpha}p^{1-\alpha}}. \tag{46}$$

- *Finally, if $A > A^{\text{PRE-MOD}}$, then there exists at least one stable equilibrium in the modern regime.*

Proof. See Appendix in Fiaschi and Fioroni (2014). ■

The transition from the Malthusian to the pre-modern regime is driven by an increase in traditional production, which makes a positive transfer to offspring possible, the source of accumulation of physical capital. Instead, the transition from the pre-modern to the modern regime is driven by the higher accumulation of physical capital generated by an increase in productivity in the modern sector. This accumulation, in turn, increases the return on the investment in human capital, and, therefore, incentivizes this type of investment. Once the accumulation of human capital starts, a self-reinforcing mechanism operates to further increase the income per worker. Decreasing returns in the accumulation of human capital (i.e. $\gamma < 1$) limit, in our model, the possibility of positive long-run growth (see Robert E. Lucas, 2004).

5 A Quantitative Exploration of the Model

In this section we explore the properties of the model by numerical simulations. Our theoretical premise is that the causes of technological progress and adult mortality are exogenous to the economic sphere (see Fiaschi and Fioroni (2014) for more details)[12]. Physical and human capital, fertility, population, income, and consumption are instead our key endogenous variables.

In all the simulations we use the parameters calibrated for the UK economy for the period in $1541 - 1914$ as discussed in Fiaschi and Fioroni (2014), and the initial population and fertility rate are always both set equal to 1, i.e. $L_0 = 1$ and $n_0 = 1$ if not differently stated.

First we will discuss how the combination of technological progress and adult survival in an economy determines which regime this economy will belong to over time. Then we will analyze how continuous changes in technological progress and adult survival move an economy through the different regimes.

[12]It would possible to extend the model to some plausible cases of endogeneity of technological progress and mortality; for example, Cervellati and Sunde (2014) link human capital accumulation to the decline in mortality. However, this feedback from income (human capital) to mortality would make the mechanics linking mortality to income less clear.

5.1 Simulation of Regimes

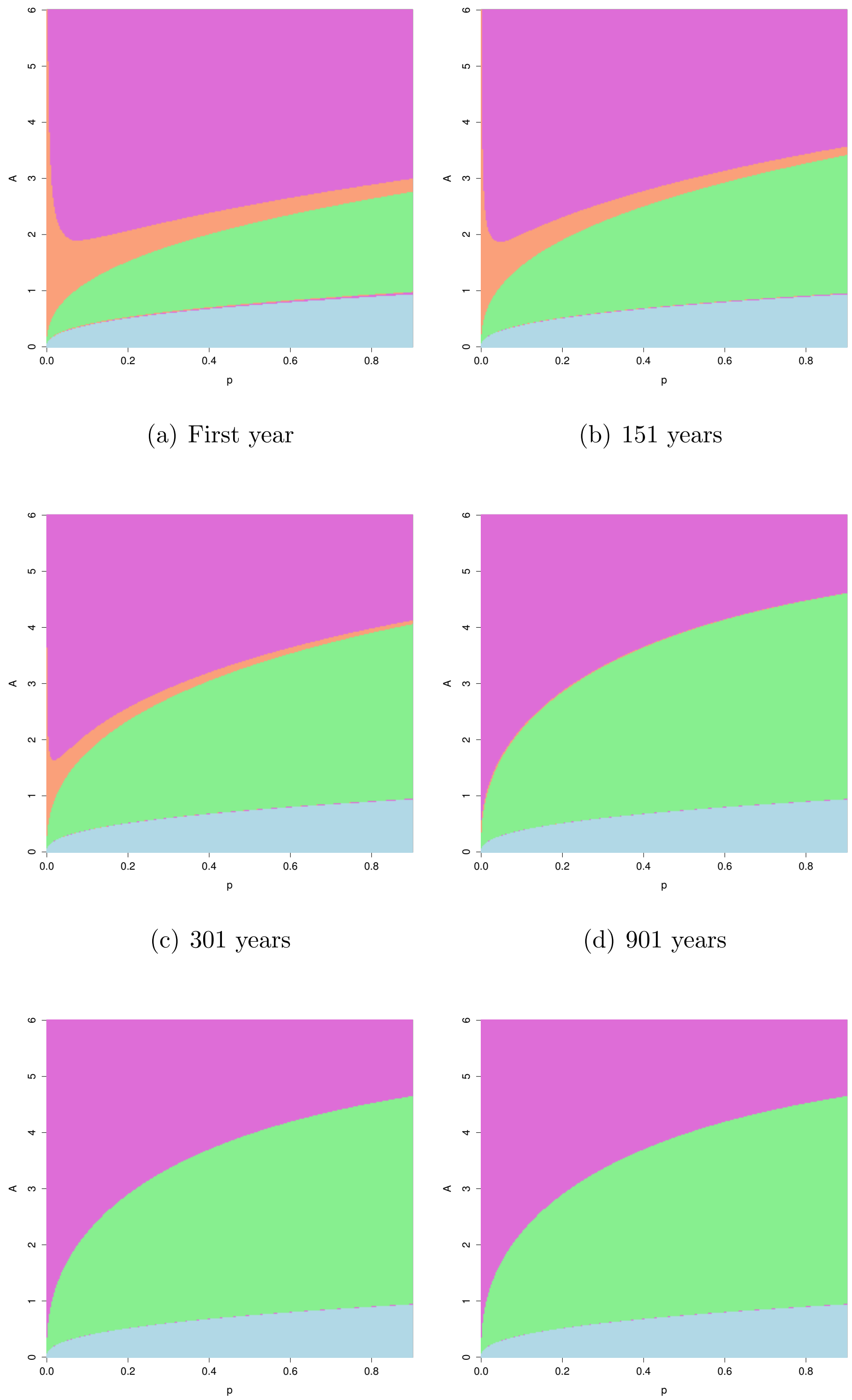
Fig. 8(a) maps the combinations of technological progress and adult survival rate in terms of the three different regimes; in particular, the violet, orange and green areas indicate the couples of (A, p) such that the economy is in the modern, pre-modern and Malthusian regimes after one year, while the light-blue area indicates the couples of (A, p) such that $y_t < y^{\text{MIN}}$ (this possibility is excluded by the assumption of the existence of a minimum consumption). Figg. 8(b), 8(c), 8(d), 8(e), and 8(f) represent the maps of combinations in terms of the three different regimes after 151, 301, 901, 3001, and 6001 years.

After 901 years the pre-modern regime disappears and the economy appears to converge to its long-run equilibrium (maps after 3001 and 6001 are substantially the same of the one after 901). In particular, combinations of (A, p) with low level of p initially in the pre-modern regime are included in the modern regime over time; by contrast, combinations with high level of p are trapped in the Malthusian regime. The key mechanism at work here is the population size, which is strongly checked at low level of p. Simulations also highlight the very long time scale of the phenomena we are considering: more than 900 years are needed to converge to equilibrium. In this view 151 years should be considered the short run, while a minimum of 901 years are needed to arrive at the long run.

Fig.8 reveals that advances in technological progress alone always allow the transition through the different regimes; by contrast, increments in adult survival can damage the possibilities of a country to transit from the pre-modern to the modern regime.

Under the set of parameters used in the simulations in the long run the higher incentives to accumulate human and physical capital determined by mortality declines cannot overwhelm the negative check on income imposed by the increase in population size; in this respect our calibrated version of the model fully agrees with Acemoglu and Johnson (2007, 2014)'s empirical findings on the existence of a negative or null effect of the improvements in health conditions on the income of countries. Of course, this conclusion heavily depends on the lack of any causal link between human capital accumulation and mortality decline and advances in technological progress.

In the short run (i.e. 151 years in our time scale) it is possible to observe for a very limited (and not very plausible) combination of (A, p) (A around 2.5 and p around 0 in Fig. 8) a positive effect of a decline in mortality, i.e. a transition from the pre-modern to the modern regime. Since this transition is also associated with an ongoing demographic transition, i.e. a decline in fertility rate, this result is compatible with Cervellati and Sunde (2011)'s empirical finding of an asymmetry of the effect of mortality declines on income depending on the demographic regime of country. However, our theoretical model suggests that in the long run this positive



(a) First year (b) 151 years

(c) 301 years (d) 901 years

(e) 3001 years (f) 6001 years

Figure 8: Simulation of regimes: the violet, orange, green areas indicate the couples of (p, A) such that the economy is in the modern, pre-modern and Malthusian regimes after t years respectively. The light-blue area indicates the couples of (p, A) such that $y_t < y^{\text{MIN}}$.

effect could disappear if not associated with an increase in technological progress.

5.2 The Role of the Mortality Rate in the Transition between Regimes

As shown in Section 2 technological progress and adult survival rate has experienced dramatic advances in the last 300 years in the most of countries. In the theoretical model these ongoing dynamics of (A, p) cause continuous changes in physical and human capital, fertility, and overall population size, which make a general analysis of regime transition very complex. In this respect, we limit our study to four trajectories of (A, p), reported in Fig. 9, with a time span of 301 years.[13]

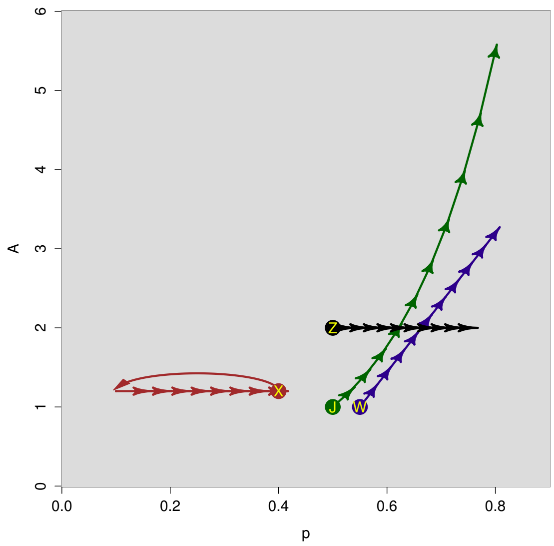


Figure 9: Four different trajectories in the plane (A, p) with a time span of 301 years.

Point J in Fig. 9 represents an economy with an initially low level of technological progress and a middle level of adult survival rate. The trajectory starting from point J (the green line) considers simultaneous improvements in technological progress and adult survival rate, but with an overall prevalence of the advances of the technological progress with respect to the ones in the adult survival rate. Fig. 10(a) reports the time path of income per worker, physical capital, human capital, consumption, fertility rate and workforce related to trajectory J, and the green, red and blue shadow areas indicate when the economy is in a Malthusian, pre-modern and modern regime respectively. Along trajectory J the economy transits through all

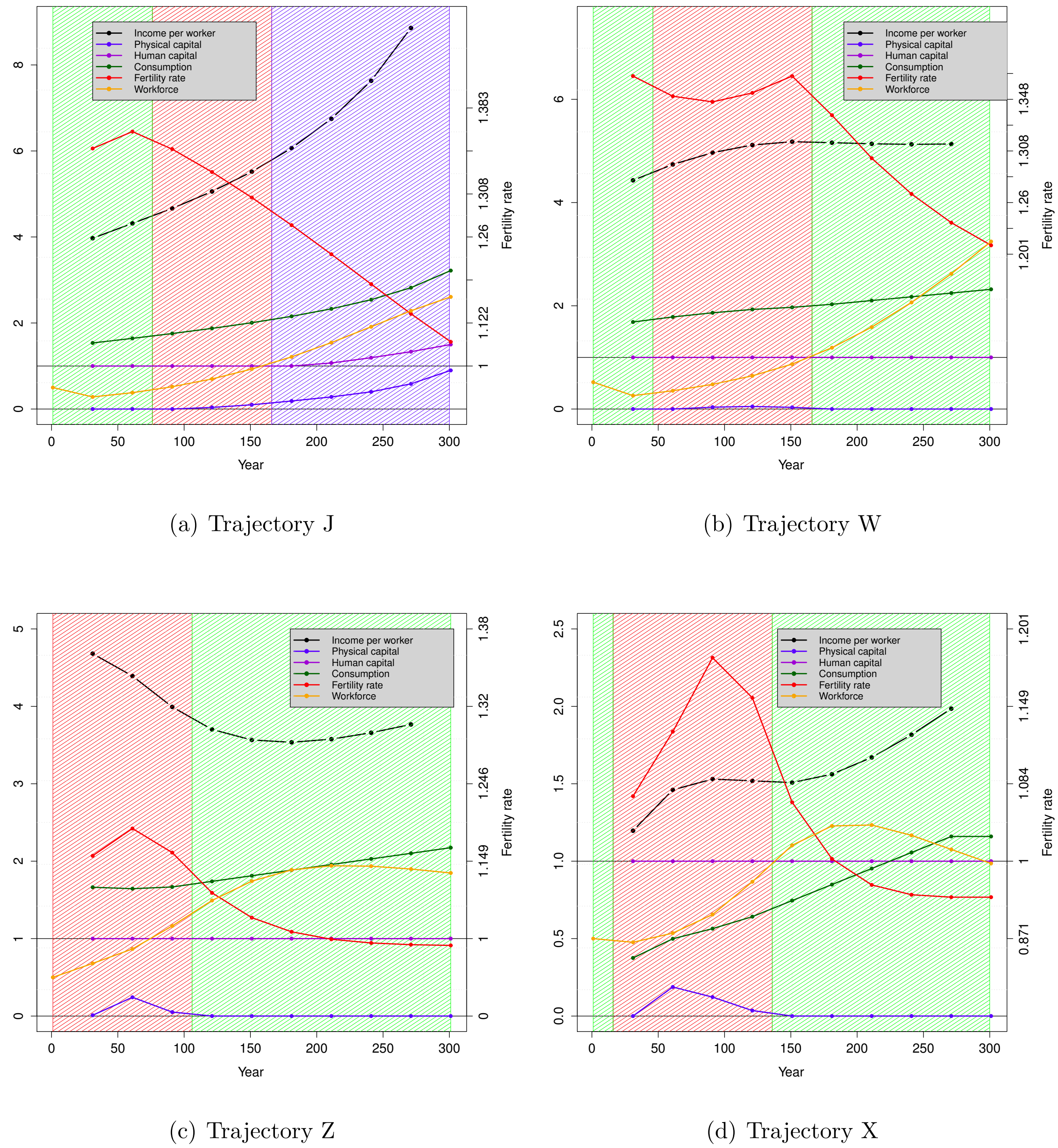
[13] In all the above simulations we set $L_0 = 0.5$ and $n_0 = 1$. We changed the initial level of population in order to make the results of the simulations clearer.

three regimes; accordingly, the income per worker is increasing at an exponential rate, reflecting the growth of technological progress and the increments in adult survival rate. Intergenerational transfers are also increasing, causing an exponential growth of the investments in physical and human capital. The fertility rate starts declining in the period where the economy transits from Malthusian to pre-modern regime and is converging to 1; population size is therefore doomed to stabilize in the long run. Trajectory J seems to match the key features of the development path followed by most western countries in the last 300 years (see Section 2)

Consider now an economy starting from point W in Fig. 9, very similar in its initial condition to the economy of point J. With respect to trajectory J, trajectory W has the same increments in adult survival rate but a limited overall increments in technological progress. In a first phase the joint improvements of A and p push the economy from the Malthusian to the pre-modern regime (see Fig. 10(b)), causing therefore a growth in income per worker and a decline in the fertility rate. The overall dynamics would suggest that this economy is doomed to transit to modern regime and therefore goes through a modern economic growth in the long run. However, the limited improvements in technological progress only partially compensate the negative pressure on income per worker deriving from the strong surge in population size. Fertility rate, indeed, as result of raising adult survival rate, starts increasing, in the pre-modern regime (see Eq. (25)). Over time, the pressure of population size on economy prevails on technological progress, and the economy moves back to the Malthusian regime, where the effect of the improvements in the adult survival rate on fertility rate becomes negative (see again Eq. (25)). The impression is therefore of a sort of "false" take-off (see Section 2.3)

The trajectory starting from point Z in Fig. 9 considers the limiting case of an economy starting in a pre-modern regime which only undergoes advances in adult survival. Fig. 10(c) highlights the negative effects of an upward trend in adult mortality rates without any technological progress: the economy is pushed from the pre-modern regime into a Malthusian regime where the increasing population size negatively affects income per worker, which declines over time converging to a Malthusian level.

Finally, the trajectory starting from point X in Fig. 9 considers a scenario in which a dramatic fall in the adult survival rate, caused for example by an epidemic such as the Black Death, moves an economy away from a Malthusian regime to a pre-modern regime. However, as the adult survival rate undertakes the recovery to its initial level, the economy starts slipping toward a Malthusian regime via a burst in population size. Along trajectory X just after the shock, income per worker displays a positive trend, to then stabilize after 100 years. However, as population size reaches a critical level the economy transits to a Malthusian regime after 150 years. In such a regime the rise of the adult survival rate leads to a strong reduction in the



(a) Trajectory J

(b) Trajectory W

(c) Trajectory Z

(d) Trajectory X

Figure 10: Simulation of the dynamics of GDP per worker, consumption, physical and human capital, fertility rate, workforce for the four trajectories of A and p reported in Fig. 9. The green, red and blue bars indicate respectively the Malthusian, pre-modern and modern regimes.

fertility rate, ultimately leading to a shrink in the population size and income per worker begins to grow (a typical Malthusian demographic cycle emerges). The lack of any (positive) feedback from the accumulation of physical (human) capital to advances in technological progress explains the difference of our results with respect to Voigtlander and Voth (2012) in the analysis of the effects of Black Death.

6 Concluding Remarks

The main message of our theoretical model is that mortality decline alone, despite the higher incentives to the accumulation of physical and human capital, has a negative impact on income in the long run (in the short run, where demographic pressure on resources is not yet fully working, an opposite result is possible). The observed positive correlation between mortality decline and income would therefore be the result of the (almost) contemporaneous advances in technological progress and health conditions (see Fiaschi and Fioroni, 2014 for a discussion on why these advances can stem from the same source). A possible positive feedback from income to health would configure a self-reinforcing mechanism which should not affect this conclusion.

However, our argument relies on the crucial assumptions of no effect of physical and human capital accumulation on technological progress (see Galor, 2011 for an opposite view), and on a theory of fertility choice which is questionable both on the theoretical and empirical level (see Easterlin, 2004).

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A List of Countries

The table below shows the list of 17 currently advanced countries. Countries for which it is possible to calculated the adult survival rate are indicated in bold.

1	**United Kingdom of Great Britain and Northern Ireland**
2	**Sweden**
3	**Finland**
4	**Denmark**
5	**France**
6	Canada
7	**Belgium**
8	**Norway**
9	**Netherlands**
10	Japan
11	**Austria**
12	Italy
13	**Germany**
14	**Switzerland**
15	Greece
16	United States of America
17	Spain

Table 5: List of 17 western countries

Algeria	Japan
Bahrain	Jordan
Benin	Lebanon
Bolivia	Malaysia
Brazil	Mauritius
Bulgaria	Mexico
Colombia	Morocco
Congo, Rep.	Mozambique
Costa Rica	Namibia
Cuba	Nicaragua
Djibouti	Panama
Ecuador	Paraguay
El Salvador	Peru
Gabon	Poland
Ghana	Portugal
Greece	Puerto Rico
Guatemala	Romania
Honduras	Saudi Arabia
Hong Kong SAR, China	Senegal
Hungary	Seychelles
Iran, Islamic Rep.	Singapore
Iraq	South Africa
Israel	Spain
Italy	Sri Lanka
Jamaica	Syrian Arab Republic
Tunisia	Turkey

Table 6: List of 52 lower-middle-income countries

Afghanistan	Lesotho
Albania	Liberia
Angola	Libya
Bangladesh	Madagascar
Botswana	Malawi
Burkina Faso	Mali
Burundi	Mauritania
Cabo Verde	Mongolia
Cambodia	Myanmar
Cameroon	Nepal
Central African Republic	Niger
Chad	Nigeria
China	Oman
Comoros	Pakistan
Congo, Dem. Rep.	Philippines
Cote d'Ivoire	Rwanda
Dominican Republic	Sao Tome and Principe
Egypt, Arab Rep.	Sierra Leone
Equatorial Guinea	Somalia
Gambia, The	Swaziland
Guinea	Tanzania
Guinea-Bissau	Thailand
Haiti	Togo
India	Uganda
Kenya	Vietnam
Korea, Dem. Rep.	West Bank and Gaza
Korea, Rep.	Zambia
Lao PDR	Zimbabwe

Table 7: List of 57 low-income countries