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Collective Household Welfare and Intra-household Inequality*

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Abstract

We investigate the relationship between the individual and household indirect utility functions in the context of a collective household model. Our analysis produces new results that explain how the rule governing the distribution of resources within household members is related to the measurement of household welfare and intra-household inequality. By examining the relationship between centralized and decentralized program, we show that, in general, income shares are equal to the product of two weights: the Pareto weight and a weight reflecting income effects across individuals. For a weighted Bergsonian representation of household utility and general assumptions about individual preferences, we derive the associated household welfare functions and intra-household inequality measures belonging to a family of entropy indexes.

Keywords: Collective household model, Sharing rule, Household welfare, Intra-household inequality, Theil index.
JEL Classification: D11, D12, D13.

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1 Introduction

In consumption analysis, the role of aggregation across individuals has been the subject of much interest (Blundell and Stoker 2005, Chiappori and Ekeland 2011, Gorman 1953, 1961, Jorgenson, Lau and Stoker 1980, 1982, Muellbauer 1975). These studies were treating households as individuals by implicitly assuming an egalitarian distribution equating the levels of well-being across family members. An established evidence of the more recent collective theory of the household (Browning and Chiappori 1998, Browning *et al.* 2014, Chiappori 1988, 1992) is that the levels of well-being of each family member differ and can be estimated. The knowledge of the individual levels of well-being entails an articulate family welfare function that relates the collection of potentially unequal levels of well-being of family members to an aggregate measure for the family as a whole (Sen 1984: 378). This "mini social choice problem," as Sen terms it, involves understanding the linkages between individual behavior, household demand and the aggregation of unequal well-beings. However, the relationships between individual and household welfare are complex and still not fully understood. This suggests a need for a refined analysis of the linkages between individual demand behavior and household welfare. The composition of this missing piece of aggregation theory is our major contribution.

In the tradition of collective household models, the efficient allocation of resources within a household is captured by a Bergsonian household utility function involving different welfare weights applied to each individual. Maximizing this Bergsonian utility function subject to a household budget constraint generates centralized household demands. We show that the Bergsonian (Pareto) weights are the outcome of a bargaining process in a household utility function but they also affect how household income gets distributed across household members generating a generalized sharing rule. Given this income sharing rule, efficient allocation within the household is consistent with decentralized utility maximization for each individual. This generates decentralized allocations and individual demands. By examining the relationship between centralized and decentralized demand, we show that income shares are equal to the product of two weights: the Pareto weight and a weight reflecting income effects across individuals. We also show how these weights play a role in the evaluation of both household welfare and intra-household inequality. Our results are in line with Chiappori and Meghir's (2015) key remark asserting that the sharing rule contains all the information required to implement the measurement of intrahousehold inequality because it is possible to construct a general inequality index as a function of individual incomes. Our analysis shows that for a weighted Bergsonian representation of household utility and general assumptions about individual preferences, the inequality index takes the form of a family of entropy indexes.

The paper is structured as follows. The next section introduces the basic model of collective consumption uncovering interesting features linking the centralized and decentralized demand representations. Section 3 examines the income sharing rules within the household. Section 4 presents an evaluation of household welfare along with the role played by individual preferences and welfare weights for different functional forms of empirical interest and derives measures of inequality describing intra-household distribution of resources in a formal way. The final section concludes.

2 The Collective Model of Household Consumption

Our household model holds under fairly general conditions. Consider a household composed of K individuals involved in the consumption of goods. The k^{th} individual consumes a bundle of n_k goods $\mathbf{x}_k = (x_{k1}, ..., x_{kn_k}) \in \mathbb{R}^{n_k}$ with associated prices $\mathbf{p}_k = (p_{k1}, ..., p_{kn_k}) \in \mathbb{R}^{n_k}_{++}$, $k = 1, ..., K$. Let $\mathbf{x} = (\mathbf{x}_1, ..., \mathbf{x}_K)$ and $\mathbf{p} = (\mathbf{p}_1, ..., \mathbf{p}_K)$. In our conceptual framework, the vector of market prices $\mathbf{p}$ is given. The k^{th} household member has preferences over $\mathbf{x}_k$ represented by a non-satiated and quasi-concave utility function u_k: $\mathbb{R}^{n_k} \to \mathbb{R}$, $k = 1, ..., K$.

Our analysis applies under two important scenarios. The first scenario is the case where the goods in $\mathbf{x}_k$ are private goods assigned to the k^{th} individual, for instance clothing.

Then the prices $\mathbf{p}_k$ are simply the market prices for private goods, $k = 1, ..., K$. The second scenario is the one where the household is involved in production activities, using a bundle of goods $\mathbf{z} = (z_1, ..., z_n) \in \mathbb{R}^n$ to produce the consumer goods $\mathbf{x} = (\mathbf{x}_1, ..., \mathbf{x}_K)$. The goods in $\mathbf{z}$ can be private goods or public goods, purchased by the household at prices $\mathbf{q} = (q_1, ..., q_m) \in \mathbb{R}^m_{++}$. The household production technology is represented by the feasible set F where $(\mathbf{x}, \mathbf{z}) \in F$ means that goods $\mathbf{x}$ can be obtained when $\mathbf{z}$ is purchased (Barten 1964, Browning and Chiappori 1998, Chavas 1989, Ferreira and Perali 1992, Lewbel 1985, Perali 2003). Assume that the production technology exhibits constant return to scale (CRS). Under efficiency, the household purchases goods $\mathbf{z}$ to produce consumer goods $\mathbf{x}$ so as to minimize expenditure $E(\mathbf{x}) = \min_z \{\mathbf{q}\,\mathbf{z} : (\mathbf{x}, \mathbf{z}) \in F\}$. Let $\mathbf{p}_k = \partial E / \partial \mathbf{x}_k$ denote the shadow price of $\mathbf{x}_k$, $k = 1, ..., K$. Under CRS, $E(\mathbf{x})$ is a linear homogenous function of $\mathbf{x}$ and can be written (at least locally) as $E(\mathbf{x}) = \sum_{k=1}^K \mathbf{p}_k \mathbf{x}_k$, yielding the household budget constraint $\sum_{k=1}^K \mathbf{p}_k \mathbf{x}_k = y$. Note that, in this case, the shadow prices $\mathbf{p}_k$ can differ across individuals because of differences in the production process of goods $\mathbf{x}_k$ from $\mathbf{z}$, $k = 1, ..., K$.

In the collective framework, the household utility function is defined as $U(\mathbf{x}) = U(u_1(\mathbf{x}_1), ..., u_K(\mathbf{x}_K))$ where $U : \mathbb{R}^K \to \mathbb{R}$ is a strictly increasing function that aggregates individual preferences into household preferences and reflects distribution issues within the household. The utility function $U(u_1, ..., u_K)$ represents household preferences across individuals[1] and may reflect the relative bargaining power of individuals within the household. . Throughout the paper, we assume that $U(\mathbf{x})$ is a strongly monotone and strictly quasi-concave function of $\mathbf{x}$. The household faces a linear budget constraint $\sum_{k=1}^K \mathbf{p}_k \mathbf{x}_k = y$, where $\mathbf{p}_k \mathbf{x}_k$ denotes the inner product of the two vectors $\mathbf{p}_k$ and $\mathbf{x}_k$, and $y \in \mathbb{R}_{++}$ is household income.

We first examine collective decisions at the household level. Efficient allocations within

[1] Household preferences can also be affected by demographic characteristics such as age or years of schooling of each household member, but we choose not to include them to simplify notation. Previous literature has analyzed this issue by allowing socio-demographic characteristics to affect household preferences (Barten 1964, Blackorby and Donaldson 1991, Lewbel 1997, 2004, Perali 2003). While we do not explore the direct role of socio-demographic effects in the present context, we allow for heterogeneity of preferences among household members.

the household are derived from

$$V(\mathbf{p}, y) = \max_{\mathbf{x}_1,...,\mathbf{x}_K} \left\{ U(u_1(\mathbf{x}_1), ..., u_K(\mathbf{x}_K)) : \sum_{k=1}^{K} \mathbf{p}_k \mathbf{x}_k = y \right\}, \tag{1}$$

where $\mathbf{p} = (\mathbf{p}_1, ..., \mathbf{p}_K)$ and $V(\mathbf{p}, y)$ is the household indirect utility function. The optimal solution to (1) gives the household centralized Marshallian demands $\mathbf{x}^C(\mathbf{p}, y) = (\mathbf{x}_1^C(\mathbf{p}, y), ..., \mathbf{x}_K^C(\mathbf{p}, y))$.

Consider a situation where the household income y is allocated among the K household members and where the k^{th} individual receives the monetary amount $\phi_k \in \mathbb{R}_{++}$ subject to the household income constraint $y = \sum_{k=1}^{K} \phi_k$. The efficient household allocation in (1) can then be decomposed into two stages as follows

$$V(\mathbf{p}, y) = \max_{\phi_1,...,\phi_K} \left\{ \max_{\mathbf{x}_1,...,\mathbf{x}_K} \{U(u_1(\mathbf{x}_1), ..., u_K(\mathbf{x}_K)) : \mathbf{p}_k \mathbf{x}_k = \phi_k\} : \sum_{k=1}^{K} \phi_k = y \right\}. \tag{2}$$

The household preference function $U(u_1, ..., u_K)$ being strictly increasing in each component, equation (2) can be written equivalently as

$$V(\mathbf{p}, y) = \max_{\phi_1,...,\phi_K} \left\{ U(V_1(\mathbf{p}_1, \phi_1), ..., V_K(\mathbf{p}_K, \phi_K)) : \sum_{k=1}^{K} \phi_k = y \right\}, \tag{3}$$

$$V_k(\mathbf{p}_k, \phi_k) = \max_{\mathbf{x}_k} \{u_k(\mathbf{x}_k) : \mathbf{p}_k \mathbf{x}_k = \phi_k\}, \ \forall k = 1, \ldots, K \tag{4}$$

where $V_k(\mathbf{p}_k, \phi_k)$ is the indirect utility function for the k^{th} individual, conditional on prices $\mathbf{p}_k$ and on the income allocation ϕ_k. The amount of shared resources ϕ_k can be interpreted as the level of income, or total expenditure, required by the k^{th} individual consuming goods privately to be as well off if living alone or living with others in a household (Browning *et al.* 2013). Equation (3) describes the optimal income allocation among all household members, with solution denoted by $(\phi_1(\mathbf{p}, y), ..., \phi_K(\mathbf{p}, y))$ where the income allocated to the k^{th} individual is $\phi_k(\mathbf{p}, y)$ which depends on all prices $\mathbf{p}$ and on household income y, $k = 1, ..., K$. And equation (4) defines the decentralized Marshallian demand for the k^{th}

individual, with solution denoted by $\mathbf{x}_k^D(\mathbf{p}_k, \phi_k)$ which depends on $(\mathbf{p}_k, \phi_k)$. Under household efficiency, the optimal income allocation to the k^{th} individual satisfies $\phi_k(\mathbf{p}, y) = \mathbf{p}_k \mathbf{x}_k^C(\mathbf{p}, y)$ and the maximization problem in (1) is equivalent to (3), (4). This yields the following identity.

Proposition 1. *Under household efficiency, the following linkages exist between the centralized household decisions* $\mathbf{x}^C(\mathbf{p}, y) = (\mathbf{x}_1^C(\mathbf{p}, y), ..., \mathbf{x}_K^C(\mathbf{p}, y))$ *and the decentralized individual decisions* $\mathbf{x}_k^D(\mathbf{p}_k, \phi_k)$

$$\mathbf{x}_k^C(\mathbf{p}, y) \equiv \mathbf{x}_k^D(\mathbf{p}_k, \phi_k(\mathbf{p}, y)). \tag{5}$$

Equation (5) establishes the general relationships between the centralized household decisions $\mathbf{x}_k^C(\mathbf{p}, y)$ and the decentralized individual decisions $\mathbf{x}_k^D(\mathbf{p}_k, \phi_k)$. Under differentiability, differentiating equation (6) with respect to $(\mathbf{p}, y)$ gives the following results.

Corollary 1. *For each $k = 1, ..., K$, we have*

$$\frac{\partial \mathbf{x}_k^C(\mathbf{p}, y)}{\partial \mathbf{p}_k} = \frac{\partial \mathbf{x}_k^D(\mathbf{p}_k, \phi_k)}{\partial \mathbf{p}_k} + \frac{\partial \mathbf{x}_k^D(\mathbf{p}_k, \phi_k)}{\partial \phi_k} \frac{\partial \phi_k(\mathbf{p}, y)}{\partial \mathbf{p}_k}, \tag{6}$$

$$\frac{\partial \mathbf{x}_k^C(\mathbf{p}, y)}{\partial \mathbf{p}_r} = \frac{\partial \mathbf{x}_k^D(\mathbf{p}_k, \phi_k)}{\partial \phi_k} \frac{\partial \phi_k(\mathbf{p}, y)}{\partial \mathbf{p}_r}, \qquad \text{for } r \neq k, \tag{7}$$

$$\frac{\partial \mathbf{x}_k^C(\mathbf{p}, y)}{\partial y} = \frac{\partial \mathbf{x}_k^D(\mathbf{p}_k, \phi_k)}{\partial \phi_k} \frac{\partial \phi_k(\mathbf{p}, y)}{\partial y}. \tag{8}$$

Equations (6) and (7) show how prices $\mathbf{p}$ affect consumption behavior at the individual level.[2] Interestingly, the cross-price effects (7) impact on the demand function of k only indirectly through the individual income allocation $\phi_k(\mathbf{p}, y)$. Equation (8) shows the effects of household income y on demand. Note that all of them identify the role of individual income ϕ_k. This indicates that the distribution of income across individuals within the household always matters. We now exploit the collective identity (5) to define the relationship between

[2] The effect of a price change on household consumption is described by a Slutsky matrix that is the sum of a symmetric, negative definite matrix and a matrix of rank at most $K-1$ (Browning and Chiappori 1998).

the decentralized income ϕ_k and welfare distribution issues.

3 Income Sharing

An important decision made by the household is how to allocate income y among its K members as described in equation (3). As expected, this allocation is related to marginal utilities of income. The properties of the associated income distribution are explored next.

Proposition 2. *Under differentiability, the distribution of income within the household* $(\phi_1, ..., \phi_K)$ *satisfies*

$$\lambda(\mathbf{p}, y) = [\partial U(u_1, \ldots, u_K)/\partial u_k] \; \lambda_k(\mathbf{p}_k, \phi_k), \tag{9}$$

where $\lambda(\mathbf{p}, y) = \partial V(\mathbf{p}, y)/\partial y$ *is the marginal utility of income for the household, and* $\lambda_k(\mathbf{p}_k, \phi_k) = \partial V_k(\mathbf{p}_k, \phi_k)/\partial \phi_k$ *is the marginal utility of income for the* k^{th} *individual,* $k = 1, ..., K$.

Proof. Consider the constrained maximization problem in (3). The associated Lagrangian is $\mathcal{L} = U(V_1(\mathbf{p}_1, \phi_1), ..., V_K(\mathbf{p}_K, \phi_K)) + \lambda\,[y - \sum_{k=1}^{K} \phi_k]$, where λ is the Lagrange multiplier of the budget constraint $y = \sum_{k=1}^{K} \phi_k$. Assuming an interior solution, the first-order necessary condition with respect to ϕ_k is

$$\partial \mathcal{L}/\partial \phi_k = [\partial U(V_1, \ldots, V_K)/\partial V_k]\,[\partial V_k(\mathbf{p}_k, \phi_k)/\partial \phi_k] - \lambda = 0. \tag{10}$$

Under household efficiency, note that equation (4) implies that $V_k(\mathbf{p}_k, \phi_k) = u_k(\mathbf{x}_k^D(\mathbf{p}_k, \phi_k))$. And at the optimum, applying the envelope theorem to (3) gives $\partial V(\mathbf{p}, y)/\partial y = \lambda$. Combining these results with (10) yields

$$\partial V(\mathbf{p}, y)/\partial y = [\partial U(u_1, \ldots, u_K)/\partial u_k]\,[\partial V_k(\mathbf{p}_k, \phi_k)/\partial \phi_k]\,, \tag{11}$$

which gives (9).Proposition (2) states that, under household efficiency, the marginal utility of household income $\lambda(\mathbf{p}, y)$ is proportional to the marginal utility of income for the k-th individual $\lambda_k(\mathbf{p}_k, \phi_k)$, with $\partial U(u_1, ..., u_K)/\partial u_k$ as proportionality factor $\square$

This result has interesting implications for the income sharing rule ϕ_k/y, that is the proportion of household income allocated to the k^{th} individual, $k = 1, ..., K$.

Proposition 3. *Equation (3) implies the following income sharing rule for the k^{th} individual*

$$\frac{\phi_k}{y} = \frac{\partial U(u_1, \ldots, u_K)}{\partial u_k} w_k, \tag{12}$$

where $w_k \equiv \left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\phi_k)}\right) / \left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln(y)}\right)$ *is a weight satisfying* $\sum_{k=1}^{K} \frac{\partial U(u_1, \ldots, u_K)}{\partial u_k} w_k = 1$.

Proof. Given $y > 0$ and $\phi_k > 0$, equation (11) can be alternatively written as $[\partial V(\mathbf{p}, y)/\partial \ln(y)][1/y] = [\partial U(u_1, \ldots, u_K)/\partial u_k]\, [\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k)]\, [1/\phi_k]$. This gives equation (12). $\square$

The sharing rule given in (12) shows that the proportion of household income ϕ_k/y received by the k^{th} individual is equal to the product of two weights: the welfare weight $\frac{\partial U(u_1, \ldots, u_K)}{\partial u_k}$ reflecting household preferences with respect to the k^{th} individual, and the weight w_k capturing marginal effects of income distribution. For the k^{th} individual, the weight w_k is the ratio between the marginal individual utility due to a proportional change in individual income $\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k)$ and the marginal household utility due to a proportional change in household income $\partial V(\mathbf{p}, y)/\partial \ln(y)$. This can be interpreted as an elasticity describing how the curvature of the indirect utility function of the household changes as the distribution of income varies across household members (Menon, Pagani, and Perali 2015). We now investigate the implications for household welfare of these general results by examining in details how household welfare varies with the distribution of income within the household.

4 Household Welfare Evaluation

There has been much interest in linking welfare at the individual level versus the aggregate level (Gorman 1953, Muellbauer 1975). This section analyzes these issues in the context of the household, with a focus on the evaluation and measurement of household welfare. While the household utility function $U(u_1, ..., u_K)$ reflects preferences with respect to distribution within the household, it does not make such preferences explicit. In general, the sharing rule given in equation (12) identifies the role played by $\frac{\partial U(u_1,...,u_K)}{\partial u_k}$ as a welfare weight. This corresponds to the intuition of close relationships between welfare weights and income distribution within the household. To analyze distribution issues, it is convenient to focus on a specific representation of the household utility function. For that purpose, consider the weighted Bergsonian specification in the tradition of the collective household literature

$$U(u_1, ..., u_K) = \sum_{k=1}^{K} \mu_k \, u_k, \tag{13}$$

where $\mu_k \in (0, 1)$ is a welfare weight[3] associated with the k^{th} individual, $k = 1, ..., K$. Without a loss of generality, these welfare weights are normalized to satisfy $\mu_k \in (0, 1)$ and $\sum_{k=1}^{K} \mu_k = 1$. The specification (13) implies that $\partial U(u_1, ..., u_K)/\partial u_k = \mu_k$, $k = 1, ..., K$. From Proposition 2, this gives the following Corollary.

Corollary 2. Under (13), for all $k = 1, ..., K,$ equation (9) reduces to

$$\lambda(\mathbf{p}, y) = \mu_k \, \lambda_k(\mathbf{p}_k, \phi_k). \tag{14}$$

Equation (14) shows explicitly how the welfare weights $\mu = (\mu_1, ..., \mu_K)$ affect the marginal utilities of income. When $0 < \mu_k < 1$, it follows that $\lambda < \lambda_k$, $k = 1, ..., K$. For the k^{th} member, then, the value of one extra unit of income when she lives inside the family is worth less as compared to the situation where she has control over her own income. Note that expression (14) is the cardinal representation of the ordinal object (9) presented in Proposition 2. The reader may refer to Proposition 3 in Browning *et al.* (2013) to find an alternative way to

[3] The weight μ_k is termed Pareto weight because it corresponds to the Lagrange multiplier of the Pareto optimal problem where agent k maximizes its own utility while ensuring that utility of agents r is larger or equal than a predetermined utility level (Pagani 2013).

derive the Pareto weight. Note that equation (14) also gives for all $k, r = 1, 2, ..., K,\ k \neq r$,

$$\mu_k/\mu_r = \lambda_r(\mathbf{p}_r, \phi_r)/\lambda_k(\mathbf{p}_k, \phi_k). \tag{15}$$

In a way consistent with Browning *et al.* (2013), this states that, under household efficiency, the relative welfare weights μ_k/μ_r are equal to the relative decentralized marginal utilities $\lambda_r(\mathbf{p}_r, \phi_r)/\lambda_k(\mathbf{p}_k, \phi_k)$. As shown below, equation (14) will provide useful insights into the economic and welfare analysis of household behavior.

For any given economic situation $(\mathbf{p}, y)$, note that the Bergsonian specification (13) can apply to general household preferences $U(u_1, ..., u_K)$ as long as $\partial U(u_1, ..., u_K)/\partial u_k = \mu_k$, $k = 1, ..., K$. It means that, conditional on given $(\mathbf{p}, y)$, it is valid to analyze the behavioral and welfare implications of alternative Bergsonian welfare weights $(\mu_1, ..., \mu_K)$. This will prove particularly convenient for our analysis: the presence of explicit welfare weights $(\mu_1, ..., \mu_K)$ in (13) can shed useful lights on their role in consumption decisions and the welfare analysis of distribution issues within the household. We will use this property extensively in this section.

But what happens when economic conditions $(\mathbf{p}, y)$ change? Then, under household efficiency, $V(\mathbf{p}, y)$ would change, so would $\partial U(u_1, ..., u_K)/\partial u_k$, $k = 1, ..., K$. In this context, the Bergsonian utility function (13) would remain valid under general household preferences $U(u_1, ..., u_K)$ but only under the condition that we allow the welfare weights $(\mu_1, ..., \mu_K)$ to adjust to changing economic conditions. In other words, the Bergsonian approach would continue to hold provided that the welfare weights $(\mu_1, ..., \mu_K)$ are treated as endogenous to satisfy $\mu_k(\mathbf{p}, y) = \partial U(u_1, ..., u_K)/\partial u_k$ at the optimum for all $(\mathbf{p}, y)$, $k = 1, ..., K$. To simplify the notation, the analysis presented below treats the dependency of the welfare weights $(\mu_1, ..., \mu_K)$ on $(\mathbf{p}, y)$ as implicit. This should be kept in mind in our discussion below.

Next, we evaluate the implications of our analysis for household welfare. In the process, we also examine the linkages between individual preferences and income sharing rules under alternative individual preferences. We start the analysis with the following indirect utility specification for the k^{th} individual

$$V_k(\mathbf{p}_k, \phi_k) = f_k\left(\frac{g_k(\phi_k, \mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\, g_k(\phi_k, \mathbf{p}_k)}\right), \tag{16}$$

where f_k is a strictly increasing function, and the functions $B_k(\mathbf{p}_k)$, $C_k(\mathbf{p}_k)$ and $g_k(\phi_k, \mathbf{p}_k)$ are chosen so that $V_k(\mathbf{p}_k, \phi_k)$ is homogeneous of degree zero in $(\phi_k, \mathbf{p}_k)$. The utility specification (16) is very flexible and includes as special cases many models commonly found in the literature. Following Lewbel (1989b, 1990, 1991, 1995), the preference specification (16) belongs to the class of rank-three demand systems that can exhibit non-linear Engel curves. For example, when $g_k(\phi_k, \mathbf{p}_k) = \phi_k - A_k(\mathbf{p}_k)$, (16) gives the Quadratic Expenditure System (QES) proposed by Howe *et al.* (1979), where demands are quadratic functions of income[4] Alternatively, when $g_k(\phi_k, \mathbf{p}_k) = \ln(\phi_k) - \ln(A_k(\mathbf{p}_k))$, (16) gives the Quadratic Almost Ideal Demand System (QAIDS) proposed by Banks *et al* (1997) where budget shares are quadratic functions of the logarithm of the income.

The general implications of individual preferences (16) are presented next.

Proposition 4. *Under the Bergsonian household utility (13) and individual preferences (16), the income sharing rule is*

$$\frac{\phi_k}{y} = \mu_k w_k,\ \ k = 1, ..., K. \tag{17}$$

and household welfare is

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k\, f_k\left[g_k(\mu_k\, w_k\, y, \mathbf{p}_k)/[B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\, g_k(\mu_k\, w_k\, y, \mathbf{p}_k)]\right]. \tag{18}$$

Proof. Under the Bergsonian household utility (13), we start with the relationship $V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\phi_k, \mathbf{p}_k)$. Using equation (16), we have

[4] Another specification of preferences is the translog utility function (Christensen *et al.* 1975, Jorgenson *et al.* 1982, Lewbel 1989a).

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \, f_k[g_k(\phi_k, \mathbf{p}_k)/[B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\, g_k(\phi_k, \mathbf{p}_k)]]. \tag{19}$$

Equation (12) gives (17). Substituting (17) into (19) gives (18). □

As a result, the income sharing rule (17) and the indirect household utility function (18) apply under fairly general conditions. As further discussed below, the specification (16) allows for non linear Engel curves. It means that the income sharing rule (17) and the household utility function (18) apply in the presence of non linear income effects that can vary across individuals. Below, we investigate in more details the implications of varying assumptions about the functions f_k and g_k as summarized in table 1 in the Appendix. The last subsection is devoted to the derivation of the measures of intra-household inequality.

4.1 Translated Sharing Rule

Consider the specification where $g_k(\phi_k, \mathbf{p}_k) = \phi_k - A_k(\mathbf{p}_k)$ in (16) corresponding to the QES preference representation (Howe *et al.* 1979). The indirect utility function for the k^{th} individual takes the form

$$V_k(\mathbf{p}_k, \phi_k) = f_k\left(\frac{\phi_k - A_k(\mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\,(\phi_k - A_k(\mathbf{p}_k))}\right) \tag{20}$$

where f_k is a strictly increasing function, the functions $A_k(\mathbf{p}_k)$ and $B_k(\mathbf{p}_k)$ are homogeneous of degree one in $\mathbf{p}_k$, and the function $C_k(\mathbf{p}_k)$ is homogeneous of degree zero in $\mathbf{p}_k$. Throughout, we assume that $\phi_k - A_k(\mathbf{p}_k) > 0$. In this context, $\phi_k - A_k(\mathbf{p}_k)$ is a measure of "translated income", where $A_k(\mathbf{p}_k)$ is a price function that can be interpreted as a committed level of subsistence income for the k^{th} individual. Specification (20) allows for flexible price effects (e.g., when $A_k(\mathbf{p}_k)$ is specified to be a quadratic function of $\mathbf{p}_k$). Indeed, using Roy's identity, the demand for x_{ki} associated with QES preferences (20) is

$$x_{ki}^D = \frac{\partial A_k(\mathbf{p}_k)}{\partial p_{ki}} + \frac{\partial \ln(B_k(\mathbf{p}_k))}{\partial p_{ki}} [\phi_k - A_k(\mathbf{p}_k)] + \frac{\partial C_k(\mathbf{p}_k)}{\partial p_{ki}} \frac{1}{B_k(\mathbf{p}_k)} [\phi_k - A_k(\mathbf{p}_k)]^2. \tag{21}$$

Note that QES preferences (20) include as special case quasi-homothetic preferences when $C_k = 0$. Under quasi-homothetic preferences, individual Engel curves are linear as $\partial x_{ki}^D / \partial \phi_k$ is independent of ϕ_k (Deaton and Muellbauer 1980a, 1980b, Gorman 1961). In turn, QES preferences (20) include as special case homothetic preferences when $C_k = 0$ and $A_k = 0$. In this case all Engel curves are linear and go through the origin. This indicates that QES preferences (20) offer some flexibility capturing income effects, including non linear Engel curves when $\partial C_k / \partial \mathbf{p}_k \neq 0$. As such, they provide a good basis to evaluate the linkages between individual welfare and household welfare.

The implications of individual preferences (20) for income sharing and household welfare are presented next.

Proposition 5. *Under the Bergsonian household utility (13) and individual preferences (20), then the income sharing rule satisfies*

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_k, \;\; k = 1, ..., K, \tag{22}$$

and household welfare is given by

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \, f_k \left(\mu_k \, W_k \, \frac{y - A(\mathbf{p})}{P_k(\mathbf{p}, y, \mu)} \right), \tag{23}$$

where $A(\mathbf{p}) = \sum_{k=1}^{K} A_k(\mathbf{p}_k)$, $W_k = \left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\phi_k - A_k(\mathbf{p}_k))} \right) / \left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln(y - A(\mathbf{p}))} \right)$ *is a weight satisfying* $\sum_{k=1}^{K} \mu_k \, W_k = 1$, *and* $P_k(\mathbf{p}, y, \mu) = B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) \, \mu_k \, W_k \, (y - A(\mathbf{p}))$, $k = 1, ..., K$.

Proof. Starting from the relationship $V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\phi_k, \mathbf{p}_k)$ and using equation (20), we have

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \, f_k \left[(\phi_k - A_k(\mathbf{p}_k)) / \left(B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) \; (\phi_k - A_k(\mathbf{p}_k)) \right) \right]. \tag{24}$$

Let $A(\mathbf{p}) = \sum_{k=1}^{K} A_k(\mathbf{p}_k)$. Under the Bergsonian household utility (13), equation (12) can be alternatively written as

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k \left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\phi_k - A_k(\mathbf{p}_k))} \right) / \left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln(y - A(\mathbf{p}))} \right) = \mu_k \, W_k, \tag{25}$$

where $W_k = \left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\phi_k - A_k(\mathbf{p}_k))} \right) / \left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln(y - A(\mathbf{p}))} \right)$ is a weight satisfying $\sum_{k=1}^{K} \mu_k \, W_k = 1$. This gives equation (22). Substituting (22) into (24) gives the desired result. □

Proposition 5 applies for any increasing function f_k in (20) and (23). The sharing rule in (22) states that the proportion of "translated income" received by the k^{th} individual $\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})}$ is equal to the product of two weights: the Bergsonian welfare weight μ_k, and the weight W_k reflecting income effects under optimal income distribution within the household. This can be important when marginal income effects are observed to vary with income levels. In this context, the term W_k in (22) captures the effects of preference heterogeneity across individuals within the household. This term also enters the household utility (23). This raises the question: Are there situations where these heterogeneity effects vanish? In other words, are there scenarios where $W_k = 1$ in (22) and (23)? Our analysis identifies such conditions below.

In a first step, we examine the special case where $f_k(z) = \ln(z)$ in (20), summarized in Panel c. of Table 1.

Proposition 6. *Under the Bergsonian household utility (13) and individual preferences (20), let $f_k(z) = \ln(z)$. Then, the income sharing rule is*

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k \, W_{0k}, \;\; k = 1, ..., K, \tag{26}$$

and the household welfare function is

$$V(\mathbf{p},y)=\ln\left(\frac{y-A(\mathbf{p})}{P(\mathbf{p},y,\mu)}\right), \tag{27}$$

where $A(\mathbf{p})=\sum_{k=1}^{K}A_k(\mathbf{p}_k)$*,* $P(\mathbf{p},y,\mu)$ *is a household price index satisfying*

$$\ln P(\mathbf{p},y,\mu)=\sum_{k=1}^{K}\mu_k\,\ln\left(B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\,\mu_k\,W_{0k}\,(y-A(\mathbf{p}))\right)-\sum_{k=1}^{K}\mu_k\,\ln(\mu_k\,W_{0k}), \tag{28}$$

and $W_{0k}=\left(1-\frac{C_k\,(\phi_k-A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\,(\phi_k-A_k(\mathbf{p}_k))}\right)/\sum_{k=1}^{K}\mu_k\left(1-\frac{C_k\,(\phi_k-A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\,(\phi_k-A_k(\mathbf{p}_k))}\right)$*.*

Proof. Under individual preferences (20), having $f_k(z)=\ln(z)$ implies that $\frac{\partial V_k(\mathbf{p}_k,\phi_k)}{\partial\ln(\phi_k-A_k(\mathbf{p}_k))}=1-\frac{C_k(\mathbf{p}_k)\,(\phi_k-A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\,(\phi_k-A_k(\mathbf{p}_k))}$. Substituting this result into (25) gives

$$\frac{\phi_k-A_k(\mathbf{p}_k)}{y-A(\mathbf{p})}=\mu_k\,\left(1-\frac{C_k(\mathbf{p}_k)\,(\phi_k-A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\,(\phi_k-A_k(\mathbf{p}_k))}\right)/\left(\sum_k\frac{\partial V(\mathbf{p},y)}{\partial\ln(\phi_k-A_k(\mathbf{p}_k))}\right). \tag{29}$$

Summing equation (29) over all k and using $\sum_{k=1}^{K}[\phi_k-A_k(\mathbf{p}_k)]=y-A(\mathbf{p})$, we obtain $\sum_k\frac{\partial V(\mathbf{p},y)}{\partial\ln(\phi_k-A_k(\mathbf{p}_k))}=\sum_{k=1}^{K}\mu_k\left(1-\frac{C_k(\mathbf{p}_k)\,(\phi_k-A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\,(\phi_k-A_k(\mathbf{p}_k))}\right)$. Substituting this result into (22) yields (26). When $f_k(z)=\ln(z)$, it follows that equation (23) becomes

$$V(\mathbf{p},y)=\sum_{k=1}^{K}\mu_k\,\ln\left[\mu_k\,W_{0k}\,(y-A(\mathbf{p}))/\left(B_k(\mathbf{p_k})+C_k(\mathbf{p}_k)\,\mu_k\,W_{0k}\,(y-A(\mathbf{p}))\right)\right], \tag{30}$$

or, using $\sum_{k=1}^{K}\mu_k=1$,

$$V(\mathbf{p},y)=\ln(y-A(\mathbf{p}))+\sum_{k=1}^{K}\mu_k\,\ln(\mu_k\,W_{0k})-\sum_{k=1}^{K}\mu_k\,\ln\left(B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\,\mu_k\,W_{0k}\,(y-A(\mathbf{p}))\right), \tag{31}$$

which gives equations (27) and (28). □

Proposition 6 gives the income sharing rule (26) and the household indirect utility function (27) when $f_k(z) = \ln(z)$, allowing for departures from quasi-homothetic preferences. Again, the weights W_{0k} affect both the sharing rules and the indirect utility function, reflecting income effects under optimal income sharing. The household utility function $V(\mathbf{p}, y)$ given in (27) is equal to $\ln\left(\frac{y-A(\mathbf{p})}{P(\mathbf{p},y,\mu)}\right)$. It is a monotonic transformation of the household income. Interestingly, the logarithm of the price index of equation (28) involves the terms $\sum_{k=1}^{K} \mu_k \ln\left(B_k(\mathbf{p}_k) + C_k\, \mu_k\, W_{0k}\,(y - A(\mathbf{p}))\right)$ and $-\sum_{k=1}^{K} \mu_k \ln(\mu_k\, W_{0k})$. The first term is a weighted sum of $\ln\left(B_k(\mathbf{p}_k) + C_k\, \mu_k\, W_{0k}\,(y - A(\mathbf{p}))\right)$ reflecting the cost of living for the k^{th} individual with μ_k as weight. The second term captures the effects of the Bergsonian welfare weights μ_k and income distribution weights W_{0k} on the household price index.

While Proposition 5 allows for departures from quasi-homotheticity (when $C_k \neq 0$), it includes as special case situations where individual preferences exhibit quasi-homotheticity (when $C_k = 0$, $k = 1, ..., K$). Browning *et al.* (2014) provide an example of household decisions under quasi-homothetic preferences for centralized demands. The link with the decentralized demands can be readily applied in this context using our results. Here, we say that individual preferences in (20) are "log-quasi-homothetic" if they exhibit quasi-homotheticity with $C_k = 0$) and if $f_k(z) = \ln(z)$, $k = 1, ..., K$. Note that $C_k = 0$ for all $k = 1, ..., K$ implies that $W_{0k} = 1$ for all $k = 1, ..., K$ in Proposition 6. This generates the following result.

Corollary 3. *Under the Bergsonian household utility (13) and individual preferences (20) satisfying log-quasi-homotheticity (with $C_k = 0$ and $f_k(z) = \ln(z)$), the income sharing rule is*

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k, \tag{32}$$

and the household welfare function is

$$V(\mathbf{p}, y) = \ln\left(\frac{y - A(\mathbf{p})}{P_0(\mathbf{p}, y, \mu)}\right), \tag{33}$$

where $A(\mathbf{p}) = \sum_{k=1}^{K} A_k(\mathbf{p}_k)$*,* and $P_0(\mathbf{p}, y, \mu)$ *is a household price index satisfying*

$$\ln[P_0(\mathbf{p}, y, \mu)] = \sum_{k=1}^{K} \mu_k \ln\left(B_k(\mathbf{p}_k)\right) - \sum_{k=1}^{K} \mu_k \ln(\mu_k). \tag{34}$$

Corollary 3 reports the income sharing rule (32) and the household welfare function (33) obtained when individual preferences exhibit log-quasi-homotheticity. First, comparing (27) and (33) is instructive. It shows that the evaluation of household welfare is similar with or without quasi-homotheticity, with one notable exception: the price indices P in (28) and P_0 in (34) differ. Indeed, the price index P in (28) includes the terms involving C_k, while the price index P_0 obtained under quasi-homotheticity does not. This illustrates that non-homothetic preferences (or non-linear Engel curves) do affect the evaluation of household welfare. Second, Corollary 3 shows that the preference heterogeneity weight W_k no longer plays a role under log-quasi-homotheticicty (as $W_{0k} = 1$ for all $k = 1, ..., K$). This establishes that, under log-quasi-homotheticicty, heterogeneity in income effects across individuals no longer affects income sharing nor household welfare. Note that, while individual Engel curves are linear under quasi-homotheticity, the results in Corollary 3 still allow Engel curves to have different slopes across individuals.

Under log-quasi-homotheticity, the income sharing rule (32) becomes simpler: the proportion of household "translated income" allocated to the k^{th} individual $(\phi_k - A_k(\mathbf{p}_k))/(y - A(\mathbf{p}))$ is equal to the Bergsonian welfare weight μ_k. This is an intuitive result: increasing the Bergsonian welfare for an individual means a proportional increase in her share of "translated income". Also, under log-quasi-homotheticity, household welfare (33) takes a simple form: it is a monotonic function of translated income $(y - A(\mathbf{p}))$ deflated by the household price index P_0 in (34). The price index P_0 in (34) shows that the welfare weights $(\mu_1, ..., \mu_K)$ have two effects on household welfare: as weights on the terms involving $B_k(\mathbf{p_k})$, and through the term

$$-\sum_{k=1}^{K} \mu_k \ln(\mu_k). \tag{35}$$

This term is the Shannon entropy index. It reflects the impacts of individual welfare weights μ_k on household utility. We investigate the consequences on intra-household inequality in Subsection 4.3.

Finally, when $C_k = 0$, note that letting $A_k(\mathbf{p}_k) = 0$ moves preferences of the k^{th} individual from being quasi-homothetic (with linear Engel curves) to being homothetic (with linear Engel curves going through the origin). In this context, we say that individual preferences in (20) are "log-homothetic" if they are log-quasi-homothetic (with $C_k = 0$ and $f_k(z) = \ln(z)$) and if $A_k = 0$ for all $k = 1, ..., K$. This gives the following result.

Corollary 4. *Under the Bergsonian household utility (13) and individual preferences (20) satisfying log-homotheticity (with $A_k = 0$, $C_k = 0$ and $f_k(z) = \ln(z)$), the income sharing rule is*

$$\frac{\phi_k}{y} = \mu_k, \tag{36}$$

and the household utility function is

$$V(\mathbf{p}, y) = \ln\left(\frac{y}{P_0(\mathbf{p},y,\mu)}\right), \tag{37}$$

where $P_0(\mathbf{p}, y, \mu)$ is given in (34).

Corollary 4 shows that, under log-homotheticity, the income sharing rule given (36) becomes very simple: the proportion of household income allocated to the k^{th} individual (ϕ_k/y) is equal to the Bergsonian welfare weight μ_k, $k = 1, ..., K$.[5]

We now present a simple example illustrating the results presented in Corollary 4.

Example 1. Consider the case where $K = 2$ under individual Cobb-Douglas utility functions where $\ln(u_1) = \alpha_1 \ln(x_{11}) + \alpha_2 \ln(x_{12})$ and $\ln(u_2) = \beta_1 \ln(x_{21}) + \beta_2 \ln(x_{22})$, with

[5]The interested reader may observe that because of the logarithmic transformation, Euler's theorem specializes to $h = x_1^k(\partial u^k/\partial x_1) + x_2^k(\partial u^k/\partial x_2)$, where h is the degree of homogeneity. From equating the first order conditions of the centralized and decentralized model, h is equal to $\phi^k\lambda^k$. Using $\lambda/\mu^k = \lambda^k$ and the fact that the centralized Lagrange multiplier is $\lambda = \frac{h}{y}$, it follows that $\phi^k = \mu^k y$. This derivation shows how the Lagrange multipliers are associated with the sharing rules for log-homothetic preferences.

$\alpha_1 + \alpha_2 = \beta_1 + \beta_2 = 1$. This is a case of log-homothetic individual preferences (as presented in Corollary 4). The associated collective household welfare function is $U = \mu_1 (\alpha_1 \ln(x_{11}) + \alpha_2 \ln(x_{12})) + \mu_2 (\beta_1 \ln(x_{21}) + \beta_2 \ln(x_{22}))$. The necessary conditions for the centralized and decentralized problems are, respectively

$$\begin{cases} x_{1i}^C = \alpha_i \mu_1 y / p_{1i}, \\ x_{2i}^C = \beta_i \mu_2 y / p_{2i}, \quad i = 1,2 \\ \lambda = 1/y \end{cases} \text{and} \quad \begin{cases} x_{1i}^D = \alpha_i \phi_1 / p_{1i}, \\ x_{2i}^D = \beta_i \phi_2 / p_{2i}, \quad i,k = 1,2 \cdot \\ \lambda_k = 1/\phi_k, \end{cases}$$

This implies $\phi_k = \mu_k y, \; k = 1,2$, which is equation (36). As expected, the sharing rule reflecting the distribution of resources inside the family states that the level of individual income ϕ_k is the product of household income y times the Bergsonian welfare weight μ_k.

For the sake of completeness, we find interesting to examine also the translated case corresponding to $g_k(\phi_k, \mathbf{p}_k) = \phi_k - A_k(\mathbf{p}_k)$ and $f_k(z)$ equal to the identity function, summarized in Panel a. of Table 1.

Proposition 7. *Under Bergsonian household utility (13) and individual preferences (20) such that $f_k(z) = z$ and $g_k(\phi_k, \mathbf{p}_k) = \phi_k - A_k(\mathbf{p}_k)$, then the income sharing rule satisfies*

$$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_k \tag{38}$$

and household welfare is given by

$$V(\mathbf{p}, y) = \frac{y - A(\mathbf{p})}{P^*(\mathbf{p}, y, \mu)}, \tag{39}$$

where $1/P^* = \sum_k \mu_k \frac{\mu_k W_k}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) \mu_k W_k (y - A(\mathbf{p}))}$.

Proof. Recall that $V_k(\mathbf{p}_k, \phi_k) = \left(\frac{\phi_k - A_k(\mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\,(\phi_k - A_k(\mathbf{p}_k))} \right)$, then, using (38), we have $V(\mathbf{p}, y) = \sum_k \mu_k V_k = \sum_k \mu_k \frac{\mu_k W_k (y - A(\mathbf{p}))}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) \mu_k W_k (y - A(\mathbf{p}))}$, and hence, $V(\mathbf{p}, y) = (y - A(\mathbf{p})) \sum_k \mu_k \frac{\mu_k W_k}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) \mu_k W_k (y - A(\mathbf{p}))}$ =

$\frac{y-A(\mathbf{p})}{P^*}$. □

Note that the indirect utility function depending on $1/P^* = \sum_k \mu_k \frac{\mu_k W_k}{B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\mu_k W_k(y-A(\mathbf{p}))}$ would be linear in income if μ_k is constant and $C_k = 0$, giving corner solutions for the sharing rule.

4.2 Deflated Sharing Rule

We now consider the specification where $g_k(\phi_k, \mathbf{p}_k) = \ln(\phi_k) - \ln(A_k(\mathbf{p}_k))$, $k = 1, ..., K$, corresponding to QAIDS preferences as reported in Panels b. and d. of Table 1. Following Banks *et al.* (1997), under a QAIDS model for the k^{th} individual, the indirect utility function (16) takes the form

$$V_k(\mathbf{p}_k, \phi_k) = f_k\left(\frac{\ln(\phi_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\ (\ln(\phi_k) - \ln(A_k(\mathbf{p}_k)))}\right), \tag{40}$$

where f_k is a strictly increasing function, the functions $\ln(A_k(\mathbf{p}_k))$ and $B_k(\mathbf{p}_k)$ are each homogeneous of degree one in $\mathbf{p}_k$, and the function $C_k(\mathbf{p}_k)$ is homogeneous of degree zero in $\mathbf{p}_k$. Note that $g_k(\phi_k, \mathbf{p}_k) = \ln(\phi_k) - \ln(A_k(\mathbf{p}_k)) = \ln\left(\frac{\phi_k}{A_k(\mathbf{p}_k)}\right)$. In this context, $\frac{\phi_k}{A_k(\mathbf{p}_k)}$ measures the "deflated income" and $A_k(\mathbf{p}_k)$ is a price function that can be interpreted as the weight that scales the individual income. Using Roy's identity, the budget share associated with x_{ki} under the QAIDS model (40) is

$$\frac{p_{ki}x_{ki}^D}{\phi_k} = \frac{\partial \ln(A_k(\mathbf{p}_k))}{\partial \ln(p_{ki})} + \frac{\partial \ln(B_k(\mathbf{p}_k))}{\partial \ln(p_{ki})}\ (\ln(\phi_k) - \ln(A_k(\mathbf{p}_k))) + \frac{\partial C_k(\mathbf{p}_k)}{\partial \ln(p_{ki})}\ \frac{(\ln(\phi_k) - \ln(A_k(\mathbf{p}_k)))^2}{B_k(\mathbf{p}_k)}. \tag{41}$$

The share equations (41) show that the QAIDS specification (40) allows for flexible price effects (e.g., when $\ln(A_k(\mathbf{p}_k))$ is specified to be a quadratic function of $\ln(\mathbf{p}_k)$) as well as quadratic income effects (when $\partial C_k/\partial \ln(\mathbf{p}_k) \neq 0$). Note that the QAIDS model reduces to the Almost Ideal Demand System (AIDS) proposed by Deaton and Muellbauer (1980a) when

$C_k = 0$, where AIDS budget shares are linear functions of the log of income. This indicates that, like the QES specification evaluated in Section 4, the QAIDS specification (40) is also a good candidate to evaluate the linkages between individual welfare and household welfare.

The implications of QAIDS individual preferences (40) for income sharing and household welfare are presented next.

Proposition 8. *Under the Bergsonian household utility (13) and QAIDS individual preferences (40), let* $f_k(z) = z$ *,* $k = 1, ..., K$*. Then the income sharing rule satisfies*

$$\frac{\phi_k}{y} = \mu_k w_k, \;\; k = 1, ..., K, \tag{42}$$

and household welfare is given by

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \frac{\ln(y) + \ln(\mu_k \, w_k) - \ln(A_k(\mathbf{p}_k))}{\tilde{P}_k(\mathbf{p}, y, \mu)}, \tag{43}$$

where $w_k = \left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\phi_k)}\right) / \left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln(y)}\right)$ *is a weight satisfying* $\sum_{k=1}^{K} \mu_k \, w_k = 1$*, and* $\tilde{P}_k(\mathbf{p}, y, \mu) = B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) \left[\ln(y) + \ln(\mu_k \, w_k) - \ln(A_k(\mathbf{p}_k))\right]$*,* $k = 1, ..., K$*.*

Proof. Under the Bergsonian household utility (13), start with the relationship $V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\phi_k, \mathbf{p}_k)$ and equation (40). When $f_k(z) = z$, we have

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \frac{\ln(\phi_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k) \; (\ln(\phi_k) - \ln(A_k(\mathbf{p}_k)))}. \tag{44}$$

Equation (42) is obtained from (12). And substituting (42) into (44) gives the desired result. □

The last case that we consider is $f_k(z) = \ln(z)$.

Proposition 9. *Under the Bergsonian household utility (13) and QAIDS individual preferences (40), let* $f_k(z) = \ln(z)$*,* $k = 1, ..., K$*. Then the income sharing rule satisfies (42) and* *household welfare is given by*

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \ln \left[\frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{\tilde{P}_k(\mathbf{p}, y, \mu)} \right]. \tag{45}$$

Proof. It easily follows from substituting (42) in $V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\mathbf{p}_k, \phi_k)$, where $V_k(\mathbf{p}_k, \phi_k) = \ln \left(\frac{\ln(\phi_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\,(\ln(\phi_k) - \ln(A_k(\mathbf{p}_k)))} \right)$. □

In this context, the term w_k in (42) captures the effects of heterogeneous income effects across individuals within the household. This term also enters the household utility (43).

Next, we examine the case of AIDS preferences where $C_k = 0$ in (40). When $f_k(z) = z$ and from Proposition 8, equations (42) and (43) still apply under the AIDS model, where $w_k = \left(\frac{1}{B_k(\mathbf{p}_k)} \right) / \left(\sum_{k=1}^{K} \frac{\mu_k}{B_k(\mathbf{p}_k)} \right)$. This shows that, under the AIDS specification, unless $B_k(\mathbf{p}_k)$ is a constant across household members, the weights w_k's (reflecting income effects) play a role and affect both the sharing rule (42) and the household utility (43). In other words, the AIDS model (or its QAIDS generalization) typically implies nonlinear sharing rules which depend on two sets of weights: the Bergsonian weights μ_k's and the weights w_k's. And this property holds as well for the household utility function $V(\mathbf{p}, y)$.

4.3 Intra-household Inequality

A desirable property of social evaluation functions is the possibility to express social welfare in terms of an efficiency and equity component (Lambert 1993). We now show that the collective (indirect) household welfare function derived so far can be "abbreviated" into an efficiency component, as if the household were in a unitary framework that assumes an equal distribution of resources, and an equity component that accounts for the dispersion in individual prices and resources across family members. The collective household welfare function is increasing in income and decreasing in the inequality index.

For example, collective household welfare under the QES specification (33) is a monotonic function of household income y deflated by the household price index P_0 as defined in (34)

$$V(\mathbf{p}, y) = \ln(y) - \ln P_0(\mathbf{p}, y, \mu) = \left(\ln(y) - \sum_{k=1}^{K} \mu_k \ln(B_k(\mathbf{p}_k))\right) + \sum_{k=1}^{K} \mu_k \ln(\mu_k).$$

Again, the price index P_0 is composed of two additive terms: the weighted sum of the $B_k(\mathbf{p}_k)$, $\sum_{k=1}^{K} \mu_k \ln(B_k(\mathbf{p}_k))$ describing the dispersion in individual prices, and the Shannon entropy index $-\sum_{k=1}^{K} \mu_k \ln(\mu_k)$ capturing inequality in the distribution of household resources.

Interestingly, for each individual, this term has an inverted U-shape relationship with μ_k: it first increases as μ_k rises from 0 to 0.368, it reaches a maximum when $\mu_k = 0.368$, and then decreases as μ_k rises between 0.368 and 1. Note that, in this case, the Shannon index is closely related to the Theil inequality index. Indeed, using equation (36), the Theil T inequality index (TT) applied to a K-member household can be written as

$$TT \equiv \frac{1}{K}\left(\sum_{k=1}^{K} (\phi_k/\bar{y}) \ln(\phi_k/\bar{y})\right) = \sum_{k=1}^{K} \mu_k \ln(\mu_k) + \ln K \tag{46}$$

where $\bar{y} = y/K$. The TT index measures the distance from the situation where every member of the household attains the same resource share associated with the ideal condition of maximum disorder. If all the power is concentrated in only one member, then the index gives the minimum value corresponding to maximum order. Being negative, it is a measure of inequality rather than equality. A distribution of resources skewed towards one member of the household implies scarcity for the other members (Jorgenson et al. 1980, 1982). Interestingly, if $\mu_k = 1/K$, then TT=0 as in the unitary model. Note also that in the unitary framework individual prices are equal to the market prices and the collective indirect household welfare function degenerates into the traditional unitary indirect welfare function.

If we now consider the specification of the household indirect utility function in equation (39)

$$V(\mathbf{p}, y) = \frac{y - A(\mathbf{p})}{P^*},$$

where the price aggregator is $1/P^* = \sum_k \mu_k \frac{\mu_k W_k}{B_k(\mathbf{p}_k)+C_k(\mathbf{p}_k)\mu_k W_k(y-A(\mathbf{p}))}$ we realize that we can not longer exploit the additive separability property carried by the logarithmic transformation of z, $f_k(z) = \ln(z)$. This structural feature limits both the theoretical and empirical appeal of this representation of collective household welfare. However, it may be interesting to note that in absence of price variation, as it is often the case in cross-sectional demand analysis, we can obtain the following interesting analogy with the intra-household inequality measures obtained in the logarithmic space

$$\sum_k \mu_k \mu_k W_k \tag{47}$$

that is the weighted Simpson diversity index. This means that household inequality is relatively more influenced by the members having greater weight in the family.

5 Conclusions

We model a household in terms of the utility functions of its members using a weighted Bergsonian household utility. In a Pareto efficient household environment, we recover the relationships between centralized and decentralized programs where the maximization of the utility of each household member is subject to an income sharing rule and investigate the relationship between the sharing rules, household welfare and intrahousehold inequality. The established linkages between centralized and decentralized demands provide new and useful information on the economics of intra-household allocations. Our analysis describes the general properties of the income sharing rule as a function of two sets of weights: the Bergsonian weights, and weights reflecting income effects across household members. We show that the latter weights play a role in the evaluation of both household welfare and inequality. We also examine how individual preferences affect the relationship between centralized and decentralized demand. The results provide insights on intra-household decisions and implications for the distribution of household welfare. Intra-household inequality is described by a family of entropy indexes that are function of the sharing rule.

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Appendix

Here, we summarize the transformations analyzed starting from our general specification of the indirect utility function

$$V_k(\mathbf{p}_k, \phi_k) = f_k \left(\frac{g_k(\phi_k, \mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)\, g_k(\phi_k, \mathbf{p}_k)} \right).$$

Note the following features. Panels a.ii) and a.iii) lead to corner solutions for the sharing rule. Interestingly, changing notation Panels b. and d. give the same functional representations of household welfare as in a. and c., respectively. Consider the following substitutions: $\tilde{\phi}_k = \ln(\phi_k)$, and $\tilde{A}_k(\mathbf{p}_k) = \ln A_k(\mathbf{p}_k)$, with $\tilde{y} = \sum_k \ln(\phi_k)$ and $\tilde{A}(\mathbf{p}) = \sum_k \ln A_k(\mathbf{p}_k)$. Rewriting $\ln(\phi_k / A_k(\mathbf{p}_k)) = \ln \phi_k - \ln A_k(\mathbf{p}_k) = \tilde{\phi}_k - \tilde{A}_k(\mathbf{p}_k)$, one obtains

$$\frac{\tilde{\phi}_k}{\tilde{y}} = \mu_k w_k, \tag{48}$$

where $w_k \equiv \left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\ln(\phi_k))}\right) / \left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln(\ln(y))}\right)$ is a weight satisfying $\sum_{k=1}^{K} \mu_k\, w_k = 1$. Note that $\ln y = \sum_k \ln(\phi_k)$ is admissible if $\ln(\phi_k) = (\mu_k\, w_k) \ln y$, and summing over all k, one gets $\sum_k \ln(\phi_k) = \sum_k \mu_k\, w_k \ln y = \ln y$.

Table 1: Sharing Rule and Household Welfare Specifications

$f_k(z) =$	$g_k(\phi_k, \mathbf{p}_k) =$		Sharing rule	Household welfare $V(y, \mathbf{p})$
z	a. $\phi_k - A_k(\mathbf{p}_k)$	i) $C_k \neq 0$	$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_k$	$\frac{y - A(\mathbf{p})}{P^*}$
		ii) $C_k = 0$	$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k$	$\frac{y - A(\mathbf{p})}{P_0^*}$
		iii) $C_k = 0$ and $A_k = 0$	$\frac{\phi_k}{y} = \mu_k$	$\frac{y}{P_0^*}$
	b. $\ln(\phi_k / A_k(\mathbf{p}_k))$	i) $C_k \neq 0$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_{k=1}^{K} \mu_k \frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{\tilde{P}_k(\mathbf{p}, y, \mu)}$
		ii) $C_k = 0$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_{k=1}^{K} \mu_k \frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}, y, \mu)}$
		iii) $C_k = 0$ and $A_k = 1$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_{k=1}^{K} \mu_k \frac{\ln(y) + \ln(\mu_k w_k)}{B_k(\mathbf{p}, y, \mu)}$
$\ln(z)$	c. $\phi_k - A_k(\mathbf{p}_k)$	i) $C_k \neq 0$	$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k W_{0k}$	$\ln\left(\frac{y - A(\mathbf{p})}{P(\mathbf{p}, y, \mu)}\right)$
		ii) $C_k = 0$	$\frac{\phi_k - A_k(\mathbf{p}_k)}{y - A(\mathbf{p})} = \mu_k$	$\ln\left(\frac{y - A(\mathbf{p})}{P_0(\mathbf{p}, y, \mu)}\right)$
		iii) $C_k = 0$ and $A_k = 0$	$\frac{\phi_k}{y} = \mu_k$	$\ln\left(\frac{y}{P_0(\mathbf{p}, y, \mu)}\right)$
	d. $\ln(\phi_k / A_k(\mathbf{p}_k))$	i) $C_k \neq 0$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_{k=1}^{K} \mu_k \ln\left[\frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{\tilde{P}_k(\mathbf{p}, y, \mu)}\right]$
		ii) $C_k = 0$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_{k=1}^{K} \mu_k \ln\left[\frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}, y, \mu)}\right]$
		iii) $C_k = 0$ and $A_k = 1$	$\frac{\phi_k}{y} = \mu_k w_k$	$\sum_{k=1}^{K} \mu_k \ln\left[\frac{\ln(y) + \ln(\mu_k w_k)}{B_k(\mathbf{p}, y, \mu)}\right]$