



Working Paper Series  
Department of Economics  
University of Verona

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WP Number: 1

January 2015

ISSN: 2036-2919 (paper), 2036-4679 (online)

# Return Expectations and Risk Aversion Heterogeneity in Household Portfolios

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## Abstract

We develop a structural econometric model to elicit household-specific expectations about future financial asset returns and risk attitudes by using data on observed portfolio holdings and self-assessed willingness to bear financial risk. Our framework assumes that household portfolios are subject to short-selling constraints in stocks and bonds, and that financial investment decisions are taken conditional on real estate and business wealth. We derive an explicit solution for the model, and estimate its parameters using the US Survey of Consumer Finances from 1995 to 2010. The results show that our modified mean-variance model fits the data adequately, and that the demographic, occupational and educational characteristics of the investors are relevant in shaping risk aversion and return expectations. In contrast, wealth, income, and past market performance have limited impacts on expectations and risk aversion.

*JEL codes:* C34; D14; D81; G11.

*Keywords:* Household Finance; Risk Aversion; Expectations; Mean Variance Analysis; Truncated and Censored Models.

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## 1 Introduction

Only around 50% of the US households hold stocks, either directly or indirectly (Bricker et al. 2012); the rates of participation in the stock market are even lower in Europe (Guiso et al., 2002), and these have recently fallen during the financial crisis (Bricker et al., 2012). There are several reasons for this low participation rate, including entry barriers and transaction costs (e.g. Vissing-Jorgensen, 2002), high borrowing costs (e.g. Davis et al., 2006), crowding-out effects due to real asset holdings (e.g. Cocco, 2005) and financial illiteracy (e.g. van Rooij et al., 2011). In addition, lifecycle portfolio models predict that investment in stocks correlates with age, income and entrepreneurial risk (e.g. Gomes and Michaelides, 2005, and Heaton and Lucas, 2000). However, among those who invest in stocks, large differences in the portfolios held by similar households are quite common (Guiso et al., 2002), and poorly diversified portfolio allocations are also frequently observed (Calvet et al., 2007).

The wide variety of portfolio types held by the households is at odds with the stripped down version of the optimal portfolio allocation. Households make their own financial decisions based on their expectations about assets return and their financial risk attitude, conditional on the background risk and the limits to their investment choice they face. This paper investigates how the heterogeneity in terms of these variables determines their various portfolio holdings.

The heterogeneity of households' expectations regarding the future performance of financial markets has been widely documented in the literature using probabilistic expectations data. Recent research has focused on the likely presence of various *expectation types* (Dominitz and Manski, 2011), on how expectations quickly react to sudden downturns in the market (Hoffmann et al., 2013; Hudomiet et al., 2011), on their relationship with past performance in financial markets (Hurd et al., 2011) and on the quality of the information regarding past performance (Arrondel et al., 2012). Overall, expectations have been found to only moderately correlate with financial market indexes, and to be strongly negatively correlated with model-based expected returns (e.g. Greenwood and Shleifer, 2014). Households' expected financial returns are often negative (Hurd et al. 2011), and not consistent with the rule of thumb that higher expected return should come together with higher expect volatility (Amromin and Sharpe, 2014). Furthermore, households' expectations largely depend on personal investment experience and characteristics, and are known to correlate with non-economic sentiment-creating factors, such as individuals' moods and their perceptions of the weather, and even the results of their favourite sports teams (Kaplanski et al., 2014).

There is also established empirical evidence documenting the heterogeneity of risk attitudes among households. This evidence sometimes comes from laboratory and field experiments (e.g., Andersen et al., 2008, and von Gaudecker et al., 2011) and more frequently from survey data.

Specifically, some authors derive proxies for risk attitudes using various types of survey data: the observed portfolio composition in the form of risky assets holdings (e.g. Riley and Chow, 1992) or the variance in portfolio returns (Buccioli and Miniaci, 2011), choices in a hypothetical lottery (e.g. Donkers et al., 2001; Guiso and Paiella, 2008; Kimball et al., 2009), and self-assessed attitudes toward taking risks (the reliability of which has been assessed by Dohmen et al., 2011). This research generally finds that risk aversion is negatively correlated with wealth and high levels of education, while its correlation with age is unclear – possibly because most of these studies are based on a single cross-section of data, which does not allow to disentangle age from time and cohort effects.

Finally, households' financial portfolio allocation is correlated with the amount and nature of households' illiquid assets, such as real estate investments and businesses. Households with real wealth investments may choose to allocate financial wealth to hedge against the risk embedded in their illiquid assets. The role of homeownership in financial portfolio choice is considered for instance by Flavin and Yamashita (2002), Pelizzon and Weber (2008) and Cocco (2005), while the effect of the presence of significant business investment on the portfolios of entrepreneurs is investigated by Heaton and Lucas (2000), and Faig and Shum (2002) among others.

Data on expectations, risk attitudes and illiquid assets are often used as predictors in reduced-form models of financial market participation (e.g., Amromin and Sharpe, 2014; Arrondel et al., 2012; Dohmen et al., 2011; Hurd et al., 2011), but to the best of our knowledge they have not been linked to a structural model of household portfolio choice in the past. In this paper, we elicit households' expectations of future financial market performance *jointly with* their risk attitudes by combining information from two types of data: observed financial and real portfolio holdings and self-assessed willingness to bear financial risks. We assume that households choose their financial portfolio allocation by maximizing a mean-variance expected utility, subject to a variety of constraints: (i) they cannot take short positions in risky assets, (ii) they cannot trade their real assets, and finally (iii) they differ in terms of risk aversion and expectations on the distribution of excess returns. Accounting for heterogeneity in expectations, preferences and investment limits allows the standard mean-variance approach to generate the observed variety of portfolio types.

More in detail, we consider an investment environment with one riskless asset (*deposits*) and three composite risky assets (stocks, net bonds and real assets) in which households cannot hold short positions in stocks or deposits and the net position in bonds (which includes outstanding mortgages and loans) cannot be larger than the negative value of their real assets (taken as given). This combination of assets and constraints gives rise to seven possible regimes. Each household's choice of the portfolio regime and of the asset demands depend on its perception of the expected

values of the returns on stocks and net bonds and of their correlation with the return on real assets, together with its risk aversion. The heterogeneity of these quantities across households is partly observable and partly unobservable. Given an assumption on the distribution on the unobservable heterogeneity component, the likelihood function of our structural econometric model turns out to be a combination of the likelihoods of a bivariate tobit model and of an ordered probit one. We estimate the model via weighted maximum likelihood using repeated survey data from the US Survey of Consumer Finances from 1995 to 2010. We then use the model to evaluate the effects of demographic, wealth and income changes on risk attitude, expectations and portfolio composition.

Miniaci and Pastorello (2010) already use a related framework to analyze households' portfolio choices with individual investors' data. This work extends their paper in three important directions. First, we combine information on observed portfolio holdings with the self-assessed willingness to take financial risk. This allows us to disentangle observed heterogeneity in expectations and risk attitudes, something that was not possible in previous works that relied on portfolio data only. Second, we explicitly model the crucial role played by households' investments in real estate and business wealth. Housing is the main and often the only investment in risky assets for many households; its risk is potentially correlated with the risk of financial assets, and this can crowd out some households' financial investments. The distribution of business wealth is heavily concentrated, but it plays a crucial role for the few households who invest in their own businesses. Finally, we use six repeated cross-sectional datasets (from 1995 to 2010) rather than one single survey. This allows us to remove the dependency of the results on specific events occurring in a given year and to study how recent financial market performances, and the early experience of the investors, influence households' expectations of future returns.

Our main results can be summarized as follows. We find that an age increase is associated with a rise in risk aversion and in the expected Sharpe performance of net bonds, as well as a fall in the Sharpe performance of stocks. Female, white, married, graduate and retired investors are more risk-averse but also more optimistic about the Sharpe performances of both net bonds and stocks. Self-employed workers and homeowners are more risk-averse and more optimistic about net bond performance, but are less optimistic about stocks. In contrast, expectations do not seem to change across cohorts or following periods of either expanding markets or market crashes. The effects of the observable characteristics are always accurately estimated, and the model predicts net bond and stock shares much better than standard reduced-form models that are conditional on portfolio types.

The remainder of this paper is organized as follows. Section 2 presents the framework, and Section 3 describes the data. Section 4 shows the model estimates and discusses their implications.

Finally, Section 5 concludes. The Appendixes in a separate supplementary document<sup>1</sup> provide further details regarding the solution of the model, the components of the likelihood function and the conditional expected values of the net bond and stock shares.

## 2 The framework

Our framework is made of two components. The first one, introduced in Section 2.1, focuses on household portfolio choice based on the maximization of a mean-variance expected utility function, subject to short-selling and inequality restrictions for financial assets and equality constraints for real assets. The second component, in Section 2.2, deals with self-assessed risk attitudes. Section 2.3 presents our assumptions regarding parameter heterogeneity; Section 2.4 sketches the components of the likelihood function and, finally, section 2.5 discusses the identification restrictions.

### 2.1 Mean-variance portfolio allocation with short-selling and equality constraints

We consider a variant of the standard Markowitz (1959) mean-variance framework in which households with various degrees of risk aversion face restrictions on asset allocation, and we study which expectations would make their actual portfolios consistent with this simple model. While we do not advocate the static mean-variance model as the most suitable one to describe households' portfolio choice, we argue that our alterations of its basic version allow it to fit the data adequately. We will show in Section 4 that this is indeed the case.

We assume the household is endowed with a total wealth  $W_0$ , which includes financial (deposits, stocks and net bonds) and real assets (real estate and business wealth). Following Flavin and Yamashita (2002) and Pelizzon and Weber (2008), we assume the household cannot trade its real assets; moreover, we assume that its long position in real assets can be financed with mortgages, which we assimilate to a short position in bonds. We refer to the difference between the long position in bonds and the outstanding debt as "net bonds". In this framework, therefore, the net position in bonds has a lower bound given by the negative value of the real assets. Moreover, the household faces two non-negativity constraints corresponding to the shares of wealth allocated to stocks and deposits. We further assume that the household is price taker and that a standard budget constraint holds.

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<sup>1</sup> Downloadable at [www.sites.google.com/site/abucciol/assets/bmp\\_appendix.pdf](http://www.sites.google.com/site/abucciol/assets/bmp_appendix.pdf).

Let  $r_0$  denote the interest rate on the riskless asset and  $\mathbf{e} = (e_b, e_s, e_r)'$  be the random vector of the excess returns on net bonds, stocks and real assets, respectively. If  $\mathbf{a} = (a_b, a_s, a_r)'$  is the vector of the shares of wealth invested in the risky assets, then the random wealth at the end of the period is given by  $W_1 = W_0(1 + r_0 + \mathbf{a}'\mathbf{e})$ . Investors' preferences are described by a standard mean-variance expected utility function,  $E[U(\mathbf{a})] = E(W_1) - V(W_1)\eta/2$ , where  $\eta > 0$  represents the absolute risk aversion index varying across households. According to this construction, the moments of  $W_1$  depend on those of the excess returns vector  $\mathbf{e}$ . If we let

$$\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_f \\ \mu_r \end{pmatrix}, \quad \boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{ff} & \boldsymbol{\omega}_{fr} \\ \boldsymbol{\omega}'_{fr} & \omega_r^2 \end{bmatrix} \quad (1)$$

with

$$\boldsymbol{\mu}_f = \begin{pmatrix} \mu_b \\ \mu_s \end{pmatrix}, \quad \boldsymbol{\Omega}_{ff} = \begin{bmatrix} \omega_b^2 & \omega_{bs} \\ \omega_{bs} & \omega_s^2 \end{bmatrix}, \quad \boldsymbol{\omega}_{fr} = \begin{pmatrix} \omega_{br} \\ \omega_{sr} \end{pmatrix} \quad (2)$$

we can rewrite the mean-variance expected utility function as

$$E[U(\mathbf{a})] = (1 + r_0)W_0 + W_0 \left( \mathbf{a}'\boldsymbol{\mu} - \frac{\gamma}{2} \mathbf{a}'\boldsymbol{\Omega}\mathbf{a} \right) \quad (3)$$

where  $\gamma = \eta W_0$  denotes relative risk aversion. The household portfolio selection problem can then be formally stated as follows:

$$\begin{aligned} \max_{\mathbf{a}} \left\{ \mathbf{a}'\boldsymbol{\mu} - \frac{\gamma}{2} \mathbf{a}'\boldsymbol{\Omega}\mathbf{a} \right\} \\ \text{s. t.} \quad a_r = \bar{a}_r, \quad a_b \geq -\bar{a}_r, \quad a_s \geq 0, \quad a_b + a_s \leq 1 - \bar{a}_r = \bar{a}_f. \end{aligned} \quad (4)$$

Given the equality constraint on the real assets share, we can reformulate the problem in Equation (4) as a maximization problem with respect to the financial assets shares only,  $\mathbf{a}_f = (a_b, a_s)'$ ,

$$\begin{aligned} \max_{\mathbf{a}_f} \left\{ \mathbf{a}'_f \boldsymbol{\nu}_f - \frac{\gamma}{2} \mathbf{a}'_f \boldsymbol{\Omega}_{ff} \mathbf{a}_f \right\} \\ \text{s. t.} \quad a_b \geq -\bar{a}_r, \quad a_s \geq 0, \quad a_b + a_s \leq 1 - \bar{a}_r = \bar{a}_f \end{aligned} \quad (5)$$

where  $\boldsymbol{\nu}_f = \boldsymbol{\mu}_f - \gamma \boldsymbol{\omega}_{fr} \bar{a}_r$ .

The problem in Equation (5) makes it explicit that the optimal allocation of the financial component of the portfolio will be unaffected by the presence of real investments only if the returns on the latter are uncorrelated with those of stocks and net bonds, that is, when  $\boldsymbol{\omega}_{fr} = \mathbf{0}$ . In all the other cases, the presence of real assets (that is,  $\bar{a}_r \neq 0$ ) causes the optimal financial portfolio to differ from the one without real assets, the difference being larger for larger real asset weights in the overall portfolio and for higher relative risk aversion among the households. That is, the crowding

out effect of real asset investments is larger for households that are more heavily invested in housing and more risk averse, and it is also larger given stronger correlations between financial and real assets. Before studying the solution to the problem in Equation (5), let us define  $\rho = \omega_{bs}/\omega_b\omega_s$  the correlation coefficient between net bonds and stocks excess returns,  $\pi_b = \mu_b/\omega_b$  and  $\pi_s = \mu_s/\omega_s$  the associated Sharpe performances and

$$\tilde{\pi}_b = \pi_b - \gamma \frac{\omega_{br}}{\omega_b} \bar{a}_r = \pi_b - \gamma \xi_b \bar{a}_r, \quad \tilde{\pi}_s = \pi_s - \gamma \frac{\omega_{sr}}{\omega_s} \bar{a}_r = \pi_s - \gamma \xi_s \bar{a}_r, \quad (6)$$

where the ratios  $\xi_b = \omega_{br}/\omega_b$  and  $\xi_s = \omega_{sr}/\omega_s$  are the hedging terms. We refer to  $\tilde{\pi}_b$  and  $\tilde{\pi}_s$  as the adjusted Sharpe performances, the adjustment being the presence of a term used to hedge against the risk of the constrained component of the overall portfolio. With two tradable risky assets and the set of constraints given in Equation (5), there are seven possible regimes:

- Portfolios in which the shares of stocks and net bonds do not hit any of the constraints:
  - (i) Holdings of deposits, net bonds and stocks,  $(a_b, a_s)$ ;
- Portfolios in which the amount of debt is at its maximum and the household does not invest in bonds:
  - (ii) Holdings of deposits but neither bonds nor stocks,  $(-\bar{a}_r, 0)$ ;
  - (iii) Holdings of deposits and stocks but not bonds,  $(-\bar{a}_r, a_s)$  ;
  - (iv) Holdings of stocks but neither deposits nor bonds,  $(-\bar{a}_r, 1)$ ;
- Portfolios in which the net position in bonds is not at its minimum:
  - (v) Holdings of net bonds and stocks but not deposits,  $(a_b, 1-\bar{a}_r - a_b)$ ;
  - (vi) Holdings of net bonds but neither deposits nor stocks,  $(1 - \bar{a}_r, 0)$ ;
  - (vii) Holdings of net bonds and deposits but not stocks,  $(a_b, 0)$ .

The seven possible combinations can be represented as a right triangle in the  $(a_b, a_s)$  plan, with unitary catheti, as depicted in Figure 1. Its base lies on the  $a_b$  axis due to the no-short-selling constraint on stocks, and its horizontal position is dictated by the debt of the investor. We therefore have various types of solutions to the problem in Equation (5): an internal solution (for case (i)), three “edge” solutions (cases (iii), (v) and (vii)) and three “vertex” solutions (cases (ii), (iv) and (vi)). In what follows, we describe cases (i), (ii), (iii) and (vii), that is, the regimes with at least some investment in deposits. The details of the three remaining regimes (iv, v and vi) are provided in Appendix A.

FIGURE 1 ABOUT HERE



*Internal solution: regime (i)*

The optimal shares are given by:

$$\begin{pmatrix} a_b^* \\ a_s^* \end{pmatrix} = \frac{1}{\gamma(1-\rho^2)} \begin{bmatrix} (\tilde{\pi}_b - \rho\tilde{\pi}_s) / \omega_b \\ (\tilde{\pi}_s - \rho\tilde{\pi}_b) / \omega_s \end{bmatrix}. \quad (7)$$

The solution in Equation (7) describes the optimal allocation for regime (i) only if its values satisfy the three conditions defining the regime, that is,  $a_b^* > \bar{a}_r$ ,  $a_s^* > 0$  and  $a_b^* + a_s^* < 1 - \bar{a}_r = \bar{a}_f$ . Equivalently, the problem in Equation (5) has an optimal internal solution only if the expectations for excess returns on net bonds and stocks are such that

$$\rho\tilde{\pi}_s < \tilde{\pi}_b + \gamma\omega_b(1-\rho^2)\bar{a}_r \quad (8)$$

$$\tilde{\pi}_s > \rho\tilde{\pi}_b \quad (9)$$

$$\tilde{\pi}_s(\omega_b - \rho\omega_s) < -\tilde{\pi}_b(\omega_s - \rho\omega_b) + \gamma\omega_b\omega_s(1-\rho^2)(1-\bar{a}_r). \quad (10)$$

Constraints (8) – (10) define the triangle in the  $(\tilde{\pi}_b, \tilde{\pi}_s)$  plan in Figure 2. If the expectations of the investors fall within the triangle, the model predicts that their optimal portfolio shares are given by Equation (7). Notice that Equation (7) is equivalent to

$$\begin{pmatrix} a_b^* \\ a_s^* \end{pmatrix} = \frac{1}{\gamma(1-\rho^2)} \begin{bmatrix} (\pi_b - \rho\pi_s) / \omega_b \\ (\pi_s - \rho\pi_b) / \omega_s \end{bmatrix} - \frac{\bar{a}_r}{(1-\rho^2)} \begin{bmatrix} (\xi_b - \rho\xi_s) / \omega_b \\ (\xi_s - \rho\xi_b) / \omega_s \end{bmatrix} \quad (11)$$

The first term is equivalent to the solution of the allocation portfolio problem in the case of having no (or non-binding) short-selling constraints on bonds and stocks but an absence of real assets (see Miniaci and Pastorello, 2010). Therefore, Equation (11) informs us that within this framework, the adjustment of the allocation due to the presence of non- tradable real assets does not depend on preferences (i.e., on  $\gamma$ ) but on expectations, particularly on the covariance between tradable risky financial assets and real assets.

FIGURE 2 ABOUT HERE

*“Edge” solution: regimes (iii) and (vii)*

When only constraint (8) is violated, the optimal net bonds share will be equal to its lower limit (i.e.,  $a_b^* = -\bar{a}_r$ ), and the problem reduces to the maximization of

$$E[U(W_1)] = a_s(v_s + \gamma\rho\omega_b\omega_s\bar{a}_r) - \frac{\gamma}{2}a_s^2\omega_s^2 \quad (12)$$

with respect to the stock share  $a_s$ . The optimal share is therefore

$$a_s^* = \frac{1}{\gamma} \frac{\tilde{\pi}_s}{\omega_s} + \rho \frac{\omega_b}{\omega_s} \bar{a}_r = \frac{1}{\gamma} \frac{\pi_s}{\omega_s} + \left( \rho \frac{\omega_b}{\omega_s} - \frac{\xi_s}{\omega_s} \right) \bar{a}_r. \quad (13)$$

Such a solution is internal to the (0,1) interval only if

$$-\gamma \omega_b \bar{a}_r < \tilde{\pi}_s < \gamma \omega_s - \gamma \rho \omega_b \bar{a}_r. \quad (14)$$

The upper and lower limits in Equation (14) define two horizontal lines through the north and south-west vertices of the triangle delimited by conditions (8) - (10) that is drawn in Figure 2. The combination of conditions (8) and (14) identifies the expectations that make the choice of regime (iii) optimal.

When only constraint (9) is violated, it is optimal not to invest in stocks (that is,  $a_s^* = 0$ ), and the problem reduces to the maximization of

$$E[U(W_1)] = a_b v_b - \frac{\gamma}{2} a_b^2 \omega_b^2 \quad (15)$$

with respect to the net bond share  $a_b$ . The optimal share is therefore

$$a_b^* = \frac{1}{\gamma} \frac{\tilde{\pi}_b}{\omega_b} = \frac{1}{\gamma} \frac{\pi_b}{\omega_b} - \frac{\xi_b}{\omega_b} \bar{a}_r. \quad (16)$$

Such a solution is internal to  $(-\bar{a}_r, 1 - \bar{a}_r)$  only if

$$-\gamma \omega_b \bar{a}_r < \tilde{\pi}_b < \gamma \omega_b (1 - \bar{a}_r). \quad (17)$$

The upper and lower limits in Equation (17) define two vertical lines through the south-west and south-east vertices of the triangle in Figure 2. The combination of conditions (9) and (17) identifies the expectations that make the choice of regime (vii) optimal. Notice that the violation of condition (9),  $\tilde{\pi}_s > \rho \tilde{\pi}_b$ , can be induced by a relatively low performance of stocks with respect to net bonds, but also by a high exposition in real assets and high expected correlation between stocks and real assets return, and/or presence of real assets and high risk aversion. In this framework, the choice of an incomplete portfolio does not necessarily coincide with an irrational behaviour, unrealistic expectations or extreme risk aversion.

*“Vertex” solution: regime (ii)*

Vertex solution (ii) emerges when the expectations fall within the remaining areas of the  $(\tilde{\pi}_b, \tilde{\pi}_s)$  plan, that is:

$$\left. \begin{array}{l} \tilde{\pi}_b \leq -\gamma \omega_b \bar{a}_r \\ \tilde{\pi}_s \leq -\gamma \rho \omega_b \bar{a}_r \end{array} \right\} \Rightarrow a_b^* = -\bar{a}_r, a_s^* = 0. \quad (18)$$

It is worth noting that excluding real assets from this framework reduces the number of portfolio regimes from seven to four and that households' choices among them would be independent of the relative risk aversion parameter  $\gamma$  (see, e.g., Miniaci and Pastorello, 2010). The number and types of regimes are instead preserved under the (testable) assumption that real assets and financial assets returns are uncorrelated (that is,  $\omega_{fr} = \mathbf{0}$ ). In this case  $\tilde{\pi}_b = \pi_b$  and  $\tilde{\pi}_s = \pi_s$ , and the conditions on the expectations determining the alternative portfolio regimes can be rewritten in terms of Sharpe performances rather than in terms of adjusted Sharpe performances.

## 2.2 Self-assessed level of risk attitude

The choice of a portfolio type and asset demands depends on the parameters of the distribution of the financial asset returns,  $\mu_f$  and  $\Omega_{ff}$ , and on their correlation with the real assets returns,  $\omega_{fr}$ , together with the relative risk aversion of the investors,  $\gamma$ . We aim to estimate the expectations parameters and risk preferences exploiting two types of data: the observed household portfolios, and the self-assessed attitude toward financial risk taking. Specifically, we use the *Survey of Consumer Finances* (SCF) from 1995 to 2010 to accurately describe the financial and real components of the portfolios of a representative sample of US households and complement this with information from a question designed to elicit their self-assessed willingness to bear financial risk.

Currently, a few household surveys include hypothetical lottery questions in an attempt to determine the respondents' attitudes toward financial risk, and some research projects already use these responses to infer the correlation between risk attitude and socio-demographic and economic characteristics (see, e.g., Barsky et al., 1997, on the US *Health and Retirement Study*; Kimball et al., 2009, on the US *Panel Study of Income Dynamics*; Guiso and Paiella, 2008, on the Italian *Survey of Household Income and Wealth*; and Donkers et al., 2001, on a precursor of the Dutch *DNB Household Survey*). An alternative strategy is to directly ask households how much risk they are willing to bear in financial investments. In the *Survey of Consumer Finances* (SCF), the relevant question reads as follows:

*“Which of the following comes closest to describing the amount of financial risk that you [and your husband/wife/partner] are willing to take when you save or make investments?”*

1. *Take substantial financial risks, expecting to earn substantial returns*
2. *Take above average financial risks, expecting to earn above average returns*
3. *Take average financial risks, expecting to earn average returns*
4. *Not willing to take any financial risks”*

This is a general question regarding the self-assessed level of risk aversion in the financial domain, and the answers are clearly influenced by the personal beliefs of the respondent about what “substantial”, “above average” and “average” financial risks mean. Although very simple, questions like this have proven to deliver information that is consistent with the one derived from more sophisticated elicitation methods, such as paid lottery choices (see Dohmen et al., 2011), and the answers are usually found to be highly correlated with a wide range of objective measures of risk-related behavior (see, e.g., Bucciol and Miniaci, 2011, for the SCF case).

In what follows we assume that, although there is no perfect matching between the declared willingness to bear financial risk and the unobserved risk aversion  $\gamma$ , households reporting a lower willingness to bear financial risks are – on average – more risk-averse than the other households. In practice, we merge the two most risk-prone categories (less frequent in the data) and denote  $c=1$  if the household chooses options 1 or 2,  $c=2$  if the chosen option is 3 and  $c=3$  otherwise. Therefore,  $c=1, 2, 3$ , where a higher value of  $c$  corresponds to a lower willingness to bear financial risks. How risk aversion maps into the reported value  $c$  is unknown. We postulate that, given the risk aversion  $\gamma$ , the reported value  $c$  depends on household’s interpretation of the question, and it is related to the risk aversion by means of a continuous latent variable defined as  $z = \log \gamma + \epsilon_z$ , where  $\epsilon_z$  is a zero mean random error independent of  $\gamma$ , which captures the heterogeneity in the answer styles. The lowest value of  $c=1$  is observed for the lowest values of the latent variable, say  $z \leq \delta_1$ , the intermediate value  $c=2$  whenever  $\delta_1 < z \leq \delta_2$ , and the highest level  $c=3$  if  $z > \delta_2$ , where  $-\infty < \delta_1 < \delta_2 < +\infty$  are two unknown threshold parameters to be estimated.

### 2.3 Heterogeneity assumptions

The above results show that the shares invested in financial assets  $(a_b, a_s)$  depend on  $\boldsymbol{\mu}_f$ ,  $\boldsymbol{\Omega}_{ff}$ , the covariances  $\boldsymbol{\omega}_{fr}$ , and the relative risk aversion of the investors  $\gamma$ , while the reported risk attitude  $c$  depends on  $z$  and hence on  $\gamma$ . Because households differ in various respects and because expectations and risk preferences may change over time, it seems natural to allow all of the relevant parameters to vary with the observable characteristics of the investors. Moreover, we introduce additional heterogeneity in expectations by assuming the presence of unobserved heterogeneity in the expected Sharpe performances of stocks and bonds and the latent variable for risk aversion. More specifically, we assume

$$\begin{bmatrix} \pi_b \\ \pi_s \\ z \end{bmatrix} | \mathbf{x} \sim N \left( \begin{bmatrix} \mathbf{x}'_b \boldsymbol{\beta}_b \\ \mathbf{x}'_s \boldsymbol{\beta}_s \\ \mathbf{x}'_\gamma \boldsymbol{\beta}_\gamma \end{bmatrix}, \begin{bmatrix} \sigma_b^2 & 0 & 0 \\ 0 & \sigma_s^2 & 0 \\ 0 & 0 & \sigma_\gamma^2 \end{bmatrix} \right) \quad (19)$$

$$\xi_k = \frac{\omega_{kr}}{\omega_k} = \mathbf{x}'_{\xi_k} \boldsymbol{\beta}_{\xi_k} \quad \ln \omega_k = \mathbf{x}'_{\omega_k} \boldsymbol{\beta}_{\omega_k} = \beta_{0\omega_k} + \sum_{year=1998}^{2010} d_{year} \beta_{year, \omega_k} \quad k = b, s$$

$$\tan\left(\frac{\pi}{2} \rho\right) = \mathbf{x}'_\rho \boldsymbol{\beta}_\rho \quad \ln(\gamma) = \mathbf{x}'_\gamma \boldsymbol{\beta}_\gamma,$$

where  $\mathbf{x} = [\mathbf{x}'_b, \mathbf{x}'_s, \mathbf{x}'_{\xi_b}, \mathbf{x}'_{\xi_s}, \mathbf{x}'_{\omega_b}, \mathbf{x}'_{\omega_s}, \mathbf{x}'_\rho, \mathbf{x}'_\gamma]'$  are explanatory variables. Notice that, in contrast to all other parameters, the net bonds' and stocks' volatility,  $\omega_b$  and  $\omega_s$ , respectively, are assumed to be constant across households at a given point in time. This choice helps in keeping the model estimation manageable. For the same reason we assume that, conditional on the entire set of observable characteristics  $\mathbf{x}$ , the unobserved heterogeneity components of  $\pi_b$  and  $\pi_s$  are uncorrelated. The zero correlation between  $(\pi_b, \pi_s)$  and  $z$  also contributes to the tractability of the model, but we believe that this assumption is fairly reasonable. In fact  $\epsilon_z$  should not be interpreted as unobserved heterogeneity in the risk aversion parameter  $\gamma$  but rather as heterogeneity in the answer styles and in the relationship between households' understanding of "financial risk" and their risk aversion. Overall, including the coefficients of the explanatory variables, the model is made of 118 parameters that need to be estimated.

## 2.4 Likelihood components

Given the full vector of observable characteristics  $\mathbf{x}$  and the fraction of wealth invested in real assets,  $a_r$ , the assumptions in Equation (19) are such that the probability of observing the triple  $(a_b^* = a_b, a_s^* = a_s, c = l)$  is the product of the probability of observing the specific financial portfolio  $(a_b^* = a_b, a_s^* = a_s)$  and the specific self-reported risk attitude  $c = l$ .

The latter is described by a standard-ordered probit model:

$$\Pr(c = l | \mathbf{x}) = \begin{cases} \Phi(\delta_1 - \mathbf{x}'_\gamma \boldsymbol{\beta}_\gamma) & \text{for } l = 1 \\ \Phi(\delta_2 - \mathbf{x}'_\gamma \boldsymbol{\beta}_\gamma) - \Phi(\delta_1 - \mathbf{x}'_\gamma \boldsymbol{\beta}_\gamma) & \text{for } l = 2 \\ 1 - \Phi(\delta_2 - \mathbf{x}'_\gamma \boldsymbol{\beta}_\gamma) & \text{for } l = 3 \end{cases} \quad (20)$$

where  $\Phi(\cdot)$  is the cumulative distribution function (cdf) of a standard normal distribution; for later use, we also denote  $\phi(\cdot)$ , the corresponding probability density function (pdf). Here and in the

following discussion, we simplify the notation by omitting the conditioning on  $\mathbf{x}$  when this will not cause confusion. We thus follow the standard practice to use an ordered probit model to predict the choice among the alternative willingness to bear financial risks. We depart from what routinely done in the literature (e.g., in Malmandier and Nagel, 2011), as the parameters  $\boldsymbol{\beta}_\gamma$  enter the probability to observe financial portfolio  $(a_b^* = a_b, a_s^* = a_s)$  and are estimated jointly with the expectations' parameters.

To study the probability of observing the alternative portfolios, it is useful to consider the distribution of the adjusted Sharpe performances:

$$\begin{bmatrix} \tilde{\pi}_b \\ \tilde{\pi}_s \end{bmatrix} \Big| \mathbf{x} \sim N \left( \begin{bmatrix} \tilde{m}_b \\ \tilde{m}_s \end{bmatrix}, \begin{bmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_s^2 \end{bmatrix} \right) \quad (21)$$

where

$$\tilde{m}_b = \mathbf{x}'_b \boldsymbol{\beta}_b - \gamma \zeta_b \bar{a}_r, \quad \tilde{m}_s = \mathbf{x}'_s \boldsymbol{\beta}_s - \gamma \zeta_s \bar{a}_r \quad (22)$$

are the expected values of the adjusted Sharpe performances.

*Internal solution: regime (i)*

The distribution of the optimal *bonds* and *stocks* shares for the internal solution (i.e., regime (i)) can be derived using Equation (7) and the normality assumption in Equation (21). We obtain

$$\begin{bmatrix} a_b^* \\ a_s^* \end{bmatrix} \Big| \mathbf{x} \sim N \left( \begin{bmatrix} m_b^* \\ m_s^* \end{bmatrix}, \begin{bmatrix} \sigma_b^{*2} & \sigma_{bs}^* \\ \sigma_{bs}^* & \sigma_s^{*2} \end{bmatrix} \right) \quad (23)$$

where

$$\begin{aligned} m_b^* &= \frac{\tilde{m}_b - \rho \tilde{m}_s}{\gamma(1-\rho^2)\omega_b}, & m_s^* &= \frac{\tilde{m}_s - \rho \tilde{m}_b}{\gamma(1-\rho^2)\omega_s} \\ \sigma_b^{*2} &= \frac{\sigma_b^2 + \rho^2 \sigma_s^2}{[\gamma(1-\rho^2)\omega_b]^2}, & \sigma_s^{*2} &= \frac{\sigma_s^2 + \rho^2 \sigma_b^2}{[\gamma(1-\rho^2)\omega_s]^2} \\ \sigma_{bs}^* &= -\rho \frac{\sigma_b^2 + \sigma_s^2}{[\gamma(1-\rho^2)]^2 \omega_b \omega_s}. \end{aligned} \quad (24)$$

The contribution to the likelihood of the portfolio  $(a_b, a_s)$  is therefore

$$p(a_b, a_s) = \frac{1}{\sigma_b^* \sigma_s^*} \phi_2 \left( \frac{a_b - m_b^*}{\sigma_b^*}, \frac{a_s - m_s^*}{\sigma_s^*}, \frac{\sigma_{bs}^*}{\sigma_b^* \sigma_s^*} \right) \quad (25)$$

where  $\phi_2(\cdot, \cdot; \boldsymbol{\omega})$  is the pdf of a bivariate standardized normal distribution with correlation  $\boldsymbol{\omega}$ ; for later use, we also denote with  $\Phi_2(\cdot, \cdot; \boldsymbol{\omega})$  the corresponding cdf.

“Edge” solution: regime (iii)

For households whose net position in bonds is at its minimum ( $a_b^* = -\bar{a}_r$ ) and who invest a fraction of their financial wealth in stocks ( $a_s^* \in (0,1)$ ) – that is, the investors in the edge regime (iii) – the contribution to the likelihood function is as follows:

$$p(-\bar{a}_r, a_s) = f(a_s) \Pr(\tilde{\pi}_b < \rho\tilde{\pi}_s - \gamma\omega_b(1-\rho^2)\bar{a}_r | a_s). \quad (26)$$

The joint distribution of the optimal stocks share – defined by Equation (13) – and the adjusted Sharpe performance for net bonds is bivariate normal,

$$\begin{bmatrix} \tilde{\pi}_b \\ a_s^* \end{bmatrix} | \mathbf{x} \sim N\left(\begin{bmatrix} \tilde{m}_b \\ m_s^* \end{bmatrix}, \begin{bmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_s^{*2} \end{bmatrix}\right) \quad (27)$$

where

$$m_s^* = \frac{\tilde{m}_s}{\gamma\omega_s} + \rho \frac{\omega_b}{\omega_s} \bar{a}_r, \quad \sigma_s^{*2} = \frac{\sigma_s^2}{\gamma^2 \omega_s^2}. \quad (28)$$

We therefore have

$$f(a_s) = \frac{1}{\sigma_s^*} \phi\left(\frac{a_s - m_s^*}{\sigma_s^*}\right) \quad (29)$$

$$\tilde{\pi}_b | a_s^*, \mathbf{x} \sim N(\tilde{m}_b, \tilde{\sigma}_b^2)$$

and

$$\begin{aligned} \Pr(\tilde{\pi}_b < \rho\tilde{\pi}_s - \gamma\omega_b(1-\rho^2)\bar{a}_r | a_s) &= \Pr(\tilde{\pi}_b < \rho\gamma\omega_s a_s - \gamma\omega_b \bar{a}_r | a_s) \\ &= \Phi\left(\frac{\rho\gamma\omega_s a_s - \gamma\omega_b \bar{a}_r - \tilde{m}_b}{\tilde{\sigma}_b}\right). \end{aligned} \quad (30)$$

“Vertex” solution: regime (ii)

Let us consider the vertex solution ( $a_b^* = -\bar{a}_r, a_s^* = 0$ ), that is, regime (ii). Given the conditions in Equation (18), the probability of such a portfolio allocation is given by the following:

$$p(-\bar{a}_r, 0) = \Pr(\tilde{\pi}_b \leq -\gamma\omega_b \bar{a}_r, \tilde{\pi}_s \leq -\gamma\omega_s \bar{a}_r) = \Phi_2\left(\frac{-\gamma\omega_b \bar{a}_r - \tilde{m}_b}{\tilde{\sigma}_b}, \frac{-\gamma\omega_s \bar{a}_r - \tilde{m}_s}{\sigma_s}; 0\right). \quad (31)$$

In a similar way, we compute the likelihood contribution of portfolios in the other four regimes (iv)-(vii); for details, see Appendix B.

## 2.5 Identification restrictions

Inspection of the above expressions shows that all the probabilities and densities involved in the likelihood components remain unchanged if the elements of  $\beta_b$ ,  $\beta_s$ ,  $\beta_{\xi_b}$ ,  $\beta_{\xi_s}$ ,  $\sigma_b$  and  $\sigma_s$  are multiplied by the same constant and at the same time the logarithm of that constant is subtracted from  $\beta_{\omega_b,0}$  and  $\beta_{\omega_s,0}$ , the intercepts of  $\ln \omega_b$  and  $\ln \omega_s$ , respectively. For this reason, we set  $\sigma_b = 1$ . The same issue arises when an arbitrary constant is added to  $\delta$  and to the intercept in  $\beta_\gamma$  and then multiplied after exponentiation to  $\beta_b$ ,  $\beta_s$ ,  $\sigma_b$  and  $\sigma_s$ . To attain identification, we set  $\delta_1 = 0$ . Finally, although  $\beta_\gamma$  and  $\sigma_\gamma$  are in principle identified, we found that the accuracy of the estimates is significantly improved by imposing the standard identification condition  $\sigma_\gamma = 1$ . Under these constraints, all the remaining parameters in  $\theta = [\beta'_b, \beta'_s, \beta'_{\xi_b}, \beta'_{\xi_s}, \beta'_{\omega_b}, \beta'_{\omega_s}, \beta'_\gamma, \sigma_s, \delta_2]'$  are identified.

## 3 The data

Our analysis is based on data from the US *Survey of Consumer Finances* (SCF). The SCF is a repeated cross-sectional survey of households conducted every three years since 1983 on behalf of the Federal Reserve Board. Its purpose is to collect detailed information on assets and liabilities, together with income and the main socio-demographic characteristics of a sample of US households. In this work, we will consider the six waves from 1995 to 2010, i.e., the most recent waves available at the time of this study. We neglect previous waves mainly to analyze observations with similar investment options.<sup>2</sup> By design, the survey over-samples relatively wealthy households; in our study, we always use the sampling weights provided by the SCF to compute descriptive statistics and estimate the model. The SCF copes with the typical item non-response issue of the wealth-related micro-data by adopting a multiple imputation approach; we base our analysis on the average of these imputations.

We focus our attention on households with a financial wealth of at least \$5,000, non-negative net real wealth, and a head of household aged between 25 and 90 years. Our final sample consists of 19,444 observations, with the sample size increasing from 2,851 in 1995 to 4,108 in 2010 (see Table 1).

TABLE 1 ABOUT HERE

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<sup>2</sup> For instance, 401(k) plans and other retirement assets that are now important to household portfolios were not widespread until the mid-90s.



Given its characteristics, the SCF has been widely used to investigate asset allocation. In our model of portfolio choice the endowment available to households is made of one risk free asset (deposits), which includes checking, savings and money market accounts, certificates of deposit and call accounts at brokerages, two risky financial assets and one risky real asset.

The first risky asset, which we label net bonds, includes the market value of directly held corporate and government bonds, savings bonds, bond mutual funds, half the value of balanced mutual funds and the cash value of life insurances. In our framework, households choose their portfolios in the presence of non-tradable real assets; we therefore take into consideration loans related to real estate and business and treat them as negative positions in bonds. More specifically, the composite asset net bonds is given by the value of bonds net of the loans on the primary residence, other real estate and businesses.

The second risky financial asset (stocks) includes directly held stocks, stock mutual funds and half the value of balanced mutual funds. The real assets are defined as the sum of the owner-occupied primary residence, other real estate and business wealth. The real estate category excludes properties used to run a household's own business, which are instead included in the business wealth category. The SCF considers "farms, professional practices, limited partnerships, private equity or any other business investments that are not publicly traded" to be businesses. Respondents also report the investment allocations of any composite asset holdings (IRA-KEOGH accounts, retirement accounts, annuities and trust-managed accounts), which allows us to split these holdings across our asset categories accordingly. In what follows, we define financial wealth as the sum of risk-free, bond and stock holdings; total wealth, in contrast, includes financial wealth, as well as real assets and the related liabilities. Current values are converted in 2010 USD using the Consumer Price Index, which is computed by the Federal Reserve Bank of St. Louis, for all urban consumers.

Financial wealth showed ups and downs over the period under investigation, going from a median 54,346 USD in 1995 to 87,439 USD in 2001 (+61%) and declining to 70,000 USD in 2010 (-20% with respect to 2001; see Table 1). The average financial wealth shows a different pattern, increasing from 213,297 USD in 1995 to 361,668 USD in 2001 and being almost constant afterward. The difference between the dynamics of mean and median financial wealth is indicative of how the downturn of the financial markets had remarkably different effects depending on the amount of the household wealth. Both average and median total wealth, which include real assets and their related loans, grew steadily from 1995 to 2007 (+81% for the mean, +69% for the median value), and they both experienced a drop in 2010 (-10% for the mean, -20% for the median value). The relative stability of total wealth with respect to financial wealth is due to several contributing

factors: the interrupted increase in real estate values until 2007<sup>3</sup>, the diffusion of real estate holdings among US households (Table 1, panel b) and the importance of real assets to the overall amount of wealth (Table 1, panel c). Indeed, the percentage of households owning some real estate has constantly been above 83%, and the share of real assets in the aggregate portfolio ranged between 61% in 2001 and 73% in 2007. Participation in the stocks and bonds markets has been less common and more discontinuous. The fraction of households holding bonds in their portfolio increased from 24% in 1995 to 31% in 2004; in the meantime, the percentage of families investing in stocks increased by 10 percentage points, reaching its peak of 77% in 2001, and then returned to the starting value of 67% in 2010.

The shares of stocks in the aggregate portfolio changed accordingly: they accounted for 20% of the value of total wealth in 1995 and 30% in 2001, and they then returned to 20% in 2010. The percentage of households with debts related to real estate and businesses moved from 49% in 1995 to 56% in 2004 and 2007. This increase was accompanied by a deterioration of the net position of bonds and loans, which was -5% of total wealth in 1995 and decreased to -8% in 2004.

The dissemination of the seven portfolio types identified in the previous section depends on the assets holdings and shares reported in panels b) and c) of Table 1. Panel d) shows that during the period considered, at least 59% of the households invested in regime (i) portfolios  $(a_b, a_s)$ . That is, they had positive positions in deposits and stocks, and their net position in bonds and loans was larger than the negative value of their real assets. The second most common type of portfolio, representing 19% to 27% of cases, falls within regime (vii)  $(a_b, 0)$ , where there is no investment in stocks. Overall, the percentage of portfolios without stocks, that is, in the edge and vertex regimes (ii), (vi) and (vii), decreased from 30% in 1995 to 23% in 2001 and then recovered to over 30% in the last few years.

More than 98% of the households in regimes (ii), (iii) and (iv), with  $a_b = -\bar{a}_r$ , do not have any real assets, and their choice not to invest in net bonds can be interpreted as the presence of binding credit constraints due to the absence of possible collaterals, limiting their investment capacity. As we may reasonably expect, few households chose portfolios with no deposits (i.e., regimes (iv), (v) and (vi)). In contrast, a non-negligible percentage of households did not invest in either stocks or bonds. This percentage grew from a minimum of 19.4% in 2001 to a maximum of 32.2% in 2010, increasing the relevance of portfolio regimes (ii) and (vii).

For our purpose, the other important piece of information conveyed in the SCF is the question regarding the willingness to bear financial risks described in Section 2.2. Most frequently,

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<sup>3</sup> See the House Price Index of the Federal Housing Finance Agency at <http://www.fhfa.gov>.

households responded by choosing options 3 (willing to “*Take average financial risks expecting to earn average financial returns*”, between 44.1% and 47.1% of the sample) or 4 (“*Not willing to take any financial risk*”, between 26.5% in 1998 and 35.4% in 2010). We interpret these answers as indicators of moderate or high levels of self-assessed risk aversion among the respondents. Given the few households willing to “*take substantial financial risks expecting to earn substantial returns*” (option 1, 4.1% overall), we merge the two most risk prone options together. In the end, we have three possible levels of stated risk aversion. Risk aversion’s distribution is relatively stable across the waves, although we do observe a higher willingness to take risks in the years 1998 and 2001, which corresponds to the highest stock market participation rates during the period under analysis.

#### TABLE 2 ABOUT HERE

Households not willing to bear any financial risk are substantially older and less wealthy than the remainder of the sample. Table 2 shows that their median financial wealth (38,000 USD) was about one-third of the median financial wealth of the most risk-prone households; these households were, on average, 12 years older than the least risk-averse households. The types of portfolios change considerably between the three groups of respondents. In Table 2, we consider the four portfolio regimes with some investments in deposits, which constitute 99.9% of the cases. About 78.5% of the households most willing to bear financial risk held a type (i) portfolio, and only 13.9% of them did not hold stocks. In contrast, among households declaring themselves unwilling to bear any risk, about 49.3% actually held some stocks, even though their share invested in stock was limited.

The analysis in Section 4 uses information on portfolio choice and the willingness to bear financial risk, exploiting further household information included in the SCF as control variables. Specifically, we consider the age, gender, race, education level, marital status and occupational status of the head of household; a dummy for the presence of children and second-order polynomials for the financial wealth, total wealth and total annual income of the household. The volatility parameters  $\omega_s$  and  $\omega_b$  are assumed to vary only with the year of the interview, which we control by introducing time dummies. We allow the expected value of the Sharpe performances,  $\pi_b$  and  $\pi_s$ , vary with age (squared polynomial), period and cohort. Regarding period effects, we mimic Ameriks and Zeldes (2004) and introduce, as control variables, the annual returns on the 10-year T-bond for the survey year in  $E[\pi_b | \mathbf{x}]$ , as well as the S&P500 annual return for the same year in

$E[\pi_s | \mathbf{x}]$ .<sup>4</sup> Regarding cohort effects, we introduce the return of the stock market when the individual was aged between 20 and 24 (using historical S&P500 data<sup>5</sup>). This choice is consistent with the approach of Malmendier and Nagel (2011), which assigns to each year-of-birth cohort a specific measure of its experienced financial markets performances. We deviate from their procedure by exogenously imposing the relevant period to consider. This variable should control for the residual heterogeneity across cohorts with respect to their expectations and risk attitudes. We conjecture that, *ceteris paribus*, most of this heterogeneity is related to how good or bad the market conditions were when the individuals began making independent financial decisions. This early experience could affect the way in which people learn how to process the available information and develop their own expectations and perception of risk. Notice that the contemporaneous presence of age and cohort but not time effects for the hedging terms  $\xi_b = \omega_{br} / \omega_b$  and  $\xi_s = \omega_{sr} / \omega_s$ , the correlation  $\rho$ , and the risk aversion  $\gamma$  implies that these components of the model are time-varying, but their variation over time is attributed to age and cohort effects only.<sup>6</sup>

#### 4 Estimation results

We present the weighted-maximum likelihood estimates of all the 118 parameters, along with their asymptotic standard errors, in Tables A1 and A2 in Appendix. The parameters are estimated with precision, but their interpretation is not straightforward, due to the nonlinear transformation applied to the covariates and/or to the parameters of households' expected returns and risk aversion. We therefore discuss our results in terms of the average percentage variations reported in Table 3.

TABLE 3 ABOUT HERE

Given the identification restrictions imposed on the variances,  $\sigma_b^2 = \sigma_\gamma^2 = 1$  and  $\delta_1 = 0$ , the average predicted values shown in the first line of Table 3 do not have direct economic

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<sup>4</sup> Data retrieved from [http://people.stern.nyu.edu/adamodar/New\\_Home\\_Page/datafile/histret.html](http://people.stern.nyu.edu/adamodar/New_Home_Page/datafile/histret.html).

<sup>5</sup> The source is <http://www.econ.yale.edu/~shiller/data.htm>.

<sup>6</sup> Estimation of age, time and cohort effects is inevitably conditional on the identifying assumptions adopted, and their interpretation in terms of life cycle should take into consideration the static nature of the model. Nevertheless, the rich specification adopted, which controls for household composition and the amount of income and wealth, and the key role played by the real assets, make us confident that what we identify are changes in expectations and risk preferences genuinely related to ageing.

interpretation. Furthermore, there is no directly observable counterpart for the predicted objects. Indeed, we have direct measures of neither the risk aversion parameter nor the expectations of the investors regarding the composite risky assets, net bonds and stocks. The latter may be influenced by the observable market performance, but they are not necessarily consistent with the actual returns and volatility. Moreover, the definition of the net bonds asset in our model (as net position in bonds and real asset-related loans) does not have a traded financial instrument to compare it with.

With these caveats in mind, we observe that the average estimated relative risk aversion is 2.10, the correlation between net bonds and stocks returns is 0.20 and the predicted Sharpe performance for net bonds is about twice the corresponding value for stocks (4.90 vs 2.43). We estimate the standard deviation of the unobserved heterogeneity in the stocks Sharpe ratio to  $\hat{\sigma}_s = 2.06$ , which is larger than the corresponding parameter for net bonds, which is set equal to one. The estimated expected standard deviation of net bonds ( $\omega_b$ ) ranges between 3.69 in 2010 to 3.82 in 1995 (see Appendix Table A2), and it is lower than that of stocks (from 4.95 in 1998 to 6.16 in 2010). These figures, together with the estimated hedging terms  $\xi_b$  and  $\xi_s$ , imply that the betas of real assets with net bonds,  $beta_b = \omega_{br} / \omega_b^2 = \xi_b / \omega_b$ , are estimated to be close to one, while the  $beta_s = \xi_s / \omega_s$  for stocks ranges between 0.16 in 1995 and 0.12 in 2010.

The average effects of the changes in the observable characteristics of the investors differ across the components of the model; their standard errors are always very small and point to average effects that are significantly different from zero. Increasing the age of every head of household in the sample by 5 years induces, on average, a 2.62% increase in the expected Sharpe performance on net bonds  $E[\pi_b]$ , as well as a 1.90% increase in risk aversion; however, the same variation also causes a reduction in the stocks counterpart  $E[\pi_s]$  and the hedging term  $\xi_b$ , as well as an increase of  $\xi_s$ . Ageing is therefore associated with a lower  $beta_b$  but a higher  $beta_s$ . The relationship between the returns correlation  $\rho$  and age is not monotonic, but on average, the effect is positive. White, female, married, graduate, and retired investors are *ceteris paribus* more risk-averse but also more optimistic about Sharpe performances. Home ownership has a remarkably positive effect on  $E[\pi_b]$ , a limited impact on the covariance between real assets and net bonds and a noticeably negative impact on stocks and real assets  $beta_s$  and on the correlation between stocks and net bonds. White, female and married investors have a lower covariance between net bonds and real wealth and higher correlation between stocks and real assets, as well as stocks and net bonds.

The same pattern holds for the retirees, except that their expectations regarding the correlation between stocks and net bonds are lower than average.

The past performances of financial markets do not seem the main driver of households' expectations updates. An increase of 10 percentage points in the annual 10-year T-bond returns, which amounts to doubling the average observed value for the sample in hand, is associated with a 2.75% increase in  $E[\pi_b]$ ; a similar change in the S&P500 returns (which correspond to a 70% increase for the average annual S&P500 return in the sample) reduces the expected stocks' Sharpe performance by 2.24%. Overall, our results support neither the idea that investors tend to have significantly lower expectations after periods of expanding markets – consistent with a mean reverting process of expectation formation for equity returns (see Dominitz and Manski, 2011) – nor the work of Hudomiet et al. (2011), who show that the 2008 market crash caused a significant increase in the population average of expectations.

The wealth and income of investors are correlated with their views, but their impact on expectations and risk aversion parameters is limited. Increasing financial wealth and income by 10% has, on average, negligible effects on all the components of the model; a similar increase in net real wealth raises the expected Sharpe performance of net bonds by 2.41% and decreases the correlation between stocks and net bonds by 3.15%. The unclear relationships between risk, income and wealth are also documented in a large sample and in a completely different framework by von Gaudecker et al. (2011). Our results are in contrast with those discussed by Hurd et al. (2011), who documented a clear positive correlation between expected returns and investors' endowment. We are cautious in interpreting the estimated relationships between wealth, income and expected returns: it is probably the case that being wealthier makes people more optimistic and therefore more likely to invest in risky assets. However, the inverse relationship may also be at work: being more optimistic (for any reason) may make people look for higher but riskier returns, which makes them richer on average. It is difficult to assess to what extent this potential endogeneity issue affects our results. The use of predetermined levels of wealth and income might help to solve the problem, but unfortunately, this information is not available, because the SCF is a repeated cross-sectional survey.

Finally, our control variable for the cohort effects seems to be insignificant; apparently different initial conditions for the financial market do not noticeably change either the expectations regarding *bonds* and *stocks* or the attitude toward risk.

The last two columns of Table 3 show statistics for the adjusted Sharpe performances  $E[\tilde{\pi}_b]$  and  $E[\tilde{\pi}_s]$ , which are the performances corrected for the hedging against real asset holdings. These

performances are, in fact, those actually determining the portfolio choice in our mean-variance model, in which real wealth is risky but not tradable. The average predicted values are remarkably different from those of the unadjusted performances  $E[\pi_b]$  and  $E[\pi_s]$ : the average predicted expected value for net bonds is negative (-2.56) and lower than that for stocks (1.09); the median values (not reported in the table) are higher but characterized by the same ranking (-0.5 for net bonds and 1.25 for stocks). The partial effects are often in contrast with those computed for  $E[\pi_b]$  and  $E[\pi_s]$  both in terms of size and sign. For instance, the effect of being graduate is positive for the stocks' Sharpe performance,  $E[\pi_s]$ , but negative and large for its adjusted counterpart,  $E[\tilde{\pi}_s]$ ; homeownership has, on average, a positive and large effect on  $E[\tilde{\pi}_b]$  (+319%), which is three times the effect on  $E[\pi_b]$ ; a 10% increase in financial wealth does not greatly change the estimated expected Sharpe performance for net bonds, but it decreases the estimated  $E[\tilde{\pi}_b]$  by 10%; and finally, the expected adjusted Sharpe performance of net bonds,  $E[\tilde{\pi}_b]$ , is significantly affected by past bonds returns, while  $E[\pi_b]$  is much less sensitive to market conditions.

FIGURE 3 ABOUT HERE

The adjusted and unadjusted Sharpe performances are depicted in Figure 3. The scatter plots show that for many households the expected Sharpe performances of the financial assets are negative. With this respect, our estimated expectations, elicited using a structural model, are similar to the subjective expectations on stock returns measured using probabilistic methods by Dominitz and Manski (2011) in the U.S. and Hurd et al. (2011) in the Netherlands. Furthermore, the figure shows that the presence of the constrained real assets, and therefore the translation from the original Sharpe performances to the adjusted versions, determines a major change in the way the net bonds and stocks investments are expected to contribute to a specific household portfolio return. Two investors with the same stocks expectations  $E[\pi_s]$  but having different risk aversion  $\gamma$  and/or a different share of wealth invested in real assets  $a_r$ , will have a different  $E[\tilde{\pi}_s]$ , and they will choose different portfolio allocations. In almost all cases, the predicted adjusted Sharpe ratios for net bonds are smaller than the corresponding predicted Sharpe performances. This is because the estimated  $\xi_b$  are almost always positive. This is not the case for stocks, where the hedging term between stocks and real assets is often estimated as negative.

#### 4.1 Predicted portfolio types and assets shares

Thus far, we have illustrated the estimation results with respect to the primitives of our investment model, that is, expectations and risk aversion parameters. We now turn to the description of our results in terms of the prediction of the observable components, mainly the portfolio types, as well as portfolio shares for stocks and net bonds. In what follows, we focus on regimes (i), (iii) and (vii), in which households invest at least part of their financial wealth in deposits. These portfolio regimes account for 94.7% of the observed cases. The asset shares predicted based on these regimes, together with their simulated partial effects, are summarized in Table 4.<sup>7</sup> Studying the expected asset shares conditional on the portfolio regime is not straightforward, because they are a non-linear combination of the parameters of interest. See Appendix C for details on the computation method.

TABLE 4 ABOUT HERE

The average estimated expected values of net bond and stock shares conditional on the observed portfolio regime are close to the corresponding averages for the actual shares (first and second lines of Table 4). We can assess the goodness of fit of the model by computing the  $R^2$  as the squared correlation between the actual and predicted shares. For stocks, the  $R^2$  is 5.2% in regime (iii) and 38% in the most common regime, (i); for net bonds, the  $R^2$  is very high, about 98%, which is driven by the high correlation between  $a_b$  and  $\bar{a}_r$ .

The analysis of how the predicted shares of stocks and net bonds react to changes in the characteristics of the households shows that, as expected, in many cases the effects on  $E[a_b^* | regime = (i)]$  and  $E[a_b^* | regime = (vii)]$  have the same sign, while those on  $E[a_s^* | regime = (i)]$  have the opposite sign. Ageing increases the share of wealth invested in net bonds and reduces the position in stocks; female, married, self-employed and retired investors have, on average, a higher predicted share of net bonds and a lower share of stocks in regime (i). In regime  $(a_b, a_s)$ , turning the households from tenants into homeowners reduces the predicted stocks share by 4.4 percentage points on average in favor of a +7.8 p.p. in net bonds. The results for  $E[a_s^* | regime = (iii)]$  are sometimes not consistent with those for regime (i). For instance, for regime  $(-\bar{a}_r, a_s)$ , in which households almost always have purely financial portfolios (i.e.,  $\bar{a}_r = 0$ ), women are associated with a higher levels of investment in stocks. Conditional on portfolio regime

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<sup>7</sup> Some asset shares are trivially known as  $a_b = -\bar{a}_r$  in regimes (ii) and (iii) and as  $a_s = 0$  in regimes (ii) and (vii).



type and indirectly on the share invested in real assets ( $\bar{a}_r$ ), wealth and income variables have a rather limited effect on the shares of net bonds and stocks.<sup>8</sup> The fractions of wealth invested in net bonds and stocks are not remarkably affected by past performances of the stocks and bonds markets; nevertheless, there are significant changes in the share of stocks over time, with levels from 2004 being consistently lower than those in 2001.

#### TABLE 5 ABOUT HERE

In Table 5, we analyze the predicted probability that a portfolio will be contained in regime (i), (iii) or (vii). The predicted probabilities are in line with the observed frequencies, but with an under-estimate regarding the internal regime, (i): the model predicts a 57% average probability, whereas we observe that 65% of the sampled households are in regime (i). In contrast, the model overestimates the probability of observing a portfolio in regime (v),  $(a_b, 1 - \bar{a}_r - a_b)$ . The probability that a specific portfolio is contained within a specific regime also correlates with the characteristics of the household. Keeping in mind that portfolios in regimes (i) and (iii) include stocks, while those in regime (vii) do not, we expect the effects to often have opposite signs. This occurs for many demographic characteristics, with white investors being more likely to hold portfolios of types (i) and (iii) and less likely to fall within regime (vii); females, self-employed workers and retirees have a lower predicted probability of holding stocks and are more likely to choose a portfolio of type  $(a_b, 0)$ . Homeownership increases the chance to hold a  $(a_b, a_s)$  portfolio and drastically reduces the probability of falling within regime (iii),  $(-\bar{a}_r, a_s)$ . The latter result is unsurprising given that almost all the households with portfolio type (iii) do not own real assets and in our framework, they are considered to be credit-constrained because of a lack of collateral. *Ceteris paribus*, changes in income and wealth are not associated with remarkable differences in portfolio type choice. Finally, age, and stock and bond market performances and cohort effects as measured by the stock market return when the relevant heads of household were 20-24 years old seem to have a marginal role in portfolio type choice. Nevertheless, due to the significant changes in the estimated volatility of net bonds and stocks, the model predicts an increase in the probability a portfolio will be contained within regime (i) after 2001.

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<sup>8</sup> Given that almost no households in regime (iii) own any real wealth, the percentage variation of their net real wealth is nil, so the effect on the share of stocks is nil as well.

## 4.2 Structural vs. non-structural modeling approaches

We conclude our analysis by contrasting the results based on the mean-variance (MV) structural model with those obtained on the basis of simpler models without any structural interpretation. We first compare the relative partial effects on risk aversion  $\gamma$ , shown in Table 3, with those obtained using a standard ordered probit. The latter are shown in the first column of Table 6, where we observe that the relative partial effects of many demographic characteristics of the investors on their risk aversion are remarkably different if estimated with the standard approach rather than with our structural model. For instance, the order probit estimates confirm the usual results (graduate investors and self-employed workers are significantly more risk-prone, and females are more risk-averse), while on average, our structural estimates in Table 3 show negligible effects for educational attainment and a positive effect for self-employment, as well as confirming the result for the gender effect. The contrast is even sharper if we consider homeownership, regarding which the two approaches indicate significant effects of opposite sign. In contrast, both models agree that income, wealth and the variable controlling for cohort heterogeneity all play a limited role.

TABLE 6 ABOUT HERE

Regarding net bonds and stocks shares, we consider three independent linear models for regimes (i), (iii) and (vii), regressing  $a_b$  and  $a_s$  on  $\mathbf{x}_b$  or  $\mathbf{x}_s$ . In Table 6, we show the partial effects that are comparable with those in Table 4. The  $R^2$  of the simple linear models are considerably lower than those obtained with the complete MV model in the most prominent regimes, (i) and (vii). This makes us prefer our estimates based on the complete MV model. Both approaches confirm that conditional on the type of portfolio held and homeownership, changes in households' wealth and income do not have a major impact on stocks and net bonds shares, and in many cases, the sign of the effects are consistent between the two sets of results. In the case of homeownership status, the sign of the effect on the predicted net bonds share is positive in the structural approach, and it becomes negative and very large in the simple linear model (about -82 percentage points in regime (i) and -94 in (vii)). This drastic change must be read while taking into consideration that in the MV framework, the portfolio shares incorporate the information on investment in real assets, while in the simple linear regression model, the homeownership dummy signals the availability of a potential collateral for loans and a possibly negative position in net bonds.

Finally, we compare the results for the regime probabilities in Table 5 with those obtained using a standard multinomial logit model. For the latter, we consider four regimes: (i), (iii), (vii) and a fourth one gathering the remaining portfolio types. The comparison shows that the structural and

multinomial logit models often deliver contrasting results. For instance, the structural model predicts a lower probability that portfolios will be contained in regime (vii),  $(a_b, 0)$ , among women and the self-employed, while the reverse is predicted by the standard model.

## 5 Conclusions

In this paper, we develop a structural econometric model within the mean-variance (MV) portfolio framework to elicit US-household-specific expectations about future financial asset returns *and* risk attitudes, using data on observed portfolio holdings and answers to questions regarding the self-assessed willingness to bear financial risk. We assume that households' portfolio choices can be approximated via an MV investment model for financial and real wealth, in which real assets cannot be traded in the short run and provide a collateral for borrowing.

More specifically, the investment set is made of one riskless asset (deposits), two risky financial assets (net bonds and stocks) and one risky real asset; households cannot take short positions in deposits and stocks, but they can finance their investment in real assets using collateralized debt, which we interpret as a short position in bonds. Our net bonds asset category is therefore a composite instrument given by position in corporate and government bonds net of the mortgages and loans for real assets, and this position is bounded from below by the value of the real assets. We derive an explicit solution to the model that is characterized by seven possible portfolio regimes and three levels of self-assessed risk attitude. We then analyze the model using a combination of structural tobit and ordered probit estimation methods.

We find that there is wide heterogeneity in the expected distributions of net bonds and stocks returns, their correlation with real assets returns, and risk aversion and that many observed household characteristics correlate with these households' actual behavior. Increased age is associated with an increase in risk aversion and net bond Sharpe performance and a decrease in stock Sharpe performance. White, female, married, graduate, and retired investors are more risk averse but also more optimistic about the Sharpe performances of both financial assets; the self-employed and homeowners are more risk averse and positive about net bond Sharpe performances, but also more pessimistic with respect to stock Sharpe performance. In contrast, expectations and risk aversion vary little with wealth, income and past financial market performance. Nevertheless, the volatility of the expected stocks return is estimated to be higher after 2001.

Overall, our model predicts the shares of net bonds and stocks better than standard atheoretical reduced-form models that are conditional on portfolio types. Our findings support the view that the simple mean-variance investment model is able to explain much of the observed

diversity in household portfolios when constraints on real assets and heterogeneity in expectations and preferences are taken into account. Moreover, the use of a structural approach and appropriate data allows us to separately evaluate the effects of a change in expectations and the effects of a change in preferences, two dimensions that would otherwise become confused with each other.

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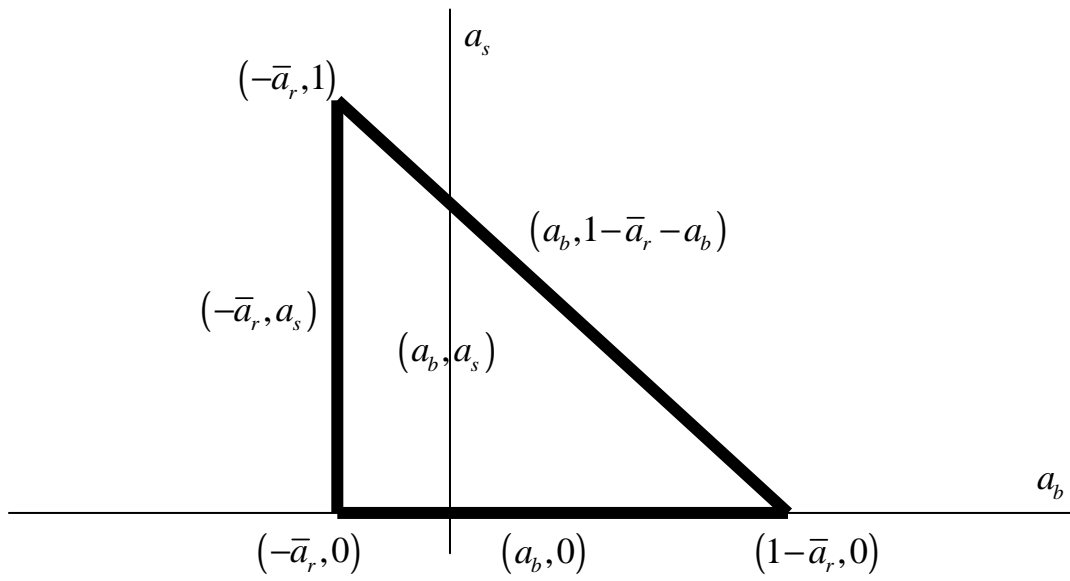
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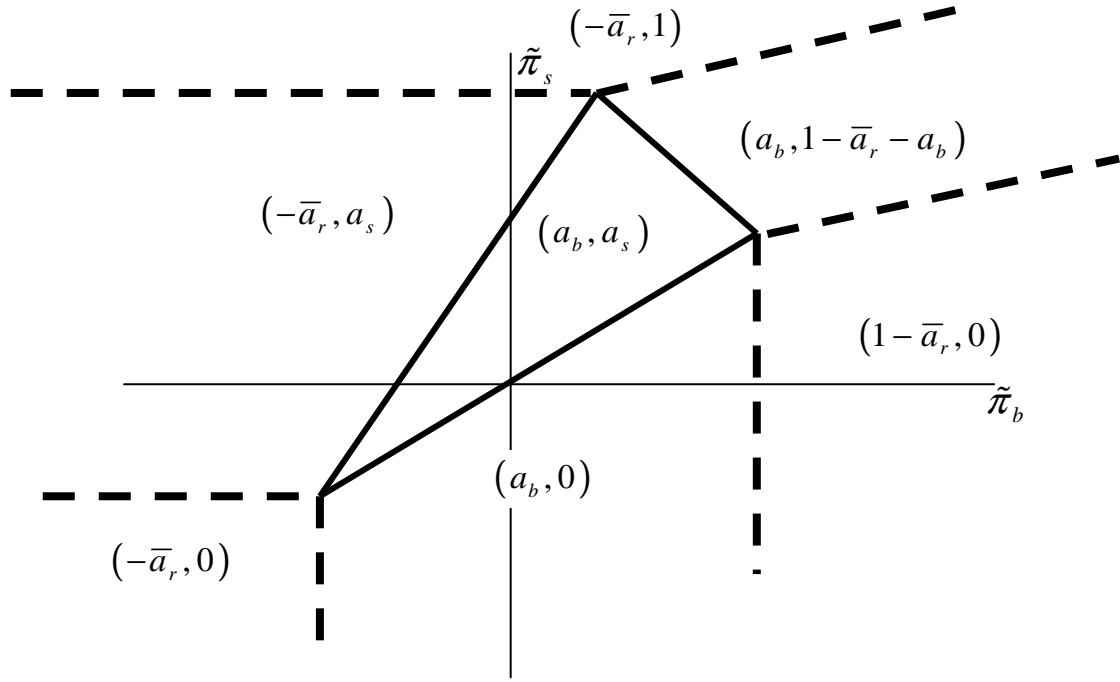
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## FIGURES

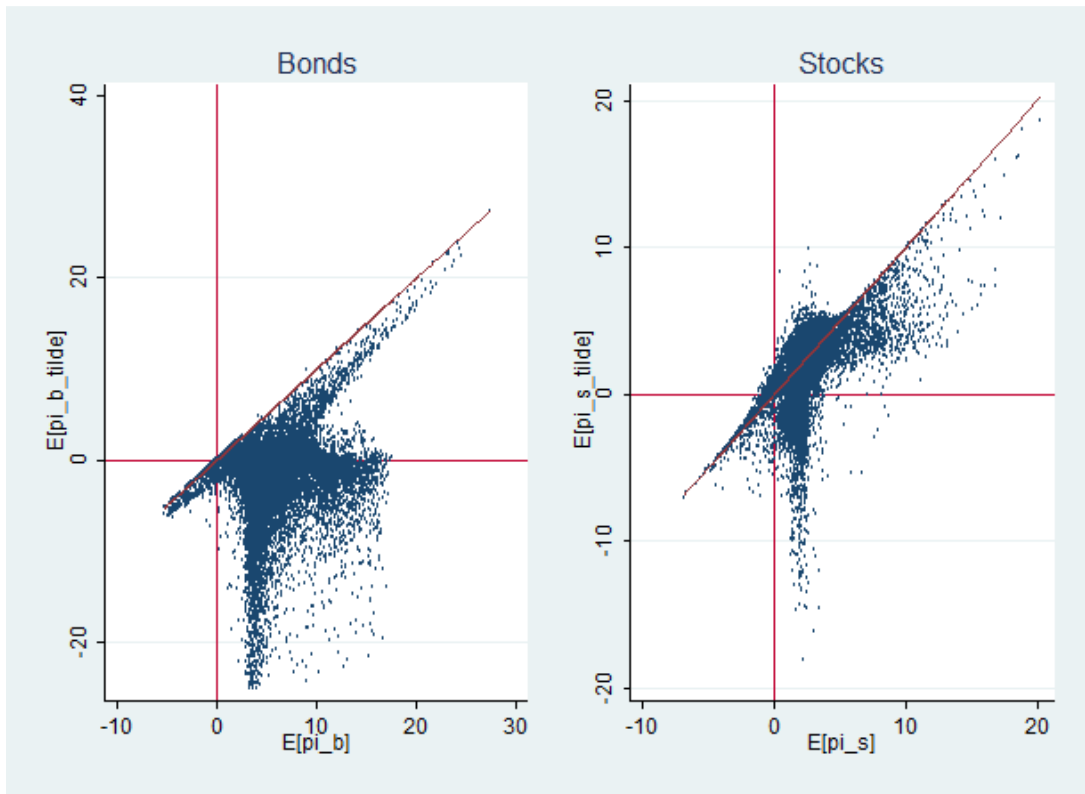


**Figure 1: Portfolio regimes.** Each pair indicates the portfolio shares in bonds and stocks, respectively. We denote with  $a_b, a_s, \bar{a}_r$  the generic shares in bonds, stocks and real assets. The latter is taken as given.



**Figure 2: Portfolio regimes in the adjusted Sharpe performance plan.** Any combination of expected adjusted Sharpe performance for *bonds*,  $\tilde{\pi}_b = \pi_b - \gamma \xi_b \bar{a}_r$ , and *stocks*,  $\tilde{\pi}_s = \pi_s - \gamma \xi_s \bar{a}_r$ , that falls in the triangle with solid edges gives origin to an internal solution, that is to the choice of a portfolio of type  $(a_b, a_s)$ . Each couple  $(\tilde{\pi}_b, \tilde{\pi}_s)$  falling outside the triangle, in areas delimited by one of the edges of the triangle and the dashed lines, generates one of the three possible “edge” solutions,  $(-\bar{a}_r, a_s)$ ,  $(a_b, 0)$ , and  $(a_b, 1 - \bar{a}_r - a_b)$ . Expectations  $(\tilde{\pi}_b, \tilde{\pi}_s)$  in the subspaces delimited only by the dashed lines originates the “vertex” solutions.





**Figure 3: Adjusted vs. unadjusted Sharpe performance**, together with a 45-degree line. The expected adjusted Sharpe performances,  $E[\tilde{\pi}_b] = E[\pi_b] - \gamma \zeta_r \bar{a}_r$  and  $E[\tilde{\pi}_s] = E[\pi_s] - \gamma \zeta_s \bar{a}_r$ , are the unadjusted performances corrected for the hedging against real asset holdings.

## TABLES

**Table 1: Sample statistics.** Median and average financial and total wealth (2010 prices), percentage of households holding various assets in their portfolios, aggregated household portfolio shares, percentage of households by portfolio regime and self-assessed risk aversion. The “low”, “moderate” and “high” categories are based on the answer to a question regarding self-assessed risk aversion. “Low” = risk options 1-2; “Moderate” = risk option 3; “High” = risk option 4. All the statistics are computed using population weights.

	1995	1998	2001	2004	2007	2010
<b>a) Wealth (USD)</b>						
Financial wealth, average	213,297	281,776	361,668	351,292	354,575	359,166
Financial wealth, median	54,346	70,766	87,439	82,013	79,311	70,000
Total wealth, average	476,755	577,494	724,083	789,034	864,703	776,956
Total wealth, median	164,969	197,983	230,297	263,113	278,019	221,400
<b>b) Asset participation (%)</b>						
Deposits	99.79	99.96	99.58	99.87	99.85	99.92
Bonds	24.02	28.79	25.23	31.72	31.55	31.24
Loans	49.14	54.93	55.82	56.20	56.45	54.17
Stocks	67.34	74.61	77.32	72.45	69.59	67.48
Real estate	83.16	85.69	85.45	86.50	86.48	84.57
Businesses	14.90	14.96	16.28	15.82	15.65	16.83
<b>c) Aggregate portfolio shares (%)</b>						
Deposits	16.44	13.59	14.23	17.24	14.67	18.53
Net bonds (net position of bonds and loans)	-5.09	-6.32	-5.02	-7.95	-7.59	-6.55
Stocks	20.41	28.32	29.78	21.05	20.37	19.75
Real assets	68.23	64.41	61.01	69.67	72.55	68.26
<b>d) Portfolio regimes (%)</b>						
(i) $(a_b, a_s)$	59.14	67.59	69.47	66.94	65.05	61.79
(ii) $(-\bar{a}_r, 0)$	5.44	3.9	3.86	5.08	5.73	6.82
(iii) $(-\bar{a}_r, a_s)$	8.03	6.98	7.58	5.41	4.45	5.65
(iv) $(-\bar{a}_r, 1)$	0.14	0.04	0.16	0.05	0	0.02
(v) $(a_b, 1 - \bar{a}_r - a_b)$	0.04	0	0.11	0.05	0.09	0.03
(vi) $(1 - \bar{a}_r, 0)$	0	0	0.06	0	0	0
(vii) $(a_b, 0)$	27.22	21.5	18.76	22.47	24.67	25.7
<b>e) Self-assessed risk aversion (%)</b>						
Low	20.65	29.02	28.71	24.12	24.59	20.5
Moderate	44.99	44.46	44.43	47.13	45.96	44.08
High	34.36	26.52	26.86	28.75	29.45	35.42
<b>Genuine observations</b>	2,851	2,946	3,116	3,206	3,217	4,108

**Table 2: Statistics by risk aversion level.** Median financial wealth (2010 prices), average age, household portfolio regimes and aggregate household portfolio shares. All the statistics are computed using population weights.

Risk aversion	Median financial wealth (USD)	Age	Portfolio regime (percent)				Portfolio shares (%)	
			(i) $(a_b, a_s)$	(ii) $(-\bar{a}_r, 0)$	(iii) $(-\bar{a}_r, a_s)$	(vii) $(a_b, 0)$	Net bonds	Stocks
Low	108,581	46.25	78.49	2.28	7.53	11.6	-9.33	26.79
Moderate	95,645	51.74	72.55	3.61	5.56	18.24	-4.62	23.51
High	38,000	58.91	43.14	9.85	6.2	40.53	-7.04	11.73
Total	73,892	52.55	65.14	5.17	6.24	23.32	-6.63	22.99

**Table 3: Predicted values and simulated partial effects.** Predicted values and partial effects of the covariates on the expected Sharpe performances  $E[\pi_b]$  and  $E[\pi_s]$ , the hedging terms  $\xi_b$  and  $\xi_s$ , the correlation  $\rho$ , the risk aversion  $\gamma$ , and the adjusted Sharpe performances  $E[\tilde{\pi}_b]$  and  $E[\tilde{\pi}_s]$ , i.e., the performances adjusted for the hedging against real asset holdings. For partial effects, we compute the percentage variation with respect to the benchmark case. The continuous explanatory variables were changed as described in the parentheses; the dummy variables were moved from 0 to 1 for the entire sample. The statistics are averages computed using the population weights. With respect to the estimation sample, we drop a few outliers whose predicted adjusted Sharpe performances were not consistent with the observed regime (0.04% of the weighted sample).

	$E[\pi_b]$	$E[\pi_s]$	$\xi_b = \omega_{br}/\omega_b$	$\xi_s = \omega_{sr}/\omega_s$	$\rho$	$\gamma$	$E[\tilde{\pi}_b]$	$E[\tilde{\pi}_s]$
<b>Average predicted values</b>	4.9026	2.4272	3.8205	0.7316	0.1953	2.1013	-2.5589	1.0882
<b>Partial effects (%)</b>								
Age (+5 years)	2.6241	-3.5437	-0.5578	0.2514	0.4009	1.9033	7.3426	18.0815
White	1.3884	36.9296	-0.3520	11.3426	0.4226	0.5103	-0.1259	13.3735
Female	0.8442	7.6357	-1.9758	11.5267	2.2601	2.3693	4.8072	5.2313
Graduate	3.9064	18.1727	1.0828	-9.3219	-7.5223	0.6132	6.0679	-26.2741
Married	5.4132	6.3114	-2.4541	17.3436	-1.8415	5.5710	3.5852	14.0882
Self-employed	12.7872	-13.7865	1.7542	-0.6918	-58.3597	2.4445	-3.0161	9.3863
Retired	3.3081	7.8492	-4.8175	20.8455	-6.4251	6.0570	16.7492	3.9711
Home owners	91.2311	-38.5681	14.4883	-83.1165	-42.7124	19.0351	319.3818	-39.5900
Financial wealth (+10%)	-1.4503	1.0264	0.9172	-0.3624	-1.7679	-0.8885	-10.0270	-1.5773
Income (+10%)	-0.2249	-0.4141	0.0276	-0.1860	-0.6738	-0.0193	-1.6304	-0.2162
Net real wealth (+10%)	2.4141	-0.4813	-0.3613	-0.3587	-3.1530	1.5544	21.0658	3.4704
Stock market return when 20-24 (+10 p.p.)	0.1539	-0.3845	0.1566	0.0855	0.4995	-0.2008	0.7808	0.2267
Annual 10 yrs T-bond return at time $t$ (+10 p.p.)	2.7474						13.0818	
Annual S&P 500 return at time $t$ (+10 p.p.)		-2.2469						1.7066

**Table 4: Predicted values and simulated partial effects conditional on the observed portfolio regime.** Predicted values and simulated partial effects for the covariates on the expected net bond ( $a_b^*$ ) and stock ( $a_s^*$ ) shares, which are conditional on the observed portfolio regimes. For simulated partial effects, we compute the percentage variation with respect to the benchmark case. The continuous explanatory variables were changed as described in the parentheses; dummy variables were moved from 0 to 1 for the whole sample. The statistics are averages computed using the population weights. With respect to the estimation sample, we drop a few outliers whose predicted adjusted Sharpe performances were not consistent with the observed regime (0.04% of the weighted sample).

	$E[a_b^*   regime = (i)]$	$E[a_s^*   regime = (i)]$	$E[a_s^*   regime = (iii)]$	$E[a_b^*   regime = (vii)]$
<b>Average observed values (percentage points)</b>	-45.15	23.49	50.77	-47.55
<b>Average predicted values (percentage points)</b>	-47.67	21.57	60.52	-46.98
<b>R<sup>2</sup></b>	0.9812	0.3802	0.052	0.9819
<b>Simulated partial effects (percentage points)</b>				
Age (+5 years)	0.6562	-0.6565	-0.0071	0.4531
White	-0.0252	1.8754	5.3325	0.8376
Female	1.0357	-0.6823	0.5817	0.9842
Graduate	-0.0040	1.4195	3.4602	0.1611
Married	0.9131	-0.5036	0.6905	0.8609
Self-employed	2.4623	-1.5770	-0.2663	1.8776
Retired	3.2328	-1.6506	-0.3131	2.5278
Home owners	7.8404	-4.4193	-6.0404	2.0488
Financial wealth (+10%)	-1.2575	0.7082	0.0228	-0.5760
Income (+10%)	-0.0524	-0.0449	-0.1059	-0.1092
Net real wealth (+10%)	1.6614	-0.7187	0.0000	0.8178
Stock market return when 20-24 (+10 p.p.)	-0.0042	0.0056	0.0460	0.0182
Annual 10 yrs T-bond return at time $t$ (+10 p.p.)	1.0764	-0.2279	0.6805	1.0831
Annual S&P 500 return at time $t$ (+10 p.p.)	0.0678	-0.2056	-0.3921	-0.0186
Year 1995	-0.8896	-0.9745	-3.1622	-1.0996
Year 1998	-0.9149	0.4388	0.8180	-0.9046
Year 2004	-0.4297	-2.3795	-7.1103	-0.8109
Year 2007	-0.3700	-2.0769	-6.2118	-0.6980
Year 2010	-0.9322	-2.6247	-7.9304	-1.3550

**Table 5: Predicted probabilities of regimes (i), (iii) and (vii) and the simulated partial effects.** Average predicted probabilities that the portfolios fall in regimes (i), (iii) or (vii), and simulated partial effects of the covariates on these probabilities. For simulated partial effects, we compute the change in probability (in percentage points) with respect to the benchmark case. The continuous explanatory variables were changed as described in the parentheses; dummy variables were moved from 0 to 1 for the entire sample. The statistics are means computed using the population weights. With respect to the estimation sample, we dropped a few outliers whose predicted adjusted Sharpe performances were not consistent with the observed regime (0.04% of the weighted sample).

	$\Pr(\text{regime} = (i))$	$\Pr(\text{regime} = (iii))$	$\Pr(\text{regime} = (vii))$
<b>Actual percentage of households in the regime</b>	65.12	6.24	23.34
<b>Average predicted probabilities (percentage points)</b>	57.38	6.72	24.03
<b>Simulated partial effects (percentage points)</b>			
Age (+5 years)	-0.2703	-0.4657	1.5448
White	2.7914	0.5135	-5.9064
Female	-1.3609	0.0558	1.8117
Graduate	3.0096	-0.3915	-4.7814
Married	1.2785	-0.3049	0.3664
Self-employed	-0.5998	-2.3012	3.6928
Retired	-1.9903	-0.6663	1.6036
Home owners	11.3304	-10.3776	3.6770
Financial wealth (+10%)	0.5070	0.4269	-0.7703
Income (+10%)	-0.0014	0.0026	0.0978
Net real wealth (+10%)	-0.7981	-0.1887	0.2456
Stock market return when 20-24 (+10 p.p.)	-0.0756	-0.0192	0.0758
Annual 10 yrs T-bond return at time $t$ (+10 p.p.)	-0.6200	-0.5491	0.0277
Annual S&P 500 return at time $t$ (+10 p.p.)	-0.3065	-0.0583	0.6226
Year 1995	1.9972	0.4313	0.1696
Year 1998	-0.1797	0.0784	0.1433
Year 2004	3.7168	0.5868	0.1325
Year 2007	3.3319	0.5299	0.1212
Year 2010	4.1353	0.7290	0.2296

**Table 6: Partial effects on risk aversion, conditional portfolio shares and portfolio type probabilities based on reduced form models.** For risk aversion  $\gamma$ , we estimate a standard ordered probit; for *bonds* ( $a_b$ ) and *stocks* ( $a_s$ ) portfolio shares, we estimate a linear model using standard OLS, restricting the estimation samples to the observations in the relevant portfolio regimes. For the portfolio type probabilities, we estimate a standard multinomial logit model with four types of regimes: regimes (i), (iii), (vii) and a regime gathering all the remaining portfolio types. For partial effects, we compute the variation with respect to the benchmark case. The continuous explanatory variables were changed as described in the parentheses; dummy variables were moved from 0 to 1 for the entire sample. For  $\gamma$ , we report percentage variation. The statistics are averages computed using the population weights. For the sake of comparability with the results shown for the structural model in terms of the shares and the portfolio probabilities, when computing the statistics, we dropped a few outliers whose predicted adjusted Sharpe performances were not consistent with the observed regime (0.04% of the total weighted sample).

	$\gamma$	$E[a_b   (i)]$	$E[a_s   (i)]$	$E[a_s   (iii)]$	$E[a_b   (vii)]$	Pr(i)	Pr(iii)	Pr(vii)
R <sup>2</sup>		0.2638	0.3097	0.0529	0.2743			
<b>Partial effects (percentage points, % for <math>\gamma</math>)</b>								
Age (+5 years)	14.3152	8.9976	-0.7148	0.0900	7.9372	-2.2348	-0.3066	2.1995
White	-20.9388	6.941	2.9416	11.6962	6.1262	5.7460	0.9986	0.5850
Female	26.0062	3.4433	-2.8605	2.3176	-2.3044	1.2354	0.4558	-5.1812
Graduate	-30.2011	-11.0327	2.4444	-0.0830	-21.3578	6.6382	-0.3923	-1.3722
Married	22.1964	-0.0724	-3.0626	-6.3123	-7.0917	2.5239	0.2388	-5.6900
Self-employed	-10.3149	2.0747	-0.4794	4.1395	7.0133	-4.2374	-0.8803	-2.8950
Retired	-3.0381	10.2175	0.1660	-1.3263	10.0327	-2.9682	-1.0758	6.7972
Home owners	-14.5056	-82.216	-3.5639	16.3428	-94.2751	4.5974	0.5279	3.0088
Financial wealth (+10%)	-1.0950	0.9378	0.7151	0.1062	0.8252	0.7872	0.0295	4.7446
Income (+10%)	-0.8823	-0.4916	-0.0151	0.1646	-1.3259	0.1123	0.0871	-0.7160
Net real wealth (+10%)	-0.0525	1.0012	-0.6002	0.0000	1.8215	-0.0424	0.0000	-0.0844
Stock market return when 20-24 (+10 p.p.)	0.1789	-0.2883	-0.0947	0.2544	0.3062	-0.0507	0.0447	0.0804
Annual 10 yrs T-bond return at time $t$ (+10 p.p.)		5.1625			13.4066	-0.2584	1.1836	0.0312
Annual S&P 500 return at time $t$ (+10 p.p.)			-0.4433	-0.3605		-0.4290	-0.3199	-0.0760