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# Welfare measures to assess urban quality of life\*

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## Abstract

The standard index of urban quality of life provides an approximated value of the quality of life, since it associates the bundles of amenities observed in urban areas with their implicit marginal prices, and not with the prices of infra-marginal units. In this paper, we adjust the standard measure to determine the monetary value of any bundle, which might substantially differ from the bundle of the marginal quantities of amenities. Our methodology relies on a welfare measure that represents the individual willingness to give up (accept) to insure (forego) a change in the current distribution of amenities across areas will take place, keeping the level of utility unchanged. We obtain a new measure, the *value-adjusted quality of life index*, that can be identified from parametric models of consumer preferences. We use this index to measure the quality of life in the city of Milan.

**Keywords:** Hedonic Models, Quality of Life, Structural models

**JEL Codes:** D6, H4; R1; R2.

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# 1 Introduction

The most common framework used by urban economists to assess quality of life in cities is the hedonic approach, firstly developed by Rosen (1979) and Roback (1982), who define a quality of life index as the estimated value of a set of urban amenities. To determine this value, the implicit prices of amenities are estimated using hedonic house and wage regressions.

In the last decade, several works have made use of similar indicators to rank cities according to the quality of life that they grant. For instance, Berger, Blomquist and Peter (2008) rank a large number of Russian cities on the basis of climate, environmental conditions, ethnic conflicts, crime rates, and health conditions. Chen and Rosenthal (2008) rank about 350 US cities according to their quality of life, as well as according to the "quality of business" environment, using an index developed by Gabriel and Rosenthal (2004). Albouy (2008) and Albouy, Leibovici and Warman (2013) rank US and Canadian cities applying the standard revealed-preference approach adjusted to account for federal taxes, non-housing costs, and non-labor income. Using Italian data, Colombo, Michelangeli and Stanca (2014) provide a ranking of Italian cities according to the quality of life they exhibit, while controlling for both residential and working choices. Finally, Brambilla, Michelangeli and Peluso (2013) reformulate the standard index to measure urban quality of life when equity concerns arise.

As Roback (1982) acknowledges, the quality of life index provides an approximated value of the quality of life in the city, since it associates any observed bundle of amenities with their implicit marginal prices, and not with the prices of infra-marginal units (Roback 1982, p. 1274).

This paper aims at contributing to this literature by proposing a new measure, the *value-adjusted quality of life index*, that provides a correct evaluation of infra-marginal units of amenities. The index is consistent with the preferences of a representative consumer in the housing market.

Our methodology, developed focusing on a single city, relies on the *compensating benefit*, a welfare measure introduced by Palmquist (2006) in the hedonic property value models to evaluate environmental values. In our framework, the compensating benefit gives the amount of money that a representative consumer living in a given neighborhood of the city would like to give up (accept) to insure (forego) a change in the current distribution of amenities across neighborhoods, keeping the level of utility unchanged. The value-adjusted quality of life index is obtained by adding the compensating benefit to the Roback (1982) quality of

life index computed at the city-wide average quantities of amenities. The latter measures the willingness to pay for the average bundle; the former provides an evaluation of the units of amenities, which differentiates the neighborhood bundle from the city-average bundle.

The new index is identified under the assumption that the consumer inverse demand for amenities admits the *inverse almost ideal demand system* representation (Eales and Unnevehr 1994) and can be evaluated from observations of housing transactions prices, housing attributes and local amenities, as well as incomes of owners.

We illustrate our methodology through an empirical application to the city of Milan, using a dataset on housing transactions occurred between 2004 and 2010. We consider a set of local amenities collected at neighborhood level, such as green areas, educational and health services, recreational activities, public transport, security services, facilities and socio-demographic composition. Our results show considerable changes in the neighborhoods scoring, when quality of life is measured according to the Roback's index rather than our index. Nevertheless the correlation between the two indices is 0.63, mainly because both indices are able to identify the highest levels of quality of life in the city centre, where neighborhoods are better-endowed.

The rest of the paper is organized as follows. In section 2 we present the theoretical framework. Section 3 discusses the identification of the value-adjusted quality of life index, the data and the empirical strategy. Section 4 shows empirical results. Finally, section 5 concludes.

## 2 Theoretical Framework

We develop a theoretical framework that considers a city where a representative agent have to choose the neighborhood to live in. The fact of focusing on a single city instead of a set of cities allows us to simplify the problem of assessing quality of life, since we can ignore intercity differences in non-housing costs, intergovernmental transfers and local taxes. Moreover we assume throughout the paper that the labour market heterogeneity across neighborhood is sufficiently low to be neglected. The next section briefly reviews the housing purchase decision in the hedonic model. In section 2.1 we introduce the compensating benefit that is used in section 2.3 to define the new index.

## 2.1 The consumption decision in the hedonic model

For a given housing unit, we denote with  $\mathbf{q} = (\mathbf{z}^A, \mathbf{z}^H) \in \mathbb{R}^{A+H}$  the vector of the unit's characteristics, where  $\mathbf{z}^A$  is a vector of local amenities and  $\mathbf{z}^H$  a vector of intrinsic characteristics of the unit. Henceforth, we use  $j = 1, \dots, A$  to indicate an amenity while  $j = A + 1, \dots, A + H$  indicates an intrinsic characteristic. All attributes in  $\mathbf{q}$  are considered as normal goods. The consumer decides how to allocate the disposable income  $m$  between housing with attributes  $\mathbf{q}$  and a composite good  $x \in \mathbb{R}_+$ . The consumer preferences over amenities, housing-specific attributes and composite good are represented, as usual, by an increasing and strictly concave utility function  $U(\mathbf{q}, x)$ , characterized by decreasing marginal rate of substitution between goods along an indifference surface. Let  $P(\mathbf{q})$  be the observed equilibrium price schedule associated to the housing unit with attributes  $\mathbf{q}$ , while the composite good is chosen to be the numéraire. The optimal bundle  $(\mathbf{q}^0, x^0)$  maximizes the utility of the consumer subject to the budget constraint, thus solving the following problem:

$$(\mathbf{q}^0, x^0) := \arg_{(\mathbf{q}, x) \in \mathbb{R}_+^{A+H+1}} \max \{ U(\mathbf{q}, x) : m \geq P(\mathbf{q}) + x \}. \quad (1)$$

First order conditions for internal solutions imply the following set of equations:

$$\begin{aligned} \frac{\partial P(\mathbf{q}^0)}{\partial q_j} &= \frac{U(\mathbf{q}^0, x^0)_{q_j}}{U(\mathbf{q}^0, x^0)_x}, \quad \forall j = 1, \dots, A + H \\ P(\mathbf{q}^0) &= m - x^0 \end{aligned} \quad (2)$$

where  $U(\cdot)_{q_j}$  is the marginal utility associated with the generic attribute  $q_j$  of the vector  $\mathbf{q}$ . At the optimum, the marginal rate of substitution between the  $j$ th attribute and the composite good must equalize the marginal price of the attribute  $j$  implicitly determined by the housing market. The marginal price gives the consumer's marginal willingness to pay for an additional amount of the  $j$ th attribute, at the consumer's optimal choice.

## 2.2 Willingness to pay welfare measures in the hedonic framework

Consider a variation in the quantities of amenities associated with the housing unit chosen at the optimum by the representative consumer, such that the preferred bundle shifts from  $(\mathbf{q}^0, x^0)$  to  $(\tilde{\mathbf{q}}, x^0)$ . We are interested in the consumer's *evaluation* of  $(\tilde{\mathbf{q}}, x^0)$ , expressed in monetary terms.

For this purpose, we adopt the *Compensating Benefit (CB)*, a welfare measure also used by Palmquist (2006) to evaluate environmental goods in an hedonic framework. It measures the maximum quantity of the composite good that the consumer is willing to give up (accept) to insure (forego) the shift in amenities.

Formally, the *CB* is defined in terms of the benefit function, originally formulated by Luenberger (1992), which maps changes in the vector of attributes  $\mathbf{q}$  into changes of the composite good quantity at the optimum,  $x^0$ , such that utility is held fixed at  $u^0$ . The benefit function  $B(\mathbf{q}, x; u^0) : \mathbb{R}_+^{A+H+1} \times \mathbb{R} \rightarrow \mathbb{R}$  is related to the distance function<sup>1</sup> and corresponds to the solution of the following program:<sup>2</sup>

$$B(\mathbf{q}, x^0; u^0) := \max\{\beta \in \mathbb{R} : U(\mathbf{q}, x^0 - \beta) \geq u^0\}. \quad (3)$$

The representation of the benefit function in problem (3) is due to the particular choice of the reference bundle, set to be  $(\mathbf{q}, x) = (\mathbf{0}, 1)$ , so that it is possible to compensate changes in the housing attributes in monetary units. Therefore, the benefit function is a monetary equivalent of the change in utility due to the change in amenities.

The compensating benefit corresponds to the difference of the benefit function computed in  $\tilde{\mathbf{q}}$  and the benefit function computed in  $\mathbf{q}^0$ , keeping the level of utility constant at  $u^0$ . In formal terms:

$$CB = B(\tilde{\mathbf{q}}, x^0, u^0) - B(\mathbf{q}^0, x^0, u^0).$$

In the next section we use the compensating benefit for assessing quality of life.

### 2.3 Quality of life measurement

The Roback's (1982) quality of life index, *QoL*, for a given area (region, city, neighborhood) is defined as the weighted sum of a set of amenities, where the weights are the implicit prices of the amenities. Usually, in the empirical applications the implicit prices are computed at the average quantities of the amenities in the

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<sup>1</sup>The *distance function* is a measure  $\delta^0 = D(\mathbf{q}, u)$  of the distance of a vector of attributes  $\mathbf{q}$  from a proportional scaled bundle  $\mathbf{q}/\delta$  providing utility  $u$  which can be attained with a unitary expenditure (after normalization). In symbols:  $D(\mathbf{q}, u) := \max_{\delta} \{\delta | U(\mathbf{q}/\delta) \geq u\} = \min_{\mathbf{p}} \{\mathbf{p} \cdot \mathbf{q} | E(\mathbf{p}, u) = 1\}$  when budget constraint is linear. Therefore distance function express a direct measure of utility by mean of budget distance.

<sup>2</sup>We assume throughout the paper that compensation of the initial quantity  $x^0$  is always possible.

sample areas, in order to minimize the predicted error.

$$QoL := \sum_{j=1}^A z_j^A \cdot \frac{\partial P(\bar{\mathbf{q}})}{\partial q_j}. \quad (4)$$

We stress that the implicit price  $\frac{\partial P(\bar{\mathbf{q}})}{\partial q_j}$ ,  $\forall j$ , corresponds to the market value of the average quantity of amenity  $\bar{z}_j^A$  and is used to assess the quantity specific to each area. This quantity can vary considerably across areas while its value is held constant to  $\frac{\partial P(\bar{\mathbf{q}})}{\partial q_j}$ .

As we mentioned in the Introduction, the quality of life literature recognizes that the vector of prices acts as a weighting scheme for amenities, which is consistent with the market evaluation and “merely shows the order of magnitude of expenditure in the average budget” (Roback 1982, p. 1274). The *QoL* index has no direct interpretation in terms of willingness to pay for bundles of amenities that are far from the average bundle, since marginal prices do not correspond to proper evaluations of these units.

To cope with this problem, we propose an adjustment of the Roback’s index, based on the correct evaluation of a non-marginal change in amenities from the sample average bundle  $\bar{\mathbf{z}}^A$  to the observed optimal choice  $\mathbf{z}^{A0}$ .

The new index, named *value-adjusted Quality of Life* index (*va-QoL*) is the sum of two components. The first component is the Roback index computed at the average amenities bundle and it also represents the level of quality of life in the city as a whole,  $\bar{\mathbf{z}}^A$ , i.e.  $\overline{QoL} = \sum_{j=1}^A \bar{z}_j^A \frac{\partial P(\bar{\mathbf{q}})}{\partial q_j}$ , which coincides with the market evaluation of the average amenity bundle. The second component is the evaluation of the residual units of amenities between  $\bar{\mathbf{z}}^A$  and  $\mathbf{z}^{A0}$ , given by the compensating benefit. The value-adjusted quality of life index for a given area is:<sup>3</sup>

$$va-QoL := \overline{QoL} - CB. \quad (5)$$

A shift from  $\mathbf{z}^{A0}$  to  $\tilde{\mathbf{z}}$  is *positively* evaluated if the latter bundle is preferred to the former. Hence, if  $\bar{\mathbf{z}}^A$  is preferred to  $\mathbf{z}^{A0}$ , then the quality of life associated to  $\mathbf{z}^{A0}$  would be overestimated by  $\overline{QoL}$ , and this evaluation should be reduced by  $CB$ . A similar reasoning applies to the case in which  $\mathbf{z}^{A0}$  is preferred to  $\bar{\mathbf{z}}^A$ .

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<sup>3</sup>Notice that the  $CB$  was defined as a shift from  $\mathbf{q}^0$  to  $\tilde{\mathbf{q}}$ . Throughout the analysis we exclusively consider shifts in amenities, therefore the component  $\mathbf{z}^H$  of  $\mathbf{q}$  is always held fixed at  $\mathbf{z}^{H0}$ .

### 3 Identification, Data and Empirical Strategy

The *va-QoL* index can be identified from the inverse demands of attributes. In the next section we derive the inverse demands and discuss the relation between them and the Compensating Benefit.

#### 3.1 Identification

The *uncompensated inverse demand* for a generic attribute  $q_j$ , either an amenity or a housing-specific characteristic, can be derived by linearizing the budget constraint around the optimal choice  $\mathbf{q}^0$ . We introduce a new income measure,  $m^{lin}$  and a vector of constant marginal prices  $\mathbf{p} \in \mathbb{R}^{A+H}$  where  $\mathbf{p}^0 := \frac{\partial P(\mathbf{q}^0)}{\partial \mathbf{q}}$ , such that:

$$m^{lin} := m + (\mathbf{p}^0 \cdot \mathbf{q} - P(\mathbf{q})).$$

The linearized income has a fixed component  $m$  and a variable part that depends on the distance between the actual price schedule (the non-linear part of the budget constraint) and its linear approximation.

By using the linearized budget constraint at the optimum and substituting price vectors in the budget constraint with equation (2) we obtain:

$$\frac{1}{U(\mathbf{q}^0, x^0)_x} := \frac{m^{lin} - x^0}{\sum_{j=1}^{H+A} U(\mathbf{q}^0, x^0)_{q_j} q_j}.$$

Substituting this result into the optimal solution (1) gives the uncompensated inverse demand system for housing attributes:  $\mathbf{p}^0(\mathbf{q}, m^{lin})$ . The uncompensated demand system depends on the quantities of attributes that are consumed and the linearized income parameter. Plugging the direct demand functions into the utility function, we get the *indirect utility function*  $V(\mathbf{p}, m^{lin})$ , which is decreasing in prices and increasing in income. For any point lying on the contour curve of  $V(\mathbf{p}, m^{lin}) = u^0$  generated by  $(\mathbf{q}^0, x^0)$ , an increase in prices makes the initial bundle unaffordable, and reduces the level of utility. Conversely, an increase in income makes available the bundles that are preferred to  $(\mathbf{q}^0, x^0)$ , and increases the level of utility. Since prices are always normalized by the price of the composite good, the indirect utility function is quasi-convex in amenities prices and linearized income. Moreover, the slope of its indifference surface must be positive.<sup>4</sup>

The *inverse compensated demand system*  $\mathbf{p}(\mathbf{q}, u^0)$  gives the willingness to pay for a bundle of attributes  $\mathbf{q}$  keeping utility as fixed at  $u^0$  (Cornes 1992, p. 79).

<sup>4</sup>To prove this, see the results on the indirect utility function in Cornes (1992) which can be

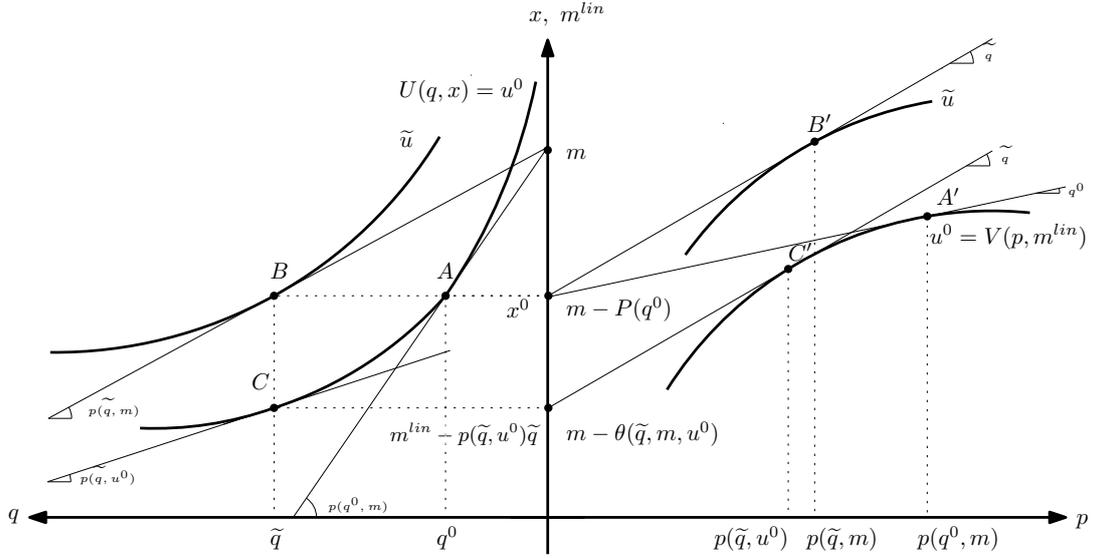


Figure 1: Compensated non-marginal changes in housing attributes and amenities and the relation with the benefit function

Making use of linearized budgets, the system is the solution of the minimization of the expenditure in the composite good, taking  $u^0$  as a constraint.

$$(\mathbf{p}(\mathbf{q}, u^0), m^{lin}) := \arg_{\mathbf{p}, m^{lin}} \min\{x : U(\mathbf{q}, x) \geq u^0, x \leq m^{lin} - \mathbf{p} \cdot \mathbf{q}\}. \quad (6)$$

At the optimal quantity  $\mathbf{q}^0$ , compensated and uncompensated demands coincide and  $x^0$  is the level of composite good that solves problem (6).

Consider again a shift from the optimal bundle  $\mathbf{q}^0$  to another bundle  $\tilde{\mathbf{q}}$ . This shift in quantities, which leaves the consumption of the composite good unchanged, cannot be evaluated at the market price of the new bundle  $\tilde{\mathbf{q}}$ , since also the utility of the consumer varies from  $u^0$  to  $\tilde{u}$ . This is well illustrated in the diagrammatic example in the left hand side panel of Figure 1, where we consider a oversimplified situation with just one amenity, denoted  $q$ .

The non-marginal shift of  $q$  from  $q^0$  to  $\tilde{q}$  induces an increase in utility. The old and new consumption bundles are denoted  $A$  and  $B$ , respectively. Both bundles can be rationalized by a budget  $m$  only if the marginal evaluation of the amenity changes from  $p(q^0, m)$  to  $p(\tilde{q}, m)$ . The marginal price of  $q$  has decreased since the utility is assumed concave. To what extent is the change in evaluation driven by

summed up by noticing that for any vector  $(\mathbf{p}_0, m_0^{lin})$  and  $(\mathbf{p}_1, m_1^{lin})$ ,

$$\max\{V(\mathbf{p}_0, m_0^{lin}); V(\mathbf{p}_1, m_1^{lin})\} \geq V(\lambda(\mathbf{p}_0, m_0^{lin}) + (1 - \lambda)(\mathbf{p}_1, m_1^{lin})), \quad \forall 0 \leq \lambda \leq 1.$$

the degree of substitution between amenities and the composite good, or by the relative rate of exchange between units of utility and composite good?

To answer the question, we decompose the shift from  $A$  to  $B$  into two movements. The first movement is from  $A$  to  $C$ , where the utility of the individual is kept fixed to  $u^0$ . Since the representative consumer likes amenities, the evaluation of  $\tilde{q}$  must be positive, so that to held  $u^0$  fixed given  $\tilde{q} - q^0 > 0$  the consumer has to reduce the overall consumption of composite good to  $\tilde{x} = m^{lin} - p(\tilde{q}, u^0)\tilde{q}$ . The price differential between  $p(q^0, m) = p(q^0, u^0)$  and  $p(\tilde{q}, u^0)$ , along with the change in income, reflect how much the consumer is willing to forego in units of composite good to receive more units of amenity.

The second movement is from  $C$  to  $B$ . The change in marginal bids associated to this shift is positive, reflecting the fact that, given  $\tilde{q}$ , the marginal utility of a unit of composite good is larger than its price, which is defined in income units. Hence, the consumer has to rise her marginal bid from  $p(\tilde{q}, u^0)$  to  $p(\tilde{q}, m)$  in order to make her budget  $m - x^0$  binding, given that utility has already reached  $\tilde{u}$  level.<sup>5</sup>

Of the two movements, only the variation between  $A$  and  $C$  is relevant for assessing a welfare consistent measure of the change in amenities. As illustrated in the figure, both marginal prices and linearized income respond to changes in quantities. Instead of looking at variations in the expenditure function (which is meaningful when the budget constraint is linear), we focus on the compensated change in the share of income that the consumer do not use for buying the housing unit. This amount is  $(m - \mathbf{p}(\mathbf{q}^0, u^0) \cdot \mathbf{q}^0) - (m^{lin} - \mathbf{p}(\tilde{\mathbf{q}}, u^0) \cdot \tilde{\mathbf{q}})$  and has a natural counterpart in the *dual setting*. It corresponds to the Rosen's (1974) *bid function*  $\theta(\mathbf{q}, m, u^0)$ , identifying the maximal (minimal) amount of money that the consumer is willing to give up (to accept) to buy  $\mathbf{q}$ , given a level of income  $m$  and utility  $u^0$ . The bid function is implicitly defined by:

$$U(\mathbf{q}, m - \theta(\mathbf{q}, m, u^0)) = u^0. \quad (7)$$

The right hand side of Figure 1 illustrates the effect of a change in amenity from  $q^0$  to  $\tilde{q}$  in the dual space, where marginal bids and composite goods are reported in the horizontal and vertical axes, respectively. The difference  $\theta(\tilde{q}, m, u^0) - P(q^0)$  measures the minimal drop in composite good consumption that leaves the consumer indifferent between the amenity quantity  $q^0$  (point  $A'$ ) and  $\tilde{q}$  (point  $C'$ ).

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<sup>5</sup>In general, the inverse demands  $p(\tilde{\mathbf{q}}, u^0)$  and  $p(\tilde{\mathbf{q}}, m)$  coincide only under a strong separability assumption on the utility function. If preferences are quasilinear in the composite good ( $U(\mathbf{q}, x) = f(\mathbf{q}) + x$  with  $f$  increasing strictly concave) the exchange rate between utility units and composite good units is unitary and so this bid differential disappears.

As shown by the graph, this quantity coincides with the expenditure variation associated to the shift from  $A$  to  $C$ .

The compensating benefit associated to a shift from  $\mathbf{q}^0$  to  $\tilde{\mathbf{q}}$  is identified by the bid function. This result, formalized in the following proposition, is also important because it allows to estimate the  $CB$  from compensated amenities and housing-specific characteristics demands, which are identified from data on housing transactions and the owners' income distribution.

**Proposition 1** *Assume the utility function  $U$  is quasi-concave, monotonic and continuous and consider the reference bundle  $(\mathbf{0}, 1)$ , then it must hold:*

$$CB := B(\tilde{\mathbf{q}}, x^0, u^0) = \theta(\tilde{\mathbf{q}}, m, u^0) - P(\mathbf{q}^0) \quad (8)$$

$$\nabla CB := \nabla_{\mathbf{q}} B(\mathbf{q}, x^0, u^0)|_{\mathbf{q}=\mathbf{q}^0} = \mathbf{p}(\mathbf{q}^0, u^0) = \nabla_{\mathbf{q}} \theta(\mathbf{q}, m, u^0)|_{\mathbf{q}=\mathbf{q}^0}. \quad (9)$$

**Proof.** The relation in (8) is a direct result of the definitions of benefit and bid functions, provided that  $(\mathbf{q}^0, x^0)$  is the reference bundle,

$$U(\tilde{\mathbf{q}}, m - \theta(\tilde{\mathbf{q}}, m, u^0)) = u^0 = U(\tilde{\mathbf{q}}, x - B(\tilde{\mathbf{q}}, x^0, u^0)),$$

and by using the fact that  $B(\mathbf{q}^0, x^0, u^0) = 0$  and  $m - x^0 = P(\mathbf{q}^0) = \theta(\mathbf{q}^0, m, u^0)$  from the dual of problem (6). The first equality in (9) is a result of Proposition 2 in Luenberger (1996) and Lemma 1 in Chambers (2001) at the optimum with linearized constraints. The second equality follows by implicit differentiating  $U(\mathbf{q}, m - \theta(\mathbf{q}, m, u^0))$  with respect to  $q_j$  for every  $j = 1, \dots, A + H$ , which gives:

$$\frac{\partial \theta(\mathbf{q}, m, u^0)}{\partial q_j} \Big|_{\mathbf{q}=\mathbf{q}^0} = \frac{U(\mathbf{q}^0, x^0)_j}{U(\mathbf{q}^0, x^0)_x} = \frac{\partial P(\mathbf{q})}{\partial q_j} \Big|_{\mathbf{q}=\mathbf{q}^0} = p_j(\mathbf{q}^0, u^0) \quad \forall j.$$

The sequence of equalities can be established only at the optimum  $\mathbf{q} = \mathbf{q}_0$ , which concludes the proof. ■

Based on the result in (8), the value adjusted quality of life index rewrites:

$$va-QoL = \overline{QoL} - \left( \theta((\bar{\mathbf{z}}^A, \mathbf{z}^{H0}), m, u^0) - P(\mathbf{q}^0) \right). \quad (10)$$

### 3.2 Implementation

As we mentioned in Section 2.3, the benefit function is related to the distance function between bundles  $\mathbf{q}^0$  and  $\tilde{\mathbf{q}}$ . The metric of the distance is defined in the space of the composite good. We can express this distance as the ratio between

the actual consumption of composite good and the level of consumption that the consumer would attain if the bundle  $\tilde{\mathbf{q}}$  is offered while utility is fixed at  $u^0$ :  $(m - P(\mathbf{q}^0))/(m - \theta(\tilde{\mathbf{q}}, m, u^0))$ . If  $\tilde{\mathbf{q}}$  is preferred to (least preferred than)  $\mathbf{q}^0$ , then this distance is greater (smaller) than one.

We assume a very flexible parametric specification for this distance, which is consistent with the *Inverse Almost Ideal Demand System (IAIDS)* developed by Eales and Unnevehr (1994). Solving the distance for  $\theta(\mathbf{q}, m, u^0)$ , we obtain the *IAIDS* consistent specification for the bid function associated to any bundle  $\mathbf{q}$ :

$$\theta(\mathbf{q}, m, u^0) = m - \left[ a(\mathbf{q})^{1-u^0} \cdot b(\mathbf{q})^{u^0} \right]^{-1} \cdot (m - P(\mathbf{q}^0)), \quad (11)$$

where, for  $i$  and  $j$  going from 1 to  $A + H$ :

$$\ln a(\mathbf{q}) = \alpha_0 + \sum_j \alpha_j \ln q_j + \frac{1}{2} \sum_j \sum_i \gamma_{ji}^* \ln q_j \ln q_i \quad (12)$$

$$\ln b(\mathbf{q}) = \beta_0 \prod_j q_j^{-\beta_j} + \ln a(\mathbf{q}) \quad (13)$$

The formulation of the bid function in (11) is consistent with the properties outlined by Rosen (1974) and Palmquist (2006): the function must be linear-homogeneous, concave, non-decreasing in quantities, decreasing in utility and linear in income. Most of the properties make intuitive sense, as already mentioned in Palmquist (2006). Linear homogeneity and concavity result from the *IAIDS* representation of preferences. The third property holds whenever the amenities are normal goods. An increase in consumption of local amenities holding income and utility level constant must increase the total bid for the housing unit, when local amenities are substitutes of consumption. It corresponds to a change in marginal rate of substitution along an indifference curve. The fourth property suggests that an increase in utility is possible, leaving the local amenity consumption and income as fixed, only increasing the composite good consumption. By the substitution effect, the marginal bid for other attributes must decrease. The last property is obviously satisfied. All these properties induce testable restrictions on the model.

Under integrability conditions, the utility level at the optimum ( $\mathbf{q}^0$ ) is obtained by solving (11) for  $u^0$ , which gives:

$$u^0 := V(\mathbf{q}^0, m) = \frac{\ln a(\mathbf{q}^0)}{[\ln a(\mathbf{q}^0) - \ln b(\mathbf{q}^0)]}. \quad (14)$$

The compensating benefit  $CB^0$  associated to a change in local quantities from

the initial observed bundle  $\mathbf{q}^0$  to  $\tilde{\mathbf{q}}$  can be formulated using result (8) in the proposition and substituting  $u^0$  with (14):

$$CB^0 = \left(1 - \left[ a(\tilde{\mathbf{q}})^{1-V(\mathbf{q}^0, m)} \cdot b(\tilde{\mathbf{q}})^{V(\mathbf{q}^0, m)} \right]^{-1}\right) \cdot (m - P(\mathbf{q}^0)).$$

Identification of the  $CB^0$  is parametric, and crucially depends on the *IAIDS* structural form. The parameters of the bid function in (11) can be estimated from inverse compensated demand functions, exploiting the result (9) in Proposition 1. Consider the elasticity  $\varrho_j$  of the demand of composite good with respect to the demand of the attribute  $j$ , when the utility is held fixed at  $u^0$ . Making use of a log-transformation of (11), we can express this elasticity as the attribute-to-consumption value ratio, for any attribute  $j = 1, \dots, A + H$ . Since attributes are treated as normal goods, this elasticity is negative, and at the optimum  $m - \theta(\mathbf{q}^0, m, u^0) = m - P(\mathbf{q}^0)$ , so that  $\varrho_j$  is:

$$\begin{aligned} \varrho_j &:= -\frac{\partial \ln(m - \theta(\mathbf{q}, m, u^0))}{\partial \ln q_j} \\ &= \frac{q_j}{m - \theta(\mathbf{q}, m, u^0)} \cdot \frac{\partial \theta(\mathbf{q}, m, u^0)}{\partial q_j} = \frac{q_j^0 \cdot p_j(\mathbf{q}^0, u^0)}{m - P(\mathbf{q}^0)} = \frac{q_j^0 \cdot \frac{\partial P(\mathbf{q}^0)}{\partial q_j}}{m - P(\mathbf{q}^0)}. \end{aligned} \quad (15)$$

Each element  $\varrho_j$  is a measure of the expenditure for attribute  $j$ , obtained by a first order approximation of the price schedule around  $\mathbf{q}^0$ , expressed in units of the composite good. Under the *IAIDS* identification assumption and using (14), the elasticity  $\varrho_j$  can be formulated as a linear function of the parameters of the *IAIDS* by differentiating the first term of (15):

$$\varrho_j = \alpha_j + \sum_i \gamma_{ji} \ln q_i + \beta_j \ln Q \quad (16)$$

$$\ln Q = \alpha_0 + \sum_j \alpha_j \ln q_j + \frac{1}{2} \sum_j \sum_i \gamma_{ji} \ln q_j \ln q_i, \quad (17)$$

where  $\gamma_{ji} = \frac{\gamma_{ji}^* + \gamma_{ij}^*}{2}$  for all  $i$  and  $j$ . This system of equations allow to identify the parameters of the bid function which enters in the formulation of the value-adjusted quality of life index.

### 3.3 Data

The empirical analysis relies on a data set of individual housing transactions provided by the real estate observatory *Osservatorio del Mercato Immobiliare* (OMI)

Table 1: Summary Statistics of Amenities

<b>Variable</b>	<b>Mean</b>	<b>Std. Dev.</b>	<b>Min.</b>	<b>Max.</b>	<b>N</b>
Green	15.16	6.88	1	29	55
Education	1.35	2.08	0	9.82	55
Transport	0.95	1.05	0	4.48	55
Security	0.44	0.8	0	5.04	55
Health	1.48	1.37	0	7.58	55
Facilities	3.23	2.3	0	11.66	55
Culture	1.74	4.88	0	34.1	55
Italians ratio	88.01	3.47	77.81	93.57	55
Housing rent	14,376	14,253	4,690	250,509	3,949
Income	23,953	12,890	12,087	78,481	55

*Source:* OMI Milan and our elaborations.

managed by the public agency *Agenzia del Territorio* within the Italian Ministry of Economy. We have selected transactions in the city of Milan from 2004 to 2010. In addition to the housing market value, the data set provides a detailed description of structural attributes, such as total floor space, age of the building in the year of sale, number of bathrooms, whether the housing unit needs to be renovated, whether the housing unit has independent heating, the floor above street level, presence of an elevator or a garage, and build quality. Transactions refer to 55 neighborhoods identified by the OMI. Neighborhood-level data on amenities and socioeconomic conditions were taken from public authority records (see Table 4 in the Data Appendix for details). We consider 8 neighborhood-level amenities. Environmental conditions are proxied by the green areas relative to the area of the neighborhood (*Green*). Education is proxied by the number of secondary schools per 10,000 inhab. (*Education*). Public transport is represented by the number of metro and railways stations per 10,000 inhab. (*Transport*). Security is measured through the number of police stations per 10,000 inhab. (*Security*). Health is proxied by the number of health centers per 10,000 inhab. (*Health*). Facilities are proxied by the number of pharmacies and post offices per 10,000 inhab. (*Facilities*). The recreational dimension is proxied by the number of cinemas, theaters, museums, art galleries, academies of music and libraries per 10,000 inhab. (*Culture*). The social composition of the neighborhood is captured by the percentage of Italians residents in the neighborhood (*Italians ratio*). Summary statistics for amenities are provided in Table 1, while a map showing the spatial distribution of income and housing prices across neighborhoods is reported in Figure 3 in Appendix.

The empirical application of the theoretical model requires a quite relevant quantity of parameters to be estimated. We derive a more parsimonious specification of the housing price equation and structural model by reducing the number of housing-specific characteristics (which are used as controls in our analysis) using principal component analysis (PCA). Given that several variables referred to housing-specific characteristics are dichotomous, we have first dichotomised continuous and count variables and then applied PCA using tetrachoric correlations. Continuous variables, such as total floor area, were converted to dichotomous variables by splitting the scale at the sample deciles. Then a continuous variable was associated with 10 dichotomous variables, one for decile. Count variables, such as floor or number of bathrooms, were dichotomised by splitting the scale at the most frequent values of the variables. We performed PCA on the resulting set of dichotomous variables and use only the first estimated factor, whose loading account for nearly 18% of the joint correlation structure among more than 50 dummies constructed from the data. The other components are not retained because they turned out to have no significant impact on housing prices.<sup>6</sup>

### 3.4 Estimation strategy

Estimation requires a two steps procedure. In the first step, implicit prices of housing attributes are estimated by considering an empirical specification of the housing price function  $P(\mathbf{q})$ . In the second step, the predicted implicit prices are used to estimate the compensating benefits associated to each neighborhood. Once the compensating benefits are estimated, the *va-QoL* index will be computed according to (10).

Our data are specified at the housing unit level. A housing unit  $h$  located in neighborhood  $n$  and sold at time  $t$  is associated with a transaction price  $P_{hnt}$  and an indicator of the housing features,  $z_{hnt}^H$ , as well as a vector of amenities of the neighborhood  $n$ , which is denoted  $\mathbf{z}_{nt}^A$ . For the housing price equation, we adopt a semilog functional representation:

$$\ln P_{hnt} = \beta_0 + \mathbf{z}_{nt}^A \cdot \boldsymbol{\beta}_1 + \beta_2 z_{hnt}^H + \beta_t + \varepsilon_{hnt} \quad (18)$$

where  $\beta_t$  is a time fixed effect. We assume that  $\varepsilon_{hnt} \sim N(0, \sigma_\varepsilon^2)$ , in which case the equation can be estimated via OLS. In order to obtain the annual implicit price of each amenity, housing prices were converted into imputed annual rents by applying

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<sup>6</sup>Results of the PCA analysis can be provided upon request to the authors.

a discount rate specific to each neighborhood. This rate has been determined by dividing for each neighborhood the average rent by the average price of housing, both in 2010 Euros.

In the second step of the estimation procedure, we retrieve the structural parameters of interest by estimating (16) and (17) as a system of simultaneous equations. The dependent variable in (16) is  $\varrho_j$  and is specified at housing unit level.<sup>7</sup> For a given amenity  $j = 1, \dots, A$ , the numerator of  $\widehat{\varrho}_j$  is the hedonic value of amenity level  $q_j$  experienced by housing unit  $h$  in neighborhood  $n$  at time  $t$ , and evaluated at the price  $\frac{\widehat{\partial P(\mathbf{q})}}{\partial q_j}$ . This price is inferred by differentiating the hedonic price equation (18) with respect to amenity  $j$ :

$$\frac{\widehat{\partial P(\mathbf{q})}}{\partial q_j} = \beta_{1j} \cdot \exp \left\{ \widehat{\ln P(\mathbf{q})} \right\}, \quad (19)$$

where  $\mathbf{q}$  is the vector of amenities of neighborhood  $n$  at time  $t$ . The denominator of (15) is computed using the average income and the average transaction price in the neighborhood where housing unit  $h$  is located.<sup>8</sup>

We adopt a standard linearization technique, widely adopted in demand analysis (Deaton and Muellbauer 1980, Eales, Durham and Wessells 1997), to estimate (16) and (17). Equation (17) is replaced by the *Stone quantity index*  $\ln Q^*$  for the housing-specific attributes and amenities, defined as:

$$\ln Q^* := \sum_{j=1}^{A+H} \frac{q_j \cdot \frac{\widehat{\partial P(\mathbf{q})}}{\partial q_j}}{\exp \left\{ \widehat{\ln P(\mathbf{q})} \right\}} \ln q_j. \quad (20)$$

The substitution of  $\ln Q$  with  $\ln Q^*$  gives a system of equations that is linear in parameters and can be estimated by iterative seemingly unrelated regression techniques, introducing symmetry ( $\lambda_{ij} = \lambda_{ji}$ ) and homogeneity ( $\sum_j \lambda_{ij} = 0$ ) constraints on (16). Additional restrictions, such as adding up ( $\sum_j \alpha_j = 1$ ,  $\sum_i \lambda_{ij} = 0$  and  $\sum_j \beta_j = 0$ ), are also introduced in the regression model to impose rationality restrictions on the behavior of the consumer.

We use the estimated parameters to calculate the values of  $\ln a(\cdot)$  and  $\ln b(\cdot)$  for each neighborhood separately according to (12) and (13), respectively. Then we calculate the reference utility level  $u^0$ , given by (14), associated with the observed distribution of amenities and housing-specific characteristics in the city.<sup>9</sup>

<sup>7</sup>To avoid cumbersome notation, in what follows we drop the subscripts indicating the housing unit, time of sale, and neighborhood.

<sup>8</sup>We use the averages at neighborhood level because we do not have data on owners' income.

<sup>9</sup>Note that the parameters  $\alpha_0$  and  $\beta_0$  are not identified. They are substituted with 0 and 1,

Finally, we compute the compensating benefit for each neighborhood and the value-adjusted quality of life index, according to (8) and (5) respectively. There are two potential sources of heterogeneity that might contribute to determine the variability of the value-adjusted quality of life index. The first is related to unobservable demographics and socio-economic factors driving the tastes of the representative consumer. They are incorporated in the error term of (16). The second type of heterogeneity is related to inequality in the distribution of income and housing prices. To control at least for observable heterogeneity, we fix income and housing price at the sample average.

## 4 Results

The hedonic model specified in (18) is estimated by OLS using 3,949 housing transactions recorded in Milan between 2004 and 2010. Robust standard errors are used with clustering at neighborhood level in order to allow for within neighborhood correlation. Columns 2 and 3 of Table 2 report OLS results.

The amenity coefficients are both individually and jointly statistically significant ( $F_{(8,3933)} = 170.10$ ,  $\text{Prob} > F = 0.0000$ ) and have the expected sign. The first principal component is mainly determined by variables positively correlated with housing prices such as the last two deciles of surface area, the first decile of age of building, the number of bathrooms or lifts equal to or higher than two, the presence of at least a parking area or a terrace.

Sociological variables such as ethnic composition of neighborhoods may be endogenous to the contemporaneous value of housing prices because of reverse causation and omitted variables. We test the endogeneity of the Italian ratio following the approach developed by Card, Mas and Rothstein (2008) and Saiz and Wachter (2011), who use a gravity model that predicts the actual ethnic composition on the basis of the settlement patterns of previous periods. The predicted ratio is used as instrument to test the endogeneity of the Italians ratio by applying the Hausman test. Results based on IV/2SLS estimation fail to reject the null hypothesis of no endogeneity for Italians ratio ( $\chi^2_{(18)} = 11.29$ ,  $\text{Prob} > \chi^2 = 0.7316$ ).

We also test for spatial error and spatial lag in model (18) using the Moran's  $I$  test and the LM test associated to the spatial error case and the spatial lag case. The robust LM tests are also performed since they correct for the presence of local

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respectively.

Table 2: Hedonic price regressions

	OLS		Spatial error	
	<i>Coeff.</i>	<i>SE</i>	<i>Coeff</i>	<i>SE</i>
<i>Housing Characteristics</i>				
First princip. comp.	0.559***	(0.011)	0.534***	(0.011)
<i>Amenities</i>				
Green	0.007***	(0.001)	0.004***	(0.001)
Schools	0.029***	(0.004)	0.016***	(0.003)
Cultural	0.012***	(0.003)	0.005*	(0.003)
Transport	0.065***	(0.010)	0.022**	(0.009)
Security	0.048***	(0.011)	0.056***	(0.011)
Health	0.049***	(0.008)	0.034***	(0.006)
Facilities	0.046***	(0.004)	0.024***	(0.004)
Italians/immigrants	0.011***	(0.001)	0.004***	(0.001)
Constant	7.082***	(0.129)	7.562***	(0.115)
$\lambda$			0.000***	(0.000)
$\sigma$			0.359***	(0.006)
<i>Time controls</i>	Yes		Yes	
R-squared	0.644			
Transactions	3949		3949	

Standard errors in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

*Source:* OMI Milan and our elaborations.

*Notes:* The coefficient  $\lambda$  is an estimate of the extent at which the spatial components of the error term in the second model are correlated with one another for nearby observations, as given by the proximity matrix. The coefficient  $\sigma$  is an estimate of the asymptotic variance of the residual term.

spatial dependence.<sup>10</sup> The Moran's test does not allow to reject absence of spatial correlation in the OLS residuals. However, both simple and robust LM tests clearly reject the absence of spatial autocorrelation.<sup>11</sup> Since we cannot exclude the presence of local spatial dependence, and since the rejection is much stronger for the autocorrelation in the error term, we adopt a spatial error model for the hedonic price equation. The estimation results of the spatial error specification are in the last two columns of Table 2.

<sup>10</sup>For details and formulas, see Anselin and Hudak (1992) and Anselin, Bera, Florax and Yoon (1996). The spatial weight matrix  $W$  used for these tests has diagonal elements  $w_{ii}$  set to 0 and  $w_{ij} = \frac{1}{d_{ij}^2}$  otherwise, where  $d_{ij}$  is the distance between the centroids of neighborhoods  $i$  and  $j$ . We assume that  $\frac{1}{d_{ij}^2} = 1$  if  $d_{ij} \in (0; 1)$ .

<sup>11</sup>The Moran  $I$  test statistic is 1.056 (p-value 0.291) with one degree of freedom. The LM test statistics (and its robust counterpart) for the spatial error model is 3326.6 (3312.2), while for the spatial lag specification is 543 (528.4).

Table 3: Parameter estimates from the structural model

	$\varrho_1$	$\varrho_2$	$\varrho_3$	$\varrho_4$	$\varrho_5$	$\varrho_6$	$\varrho_7$	$\varrho_8$	$\varrho_9$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<i>Amenities and housing-specific attributes:</i>									
Principal component	80.591***	-31.790***	-3.524***	-6.161***	-3.664***	-5.544***	-5.274***	-12.015***	-12.617***
	[40.96]	[-36.77]	[-11.94]	[-20.36]	[-14.10]	[-18.66]	[-15.25]	[-31.42]	[-14.82]
Green	-31.790***	20.555***	0.160	1.532***	0.580***	0.482***	0.200	1.880***	6.402***
	[-36.77]	[36.89]	[1.33]	[13.28]	[5.68]	[3.80]	[1.23]	[9.92]	[11.51]
Schools	-3.524***	0.160	2.022***	0.484***	0.434***	0.195***	0.130**	1.275***	-1.175***
	[-11.94]	[1.33]	[30.11]	[8.62]	[8.51]	[3.41]	[2.06]	[18.53]	[-10.34]
Cultural	-6.161***	1.532***	0.484***	1.372***	0.907***	0.397***	0.195***	1.367***	-0.093
	[-20.36]	[13.28]	[8.62]	[20.80]	[17.85]	[7.13]	[3.24]	[21.18]	[-0.90]
Transport	-3.664***	0.580***	0.434***	0.907***	1.969***	0.410***	0.192***	1.267***	-2.094***
	[-14.10]	[5.68]	[8.51]	[17.85]	[38.48]	[8.20]	[3.52]	[21.59]	[-20.29]
Security	-5.544***	0.482***	0.195***	0.397***	0.410***	2.812***	0.290***	0.952***	0.006
	[-18.66]	[3.80]	[3.41]	[7.13]	[8.20]	[34.85]	[4.54]	[13.04]	[0.05]
Health	-5.274***	0.200	0.130**	0.195***	0.192***	0.290***	3.071***	0.234***	0.962***
	[-15.25]	[1.23]	[2.06]	[3.24]	[3.52]	[4.54]	[30.50]	[2.72]	[6.13]
Facilities	-12.015***	1.880***	1.275***	1.367***	1.267***	0.952***	0.234***	4.606***	0.434**
	[-31.42]	[9.92]	[18.53]	[21.18]	[21.59]	[13.04]	[2.72]	[32.74]	[2.49]
Percentage of italians	-12.617***	6.402***	-1.175***	-0.093	-2.094***	0.006	0.962***	0.434**	8.176***
	[-14.82]	[11.51]	[-10.34]	[-0.90]	[-20.29]	[0.05]	[6.13]	[2.49]	[8.22]
<i>Additional regressors:</i>									
ln $Q^*$	1.995***	-0.345***	-0.228***	-0.306***	-0.275***	-0.176***	-0.171***	-0.094***	-0.400***
	[43.45]	[-24.48]	[-25.30]	[-33.39]	[-33.27]	[-18.45]	[-19.34]	[-10.34]	[-26.79]
Constant	0.303***	0.035***	0.074***	0.071***	0.085***	0.072***	0.069***	0.085***	0.205***
	[27.42]	[7.42]	[37.24]	[35.59]	[46.91]	[33.70]	[32.14]	[37.09]	[33.92]
Time controls	Yes								
Joint significance (Chi2)	5,786.9	3,595.7	3,080.1	2,267.9	3,516.8	2,507.6	2,312.7	2,814.4	4,028.8

$t$  statistics in parentheses. \*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

*Source:* OMI Milan and our elaborations.

*Notes:* The estimated demand system comprises: (1) First principal component, (2) green, (3) number of high schools, (4) cultural services, (5) transportation, (6) security, (7) health services, (8) other facilities, (9) percentage of Italians at the neighborhood level. Attributes coefficients are multiplied by  $10^3$ . Estimates are obtained for a sample of 3949 housing transaction observed in a seven years interval. Symmetry, homogeneity and identification constraints are imposed to the model.

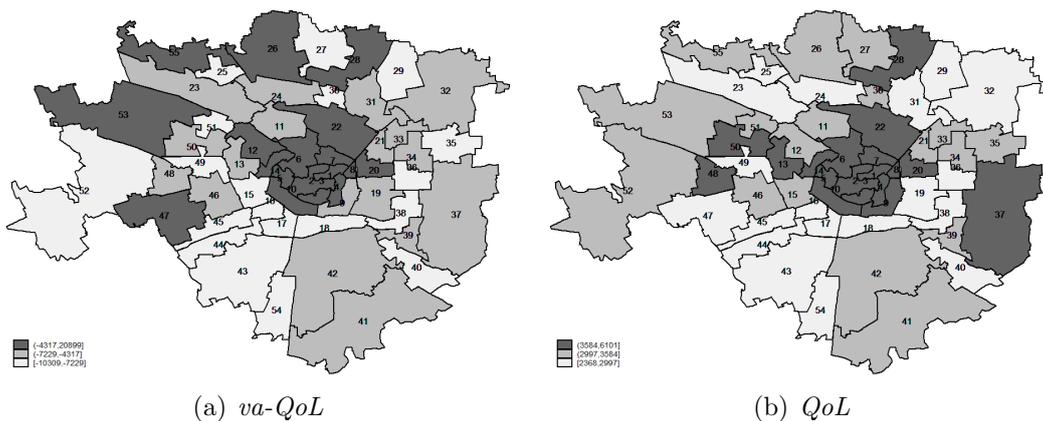


Figure 2: Quality of life evaluations across Milan neighborhoods, based on *va-QoL* estimates and on Roback’s (1982) *QoL* index.

*Source:* OMI data and our elaborations.

*Note:* The legend identifies three groups gathering the same number of neighborhoods, ranked by the most to the least preferred according to the quality of life indicator used.

The estimated coefficients of the spatial error specification are used to predict both the implicit prices of attributes for each observed housing unit  $h$  according to (19), and the price of housing at neighborhood level. We obtain the attribute-to-consumption value ratios  $\hat{q}_j$  for any attribute  $j = 1, \dots, A + H$ .

The linearized *IAIDS*, given by (16) where  $\ln Q$  is substituted with the Stone quantity index  $\ln Q^*$ , is estimated by Seemingly Unrelated Regression (SUR) methods (Eales et al. 1997). The SUR model consists of a system of 9 equations, one for each attribute, estimated on the 3949 housing transactions of our data set. Results are reported in Table 3, columns (1) to (9). For each attribute  $j$ , each equation produces eight estimates of  $\hat{\gamma}_{ij}$  coefficients referred to the amenities ( $i = 2, \dots, 9$ ) and one coefficient for the factor component describing housing-specific features ( $i = 1$ ), plus a constant. In the model, we also control for time trends. The explanatory power of the system is reasonable and practically all 81 + 18 parameters estimates are more than twice their estimated standard errors.

Making use of these estimates, we are able to identify the compensating benefit associated with each neighborhood and compute the *va-QoL* index by neighborhood according to (10). The first term of (10) is 4,123 € and corresponds to the amount that a representative household would be willing to pay to live in a neighborhood with the average quantities of the amenities. The second term of (10) is the amount of compensating benefit that ranges from -16,774 to 14,432 €. The negative values refer to neighborhoods characterized by a richer bundle of amenities than the average bundle, so that individuals living there are willing to pay

the amount of the compensating benefit to enjoy the higher endowment of amenities. A symmetric argument applies to the interpretation of positive values for the compensating benefit: they represent the monetary compensation for households who live in neighborhoods with lower endowments of amenities rather than in the neighborhood with the average bundle of amenities.

The values of the va-QoL index are shown in panel (a) of Figure 2. The map reveals that high quality of life neighborhoods are predominantly located in the city-centre. The neighborhood with the highest quality of life is 2-Brera Duomo Cordusio Torino, followed by adjacent neighborhoods, in particular 7-Turati Moscova Repubblica, 10-Porta Ticinese Porta Genova Magenta, 14-Wagner Pagano Monti, 3-Missori, Italia, Vetra, Sant'Eufemia. Households living in these neighborhoods are willing to pay an implicit premium between 16,774 € and 861 € per year to access the amenities of these neighborhoods compared to the city average. Bottom-ranked neighborhoods are 38-Ortomercato Molise Piranesi, 36-Argonne Viale Corsica, 40-Omero Gabriele Rosa Brenta, located in the East-side of the city, and 25-Largo Boccioni Aldini Lopez 49-Segesta Capecelatro Aretusa in the Nord-West and West-side, respectively. In these neighborhoods, households should receive an annual compensation that ranges between 14,432 and 13,083 € for living in more disadvantageous neighborhoods compared to the city average.

Panel (b) of Figure 2 reports the level of quality of life in each neighborhood computed through the Roback's index. The rankings of urban sections produced by the two indices are positively and significantly correlated, the Spearman's rank correlation coefficient is 0.6349,  $\text{Prob} > |t| = 0.0000$ .

## 5 Conclusion

This paper extends the standard hedonic approach to assess quality of life in urban areas using a welfare measure that quantify the gain (loss) for living in an area with a bundle of amenities higher (lower) than the average bundle. The new index adjusts the Roback's index by correctly evaluating infra-marginal units of amenities without demanding more data than those used in the standard analysis of urban quality of life. Indeed we need to know only households' income in addition to information about housing and amenities.

The theoretical framework is developed focusing on a single city, although the va-QoL could be applied to other geographical areas such as state, region or city. In that case we should include in our analysis the labor market, and other important aspects like intercity differences in non-housing costs, intergovernmental transfers

and local taxes, housing production.<sup>12</sup> Such a more general framework constitutes a promising avenue for future research.

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<sup>12</sup>These issues have been considered by Albouy (2008) and Albouy et al. (2013).

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# Data Appendix

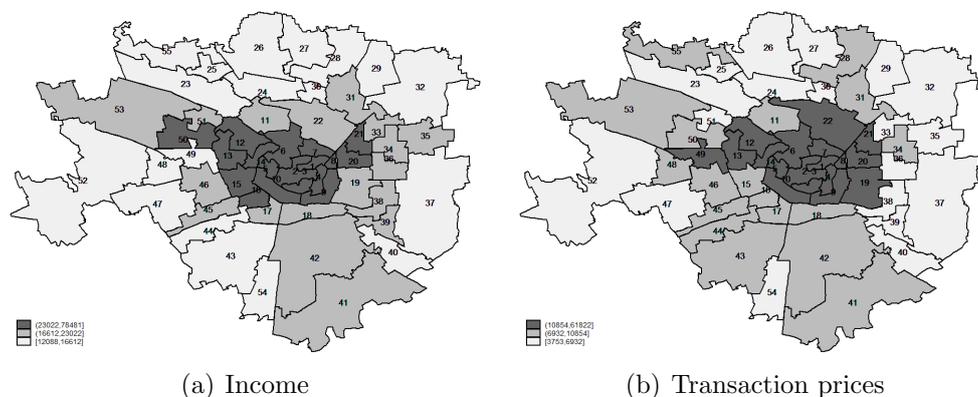


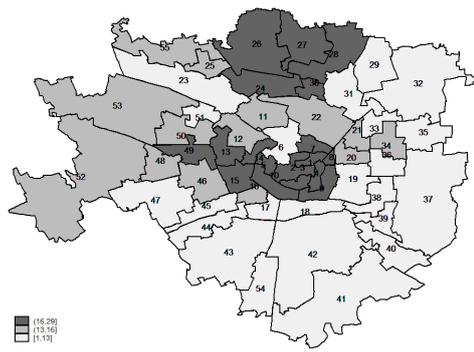
Figure 3: Average income and transaction prices across neighborhood in Milan.

*Source:* OMI and municipality data, our elaborations.

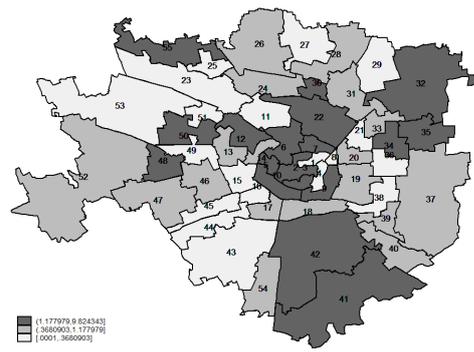
*Note:* The map reproduces the household income and the average transaction price of a housing unit, both averaged within the 55 neighborhoods of Milan.

Table 4: Description and sources of variables

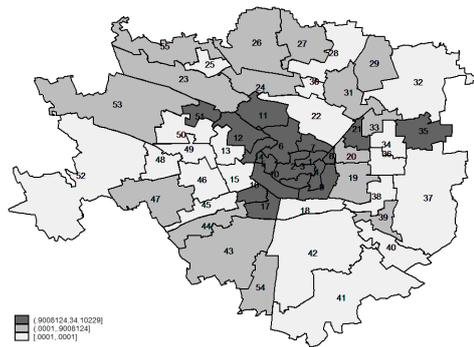
Variable	Description
<i>Green areas</i>	Pct of urban green over urban area. <i>Source:</i> Milan Municipality
<i>Secodary schools</i>	N. of secondary schools per 10,000 inhab. <i>Source:</i> Authors' computation
<i>Culture</i>	N. of cultural places per 10,000 inhab. <i>Source:</i> Authors' computation
<i>Transport</i>	N. of metro and railways station per 10,000 inhab. <i>Source:</i> Milan Transport Agency
<i>Security</i>	N. of policy stations per 10,000 inhab. <i>Source:</i> Authors' computation
<i>Health services</i>	N. of health centers per 10,000 inhab. <i>Source:</i> Authors' computation.
<i>Facilities</i>	N. of pharmacies and post offices per 10,000 inhab. <i>Source:</i> Authors' computation
<i>Italians/immigrants</i>	Percentage of Italian residents in the neighborhood. <i>Source:</i> Milan Municipality.
<i>Income</i>	Average before taxes household income at the level of the neighborhood. <i>Source:</i> Milan Municipality and University of Milan Bicocca.
<i>Housing prices</i>	Observed/imputed market prices and rents, offer prices for 3949 transactions, years 2004-2010. <i>Source:</i> OMI.
<i>Housing attributes</i>	Floor surface, second/third bathroom, renewed unit, gas central heating, second floor or above, low cost/medium/luxury building, presence of a parking slot and elevator, age of the building, distance from the city center. <i>Source:</i> OMI



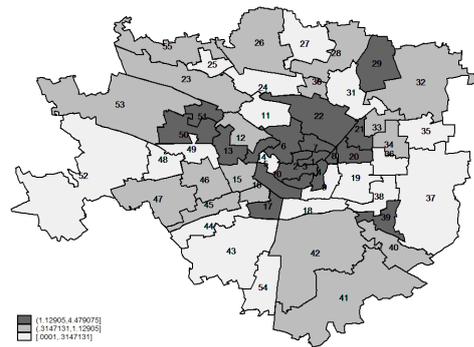
(a) Green



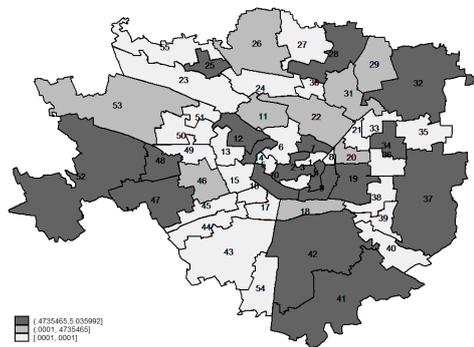
(b) Schools



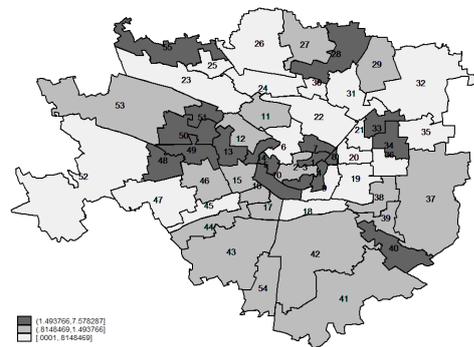
(c) Cultural



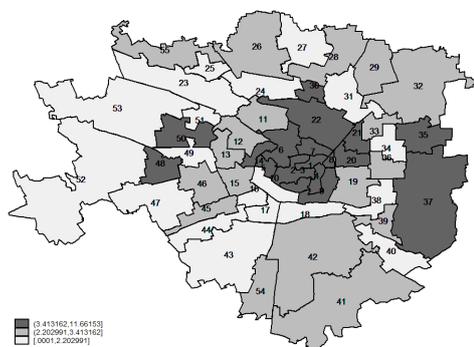
(d) Transport



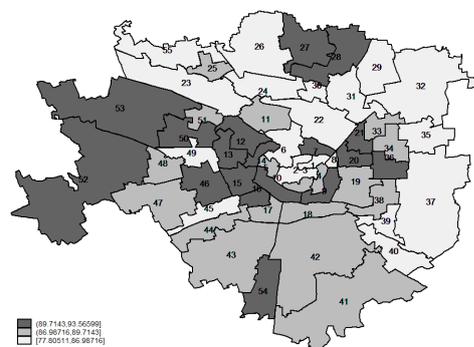
(e) Security



(f) Health



(g) Facilities



(h) Share of italians

Figure 4: Distribution of amenities across neighborhoods of Milan.

Source: OMI and municipality data, our elaborations.