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The Sharing Rule: Where Is It?*

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Abstract

The search for a robust and stable sharing rule has led us to novel results for identifying the rule governing the intra-household allocation of resources. We introduce an income proportionality property that directly connects distribution factors with individual incomes and implements this identifying condition to obtain an exact correspondence between the structural and reduced form of the collective demand equations. In our application, we appropriately exploit information about privately consumed goods, prices and distribution factor variation to exactly estimate how resources are shared between adults and children in a sample of Italian households. Our results open up the possibility of a direct structural estimation of a collective system of demand equations, and associated individual Engel curves, as easily as estimating a demand system based on a unitary framework.

JEL codes: C30, D11, D12, D13.

Keywords: Collective model, Sharing rule, Consumer demand, Identification.

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1 Introduction

Economists recognize that modeling household decisions using an aggregate unitary utility function subject to the household budget constraint is inadequate both theoretically and empirically. Because decisions are made at the individual level, they should be modeled at the level of each member of the household, especially if theory is to be empirically tested. The seminal works by Chiappori (1988, 1992) introduce a theoretical household model in which the family comprises a collection of individuals each of whom is characterized by her own preferences over market goods consumed within the family. Assuming that the decision process results in Pareto efficient outcomes, Chiappori (1988, 1992) shows that when agents are *egoistic* and consumption is purely private the collective model generates testable restrictions sufficient to recover individual preferences and the rule governing the allocation of resources within the family, provided that private consumption of exclusive or assignable goods is observed. The identification of the sharing rule is necessary to derive individual welfare functions and to assess the impact of a policy change in terms of prices, incomes or other exogenous factors of interest on the welfare level of each family member.

Judging by the numerous theoretical refinements (Browning, Chiappori and Lechene 2006, Donni 2003, 2006, 2011, Chiappori and Ekeland 2009, Chiappori, Donni, and Komunjer 2012, Vermuelen 2002, Browning, Chiappori, and Weiss 2012, Chiappori and Kim 2017) and applications (Browning *et al.* 1994, Chiappori, Fortin, and Lacroix 2002, Lise and Seitz 2011, Bargain and Donni 2009, Dunbar, Lewbel, and Pendakur 2012, Cherchye *et al.* 2015) in the areas of both consumption and labour choices, we may be inclined to think that theoretical developments and the accuracy of the applications are at a mature stage. Identification issues, though, are traditionally difficult because they involve a simultaneous blend of theory, structural assumptions about functional forms, and data quality considerations to be combined in different doses depending on the study case. This research renews the investigation of identification issues within a collective framework by taking into consideration all these three aspects to show that the search for the sharing rule is not yet over.

This study extends the collective theory of household consumption by unveiling a new restriction that extends the proportionality property to what we term the income proportionality property. This restriction proves to be crucial in achieving the postulated equality between the directly estimated structural form parameters and the indirectly estimated reduced form parameters.

The design of our experiment is based on the postulate stating that the estimates obtained from a direct estimation of a structure as in Browning *et al.* (1994) or Lise and Seitz (2011) should be the same as the estimates derived implementing an indirect least square procedure consisting in a first stage estimation of an unrestricted reduced form and a second stage that uses the collective theory restrictions to recover the structure as in Chiappori, Fortin, and Lacroix (2002) or in Rapoport, Sofer, and Solaz (2011) and Donni and Moreau (2007). Considering that the identifiability restrictions are determined exploiting the correspondence between a chosen structure and the associated unrestricted reduced form, then a direct or an indirect estimation technique should produce the same structural parameters. This is the only study that realizes this correspondence proving that all structural parameters can be identified. Our evidence opens up the possibility of a direct structural estimation of a collective demand system, and associated individual Engel curves, as easily as estimating a demand system based on a unitary framework.

In a nutshell, the paper first reexamines the collective model of household consumption and a) shows that the collective model as traditionally developed is not identified, b) illustrates the novel income proportionality condition, and c) explains how to achieve full identification using a specification that is directly anchored to the available information about exclusive consumption. The subsequent section extends the results to a functional specification, used in the applied section, which closely resembles, in a consumption rather than labor context, the specification used in Chiappori, Fortin, and Lacroix (2002), in order to facilitate comparisons across identification strategies and empirical implementations. In the applied section, we perform both an indirect estimation of the reduced form model and a direct estimation of the structural model to show that there is full correspondence between the structure and the associated reduced form, in line with the set of restrictions derived in the theory sections, obtaining the same unique set of parameters. Concluding remarks close the analysis.

2 The Collective Model of Household Consumption

We describe the consumption decisions of a family of two members or groups such as adults and children, denoted by 1 and 2, within a collective model. Each family member privately consumes an exclusive market good c^i and an ordinary composite market good c_o^i,[1] of which we only observe the whole family consumption $c_o^h = c_o^1 + c_o^2$.[2] Each member faces a market price equal to p_i to buy the exclusive good c^i and a market price of p_o for the ordinary good c_o^i that is assumed equal for both members. We normalize prices and set the price of the ordinary good c_0 to be equal to 1: $p_o = 1$. Household income is denoted by $x > 0$. Without a loss of generality, we exclude the consumption of public goods or the presence of externalities within families. We also abstract from the consumption of domestically produced goods (Apps and Rees 1997, Chiappori 1997).

The collective household model relies on a set of general assumptions, referred to as *collective rationality*, that assures the identifiability of the structure of the household model.[3] Each family member has egoistic preferences over the consumption of exclusive and ordinary goods represented by an *egoistic* quasi-concave utility function $u^i = U^i\left(c^i, c_o^i\right)$, twice differentiable and strictly increasing in its elements.[4] Family outcomes are Pareto-efficient.

[1]Exclusivity and ordinariness are two polar characteristics of goods consumed by a multiple person household. A good is ordinary when a private good is consumed without rivalry in unobserved proportions by all or some non identifiable household members. An example of ordinary good may be the consumption of food. Conversely, a good is exclusive when a private good is consumed by only one identifiable member and its price is different from the price of the other exclusive goods consumed within the family. Examples of exclusive goods are leisure and clothing, when we can observe female and male clothing expenditures and the corresponding prices.

[2]In the paper superscripts indices denote endogenous variables, subscripts denote exogenous variables.

[3]The structure of the household model is defined by preferences of the family members and the household decision process described by the sharing rule.

[4]It is important to remark that *egoistic* preferences are not necessary to recover the structure of the col-

The rationale of this assumption is that efficient allocations are likely to emerge when agents are able to make binding commitments and have full information as is often the case in a household setting.

Pareto-efficiency implies that the consumption equilibrium will be on the Pareto frontier of the family. This can be represented by the maximization problem

$$\max_{c^1,c^2,c_o^1,c_o^2} \{\mu(p_1,p_2,x,s)U^1(c^1,c_o^1)+(1-\mu(p_1,p_2,x,s))U^2(c^2,c_o^2) : p_1c^1+p_2c^2+c_o^1+c_o^2=x\}, \tag{1}$$

where $\mu(p_1,p_2,x,s) \in [0,1]$ is a welfare weight that depends on prices, income, and distribution factors s. Distribution factors are variables that only affect the decision process. Browning, Chiappori, and Lechene (2006), and Chiappori and Ekeland (2009) show the role played by distribution factors in identifying the sharing rule and present an exhaustive review of variables used as distribution factors in the collective literature.

Denote the solution of the maximization problem (1) by the Marshallian demands $c^i\left(p_1,p_2,x,s\right)$ and $c_o^i\left(p_1,p_2,x,s\right)$, $i=1,2$. It follows that the consumption expenditure associated with the i-th member is $\phi_i(p_1,p_2,x,s) \equiv p_i\, c^i(p_1,p_2,x,s)+c_o^i(p_1,p_2,x,s)$, $i=1,2$. Under non-satiation, the budget constraint in (1) implies that $\phi_1(p_1,p_2,x,s)+\phi_2(p_1,p_2,x,s)=x$. Using this equality, it will be convenient to define $\phi(p_1,p_2,x,s) \equiv \phi_1(p_1,p_2,x,s)$ as the part of household income received by member 1, while $[x-\phi(p_1,p_2,x,s)] \equiv \phi_2(p_1,p_2,x,s)$ is the part of household income received by member 2. Below we will call $\phi(p_1,p_2,x,s)$ the "sharing rule", which governs how household income is divided between the two members. This sharing rule depends in general on prices (p_1,p_2), on household income x and on the distribution factors s. Rises of the welfare weight $\mu(p_1,p_2,x,s)$ in (1) move the allocation along the Pareto utility frontier in favour of member 1, thus income gets reallocated away from member 2.

Using the sharing rule $\phi(p_1,p_2,x,s)$, consumption behaviour can be described by a two-stage process. First, the family decides how to share income among its members. Second, each member maximizes her individual utility function subject to her individual budget constraint. Given the individual-specific incomes ϕ_i , the second stage implies that the i-th member makes consumption decisions according to the following decentralized program

$$\max_{c^i,c_o^i} \{U^i(c^i,c_o^i) : p_ic^i+c_o^i=\phi_i\} \tag{2}$$

$i=1,2$. Denote the solution of the maximization problem (2) by the Marshallian demands $C^i\left(p_i,\phi_i\right)$ and $C_o^i\left(p_i,\phi_i\right)$. It is clear that the collective choice problem (1) is equivalent to the individual choice problem (2) when $\phi_i \equiv \phi_i(p_1,p_2,x,s)$, $i=1,2$. Given the condition $\phi_1(p_1,p_2,x,s)+\phi_2(p_1,p_2,x,s)=x$, this generates the following identities

$$c^i\left(p_1,p_2,x,s\right) = C^i\left(p_i,\phi_i\left(p_1,p_2,x,s\right)\right), \tag{3}$$

$$c_o^i\left(p_1,p_2,x,s\right) = C_o^i\left(p_i,\phi_i\left(p_1,p_2,x,s\right)\right), \tag{4}$$

lective model. The collective set-up may be extended to a caring utility function $W^i\left[U^1\left(c^1,c_o^1\right),U^2\left(c^2,c_o^2\right)\right]$ without altering the conclusions of the household model (Chiappori 1992, Chiappori and Ekeland 2009).

for $i = 1, 2$. Note that only the left-hand side of equations (3) and (4) are observable because the sharing rule function $\phi_i(p_1, p_2, x, s)$ is not observable. Further, for non-assignable goods, we can observe household demand $c_o^h = c_o^1 + c_o^2$, but not its components.

Throughout the paper we refer to c^i and c_o^i as the reduced demand functions and to C^i and C_o^i as the structural demand functions of member i. We use the term "reduced form" to indicate a functional relationship between the endogenous variable and the observable exogenous variables. Note that in our framework both structural and reduced form are expressed in explicit form. The structure depends on the unobservable sharing rule and incorporates the restrictions underlying the collective model. We now provide results about the identification of the sharing rule $\phi(p_1, p_2, x, s)$ adding new insights to the existing works. The presentation refers to Chiappori, Fortin, and Lacroix (2002) translated into a consumption context.

Identification of the Sharing Rule For the purpose of our theoretical investigation, we assume observation of an information set comprising two exclusive goods, associated individual prices and at least one distribution factor.

We follow the traditional treatment in Rothenberg (1973) to show the identification of the sharing rule and the demand parameters that interact with the sharing rule. This allows us to identify new conditions necessary for identification.In Appendix A, it is reported an identification proof similar to the one developed by Chiappori, Fortin, and Lacroix (2002), but for the crucial fact that we place special attention also on the identification of the demand parameters that interact with the parameters of the sharing rule.

Equation (3) can be expressed as $f(R) = g(S)$, where$f(R)$ is the reduced form specification and $g(S)$ is the structural specification. Define then the set of structural parameters as $S = \left\{C_{p_1}^1, C_\phi^1, C_{p_2}^2, C_\phi^2, \phi_x, \phi_{p_1}, \phi_{p_2}, \phi_s\right\}$, and $R = \{c_{p_1}^1, c_{p_2}^1, c_x^1, c_s^1, c_{p_1}^2, c_{p_2}^2, c_x^2, c_s^2\}$ the set of reduced parameters. We are interested in the identification of two sets of structural parameters: the sharing rule parameters $\{\phi_{p_1}, \phi_{p_2}, \phi_x, \phi_s\}$ and the income and price parameters $\left\{C_\phi^1, C_\phi^2, C_{p_1}^1, C_{p_2}^2\right\}$.

The structural parameters S are identified if, knowing R, $f(S) = g(R)$ can be solved to generate a unique value for S. This is a global characterization of identification. A local version of identification applies if, knowing R, a unique value of the structural parameters S can be obtained by solving $f(R) = g(S)$ in the neighborhood of a particular evaluation point. Assuming that the functions $c^i(p_1, p_2, x, s)$ and $C^i(p_i, \phi_i(p_1, p_2, x, s))$ are differentiable, local identification can be evaluated using a first-order Taylor series approximation of equation (3) and relying on the derivatives of $c^i(p_1, p_2, x, s)$ and $C^i(p_i, \phi_i(p_1, p_2, x, s))$.

Proposition 1 (Non Identification). *For a reduced demand specification $c^i(p_1, p_2, x, s)$ and structural demand specification $C^i(p_i, \phi_i(p_1, p_2, x, s))$, the structural parameters $S = \left\{C_{p_1}^1, C_\phi^1, C_{p_2}^2, C_\phi^2, \phi_x, \phi_{p_1}, \phi_{p_2}, \phi_s\right\}$ are not identified in the sense that $f(R) = g(S)$ does not have a unique solution S for any given $R = \{c_{p_1}^1, c_{p_2}^1, c_x^1, c_s^1, c_{p_1}^2, c_{p_2}^2, c_x^2, c_s^2\}$.*

Proof. In matrix notation, define the vector of exogenous variables $z = \{p_1, p_2, x, s\}$. Let $c^1(z)$ and $c^2(z)$ be the reduced form demands and $C^1(p_1, \phi_1(z))$ and $C^2(p_2, \phi_2(z))$ be the structural demands for the exclusive goods consumed by member 1 and member 2, respectively. Let $F = (c^1, c^2)$ and $G = (C^1, C^2)$. In generic form, the structural demand for (C^1, C^2) is given by $G(z, \phi(z))$. Since $F(z) = G(z, \phi(z))$ from (3), it follows that

$$\left[\frac{\partial F}{\partial z}\right] = \frac{\partial G}{\partial z} + \left[\frac{\partial G}{\partial \phi}\frac{\partial \phi}{\partial z}\right]. \tag{5}$$

Using a compact notation, this is the system of equations $F_z = G_z + G_\phi \phi_z$, where subscripts denote derivatives. In this context, the identification issue is: can we recover the structural information G_z, G_ϕ and ϕ_z from the knowledge of F_z? Denoting the unknown elements in S_z, S_ϕ, and ϕ_z by $S = \left\{C^1_{p1}, C^1_\phi, C^2_{p2}, C^2_\phi, \phi_x, \phi_{p1}, \phi_{p2}, \phi_s\right\}$, we can rewrite $F_z = G_z + G_\phi \phi_z$ as

$$F_z = H(S). \tag{6}$$

Let N be the number of elements in F_z and M be the number of unknown elements of S. Then a necessary (but not sufficient) condition for the local identification of S is the "order condition" requiring that $N \geq M$, that is the number of equations in $F_z = H(S)$ must be at least as large as the number of unknowns. In our case, we have $N = M = 8$, implying that the order condition is satisfied.

Note that, under differentiability, equation $F_z = H(S)$ can be linearized locally in the neighborhood of point S_0 using a first-order Taylor series approximation

$$F_z = H(S_0) + H_S[S - S_0], \tag{7}$$

or

$$F_z - H(S_0) + H_S S_0 = H_S S, \tag{8}$$

where H_S is the $(N \times M)$ matrix of derivatives of H with respect to S, evaluated at S_0. This is a system of N linear equations in M unknowns. From linear algebra, this system of equations has a unique solution if and only if the following "rank condition" holds

$$rank(H_S) = rank(H_S \mid F_z - H(S_0) + H_S\, S_0) = M. \tag{9}$$

This rank condition is a sufficient condition for local identification of S. It implies the order condition $N \geq M$. When $N = M$, the rank condition reduces to $rank(H_S) = M$. The rank condition implies the global identification of S if it holds at any evaluation point S_0.

In our context, we have $N = M = 8$, and the rank condition reduces to $rank(H_S) = M$, or equivalently to the non-singularity of the matrix H_S. Then, the identification of the structural parameters S reduces to checking whether the determinant of the matrix H_S

$$H_S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & C_\phi^1 & 0 & 0 \\ 0 & \phi_x & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -C_\phi^2 & 0 \\ 0 & 0 & 0 & (1-\phi_x) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_\phi^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -C_\phi^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C_\phi^1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_\phi^2 \end{bmatrix} \tag{10}$$

is non-zero. For the determinant of H_S to be non-zero, we need a) $\phi_x \neq 0$, $(1-\phi_x) \neq 0$, $C_\phi^1 \neq 0$ and $C_\phi^2 \neq 0$ and b) the parameters are not linearly related. The structural elements $\left\{\phi_x, C_\phi^1, C_\phi^2\right\}$ in the H_S matrix cannot be expressed only as a function of reduced form parameters. As an illustration, consider the sharing rule parameter associated with income

$$\phi_x = \frac{c_x^1 c_s^2}{c_x^1 c_s^2 - c_s^1 c_x^2} = \frac{c_x^1}{C_\phi^1} = 1 - \frac{c_x^2}{C_\phi^2}, \tag{11}$$

and the structural parameter C_ϕ^1 associated with the sharing rule of individual 1

$$C_\phi^1 = c_x^1 - c_x^2 \frac{c_s^1}{c_s^2} = \frac{c_x^1}{\phi_x}, \tag{12}$$

and individual 2

$$C_\phi^2 = \frac{c_x^2}{c_x^1} \frac{\phi_x}{(1-\phi_x)} C_\phi^1 = \frac{c_x^2}{(1-\phi_x)}. \tag{13}$$

showing that the terms ϕ_x, C_ϕ^1 and C_ϕ^2 are linearly related. This implies linear dependency and the singularity of the matrix H_S. In turn, this implies that the structural parameters S cannot be uniquely recovered from the observable data.[5] Let us examine the implications of this indeterminateness. The proportionality condition, which is fundamental because it solves the over-identification problem generated by the presence of more than one distribution factor, still holds (Bourguignon, Browning, and Chiappori 1995, Bourguignon, Browning, and Chiappori 2009, Chiappori, Fortin, and Lacroix 2002). □

Proposition 2 (Proportionality Condition. Bourguignon, Browning, and Chiappori 1995)**.** *With several distribution factors $r = 1, ..., R$, the ratio of the reduced parameters associated with the distribution factors is equal across household members*

$$\frac{c_{s_r}^1}{c_{s_1}^1} = \frac{c_{s_r}^2}{c_{s_1}^2}, \quad \forall r = 2, ..., R. \tag{14}$$

[5]Note that these type of inconsistencies, shown in detail in Appendix A, were not apparent in Chiappori, Fortin, and Lacroix 2002 because the authors' analysis was only concerned with the subset of parameters $S_\phi = \{\phi_x, \phi_{p_1}, \phi_{p_2}, \phi_s\}$ rather than $S = \left\{C_{p_1}^1, C_\phi^1, C_{p_2}^2, C_\phi^2, \phi_x, \phi_{p_1}, \phi_{p_2}, \phi_s\right\}$ as it is done in the present analysis.

Proof. Consider the case with $r = 1, 2$. Then, from expression (3) for member 1 we have

$$c^1_{s_1} = C^1_\phi \phi_{s_1} \text{ and } c^1_{s_2} = C^1_\phi \phi_{s_2}, \tag{15}$$

and analogously for member 2

$$c^2_{s_1} = -C^2_\phi \phi_{s_1} \text{ and } c^2_{s_2} = -C^2_\phi \phi_{s_2}, \tag{16}$$

from which it follows that

$$\frac{c^i_{s_2}}{c^i_{s_1}} = \frac{\phi_{s_2}}{\phi_{s_1}} \tag{17}$$

and the ratio between reduced form parameters is independent of household members. □

Further, exploiting the relationships implied by Proposition 1, we can see that the ratio associated with the distribution factors' partial effects are related to the income effect. As shown in the following proposition, this condition complements Proposition 2.

Proposition 3 (Income Proportionality Condition)**.** *The ratio of the distribution factor parameters associated with the reduced form is equal to*

$$\frac{c^1_{s_r}}{c^2_{s_r}} = -\frac{c^1_x}{c^2_x}\frac{1-\phi_x}{\phi_x}, \quad \forall r = 1, ..., R. \tag{18}$$

Proof. Consider that for member 1

$$\frac{c^1_{s_r}}{c^1_x}\phi_x = \phi_{s_r}, \tag{19}$$

and similarly for member 2

$$-\frac{c^2_{s_r}}{c^2_x}(1-\phi_x) = \phi_{s_r}, \tag{20}$$

thus it follows

$$\frac{c^1_{s_r}}{c^2_{s_r}} = -\frac{c^1_x}{c^2_x}\frac{1-\phi_x}{\phi_x} \quad \forall r = 1, ...R, \tag{21}$$

for $\phi_x \neq 0$ and $\phi_x \neq 1$. □

Proposition 3 says that distribution factors are informative about the rule governing the intra-household distribution of resources only if an exact relationship with income can be implemented. This is reasonable if we consider that distribution factors affect optimal choices only through the sharing function that describes across-household variations in the individual income levels of family members.[6] Note also that while the restrictions described

[6]It is interesting to note that the relationship of Proposition 3 is present also in Chiappori, Fortin, and Lacroix (2002), though not explic. The equalities across ratios of distribution factors in equation (8) of Chiappori, Fortin, and Lacroix (2002) summarizing the collective restrictions on labour supply are also equal to the negative of the ratio of the demand parameters α_2 and β_2 in equations (11) and (12) of Chiappori, Fortin, and Lacroix (2002).

in Proposition 2 can be imposed on the model estimation because they involve only reduced form parameters, the condition of Proposition 3 cannot be imposed on the model estimation because it involves both reduced and structural parameters. This is another evidence of identification difficulties unless further information is exploited. A solution to this identification problem can be found by investigating whether prior information is available. Closer inspection of the restrictions associated with the sharing rule parameters described in Proposition 1 reveals that it would be sufficient to know *a priori* at least one sharing rule parameter to identify all other parameters exactly. An interesting candidate is ϕ_x because of the natural interpretation of the parameter as a share. To see this, suppose for now that we do not have sufficient variation in both prices and distribution factors so that the reduced form parameters associated with prices and distribution factors are not statistically different from zero. Then, the sharing function becomes only a function of x and can be reasonably described as

$$\begin{cases} \phi_1(x) = \phi_x x \\ \phi_2(x) = (1-\phi_x)\,x \end{cases} \quad \text{for } \phi_x \neq 0,\ \phi_x \neq 1. \tag{22}$$

Note that because $0 < \phi_1(x) < x$, then also $0 < \phi_x x < x$ thus implying that

$$0 < \phi_x < 1. \tag{23}$$

Thus, the parameter ϕ_x acts as a share that scales the income parameters as shown in equation (21). Equation (22) tells us that information about total income x is not sufficient for identification, unless something is known *a priori* about how income is shared within the household. Part of this information is provided by the knowledge of at least one privately consumed good, in line with the minimal informational requirement of the collective theory, so that it is possible to construct the best approximation of individual incomes before estimation.[7] Therefore, ϕ_x is a share that, if known *a priori*, allows the derivation of an approximate measure of individual incomes.

Now, we investigate whether it is possible to achieve exact identification taking the interpretation of ϕ_x derived above into account. To do so, we suppose, for the moment, that ϕ_x is a known parameter $\xi \in (0,1)$ specific to each family.

As a consequence, the sharing rule function is separable in a function $m\,(p_1, p_2, s)$ that scales individual incomes as

$$\phi_1\,(x, p_1, p_2, s) = (\xi\, x)\, m\,(p_1, p_2, s)\,. \tag{24}$$

Note that when $m(p_1, p_2, s)$ approximates 1 the function reduces to equation (22). The separability property implies that the scaling function $m\,(p_1, p_2, s)$ is independent of income.

We can now show that this is sufficient to obtain the exact identification of all the remaining sharing rule parameters belonging to the scaling function $\{\phi_{p_1}, \phi_{p_2}, \phi_s\}$.

[7]This explains why a number of applied works use a logistic transformation to ensure that the estimated sharing function is maintained within the zero-one range.

Proposition 4 (Identification)**.** *If* $\phi_x = \xi$ *with* $\xi \in (0, 1)$*, then the sharing rule parameters* $\{\phi_{p_1}, \phi_{p_2}, \phi_s\}$ *are exactly identified.*

Proof. Recall equation (3) for $i = 1, 2$ and use $\phi_x = \xi$. Then, the elements of the Jacobian matrices become

Member 1		*Member* 2		
Reduced	*Structural*	*Reduced*	*Structural*	
$c^1_{p_1}$	$C^1_{p_1} + C^1_\phi \phi_{p_1},$	$c^2_{p_1}$	$- C^2_\phi \phi_{p_1},$	(25)
$c^1_{p_2}$	$C^1_\phi \phi_{p_2},$	$c^2_{p_2}$	$C^2_{p_2} - C^2_\phi \phi_{p_2},$	(26)
c^1_x	$C^1_\phi \xi,$	c^2_x	$C^2_\phi (1 - \xi),$	(27)
c^1_s	$C^1_\phi \phi_s,$	c^2_s	$- C^2_\phi \phi_s,$	(28)

all the sharing rule parameters are exactly identified because there exists a one-to-one correspondence ensuring that all the structural parameters are a function of reduced form parameters alone

$$\phi_{p_1} = -\frac{c^2_{p_1}}{C^2_\phi} = -(1-\xi)\frac{c^2_{p_1}}{c^2_x}, \tag{29}$$

$$\phi_{p_2} = \frac{c^1_{p_2}}{C^1_\phi} = \xi \frac{c^1_{p_2}}{c^1_x}, \tag{30}$$

$$\phi_s = \frac{c^1_s}{C^1_\phi} = -\frac{c^2_s}{C^2_\phi} = \frac{c^1_s}{c^1_x}\xi = -\frac{c^2_s}{c^2_x}(1-\xi), \tag{31}$$

and similarly for the income and price parameters of the individual demand functions

$$C^1_{p_1} = c^1_{p_1} + \frac{1-\xi}{\xi} c^2_{p_1} \frac{c^1_x}{c^2_x}, \tag{32}$$

$$C^2_{p_2} = c^2_{p_2} + \frac{\xi}{1-\xi} c^1_{p_2} \frac{c^2_x}{c^1_x}, \tag{33}$$

$$C^1_\phi = \frac{c^1_x}{\xi}, \tag{34}$$

$$C^2_\phi = \frac{c^2_x}{1-\xi}. \tag{35}$$

□

The restrictions on the right-hand side of conditions (29), (30), and (31) can be used to recover the structure only from the estimates of the reduced form parameters. For this reason, the restrictions hold globally. No functional form assumptions are necessary to identify the sharing rule. Note also that the knowledge of $\phi_x = \xi \in (0,1)$ enables the income proportionality condition described in Proposition 3 to be rewritten in a form that can be imposed on the model estimation because it involves only reduced form parameters

$$\frac{c_{s_1}^1}{c_{s_1}^2} = \frac{c_{s_r}^1}{c_{s_r}^2} = -\frac{c_x^1}{c_x^2}\frac{(1-\xi)}{\xi}, \quad \forall r = 2, ..., R. \tag{36}$$

The ratio of the reduced form parameters associated with the distribution factors must be proportional to the negative ratio of the reduced form parameters associated with income scaled by the ratio between member 2's and member 1's share. Now distribution factors are directly related to income by a restriction that can be enforced in the reduced form estimation. The structure, which embeds the reduced form restrictions by construction, is also estimable. Note that given the relationships in (36), the decentralized Slutsky equations (32) and (33) can be rewritten as

$$C_{p_i}^i = c_{p_i}^i + c_{p_i}^j \frac{c_{s_r}^i}{c_{s_r}^j}, \quad \text{with } i \neq j. \tag{37}$$

Remark 1. If resources are shared equally between the two household's members assuming that $\xi = \frac{1}{2}$, then the income proportionality condition does not depend on the share parameter ξ

$$\frac{c_{s_r}^1}{c_{s_r}^2} = -\frac{c_x^1}{c_x^2}, \quad \forall r. \tag{38}$$

In this special case, the collective household model reduces to the unitary model where it is assumed that resources are shared equally among the household's members.

3 Parametric Specification of the Collective Consumption Model

We present a collective model where the sharing rule is defined by an income modifying function scaling total expenditure. Although this structural form is often used in empirical applications (Browning *et al.* 1994, Lewbel and Pendakur 2008, Lize and Seitz 2011), we argue that scaling total expenditure at the household level captures demographic differences across households but this specification does not identify the decision process of allocating resources between household members. We show that the specification of the structural sharing rule should be defined using individuals' total expenditure because total household expenditures provide no information about the distribution of resources within the household.

We choose a double-log functional form for the collective demand equations (3) leaving the functional form of the sharing rule unspecified. The specification of the demand equation does not include cross-price effects to facilitate comparisons with the labour supply application developed by Chiappori, Fortin, and Lacroix (2002).

In order to recover the underlying structure of the collective model, we use the information available for the private consumption of clothing for adults and children. In line with Rothbarth's observation (Rothbarth 1941, 1943), the presence of a child induces a reallocation of consumption within the household. Assuming that income remains unchanged before and after the birth of a child, the associated new demand for children's goods is met by reducing the expenses for adult goods. At the same level of total expenditure, larger families spend less on adult goods. As shown in Perali (2003:183, Figure 6.3), food and clothing for children are positively associated with the presence of a child. Greater expenditure allocated on children's goods is matched by a reduction in the budget share devoted to clothing for adults and, to a lesser extent, all other goods. These income reallocation effects,[8] present also in our data, are what our empirical identification strategy intends to capture.

The reduced and structural equations of the exclusive good $c^i(p_1, p_2, x, s)$ are specified according to the following differentiable equations,[9] where parameters in greek style are associated with the structural form while the latin style notation is associated with the reduced form parameters,

Reduced Form | **Structural Form**

Member 1

$$\ln c^1 = a_0 + \sum_{j=1}^{D} a_{d_j} \ln d_{1j} + a_{p_1} \ln p_1 + a_{p_2} \ln p_2 + a_x \ln x + \sum_{r=1}^{R} a_{s_r} \ln s_r, \quad (1.R)$$

$$\ln C^1 = \alpha_0 + \sum_{j=1}^{D} \alpha_{d_j} \ln d_{1j} + \alpha_{p_1} \ln p_1 + \alpha_x \phi(\ln x, \ln p_1, \ln p_2, \ln s), \quad (1.S)$$

Member 2

$$\ln c^2 = b_0 + \sum_{j=1}^{D} b_{d_j} \ln d_{2j} + b_{p_1} \ln p_1 + b_{p_2} \ln p_2 + b_x \ln x + \sum_{r=1}^{R} b_{s_r} \ln s_r, \quad (2.R)$$

$$\ln C^2 = \beta_0 + \sum_{j=1}^{D} \beta_{d_j} \ln d_{2j} + \beta_{p_2} \ln p_2 + \beta_x (\ln x - \phi(\ln x, \ln p_1, \ln p_2, \ln s)), \quad (2.S)$$

where d represents a set of exogenous socio-demographic characteristics capturing observable heterogeneity, p_1 and p_2 are the prices of the two exclusive goods, x is total family expenditure, s_r is the r-th element of the $r = 1, ..., R$ set of distribution factors. As defined in Section 2, ϕ is the sharing rule in levels not in share and $\phi_2 = \ln x - \phi$ is the amount of family resources allocated to member 2 given by the difference between overall family resources and the sharing rule. The parametric specification is a double-log.[10] We choose a log-linear specification for the unrestricted equations (1.R) and (2.R). There are no interaction or quadratic terms because our intention is to have a parsimonious specification associated with the minimum amount of information sufficient to uniquely recover the structural equations (1.S)-(2.S) and ensure a global identification of the partial effects.

The objective is identification of the structural demand parameters $(\alpha_{p_1}, \alpha_x, \beta_{p_2}, \beta_x)$

[8] Clothing for children is also a public good when the same clothes are used for more than one child. The sharing of children's clothes mitigates the income reallocation effect and can be modeled using a Barten-like price substitution effect as in Dunbar, Lewbel, and Pendakur (2012). The modeling of the public nature of children's goods, which is fully compatible with the present approach, would require the estimation of both a price and an income scaling function that, while not adding estimation difficulties, may distract attention from the core identification issues discussed in the present work. For this reason, the empirical application of our work is limited to families with only one child, making public or joint consumption effects less relevant.

[9] Note that Chiappori, Fortin, and Lacroix (2002) use a semi-log specification for the spouses' labour equations.

[10] Note that the sharing rule maintains the same interpretation if it is linear in the logarithm $\phi_1 = \phi_x \ln x$.

and of the unobservable function ϕ of equations (1.S) and (2.S). We use the same technique described in Proposition 1. The identifying information set comprises two exclusive goods and R distribution factors as in the initial theoretical section.

Following Proposition 1, we obtain the following Jacobian matrices for member 1

Reduced Form	Structural Form	
$\ln c^1_{d_i} = a_{d_i}$	$\ln C^1_{d_i} = \alpha_{d_i}, \quad i = 1, ..., D,$	(39)
$\ln c^1_{p_1} = a_{p_1}$	$\ln C^1_{p_1} = \alpha_{p_1} + \alpha_x \phi_{p_1},$	(40)
$\ln c^1_{p_2} = a_{p_2}$	$\ln C^1_{p_2} = \alpha_x \phi_{p_2},$	(41)
$\ln c^1_x = a_x$	$\ln C^1_x = \alpha_x \phi_x,$	(42)
$\ln c^1_{s_r} = a_{s_r}$	$\ln C^1_{s_r} = \alpha_x \phi_{s_r}, \quad r = 1, ..., R,$	(43)

and, similarly, for member 2

Reduced Form	Structural Form	
$\ln c^2_{d_i} = b_{d_i}$	$\ln C^2_{d_i} = \beta_{d_i}, \quad i = 1, ..., D,$	(44)
$\ln c^2_{p_1} = b_{p_1}$	$\ln C^2_{p_1} = -\beta_x \phi_{p_1},$	(45)
$\ln c^2_{p_2} = b_{p_2}$	$\ln C^2_{p_2} = \beta_{p_2} - \beta_x \phi_{p_2},$	(46)
$\ln c^2_x = b_x$	$\ln C^2_x = \beta_x (1 - \phi_x),$	(47)
$\ln c^2_{s_r} = b_{s_r}$	$\ln C^2_{s_r} = -\beta_x \phi_{s_r}, \quad r = 1, ..., R.$	(48)

Note that all the sharing rule parameters are a function of α and β as we would expect from Proposition 4. Recalling the restriction $\phi_x = \xi$ for $\xi \in (0, 1)$, the income and price parameters are identified as follows

$$\alpha_x = \frac{1}{\xi} a_x, \tag{49}$$

$$\beta_x = \frac{1}{(1-\xi)} b_x, \tag{50}$$

and

$$\alpha_{p_1} = a_{p_1} + \frac{1-\xi}{\xi} b_{p_1} \frac{a_x}{b_x}, \tag{51}$$

$$\beta_{p_2} = b_{p_2} + \frac{\xi}{1-\xi} a_{p_2} \frac{b_x}{a_x}. \tag{52}$$

The identifying conditions of the parameters of the sharing rule simplify to

$$\phi_{s_r} = -\xi \frac{a_{s_r}}{a_x} = -(1-\xi) \frac{b_{s_r}}{b_x}, \tag{53}$$

$$\phi_{p_1} = -(1-\xi)\frac{b_{p_1}}{b_x}, \tag{54}$$

$$\phi_{p_2} = \xi\frac{a_{p_2}}{a_x}. \tag{55}$$

The simplicity of this set of restrictions is evidence of the informational value of knowing ϕ_x. The income proportionality condition then becomes

$$-\frac{a_{s_r}}{b_{s_r}} = \frac{a_x}{b_x} = \frac{(1-\xi)}{\xi}, \quad \forall r. \tag{56}$$

Solving the system of differential equations (53) to (55) of the partial effects $(\phi_{p_1}, \phi_{p_2}, \phi_{s_r})$, we obtain a logarithmic specification of the sharing rule function analogous to the one obtained by Chiappori, Fortin, and Lacroix (2002) in equation (10)

$$\phi_1 = \xi \ln x + (\phi_{p_1}\ln p_1 + \phi_{p_2}\ln p_2 + \phi_{s_1}\ln s_1 + \phi_{s_2}\ln s_2), \tag{57}$$

which can be aggregated as

$$\phi_1 = \ln x_1 + \ln m(p_1, p_2, s), \tag{58}$$

and

$$\phi_2 = \ln x - \phi_1 = \ln x_2 - \ln m(p_1, p_2, s), \tag{59}$$

where $\ln x_1 = \xi \ln x$, $\ln x_2 = (1-\xi)\ln x$. The value of the scaling function $m(p_1, p_2, s)$ must be $0 < \ln m(p_1, p_2, s) < \ln \frac{x}{x_i}$.[11] This functional assumption shows that the sharing rule specified as in (57) is an income scaling function in the style described by Lewbel (1985, Theorem 8) and in the tradition of the demographic functions modifying either prices or income (Pollak and Wales 1981, Lewbel 1985, Bollino, Perali, and Rossi 2000, Perali 2003). The sharing rule and demographic functions have in common the fact that both are unobservable functions.

In line with the interpretation of ξ as a resource share, also adopted by Dunbar, Lewbel and Pendakur (2012), it is appropriate to redefine the sharing function as the product of the best approximation for individual income and an income scaling function that includes all the exogenous variables in the model. It is worth remarking that the scaling function $\ln m(p_1, p_2, s)$, which must be different from zero, depends on individual prices p_1, p_2 and distribution factors s only, not on total expenditure.[12]

[11]For the scaling function $m(p_1, p_2, s)$ to be bounded between 0 and 1, thus assuring that members do not receive either negative expenditure or an income share that exceeds the household resource endowments, in general a logistic functional form is adopted (Browning *et al.* 1994, Lise and Seitz 2011, Browning, Chiappori, and Lewbel 2010, Lewbel and Pendakur 2008, and Donni and Bargain 2009). In the present work, the scaling function $m(p_1, p_2, s)$ is estimated without imposing this functional form.

[12]This separability property is analogous to the properties of independence of income of the sharing rule

Note that if $\xi = 1$, which would assign all household resources to member 1, the sharing rule specification becomes

$$\phi_1(x, p_1, p_2, s) = \ln(xm(p_1, p_2, s)). \tag{60}$$

This specification has been adopted in previous papers (Browning *et al.* 1994, Lise and Seitz 2011, Browning, Chiappori, and Lechene 2006, and Chiappori, Donni, and Komunjer 2012) either in consumption or labour supply frameworks. However, our analysis shows that rather than learning something about the rule governing the intra-household allocation of resources, in fact it explains only variations in income across households. The assumption $\xi = 1$ is therefore observationally equivalent to the unitary model.

Given the log-linear specification of the sharing function as in (57) and that the resource share ξ is assumed to be known we obtain the following directly estimable structural specification of the demand function and the sharing rule is[13]

$$\ln C^1 = \alpha_0 + \sum_{j=1}^{D} \alpha_{d_j} \ln d_{1j} + \alpha_{p_1} \ln p_1 + \alpha_x \ln x_1 + \alpha_x (\phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2), \tag{61}$$

$$\ln C^2 = \beta_0 + \sum_{j=1}^{D} \beta_{d_j} \ln d_{2j} + \beta_{p_2} \ln p_2 + \beta_x \ln x_2 - \beta_x (\phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2). \tag{62}$$

Individual shares of total household expenditure $\xi_i \in (0,1)$, with $\xi_1 = \xi$ and $\xi_2 = 1 - \xi$, are constructed using all the observable information about private consumption in the following way $\xi_i = \frac{x_i}{x} = \frac{0.5 p_o c_o + p_i C^i}{x}$ for $i = 1, 2$ and $\xi_1 + \xi_2 = 1$. We allocate the expenditure of non-assignable goods to each household member in equal proportion[14] and add the assignable expenditure to each individual. The modifying function $m(p_1, p_2, s)$ can be interpreted as a correction factor. Individual incomes obtained by multiplying the resource share ξ_i by $\ln x$ vary across households in relation to the levels of non-assignable expenditure and the levels of individual consumption. The latter variability explains the variation in the intra-household resource allocation across households.

Proposition 5 (Neutrality of non-assignable expenditure). *The assumption of fair division of the non-assignable goods between household members is neutral because it does not affect the relative magnitude of the parameters of the scaling function $m(p_1, p_2, s)$.*

Proof. See Appendix B. □

used by Dunbar, Lewbel, and Pendakur (2012) and tested by Menon, Pendakur, and Perali (2012).

[13] If the resource share ξ were not known, the specification of the income term $\alpha_x \ln \phi_1$ would be $\alpha_x \ln \phi_1(p_1, p_2, x, s) = \alpha_x (\phi_x \ln x + \phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2)$ making the source of the parametric identification problem apparent. It would in fact be possible to identify the parameters' product $A_x = \alpha_x \phi_x$ but neither α_x nor ϕ_x, thus precluding the direct estimation of the structural model.

[14] This is like assuming that in absence of economies of scale the non-assignable good is a purely public good with one half the market price associated with a Barten technology with coefficient equal to one half (Browning, Chiappori and Lewbel 2003).

The assumption of a division in equal shares is innocent because both the parameters associated with the approximation of individual incomes and the parameters of the scaling function capture the difference between the observed individual allocations as required for the identification of the sharing rule that properly accounts for the exclusive information. As it is natural to anticipate, if the assignable information is distributed uniformly across household members, the information about private consumption is not informative about the intra-household distribution of resources. To obtain identification of the structure of the collective model the individual demand equations of the assignable or exclusive goods must significantly vary across members of the same household.

4 Data

We work in the context of a consumption model based on the observation of the private consumption of two exclusive goods, adult and children clothing. The household data used in the empirical application are drawn from the 2007 Italian household budget survey (Italian Statistics 2007). For the purposes of our study, we select families with two parents and a young dependent child. The sample size is thus reduces to 1,795 households.

In the Italian household survey, expenditures of highly detailed goods are often censored in a non negligible size. The realization of zero expenditures can be explained by the short duration of the recall period of the survey design or by a stringent budget constraint (Pudney 1989).[15] Censored expenditure categories, including clothing, have been adjusted with the specification of a bivariate model as suggested in Blundell and Meghir (1987). We compute total expenditure from the imputed expenditures of the censored goods and adjust for remaining measurement errors and possible simultaneity bias by instrumenting total expenditure using, among other instruments, total household income as shown in the next section.

A limitation of using Italian household budgets is the absence of information on quantities consumed by each household necessary for the computation of household-specific unit values. We approximate unit values with a method suggested by Lewbel (1985) and empirically developed by Atella, Menon, and Perali (2003), Hoderlein and Mihaleva (2008), and Menon, Perali, and Tommasi (2017). This technique captures the spatial and quality variability typical of unit values as it can be deduced from household socioeconomic characteristics. We then add this variability to the price indices published monthly at the provincial level and construct nominal unit values from prices at 2007 levels.

Table 1 shows the definition of the variables used in the empirical analysis and the descriptive statistics for our sample. The dependent variables are the log of clothing expenditures of parents and children divided by respective prices. Distribution factors are the age and education ratio of the parents. In the estimation we control for geographic location (North, Center and South Italy), seasonality effects using a dummy equal to one if the family has been interviewed in December, the presence of double-earner families and

[15]The choice for the recall period for the expenditure of semi-durable goods, such as clothing, is one of the most important problems encountered by researchers when designing an expenditure survey (Grosh and Glewwe 2000). An excessively long recall period can lead to underestimation of the actual expenditure.

for parents' and child's age expressed in classes. We also introduced a dummy if the child is in the 15-17 age group to account for the fact that more grown-up children can participate, to some degree, in the allocation of family resources (Dauphin *et al.* 2011). Note that some demographic attributes, such as age, level of education and working condition, are individual-specific, while other characteristics, such as location and working condition of the couple, are the same across household members.

5 Empirical Strategy and Results

The collective consumption model can be empirically implemented using a) a two stage indirect estimation where the first stage consists in the estimation of the reduced form followed by the recovery of the structural form as in Chiappori, Fortin, and Lacroix (2002), or b) a direct estimation of the structural form using maximum likelihood. Because of the correspondence theorem described in Proposition 4, the indirect and direct method lead to the same estimates. By implementing both methods, we control our identification experiment ensuring that the theoretical results are matched in the empirical application. In our experimental design, we also measure the accuracy of the parameters of the sharing rule in capturing the inter-household variation in assignable expenditure by comparing the estimation with $\xi = 1$, which presumes a uniform distribution of resources within the household as in the unitary model, and ξ is the resource share. The following section is devoted to the description of data.

We consider couples with a young dependent child. In these families, the presence of a child generates a budget reallocation effect from the parents to the child. Economies of scale in clothing use can reasonably be ruled out. Our main interest is to estimate the sharing rule within a collective consumption model whose specification resembles as closely as possible the collective labour supply model implemented in Chiappori, Fortin, and Lacroix (2002) in order to make the set of theory restrictions as comparable as possible. Unlike Chiappori, Fortin, and Lacroix (2002), we use the finding shown in Proposition 1 that the parameter associated with the income term ϕ_x cannot be identified and, as a best estimate of the information content of this parameter, we construct a resource share from the available data on private clothing consumption, in order to implement the income proportionality property and the correspondence described in Proposition 2. The collective model is specified in quantity space in an *ad hoc* fashion in the sense that it is not directly derived from a known utility or expenditure function. This functional choice is similar to the one adopted in the seminal work of Browning *et al.* (1994) on collective consumption with the difference that we do not add a quadratic income term. In general, the simplicity of the demand specification allows us to concentrate on the identification issues associated with estimation.[16]

The indirect and direct methods to estimate the system of collective demand equations (61)-(62) for clothing for adults and children in Italy are described below.

[16] If the main interest were behavioural and applied welfare, then a theoretically plausible flexible collective demand system would be required (Menon, Perali, and Piccoli 2017).

Empirical Strategy

Indirect Estimation of the Collective Structure

Indirect estimation uses a Minimum Chi Square (MCS) estimator or minimum distance method, implemented in two stages as done in Chiappori, Fortin, and Lacroix (2002).

Stage I - Estimation of the reduced form

$$\ln c^1 = a_0 + \sum_{j=1}^{D} a_{d_j} \ln d_{1j} + a_{p_1} \ln p_1 + a_{p_2} \ln p_2 + a_x \ln x_1 + \sum_{r=1}^{2} a_{s_r} \ln s_r + \epsilon_1, \tag{63}$$

$$\ln c^2 = b_0 + \sum_{j=1}^{D} b_{d_j} \ln d_{2j} + b_{p_1} \ln p_1 + b_{p_2} \ln p_2 + b_x \ln x_2 + \sum_{r=1}^{2} b_{s_r} \ln s_r + \epsilon_2, \tag{64}$$

where we have introduced additive unobserved individual heterogeneity ϵ_i which is assumed to be independent and identically distributed. The notation $i = 1$ represents the parents and $i = 2$ the child. The system is estimated with the income proportionality restriction imposed. The parameters of interest $\{\phi_{p_1}, \phi_{p_2}, \phi_s\}$ and $\{\alpha_x, \alpha_{p_1}, \beta_x, \beta_{p_2}\}$ are recovered in the second stage.

Stage II - Recovery of the collective structure The collective structure can be fully recovered by imposing the cross-equation restrictions described in equations (49) to (56) the use of the MCS estimator. Since the estimates of the unrestricted $\{a, b\}$ parameter vectors are consistent, the MCS method yields consistent asymptotically normal efficient estimates (Berkson 1949 and 1980, Ferguson 1958, Rothenberg 1973). The minimum distance technique is used to impose collective theory restrictions only. In the spirit of Malinvaud (1970), minimum distance is utilized to recover the demand structure and the sharing rule parameters.

The vector of restricted parameters is denoted by $\gamma = \{\alpha \mid \beta\}$, where the sub-vectors α and β are vertically concatenated. The collective theory restrictions can be summarized via the mapping $\gamma = F(g)$ relating the restricted parameters $g = \{a \mid b\}$ to the unrestricted structural parameters. The function $F(g)$ incorporates the prior information provided by the collective demand theory as well as the restrictions that identify the sharing rule parameters. Let the Jacobian of this function be denoted by $G\{g\} = \partial F(g) / \partial g$. The Jacobian G has full column rank. The MCS method chooses the estimator g that minimizes the following quadratic form (Rothenberg 1973)

$$\left\{ \hat{a, b} \right\} = \text{argmin}_{\{a,b\}} \left\{ [\gamma - F(g)]' \Sigma_g^{-1} [\gamma - F(g)] \right\}, \tag{65}$$

where Σ_g is a consistent estimate of the variance-covariance matrix of g obtained from the jackknife technique. Note that the computational algorithm that estimates the unrestricted parameters g along with its covariance matrix Σ_g in the first stage, and then recovers the structure in the second stage, is an indirect feasible generalized least square procedure. To the extent that it generates consistent and asymptotically efficient estimates, it provides a computationally convenient alternative to the full-information maximum likelihood estimation method. This approach has the troublesome feature that the partial effects of the sharing rule are specific to each structural specification of the collective model and corresponding reduced functional form, making the practical derivation of the partial effects cumbersome in demand systems with many equations.

Direct Estimation of the Collective Structure

We directly estimate the structural demand equations using Full Information Maximum Likelihood

$$\ln C^1 = \alpha_0 + \sum_{j=1}^{D} \alpha_{d_j} \ln d_{1j} + \alpha_{p_1} \ln p_1 + \alpha_x \left(\ln x_1 + \ln m\left(p_1, p_2, s\right)\right) + \varepsilon_1, \tag{66}$$

$$\ln C^2 = \beta_0 + \sum_{j=1}^{D} \beta_{d_j} \ln d_{2j} + \beta_{p_2} \ln p_2 + \beta_x \left(\ln x_2 - \ln m\left(p_1, p_2, s\right)\right) + \varepsilon_2, \tag{67}$$

where ε_i is the error term assumed to be independent and identically distributed. The restrictions required by the collective household model are implicitly embodied in the functional form. The direct estimation of the scaling income function $m\left(p_1, p_2, s\right)$ is akin to the problem of estimating a regression containing unobservable independent variables (Goldberger 1972). For the sake of comparison, we also estimate the above sharing rule assuming that $\ln x_i = \xi_i \ln x$ with $\xi_i = 0.5$. We do so in order to appreciate the contribution of private consumption of clothing in the construction of the resource share which is at the core of the empirical identification strategy.

The log of total expenditure is likely to be endogenous because it depends on economic behaviour and due to measurement error, either because of infrequency of purchases or because of recall errors. To adjust for the possibility of endogeneity of the log of total expenditure, we apply the control function method (Blundell and Robin 1999). In line with this method, we regress the log of total expenditure against the exogenous explanatory variables and the log of net household income. The residual r of the auxiliary regression is then included into the clothing equations (66)-(67). The log of total expenditure is exogenous, conditional on the additional regressor. Thus, the error term ε_i can be written as the orthogonal decomposition $\varepsilon_i = \nu_i r + \upsilon_i$, where ν_i is a parameter to be estimated and υ_i is an i.i.d error term. Testing if ν_i is significantly different from zero for $i = 1, 2$ corresponds to testing the exogeneity of the log of total expenditure.

Estimation Results

The estimation results for the collective consumption models specified in terms of total household income $\ln x$, which we call model (a),[17] and total individual income $\ln x_i = \xi_i \ln x$, which we call model (b), are given in Table 2. Model (a) embeds the specification of the sharing rule traditionally used in the collective literature, while our model (b) exploits

[17] Because of the income proportionality restriction, model (a), expressed in terms of $\ln x$, means that $\xi_i = 0.5$, as explained in Remark 1.

individual specific information included in the resource share ξ_i. We estimate model (a) and (b) using both the indirect and direct method as illustrated above. We choose not to show the parameter estimates with the imposed income proportionality restriction for either the first or second stage of the indirect method, because the results of the second stage are the same as the parameters obtained from the direct structural estimation. Further, we do not show the parameter estimates of the indirect estimation of the model with only the imposed proportionality restriction because the parameters of the sharing rule are not identified, as explained in Proposition 1.

Most parameters are statistically significant at conventional levels. The parameters ν_i $\forall i$ associated with the control factor r in both models are significantly different from zero showing that the log of total expenditure without correction would be endogenous. Interestingly, even though restricted to be equal to the ratio of distribution factors, the coefficients associated with the log of household income are statistically significantly different from zero. The comparison of the parameters associated with the income term of model (a) and (b) reveals that they differ significantly both statistically and economically. The income effect is much higher for children's clothing, especially in model (b), which estimates Engel effects at the individual level.

The parameters of the control variables, such as region, month of interview, number of labour income recipients, age of the child and presence of an adult-child, are equal in sign and magnitude in model (a) and model (b) because the parameters associated with the control variables enter the structural specification linearly. The parameters associated with the own-prices are statistically significantly different from zero at conventional levels in both specifications. The sign is correct.

The coefficients of the income scaling function $m(p_1, p_2, s)$ have the same sign in model (a) and (b), but differ significantly in magnitude with the exception of the age and education ratio, which are not significantly different from zero in either specification. This is evidence that the estimation expressed in terms of individual expenditures effectively captures inter-household variations in the intra-household Pareto allocation of resources as described by the clothing consumption habit of the adult and child member of each household. Model (a) is in fact describing inter-household variations in total household expenditure and we learn nothing about the allocation of resources within the household. The modifying function $m(p_1, p_2, s)$ in model (a) is simply an income scaling function, not a sharing rule.

Therefore, we comment on the impact on the sharing rule referring only to the parameters of the scaling function presented in model (b). A change in children's clothing prices affect the allocation rule more than a change in adult clothing price. The sign of the price effect directly affecting clothing consumption is the opposite of the price effect exerted indirectly through the sharing rule. As the price of both clothing for adults and children increases, adults retain more resources for themselves. An increase in age ratio, in the age gap between father and mother, is often associated with higher wages due to a higher position of the father in the life cycle, and higher bargaining power leading to more resources available to children, as the sign shows. The effect associated with the education ratio, though not estimated precisely, is positive. This lack of significance rules out verification of whether more educated mothers are more, or less, "altruistic" towards children in the context of our experiment.

Figure 1 shows the sharing rule pattern as total household expenditure rises. The rule governing the intra-household allocation of resources gives more resources to children at low income levels and less at higher levels, though in general the function is constant across income distributions.

6 Conclusions

This study introduces an income proportionality property that contributes to complete the collective theory of the household and is crucial in achieving full identification. The study shows how to obtain an exact correspondence between the structural and reduced form of the collective demand equations by implementing the income proportionality property. This property is fundamental because the effect of distribution factors is directly linked to the resource levels assignable to each individual. Failing to do this would be like trying to estimate the sharing rule without exploiting the necessary information, that is the information on exclusive goods, to implement a collective model.

Because of the incompleteness of the theory, previous works may not have fully identified the rule governing the allocation of resources within the household. This is the only study that exploits the extended results of collective theory and estimates the same sharing rule parameters both using a structural direct and a reduced form indirect approach. This is a clear evidence that full identification is achieved.

To estimate the rule governing the allocation of resources between adults and children, we implement the collective consumption model using Italian household budget data limited to young couples with one child. We perform both an indirect and direct estimation of the structural model to show that there is full correspondence between the structure and the associated reduced form, in line with the set of restrictions derived in the theoretical section. We also compare the sharing rule estimated using our novel approach with the traditional one. We conclude that the traditional model describes inter-household variations in total household expenditure and is not informative about the allocation of resources within the household. We then maintain that our approach opens up the possibility of a direct structural estimation of a collective system of demand equations (Arias *et al.* 2003, Caiumi and Perali 2014, and Menon, Perali, and Piccoli 2017) as easily as estimating a demand system based on a unitary framework. Further, it allows a neat estimation of individual Engel curves resulting from the decomposition of the income term of the Slutsky equation of a collective decentralized program.

It should be stressed that the quality of the estimation improves with the quantity and quality of the information available about the consumption of private goods and with the ability to describe economies of scale derived from the sharing of household public goods by properly modeling household technologies. This is the objective of our future research.

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Appendices

A: Proof of Proposition 1 (Chiappori, Fortin, Lacroix 2002 extended)

This appendix describes the proof of Proposition 1 using the same notation and general line of proof used in Chiappori, Fortin, and Lacroix (2002) to facilitate comparisons. Here we propose an extended proof that differs from the proof described in Chiappori, Fortin, and Lacroix (2002) essentially because we study the identification of all demand parameters $\left\{C^1_{p_1}, C^1_{\phi}, C^2_{p_2}, C^2_{\phi}, \phi_x, \phi_{p_1}, \phi_{p_2}, \phi_s\right\}$ without limiting our attention only to the sharing rule parameters $\{\phi_x, \phi_{p_1}, \phi_{p_2}, \phi_s\}$. Our extended line of proof, which starts from the explicit specification of the Jacobian matrix of both members demands, clearly shows the sources of non identification of the sharing rule parameters.

Proof. Recall first that $c^i(p_1, p_2, x, s) = C^i(p_i, \phi_i(p_1, p_2, x, s))$ for both $i = 1, 2$. This equality drives the equality of the corresponding element of the reduced and structural matrices of first derivatives. Then, the Jacobians of the demand equations of the two members are

Member 1		*Member 2*		
Reduced	*Structural*	*Reduced*	*Structural*	
$c^1_{p_1}$	$C^1_{p_1} + C^1_{\phi}\phi_{p_1}$,	$c^2_{p_1}$	$-C^2_{\phi}\phi_{p_1}$,	(68)
$c^1_{p_2}$	$C^1_{\phi}\phi_{p_2}$,	$c^2_{p_2}$	$C^2_{p_2} - C^2_{\phi}\phi_{p_2}$,	(69)
c^1_x	$C^1_{\phi}\phi_x$,	c^2_x	$C^2_{\phi}(1-\phi_x)$,	(70)
c^1_s	$C^1_{\phi}\phi_s$,	c^2_s	$-C^2_{\phi}\phi_s$,	(71)

where c^i_j represents the derivative with respect to the j-th variable of the left-hand side of equation (3) and C^i_j the derivative of the right-hand side for $i = 1, 2$ and $j = p_1, p_2, x, s$.

Using equations (69), (70), and (71) of member 1, we obtain

$$A = \frac{c^1_{p_2}}{c^1_x} = \frac{\phi_{p_2}}{\phi_x}, \qquad C = \frac{c^1_s}{c^1_x} = \frac{\phi_s}{\phi_x}, \tag{72}$$

for $c^1_x \neq 0$.

From equation (68), (70) and (71) of member 2, we have

$$B = \frac{c^2_{p_1}}{c^2_x} = -\frac{\phi_{p_1}}{(1-\phi_x)}, \qquad D = \frac{c^2_s}{c^2_x} = -\frac{\phi_s}{(1-\phi_x)}, \tag{73}$$

for $c^2_x \neq 0$.

From the terms C and D we derive the partial effects ϕ_x and ϕ_s

$$\phi_x = \frac{c_x^1 c_s^2}{c_x^1 c_s^2 - c_s^1 c_x^2} = \frac{D}{D-C}, \tag{74}$$

$$\phi_s = \frac{c_s^1 c_s^2}{c_x^1 c_s^2 - c_s^1 c_x^2} = \frac{CD}{D-C}, \tag{75}$$

assuming that $D \neq C$.

Using the definitions of A, D and C, we recover the partial effect ϕ_{p_2}

$$\phi_{p_2} = \frac{c_{p_2}^1 c_s^2}{c_x^1 c_s^2 - c_s^1 c_x^2} = \frac{AD}{D-C}, \tag{76}$$

and from B, C and D, we get the partial effect ϕ_{p_1}

$$\phi_{p_1} = \frac{c_{p_1}^2 c_s^1}{c_x^1 c_s^2 - c_s^1 c_x^2} = \frac{BC}{D-C}, \tag{77}$$

for any point such that $D \neq C$ where the unobservable parameters of the sharing rule are all expressed in terms of observable reduced form information.

The relationships (74)-(77) are considered sufficient evidence for identification by Chiappori, Fortin, and Lacroix (2002) because each structural parameter of the sharing rule is expressed as a function of only reduced form parameters. However, we show below that the sharing rule parameters can still be expressed also as a function of structural parameters thus showing the incompleteness of Chiappori, Fortin, and Lacroix (2002) proof.

Consider that

$$\phi_x = \frac{c_x^1 c_s^2}{c_x^1 c_s^2 - c_s^1 c_x^2} = \frac{D}{D-C} = \frac{c_x^1}{C_\phi^1} = 1 - \frac{c_x^2}{C_\phi^2}, \tag{78}$$

and

$$\phi_s = \frac{c_s^1 c_s^2}{c_x^1 c_s^2 - c_s^1 c_x^2} = \frac{CD}{D-C} = \frac{c_s^1}{C_\phi^1} = -\frac{c_s^2}{C_\phi^2}, \tag{79}$$

implying also that

$$\phi_x = \frac{c_x^1}{c_s^1}\phi_s = -\frac{C_\phi^2 - c_x^2}{c_s^2}\phi_s \tag{80}$$

Further, for the demand parameters not included in the sharing function that were not explicitly analyzed in Chiappori, Fortin, and Lacroix (2002), we have

$$C_{p_1}^1 = c_{p_1}^1 - c_{p_1}^2 \frac{c_s^1}{c_s^2} = c_{p_1}^1 - C_\phi^1 \phi_{p_1}, \tag{81}$$

$$C_{p_2}^2 = c_{p_2}^2 - c_{p_2}^1 \frac{c_s^2}{c_s^1} = c_{p_2}^2 + C_\phi^2 \phi_{p_2}. \tag{82}$$

From the Jacobian matrix above, we observe that $C_\phi^1 = c_x^1/\phi_x$ and $C_\phi^2 = \frac{c_x^2}{(1-\phi_x)}$. Using the relations that $\phi_x = \frac{c_x^1}{c_s^1}\phi_s$ and $\phi_s = -\frac{c_s^2}{C_\phi^2}$, we establish the following equalities

$$C_\phi^1 = c_x^1 - c_x^2 \frac{c_s^1}{c_s^2} = \frac{c_x^1}{\phi_x} = \frac{c_x^1}{\frac{c_x^1}{c_s^1}\phi_s} = -C_\phi^2 \frac{c_s^1}{c_s^2}, \tag{83}$$

and

$$C_\phi^1 = \frac{c_x^1}{c_x^2}\frac{(1-\phi_x)}{\phi_x} C_\phi^2. \tag{84}$$

□

Combining (83) and (84), we obtain the fundamental income proportionality relationship described in Proposition 3

$$\frac{C_\phi^1}{C_\phi^2} = \frac{c_x^1}{c_x^2}\frac{(1-\phi_x)}{\phi_x} = -\frac{c_s^1}{c_s^2}. \tag{85}$$

This set of expressions implies that identification is not achieved because it is not possible to establish a one-to-one correspondence between the structural and reduced form parameters. All parameters of the sharing rule can be written as a function of other sharing rule parameters.

All unobservable sharing rule parameters $(\phi_x, \phi_{p_1}, \phi_{p_2}, \phi_x)$ can be expressed in terms of both reduced form parameters and a mixed expression with at least one structural sharing rule parameter. In general, the above equations describe a contradictory logical statement. For example, the parameter C_ϕ^1 can be expressed both as a function of reduced form parameters as apparent in the first term and as a function of at least one structural parameter as shown in all other terms of(83) . The first term of the statement in (83) shows that identification can be achieved, while the other relationships contradict it. The general statement is therefore false. These conditions are critical because the parameters C_ϕ^1 , and similarly C_ϕ^2 , multiply all other parameters of the sharing rule. Note that the existing collective literature only presents the first part of the statement, because the results refer to the sharing rule parameters without considering the interaction with the parameters C_ϕ^1 , C_ϕ^2 , $C_{p_1}^1$, $C_{p_2}^2$ external to the sharing rule.

B: Neutrality of Non-Assignable Expenditure

We show that the assumption of a fair division by one half of non-assignable goods, which guarantee that each household member has equal access to the good that is not assignable such as food, is an innocent assumption as stated in Proposition 5. We do so by describing how non-assignable and exclusive consumption can be separated so that we can learn how resources are distributed within the households from exclusive consumption alone.

Proof. Consider a two-person household with $i = 1, 2$. Individual expenditure x_i can be decomposed into two components: an assignable and observable expenditure x_i^a and a non-assignable individual expenditure x_i^{no}

$$x_i = x_i^a + x_i^{no}. \tag{86}$$

Let us define x_i^{no} as an unknown proportion of the non-assignable household expenditure $x^{no} = \left(x - \sum_{i=1,2} x_i^a\right)$

$$x_i^{no} = \lambda_i x^{no} \text{ with } 0 \leq \lambda_i \leq 1, \tag{87}$$

and individual expenditure can then be rewritten as

$$\begin{aligned} x_i &= x_i^a + x_i^{no} = x_i^a + \lambda_i \left(x - \sum_{i=1,2} x_i^a \right) = \\ &= x_i^a + \lambda_i x - \lambda_i x_1^a - \lambda_i x_2^a. \end{aligned} \tag{88}$$

Assuming a fair division rule $\lambda_1 = \lambda_2 = \frac{1}{2}$, in the case of a two-person household we can rewrite the above expression for $i = 1$ as

$$x_1 = x_1^a + \frac{1}{2}x - \frac{1}{2}x_1^a - \frac{1}{2}x_2^a = \frac{1}{2}x + \frac{1}{2}\left(x_1^a - x_2^a\right). \tag{89}$$

Let us now recall the structural equation for clothing consumption of member i without demographic terms

$$\ln C^i = \alpha_0 + \alpha_x \left[\ln x_i + \ln m^\star \left(p_1, p_2, s\right)\right],$$

with

$$\ln x_i = \frac{x_i^a + \lambda_i x^{no}}{x} \ln x = w_i \ln x,$$

where $x^{no} = x - x_1^a - x_2^a$, and w_i measures the total household expenditure share allocated to member i. Then for $\lambda_1 = \lambda_2 = \frac{1}{2}$

$$\begin{aligned} \ln C^i &= \alpha_0^i + \alpha_x^i \left(\frac{x_1^a + 0.5\left(x - x_1^a - x_2^a\right)}{x} \right) \ln x + \alpha_x^i \ln m\left(p_1, p_2, s\right) = \\ &= \alpha_0^i + \alpha_x^i \left(\frac{0.5\left(x + x_1^a - x_2^a\right)}{x} \right) \ln x + \alpha_x^i \ln m\left(p_1, p_2, s\right) = \\ &= \alpha_0^i + \alpha_x^i \left(0.5 + \frac{0.5\left(x_1^a - x_2^a\right)}{x} \right) \ln x + \alpha_x^i \ln m\left(p_1, p_2, s\right) = \\ &= \alpha_0^i + \alpha_x^i 0.5 \ln x + \alpha_x^i \left(0.5\left(x_1^a - x_2^a\right)\right) \frac{\ln x}{x} + \alpha_x^i \ln m\left(p_1, p_2, s\right), \end{aligned}$$

where total household income and the term that includes individual specific expenditures are separate and the assumption of a fair division of total household income $\lambda_i = \frac{1}{2}$ is neutral because it does not affect the relative magnitude of the parameters of the scaling function. On the other hand, if the second term describing intra-household allocations were close to zero, then the scaling function would describe only inter-household rather than intra-household allocation differences. □

The assumption of a fair division is innocent because both α_x and the parameters of the scaling function do capture the difference between the observed individual allocations $(x_1^a - x_2^a)$ as is required for the identification of the sharing rule that properly accounts for the exclusive information. Note further that the fair division of income of the first term is corrected by the scaling function $m(p_1, p_2, s)$.

C: Sample Statistics and Estimation Results

Table 1: Descriptive Statistics and Variable Definition (1,795 Households)

Variable	Definition	Mean	Std. Dev.
$\ln q_1$	Log of parent clothing demand	5.752	0.492
$\ln q_2$	Log of child clothing demand	5.941	0.716
$\ln p_1$	Log of parent clothing price	−0.831	0.773
$\ln p_2$	Log of child clothing price	−1.708	1.019
$\ln x_1$	Log of parent expenditure	5.016	0.326
$\ln x_2$	Log of child expenditure	2.512	0.157
$\ln x$	Log of total household expenditure	7.527	0.479
North	= 1 if family located in Northwest regions	0.257	
South	= 1 if family located in South or Islands regions	0.337	
December	= 1 if family interviewed in December	0.087	
Double Earner	= 1 if double-earner family	0.588	
Parent age	Parents' age in classes	7.505	1.427
Child age	Child's age in classes	1.684	0.706
Child 1517	= 1 if dependent child 15-17 years old	0.141	
Age ratio	Age ratio: age wife/(age wife + age husband)	0.478	0.034
Educ. ratio	Education ratio: educ. wife/(educ. wife + educ. husband)	0.488	0.084

Note: The Italian National Statistical Institute aggregates the variable "age" in fifteen classes: the child age classes are 1(=0-5 years), 2(=6-14 years) and 3(=15-17 years); the age classes of parents go from 4(=18-24 years) to 15(=75 years or more).

Table 2: Estimation Results of the Collective Consumption Models

	Model (a) $\xi = 0.5$		Model (b) $\xi \in (0,1)$	
	Adults	Child	Adults	Child
Intercept	1.641***	−0.044	1.630***	−0.071
	0.356	*0.149*	*0.378*	*0.169*
North	−0.035*	0.044**	−0.035*	0.044**
	0.016	*0.015*	*0.016*	*0.015*
South	0.051***	0.089***	0.051***	0.089***
	0.016	*0.015*	*0.015*	*0.015*
December	−0.006	0.125***	−0.006	0.125***
	0.023	*0.020*	*0.019*	*0.019*
Double Earner	0.004	0.034**	0.003	0.033**
	0.023	*0.019*	*0.018*	*0.016*
Age	−0.032***	-0.004	−0.033***	−0.004
	0.010	*0.003*	*0.010*	*0.003*
Child 1517	−0.071**	−0.053**	−0.071**	−0.053**
	0.023	*0.018*	*0.023*	*0.017*
$\ln p_i$	−1.101***	−0.648***	−1.074***	−0.502***
	0.047	*0.040*	*0.033*	*0.066*
$\ln x$ - model (a)	0.920***	1.224***		
	0.095	*0.032*		
$\ln x_i$ - model (b)			0.693***	1.845***
			0.076	*0.080*
$\hat{\upsilon}$	−0.121***	0.187***	−0.122**	0.185***
	0.035	*0.050*	*0.042*	*0.052*
ϕ_{p_1}	0.059**		0.039**	
	0.024		*0.015*	
ϕ_{p_2}	0.120***		0.159***	
	0.022		*0.030*	
ϕ_{s_1} (age ratio)	−0.053		−0.061	
	0.061		*0.053*	
ϕ_{s_2} (educ. ratio)	0.029		0.027	
	0.028		*0.021*	

Note: *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Standard errors are in italics.

Figure 1: Child's Sharing Rule

SRchild.pdf