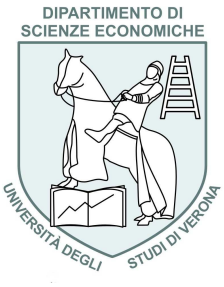

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Production of commodities by means of labour. A theory of international relations.

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Production of commodities

by means of labour –

A theory of international relations

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Abstract: Since (at least) Ricardo, international trade has been perceived as a positive-sum-gain – any partner involved in the international activity of exchange would be at the end better off, no matter how bad was its previous economic position. The Ricardian principle of comparative advantages (RPCA) allows to explain why this would be so. Though the principle is still perceived as being a non-trivial-truth-for-certain, the paper discusses critically its application by adopting a dynamic multisectoral production model of a pure labour economy. Though abstract, the features of the model seem sufficiently realistic: differentiated sectors, with different technological, demand, and productivity levels; unemployment threats, demand constraints, uneven dynamics of all the above variables. If the unit of analysis is the individual country, rather than the international system as a whole, we show that the RPCA assures only a static "once for all" drop of the level of prices. It may not assure gains from trade in seven out of eight variables examined. Employment, productivity, per capita income and their respective rates of change, plus the rate of inflation, may turn out after trade worst than they would have been without trade. Hence, the RPCA is in many respects inconclusive. An individual country may specialize according to this principle and still arm its economy. Static as well as dynamic losses from trade are just a possibility as they are static and dynamic gains.

JEL Classification: O41, F41, F43.

Keywords: comparative advantages, structural change, international specialization, economic growth.

1. Introduction

Economics, as any other science, looks for truth. Unfortunately, its investigation – as in any other *social* science - faces a dilemma. What is certainly true is most of the time obvious, and what is not obvious is not true for certain. That is why a dose of common sense is still used in practicing economics. At least in practicing real life economics.

Samuelson (1969), once challenged by mathematician Stanislaw Ulam on the point, brought the principle of comparative advantages as an example to argue that in social sciences is not all like this. The Ricardian principle – a theoretical landmark in the last two centuries – shows precisely that in economics, like in physics, it is possible to assert something certainly true, that it is not obvious. It is not obvious that a country has always an advantage to trade, even when its sectoral productivities are all around better (or worse) of the trading partner(s). But, despite being not obvious, the Ricardian principle is highly regarded, in economic theory, for being a truth-for-certain.

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Is it? In this paper I put under scrutiny precisely the Ricardian principle of comparative advantages (RPCA).[1] I do this by relaxing many of the restrictions (i.e. assumptions) that are usually connected with the traditional formal models of international trade based on it. In the foregoing analysis there is no fix technology, no fix prices, no full employment; there may be imbalances of trade, and asymmetries between the number of sectors (and goods) held by the trading partners. There is an uneven demand side, as well as an uneven supply side to consider. Unrealistic concepts, like utility functions, are avoided. There is a static stage, and a dynamic one.

Apart of reconsidering critically the RPCA, the model attempts to fulfil also a *pars construens* which is quite independent from the Ricardian principle. It consists of showing the relations between two or more economies in a typical (realistic) dynamic and multisectoral setting.

The model presented is rather self-contained. In this sense, one may regard it, as a general disequilibrium model. General, because interdependence between endogenous variables are explicitly considered, and results are generated without requiring external claims. Disequilibrium, because every crucial variable in the model is in movement and contributes to the changing structure of the economy. More explicitly, the framework in building the theoretical argument will be that of a pure labour economy, in which labour is the only factor of production. This is just like Ricardo's example and more systematically Pasinetti's (1981, 1993) work in the field of international relations.

2. Theoretical background

Before moving to the essence of the paper, it is worth giving a brief review of the literature that relates with it. The principle of comparative advantages has received much attention, for being counterintuitive, as we said, but also for being conducive with liberalism, the open up of the markets, and the commercial expansion of the developed economies. It was another concept that, apart of being powerful, was fortunate. It appeared at right time (just after the First industrial revolution) in the right place (the centre of what would have been the biggest empire of the XIX century) (Magnusson 2004). It was an open principle, ready to accept a theory on scarcity as the driving force of specialization (the Hecksher-Ohlin-Samuelson theory). It conveyed the idea that markets and trade optimize wealth, a conclusion that fits well within the Neoclassical theoretical framework. [2]

But as any concept of success, it has also been a subject of criticisms. These criticisms began quite early on (Friedrich List's attack is dated 1848), they were reinforced by some empirical evidence (Portugal and England did not specialized as Ricardo foresaw[3], Leontief's Paradox (1953) contradicted H-O-S theory) and it has been rejected sometimes also in its strategic implications (Japan refuted after the Second World War the American suggestion to

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[1] To begin with, I should have put under scrutiny the attribution itself. Ascribing the principle of comparative advantages to the author of the *Principles of Political Economy and Taxation* (1817) is a matter of contention. The "Ricardian principle" – it has been argued, Maneschi (1998) – was formulated clearly and exhaustively by Robert Torrens (1808, 1815) at least ten years before Ricardo. But the new attribution has been counter-questioned too (Ruffin 2005). Being not engaged into a history of ideas, the paper sticks – somewhat uneasy – with the majority role, and to avoid stressing the controversial point too much, it adopts an acronym.

[2] The principle of comparative advantage is at the foundation of the Classical theory (apart Ricardo 1817, see Mill 1848) as well as the Neoclassical theory of international trade, usually labelled with the acronym H-O-S (Hecksher 1919, Ohlin 1933 and Samuelson 1948). Also the "new" trade theory, à la Krugman (1979, 1991), does move from the same premises, though it adds a dynamic flavour with the concept of "increasing returns".

[3] Louca 1997:37.

specialize according to the existent comparative advantages). Here, I shall confine only to the theoretical criticisms that are incidental to the later model.[4]

Pasinetti has dealt with this principle in his two key books on structural change (Pasinetti 1981: Ch.XI and 1993: Ch.IX). The analysis he carried out is not (mathematically) formalized, but the conclusions that he reaches are logically constructed and may be summarized as follows.

a) First, the primary benefits of international trade are not based on the gains from trade, derived from comparative advantages. They are based on the international learning. It is the opportunity that the backwards countries have to learn (in terms of technology and economic organization) from the leaders that constitutes the real gains from trade. In fact, the backwards countries have a more expeditious way of acquiring new knowledge: that of obtaining it from the stock of knowledge already in use in the more advanced countries. Everything equal, it is less costly to learn technological knowledge than to create it. The exchange of goods is one of the channels that may foster this acquisition.

b) Second, the emphasis on international learning makes the situation – Pasinetti argues - asymmetric between nations at different stage of development. The asymmetry consists on the fact that the less developed countries see international relations with interest (they can gain the primary benefits), while there is less interest (or there may be no interest at all) for the advanced countries.

c) Third, gains from comparative advantages in this context do not play a pivotal role. Obviously, they are a source of growth, but this growth is only seen as the secondary benefits from trade. This is because such a gains are obtained "once for all" -- something that happens when the country opens up its frontiers, but that afterwards do not produce further economic growth.

d) Fourth, contrary to the traditional set of hypotheses, which assume that only goods in the commodity market may be subject to international movement, there exists high incentives for people to move cross borders. Countries are intrinsically bounded with reference to the levels of productivities, so people from poor countries by moving to rich countries could capture immediately (through higher salaries) the advantages of a high income country. From an individual point of view this appears a positive gain of wealth. From an aggregate point of view, being the poor country depauperated of its own labour force, it may not be so.

e) Fifth, the conclusion is that international relations should focus on the transmission, diffusion and acquisition of knowledge, more than on comparative advantages. It is through the acquisition of knowledge, more than through specialization that those nations with lower income have the chance to speed up their processes of learning. This is the way they may allow to grow.

A group of economists, mainly of Brasilian origin, and collected under the direction of Joanilio Texeira have taken on and develop the Pasinettian approach in two directions.

First they formalized the above main propositions in a mathematical form. The crucial reference in this respect is Araujo and Teixeira (2003 and 2004a), where the basic equations of the price and quantity system in an open economy are re-formulated and the new condition for fulfilling the condition of full employment is set.

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[4] For a broader discussion on theory and evidence, see Leamer (1985).

Second, they have inquired the implications of the Pasinettian approach from a South (rather than a North) point of view. As we have seen, the asymmetry between develop and underdeveloped countries in the field of international learning offers a powerful and an expeditious way for the backward countries to organize a process of catching up with the economic leaders (See Araujo and Teixeira 2004b).

There are moreover at least other two approaches of international relations out of mainstream which have some points of contact with the present analysis. I refer to the "neo-technological approach" of Freeman (1982, 1997), Dosi, Pavitt, and Soete (1990). The point of contact between their approach and the present paper is on the emphasis put on technology in delivering and promoting international competitiveness. To achieve the latter, they argue, a country should be confronted with absolute advantages, and not just with comparative advantages. The analysis does not offer a formal model, but it offers an appreciative theorising enriched with an abundant host of empirical analysis (Fagerberg 1988).

For the emphasis deserved to the role of international demand, this work share many views also with the Kaldorian approach that has been renamed after the works of MacCombie and Thirlwall (1997) on "the balance of payment constraints". Exports are seen as a relatively independent component of demand that can stimulate the supply side and promote growth. This role of demand emerges in the following model quite clearly, although we are not entering to the problems that an excess of exports over import (or vice versa) may cause to the balance of payments. Our only focus will be on discussing the economic consequences of the application of the principle of comparative advantages. With this aim in mind we explicitly focus on some macroeconomic variables, and we set to define (or re-define) them first.

3. The model

3.1. Assumptions

The model presents an open pure labour economy subject to structural change. It is a pure labour economy, in the sense that only labour is used in production, without any assistance of intermediate goods. This simplification may sound unusually unrealistic, because the modern economies (both developed and underdeveloped) do use many other factors of production, not just labour. In fact it is an abstraction of reality, which aims to highlight some of the crucial aspects of it. One may figure out at least three reasons that induces to keep the abstraction as such.

The first is logical consistency. Working with an economy of pure labour, allows, analytically to avoid all the complications of capital theory, without significant lost of generality (labour productivity will capture the beneficial effects of capital).

The second reason is essentiality. Occam's razor suggests to accept the simplest theory that works, without unnecessary complications. Focusing on the labour alone allows us to concentrate on the key factor of an economy which is behind (directly or indirectly) all productive processes and productive changes. As long as a factor of production is itself produced (like capital equipment) it can be reduced to dated labour.

The third reason is relevance. Not only labour is the only factor that *directly* and *indirectly* affects all productive activities. It is in the current industrial evolution also much more relevant than it is used to be. Therefore the *direct* error that arises from excluding other factors is smaller than it would otherwise be.

From these premises it should be clear, therefore, that the pure labour economies we shall consider are not those of a "primitive state of society" to use Adam Smith's (1776) words. They are rather economic systems with two crucial features of modernity. Namely:

a) a division of labour which is here captured by many and differentiated industrial sectors;

b) a uneven process of learning across sectors, that affects both the demand side (per capita consumption will change over time) and the supply side (a process of differentiated technical change is undergoing).

These two features give to the economic system the typical flavour of structural change, with sectoral variables (demand, output, employment, prices, productivity) that are changing over time at their own pace. Even the number of sectors themselves may be subject to change. To keep the analysis manageable (yet, hopefully, relevant) we shall suppose that there are just two trading parties (one may say a developed and a less developed country or region) and that in each of them will be no change of population.

In short, we may identify the model as 2 x n(t) x 1 model, i.e. two countries, n goods (changeable over time), one factor of production. As we have already noticed, to the extent that the factors of production are themselves produced (like capital equipment) the model can be extended in dealing with a j(t) x n(t) x m(t) situation. In any case, the bulk of the arguments can be carried out substantially intact also in our simplified framework.

The table below, succinctly, lists the symbols employed in the foregoing analysis.

Table 1 Legend of symbols

Symbols	Definitions
BASIC VARIABLES	
$t, T, T+1$	time (in the second part of the paper, whenever possible, the symbol is omitted for shortening), time in which trade begins (*T*), period sub-sequent to trade (*T+1*)
$l_i(t), l_{\#}(t)$	Labour coefficient of sector i, and average labour coefficient of the economy.
$L_i(t), L_{Tot}(t)$	Labour employed in sector i, and Total Labour employed.
$N(t)$	Population of country A, considered equal to the Labour Force
$\xi_i(t)$	Ratio between population of country A over B
$\pi_i(t), \pi_{\#}(t)$	Labour productivity of sector i, and average labour productivity of the economy
$Y(t), y_{\#}(t), y_{\#r}(t)$	Total income and average per-capita income, and real average per-capita income
$c_i(t)$	Per capita consumption of good i
$C_i(t)$	Total consumption of good i
$Q_i(t)$	Production of sector i
$p_i(t), p_{\#}(t)$	Price of good i, and average level of price of the economy
$w(t)$	Uniform unit wage.
$\phi_i(t), \phi(t)$	Share of labour force in sector i (rate of employment in sector i), and rate of employment in the economy
$u(t)$	Unemployment rate, with $u(t) = 1 - \phi(t)$
$\lambda_i(t)$	Share of labour employed in sector i $\lambda_i(t) = \frac{\phi_i(t)}{\phi(t)}$

$\varepsilon(t)$	Rate of exchange (quantity of foreign currency necessary to buy a unit of domestic currency)
COEFFICIENTS of VARIATION	
$\rho_i, \rho_{\#}(t)$	Rate of growth of labour productivity in sector *i* and average rate of growth of labour productivity of the economy
$r_i, r_{\#}(t)$	Rate of change of per capita consumption of good *i*, and average change of per capita consumption of the economy
σ_w	Rate of change of the wage rate
$\eta_i(t), \eta(t)$	Rate of change of employment rate in sector *i*, and rate of change of the overall employment rate in the economy
g	Rate of change of population (in the present analysis usually $g = 0$)
$\gamma_{Tot}(t), \gamma(t), \gamma_r(t)$	Rate of change, in nominal terms, of total (γ_{Tot}) and per capita income (γ), and rate of change of the latter in real terms (γ_r)
$\iota_i(t), \iota_{\#}(t)$	Sectoral and average inflation rate
SUPERSCRIPTS and SUBSCRIPTS	
No apex, *	Variables and coefficients of country A (No Apex) and country B (*)
i, k, m	A casual sector (*i*), the last sector in which country A has a comparative advantage (*k*), the number of sectors present in both countries (*m*)
NT, AT	Variables and coefficients in case of No Trade (NT) and After Trade (AT), taken at the same time period (AT is omitted in all cases in which there is no ambiguity)
DO, EX, IM	Variables and coefficients of sectors of country A devoted to the production for the domestic market (DO), for the export market (EX), and (if it would still be in existence) for what is AT the import market (IM).

3.2. No international trade

To begin with, let us examine the economy with no international trade, or if one wants to use a time line, before the open up of the economy to international trade. This allows us to clarify the mechanics of the model in a closed economy[5]. It gives us the necessary elements to study the possible evolution that an economy would face in a pure production system. In this way a comparison with a situation without or with trade will be later possible. The situation at time, t, in terms of production, demand, prices, and employment is the following.

The sectoral demand, expressed in terms of total consumption, $C_i(t)$ is given by:

$$C_i(t) = c_i(0) \cdot e^{r_i t} \cdot N(0) \cdot e^{gt} = c_i(0) \cdot N(0) \cdot e^{(r_i + g)t} \qquad 1$$

While the sectoral level of production from the supply side is expressed by:

$$Q_i(t) = \frac{L_i(t)}{l_i(t)} = \pi_i L_i(t) = \pi_i(0) L_i(0) e^{(\eta_i + \rho_i)t} \qquad 2$$

[5] The starting point of the model here expressed from equation 1 to equation 6 is presented in greater detailed and breath of explanations in Pasinetti 1993. See also, for the case with capital, Pasinetti 1981.

According to the Keynesian principle of effective demand (Keynes 1936), the quantity of consumption goods determines the level of sectoral production:

$$Q_i(t) \leftarrow C_i(t) \qquad 3$$

Moreover, the overall level of production (conceived as a vector of physical quantities) is subject to an additional constraint: the threshold of full employment. The latter is an inequality constraint, being unemployment always possible in our system. For simplicity if we assume that labour force coincides with population (*i.e.* there is no passive population and the participation rate of the active population is equal to 1), then the weak inequality is given by:

$$\sum_{i=1}^{m} c_i(t) l_i(t) = c_i(0) l_i(0) e^{(r_i - \rho_i)t} \leq 1 \qquad 4$$

Two crucial variables of our dynamic multisectoral economy, belongs to the price system. They are the sectoral prices, and the nominal level of wages. The sectoral prices can be written in terms of cost of production :

$$p_i(t) = l_i(t) w(t) = l_i(0) w(0) e^{(\sigma_w - \rho_i)t} \qquad 5$$

while unit wages, which are expressed in terms of the domestic monetary unit, can be taken in nominal value as exogenous:

$$w(t) = w(0) \cdot e^{\sigma_w t} \qquad 6$$

The absence of subscript, i, indicates that the unit wage is considered uniform across sectors. The level is fixed at time zero and follows a dynamic path according to institutional factors (usually, arrangements between trade unions and their counterparts). By knowing exogenously the level (at time zero) of wages and its rate of change σ_W the price system is fully determined.

If by chance condition 4 is not fulfilled because the overall demand is higher than the maximum capacity level, the price system, will reflect this aspect. Contrary to the quantity system that simply leaves the excess demand (ex-post) unfulfilled, the price system reacts to the excess demand, so to equate, in nominal terms, demand and supply. As a result the sectoral prices in this case will be made up by a further component as compared to equation 5, which reflects the overall relative scarcity of the supply side, namely:

$$p_i(t) = l_i(t) w(t) \cdot \sum_{i=1}^{m} c_i(t) l_i(t) = l_i(0) w(0) e^{(\sigma_w - \rho_i)t} \cdot \sum_{i=1}^{m} c_i(0) l_i(0) e^{(r_i - \rho_i)t} \qquad 7$$

with ex-ante $\sum_{i=1}^{m} c_i(t) l_i(t) > 1$

The degree of overheating – which reflects the scarcity of labour, as compared to the ex-ante level of demand – is represented in equation by the additional factor $\sum_{i=1}^{m} c_i(t) l_i(t) > 1$. Its appearance allows equating at nominal level demand and supply side also in the case of an excess demand over maximum supply. In the foregoing analysis

however, we shall focus on a world where the constraint comes from the demand side not the supply side: unemployment, not the "scarcity" of labour, is the problem to cure.[6]

Turning our attention to the quantity system again, we are able, from the above equations 1-3, to derive the level of sectoral employment, and total employment which are respectively:

$$L_i(t) = c_i(t)l_i(t)N(t) = c_i(0)l_i(0)N(0)e^{(r_i+g-\rho_i)t} \quad 8$$

$$L_T(t) = N(t)\sum_{i=1}^{m} c_i(t)l_i(t) = N(0)\sum_{i=1}^{m} c_i(0)l_i(0)e^{(r_i+g-\rho_i)t} \quad 9$$

Being weak inequality 4 in place, on aggregate it should always be that the occupation is lower or equal to the level of the labour force available, i.e. $L_T(t) \leq N(t)$. Dividing both sides of equation 9 by $N(t)$ we obtain the level of employment, that coincides with inequality 4.

$$\frac{L_T(t)}{N(t)} = \phi(t) = 1 - u(t) = \sum_{i=1}^{m} c_i(0)l_i(0)e^{(r_i+g-\rho_i)t} \quad 10$$

In addition to the level of employment, at the aggregate level there are at least other three more variables to be determined: the average level of labour productivity, the average level of prices, the per-capita income, and their respective rates of change.

It is convenient to start determining the average level of prices, defined as the individual price that leaves unchanged the overall value of production. Hence:

$$p_{\#} = \frac{\sum_{i=1}^{m} p_i(t)Q_i(t)}{\sum_{i=1}^{m} Q_i(t)} \quad 11$$

If constrained 4 is not violated ex-ante[7], then we may substitute in equation 11 both the quantity and the price variables. As a result we obtain:

$$p_{\#}(t) = \frac{w(t)\phi(t)}{\sum_{i=1}^{m} c_i(t)} = \frac{w(0)\sum_{i=1}^{m} l_i(0)c_i(0)e^{(\sigma_w+r_i-\rho_i)t}}{\sum_{i=1}^{m} c_i(0)e^{r_i t}} \quad 12$$

which gives precisely the average level of prices in the economic system, at any point in time as an expression of its initial level and the evolution path given by the rates of change of wages, per capita consumption, and technical progress.

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[6] Equation 7 suggests, in any case, that the framework here presented is not completely unprepared in dealing with the problem of "scarcity".

[7] Ex-post it will never happen, of course.

Other crucial variable is the average level of labour productivity, which is determined as the ratio between total production (expressed in real terms) and total labour employed by the economic system. More precisely the ratio should be expressed as follows:

$$\pi_{\#}(t) = \frac{\frac{\sum_{i=1}^{m}[Q_i(t)\,p_i(t)]}{p_{\#}}}{\sum_{i=1}^{m} L_i(t)} = \frac{\sum_{i=1}^{m} c_i(0)e^{r_i t}}{\sum_{i=1}^{m} l_i(0)c_i(0)e^{(r_i-\rho_i)t}} \qquad 13$$

The same result can also be obtain as a weighted average of the sectoral productivities, with the weight expressed in terms of sectoral labour shares:

$$\pi_{\#}(t) = \frac{\sum_{i=1}^{m}\left[\phi_i(t)\cdot\frac{1}{l_i(t)}\right]}{\phi(t)} = \frac{\sum_{i=1}^{m} c_i(0)e^{r_i t}}{\sum_{i=1}^{m} l_i(0)c_i(0)e^{(r_i-\rho_i)t}} \qquad 14$$

Being the economic system not always (and not necessarily) in a condition of full employment, equation 14 has required a factor of normalization, which disappears in the ideal case of $\sum_{i=1}^{m}\phi_i(t) = 1$.

By combining the average level of productivity with the average level of prices, another macroeconomic relation can be singled out:

$$p_{\#}(t) = \frac{w(0)e^{\sigma_w t}}{\pi_{\#}(0)e^{\rho_{\#} t}} \qquad 15$$

The average level of prices will be affected by other two aggregate variables: the level of wages and the average level of productivity. Equation 15 allows to study the level of prices as relationship between an institutional variable (the level of wages) and a technological variable (the average productivity). Therefore also the rate of inflation can be studied by detecting the dynamics of the above two variables. The implication is simple, but relevant. Nominal wages can always be fixed exogenously, and it can be fixed at any level and with whatever dynamic path institutions decide to implement. However, real wages are always determined endogenously by the level of average productivity, they are not a variable decided by the institutional side.

There is still to be determined the most important variable of all –the level of per-capita income. In nominal terms, it will be the value of the production, which (through equation 3) is determined by the per capita consumption. After some obvious substitution, it can be turned as follows:

$$y_{\#}(t) = \sum_{i=1}^{m} c_i(t)\,p_i(t) = \pi_{\#}(t)\,p_{\#}(t)[1-u(t)] \qquad 16$$

Equation 16 expresses that the nominal level of per capita income, as product of three variables: average labour productivity, average level of prices, and the rate of employment. It should be mention at this stage, that in practice it will be a fourth set of factors – not considered in this analysis – that affect per capita income. Namely, the ratio between labour force and total population (and the length of working hours), which, by hypothesis, we did not take into account.

To single out the total income in the case of full or less than full employment, we just need to multiply by *N(t)*:

$$Y_{\#}(t) = y_{\#}(t)N(t) = \pi_{\#}(t)p_{\#}(t)L_{Tot}(t) \qquad 17$$

3.2.1. Rate of changes

Each of the four aggregate variables examined so far, namely employment, productivity, prices, and income, can be examined also dynamically. Let us see, therefore, how each of these aggregate variable moves through time, by taking derivatives to the time dimension.

The level of employment will change as a result of a weighted difference between the rate of sectoral growth of per capita demand, and the rate of change of sectoral productivity. Namely:

$$\eta(t) = \sum_{i=1}^{m} \lambda_i(t)(r_i - \rho_i) \qquad 18$$

with $\lambda_i(t) = \dfrac{c_i(t)l_i(t)}{\sum_{i=1}^{m} c_i(t)l_i(t)}$

for small changes of unemployment we could also write:

$$\lambda_i(t) = \phi_i(t)[1 + u(t)]$$

The level of employment will increase only if, on a weighted average, the per capita demand of the various sectors increase faster than sectoral productivities do. If demand does not increase on average, and the system experiences technical progress, equation 18 becomes negative and a higher level of unemployment is the outcome.

Other crucial variable (which is necessary in the calculation of the other two variables) is labour productivity. Its rate of change is somewhat laborious to calculate. Its simplified result is:

$$\rho_{\#}(t) = \frac{\phi(t)\sum_{i=1}^{m} r_i c_i(t) - \sum_{i=1}^{m} c_i(t)\sum_{i=1}^{m} (r_i - \rho_i)\phi_i(t)}{\phi(t)\sum_{i=1}^{m} c_i(t)} \qquad 19$$

After some further work in adjusting and ordering the addenda we come up with a rather simple, but meaningful equation:

$$\rho_{\#}(t) = \sum_{i=1}^{m} r_i(\alpha_i - \lambda_i) + \sum_{i=1}^{m} \rho_i \lambda_i \qquad 20$$

with $\alpha_i = \frac{c_i(t)}{\sum_{i=1}^{m} c_i(t)}$ and again $\lambda_i = \frac{c_i(t)l_i(t)}{\sum_{i=1}^{m} c_i(t)l_i(t)}$

Equation 20 can be further transformed to offer even a more straightforward interpretation, by substituting c_i with the equivalent $\pi_i c_i l_i$. After some simplifications:

$$\rho_{\#}(t) = \sum_{i=1}^{m} r_i \left(\frac{\pi_i(t)}{\pi_{\#}(t)} - 1 \right) \lambda_i + \sum_{i=1}^{m} \rho_i \lambda_i \qquad 21$$

According to equation 21, the average rate of change of labour productivity is dependent from two main components both weighted by the relative size of each sector in terms of labour employed:

a) the rate of change of per capita demand multiplied by the position of the sector in terms of relative level of labour productivity, and

b) the rate of change of sectoral productivity, which represents the degree of technical progress, within each sector.

We could call component a) of the rate of change of productivity as the *Keynesian component*, being linked with the dynamics of demand, and the component b) of the rate of change of productivity as the *Schumpeterian component*, being connected with the degree of innovation of the individual sector.[8]

While this latter component, as long as each coefficient of technical progress ρ_i is positive (or at least the majority of them is positive), will always produce a positive effect on $\rho_{\#}(t)$, the former component, i.e. the Keynesian component, even if all r_i are positive, produces mixed results. If the rate of growth of demand is addressed mainly towards the sectors that have an higher level of productivity as compared to the average (so that $\frac{\pi_i(t)}{\pi_{\#}(t)} > 1$), that impact on the rate of change of productivity will be positive. On the contrary if the rate of change of demand is addressed mainly towards those sectors with lower productivity than the average (so as $\frac{\pi_i(t)}{\pi_{\#}(t)} < 1$), the effect on $\rho_{\#}(t)$ will be negative. Hence, the rate of productivity change is usually affected positively by the Schumpeterian component, it may be affected either positively or negatively by the Keynesian component.

The dynamics of prices, is given by the rate of change of equation 15: Hence we will have level of inflation in the economy, which is determined by the dynamics of wages and average productivity:

$$\iota_{\#}(t) = \sigma_w - \rho_{\#}(t) \qquad 22$$

It may be interesting to observe that individual prices move constantly, and are affected by two components. The first component is due to a monetary effect (see equation 22), the second component is due to the structural effect, which is of technological nature, and cannot be avoided[9]:

$$\iota_i = \sigma_w - \rho_i = (\sigma_w - \rho_{\#}(t)) + (\rho_{\#}(t) - \rho_i) = \iota_{\#}(t) \pm \Delta\rho_i \qquad 23$$

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[8] Equation 21 gives solution also to the problem of the dynamic standard commodity. The problem is one of the main subjects of discussion in Pasinetti 1993. Our answer is different from his, yet if our result holds the outcome will not displease him: there is a Keynesian component altogether with the Schumpeterian component in its determinants. I am dealing with it elsewhere.

[9] See on this precise point Pasinetti 1993: 107. There is a substantial difference in the outcome however: see the previous footnote.

We shall turn now to the (nominal) rate of growth of per capita income. By knowing the rate of change of average productivity, the average rate of inflation and the rate of change of employment, we can calculate the rate of growth of per-capita and total income:

$$\gamma = \rho_{\#} + \iota_{\#} + \eta = \rho_{\#} + \iota_{\#} - \frac{\dot{u}}{\phi} \qquad 24$$

and the rate of growth of total income, will be also affected by the rate of growth of the population, *g*, that in the foregoing analysis we have assumed constant.

$$\gamma_{Tot} = \rho_{\#} + \iota_{\#} - \frac{\dot{u}}{\phi} + g \qquad 25$$

Obviously speaking about performance of a country, we are interested in the level of (total and per-capita) income, and to its rate of change, expressed in real terms not in nominal ones. To clean up our variables from variations due only to nominal changes, we may either subtract from equations 24 and 25 the inflation rate, or we may set the exogenous variables (wages) in such a way that the system enjoys a stable level of prices. The latter outcome can be imposed elegantly by choosing the average level of prices as the numéraire of the system. In that case one needs two conditions: *a)* a level of unit wages equal to the average level of productivity at time zero, and *b)* a wage-increase equal to the average rate of change of productivity. To avoid inflation, with a different numéraire, condition b) is sufficient.

This simplification does not alter in any way the real variables, such as the real wage rate, or its rate of change, but it allows nominal and real values of our (monetary) variables to coincide, avoiding inflation or deflation disruptions.

Formally, we can write

Imposed conditions: $$w(0) = \pi_{\#}(0) \qquad 26a$$

$$\sigma_w = \rho_{\#} \qquad 26b$$

Consequence of the conditions: $$p_{\#}(0) = 1 \qquad 27a$$

$$\iota_{\#} = 0 \qquad 27b$$

Therefore, if the level of wage and its dynamic path will follow equation 26a and b, not only all monetary variables at aggregate level, will be expressed in real terms, but also the average level of prices will become our numéraire. In other words, the monetary measure (called it Euro, or Dollar, for instance), with which all prices are stated, is expressed precisely by the average level of prices of the economic system.

3.3. After international trade

The country, (let call it A) examined so far, will now open its economy to international trade with another country or group of countries (let call it B). The latter country or group of countries could be represented with the same set of equations that we have developed for country A. There could be a potential difference in the new set of equations. Namely, the number of sectors of country B may not coincide with the number of sectors of country A:

$$m(t) \gtreqless n(t) \qquad 28$$

where *n(t)* is the number of sectors of country B. Usually, the most develop country, among the two, will have a number of sectors higher than the less developed country. In other words, some new sectors present in the former country could not yet be present to the latter country. The new sectors may be highly innovative, which require a more advanced level of productive knowledge not (yet) present in country B.

This is a very important avenue to extend the present model, which would bring richer conclusions. Yet, for limit of space, and to focus closely to the aim we have set at the beginning, let us suppose that the number of sectors are in the two countries for the while the same. In some comments we can remove explicitly this restriction.

Before starting the comparison other four notational conventions are helpful in understanding the analysis. They are all made in the attempt to facilitate the interpretation of the results.

First, the point of view of the analysis is country specific. It refers always to country A. The variables which refer to country B (the foreign country or countries), they will be identified by the addition of an asterisk "*". The variables which refer to country A have no an explicit label that identifies it.

Second, there is the necessity of comparing country A before and after trade (AT), or to put it better, of making a distinction between the situation of *no-trade* and the situation *with-trade*. In the ambiguous cases the variables that refer to a situation of no-trade (*NT*) will be labelled as such (see Table 1) with a superscript, while the absence of subscript will mean a situation AT. In those cases in which there will be no ambiguities also the label *NT* could be omitted.

Third, being our point of view that of country A, we call export (EX) and import (IM) goods or sectors always with reference to that country. So for instance the labour shares of the import sectors refer to the shares of labour that would have been necessary in country A in case of no-trade, to produce the same quantity of goods that are now imported.

Fourth, we shall assume that sectors are numbered incrementally according to the RPCA, and that between the *k* sector and the *k+1* sector there is the switch of comparative advantages between the two countries.

Fifth, the comparison between a situation AT and a situation of no-trade, will be taken always with reference to the same time dimension. To avoid therefore to carry on too complicated notational conventions we shall not explicit mentioning the relation of the variables with time, that has been used up to now. In the following sections, we shall indicate only whether the main variable that is under analysis will be depend on time or not.

About timing we could be more explicit in sequencing the passage from a situation of no-trade to a situation AT. We may assume that a country opens up its frontiers at time t=T. The process of specialization that follows is understood as instantaneous. This process will give the comparative *static* gains (or losses) from trade.

However, the analysis is carried out also dynamically henceforth. To avoid unnecessary complications we analyze therefore what happens between the time t=T and t=T+1. Obviously, the analysis could be carried out in the further periods too. We shall refer to it as the comparative *dynamic* gains (or losses) from trade. Figure 1 illustrate the sequencing, that we have just described.

[Figure 1 around here]

3.4. Fixing the rate of exchange in PPP

When two economies open up their frontiers to trade, there is the immediate need to fix the rate of exchange between the monetary units of each country. In our case the monetary units, the numeraires, could be express by $p_{\#}$ for the economy A, and $p^*_{\#}$ for the economy B, if conditions 27*a* and *b* are in place. But it is not necessary that they are – in real life in fact they don't.[10] Instead, it is only that prices and wages are expressed with the same monetary unit, whatever it is.

How should the rate of exchange be settled between the two monetary units? The literature and the debate on the point is countless. But still the more reliable and consistent approach in dealing with it is the one that expresses the rate of exchange in terms of parity purchasing power (PPP).

A rate of exchange fixed in PPP is obtained in such a way that the monetary unit of country A (B) will be able to buy, on average, the same bundle of goods domestically as well as in the foreign country. This means more in general that the composite good (the bundle of goods), on which the average price is made, will be exchanged between the two countries at the ratio of 1:1. If in each country the average prices represent the numeraire, two elegant simplifications can be made: first, the nominal and real exchange rate will coincide; second, the ratio of wages in these two countries will be exactly equal to the ratio of their average productivities. Formally:

$$\varepsilon_R = \varepsilon = \frac{p^*_{\#}}{p_{\#}} = 1 \rightarrow \frac{\pi_{\#}}{\pi^*_{\#}} = \frac{w}{w^*} \qquad 29a$$

with $p_{\#}, p^*_{\#}$ chosen as numéraires.

There is another way of putting the same point (Pasinetti 1981, 1993). Let us suppose that the average productivity is produced by an average sector, which commodity (called it gold) works as the international mean of exchange. Since its price should internationally be equal, we could equate the two prices in the two countries, and we are again with equation 29a.

In reality each country holds different monetary units, and different level of prices. Hence if condition 27*a* and *b* is not in place, we could simply write the exchange rate as the ratio between the two levels of average price, each expressed in its own currency. Formally:

$$\varepsilon = \frac{p^*_{\#}}{p_{\#}} = \frac{w^* l^*_{\#}}{w l_{\#}} = \frac{w^*}{w}\frac{\pi_{\#}}{\pi^*_{\#}} \rightarrow \frac{\pi_{\#}}{\pi^*_{\#}} = \frac{w/p_{\#}}{w^*/p^*_{\#}} \qquad 29b$$

In this way, the rate of exchange is the price that country B should pay to obtain the average bundle of goods of the domestic unit of currency of country A. As with equation 29a, this implies that the ratio between average productivities will equal the ratio between (now real) wages. The advantage of equation 29b consists that the exchange rate will take into account fluctuations of the average level of prices in each country. The real exchange rate will always be equal to 1, by the same definition of PPP, but now the nominal exchange rate may fluctuate. Lowering $p_{\#}$ in country A, and not

[10] It is very unusual the case in which the average level of prices in a country is 1$ or 1€ or whatever other currency is in place in the economy under scrutiny.

in country B, will produce a revaluation of the exchange rate, so that the average bundle of goods in the former country will continue to have the same price for the latter country. Equation 29b suggests explicitly why an exchange rate, based on PPP, may change in a pure labour economy. There are two factors: either because of changes in nominal wages, or because of changes in the average labour productivity in one of the two economies or, unevenly, in both of them. With an increase of wages the exchange rate devaluates, with an increase of average productivity the exchange rate revaluates, everything else remain equal.

3.5. International Prices and comparative advantages

Knowing the rate of exchange allows to express the price of each good with a single currency, and hence to compare internationally the prices of any good. Being the level of wages uniform in each country, any difference between internal and external prices, once they are compared with the same currency, will be affected only by relative differences of labour coefficients, i.e. only by differences between average and sectoral productivities. Formally:

$$p_i \gtreqless \frac{p_i^*}{\varepsilon} \text{ and after substitution:}$$

30

$$\frac{l_i}{l_\#} \gtreqless \frac{l_i^*}{l_\#^*} \rightarrow \frac{\pi_i}{\pi_i^*} \lesseqgtr \frac{\pi_\#}{\pi_\#^*}$$

Hence, comparative advantages can be singled out indifferently in terms of sectoral productivities or (inversely) in terms of sectoral prices. In the first case, country A will have comparative advantages in those sectors in which the ratio $\frac{\pi_i}{\pi_i^*}$ is higher than the ratio between average productivities. In the second case, the comparative advantages will lie in those sectors (the same sectors) in which the ratio of price of the foreign good to the domestic good $\frac{p_i^*}{p_i}$ is higher than the exchange rate, so as the foreign good expressed in the domestic currency is more expensive $\frac{p_i^*}{\varepsilon} > p_i$.

The open up of the economies will impose the lower price between the two countries, inducing through competition an international specialization of production. Take, for instance, one sector which production is present in both countries. If the ratio between the sectoral productivity of country A versus country B is higher than their average productivities, it would imply a price p_i lower than $\frac{p_i^*}{\varepsilon}$. Consequently, such a good will be produced in country A, and not in B. If the ratio between productivities shows the opposite inequality, it will be produced in country B and not in A. In both cases, the cheaper price will capture the international market and specialization will follow.

With this logic, it is possible to compare all sectors that are in activity in both countries at time t=T. One would expect that some sectors will have their productivity ratios above the level of the ratio of average productivities, and other sectors below the average ratios, as shown by inequality 30, so that both countries have something to trade. Yet, in real life not always expectations are fulfilled.

3.6. Existence and non existence of the comparative advantages

What has been discussed, in the previous section, is a quite standard prototype of a textbook analysis. The RPCA, as against the principle of absolute advantages, show always a theoretical feature of symmetry. It is a standard taking that one country enjoys comparative advantages in the production of some goods, and another country enjoys comparative advantages in the production of other goods. This would imply a *do ut des* between the two (or more) countries.

Translating this feature in our model, it means that according to 30, the ratio between average productivities falls always inside the range of values made up by the ratio between sectoral productivities. But can an unbalanced trade exist? At the extreme, does a possibility exist that a country is relatively better in producing everything, and another country does not have any comparative advantage in producing anything at all? When the average productivity, and the average prices, are calculated –as it should be, and as it is done in our case – in terms of a weighted average, there is no assurance that the ratio between average productivities (and average prices) always falls within the range of ratios of sectoral productivities (and sectoral prices). Hence a completely unbalance trade between countries can exist.

In general a distinction can be made between three possible cases that can emerge from the application of the RPCA:

1. Case of absence of trade with no comparative advantages

2. Case of trade with multilateral comparative advantages

3. Case of trade with unilateral comparative advantages.

3.6.1. Absence of trade: no comparative advantages for any country

There is just one artificial case in which, despite being the economies open, there is no incentive to trade. The case has been analyzed by Pasinetti (1981, 1993), after assuming the crucial hypotheses of an equal structure of relative prices (and productivities), so as:

a) country A has τ times the average productivity of country B;

b) country A has still τ times the sectoral productivity of country B in producing each individual good.

Formally:

$$\frac{\pi_i(t)}{\pi_i^*(t)} = \frac{\pi_j(t)}{\pi_j^*(t)} = \cdots \frac{\pi_m(t)}{\pi_m^*(t)} = \frac{\pi_\#(t)}{\pi_\#^*(t)} = \tau \qquad 31$$

$$\text{hence } p_i = \frac{p_i^*}{\varepsilon} \quad \text{for} \quad i = 1....m.$$

Being the ratio between all productivities (and in particular between the ratio of the average productivity of the two economies and the individual productivities of each sector) equal to τ times, the international prices will be exactly the same, and there is no advantage to trade for any country.

Pasinetti (1981, 1993) utilizes the assumption to show that despite the absence of trade, there are incentives to leave open an economy towards the international relations. This will be particularly true for the underdeveloped countries.

They can burst their productivity, through international learning, regardless of the presence of traditional (Ricardian) gains from trade.

3.6.2. Multilateral comparative advantages

The case of no comparative advantages analyzed in the previous section (3.6.1) is a useful exercise, since it allows us to focus on other aspects of international relations, not just trade. But it is a very artificial case.

In practice it will be virtually impossible to have a perfectly identical relative level of prices between all goods produced abroad and those produced at home. If differences of relative prices exist, the law of comparative advantages suggests that the *m* sectors of the economy A and economy B, where a comparison is possible, will be split in two. Those that will have a level of international price lower than the competition, which production will increase, by acquiring also the foreign demand; and those that will have an international price higher than the international competition, which will be no longer domestically produced. They will be imported from abroad (i.e. from country B).

Formally:

$$\frac{\pi_i(t)}{\pi_i^*(t)} > \frac{\pi_j(t)}{\pi_j^*(t)} > \dots \frac{\pi_k(t)}{\pi_k^*(t)} > \frac{\pi_{\#}(t)}{\pi_{\#}^*(t)} > \frac{\pi_{k+1}(t)}{\pi_{k+1}^*(t)} > \dots \frac{\pi_m(t)}{\pi_m^*(t)} \qquad 32$$

Country A will specialize to the first *k* sectors, country B on the other *k+1* up to *m* sectors. If there is a sector *n*, where the production is made only in one of the two countries, that country will continue to produce it, being the comparative advantage abstractly infinite.

3.6.3. Unilateral comparative advantages

Albeit being largely accepted in theory, the previous case may not cover all possible situations, and hence it is incomplete. It is indeed possible that an economy shows in each sector a level of comparative advantage superior than any foreign country, so as its sectoral international prices will always be lower than the price applied by its competitors. Formally:

$$\frac{\pi_i(t)}{\pi_i^*(t)} > \frac{\pi_j(t)}{\pi_j^*(t)} > \dots \frac{\pi_m(t)}{\pi_m^*(t)} > \frac{\pi_{\#}(t)}{\pi_{\#}^*(t)} \qquad 29b$$

This is a situation very easily to occur, when two countries find themselves at two different levels of income. In this situation the emergence of unilateral comparative advantages is not just an abstract case. One is tented to justify this phenomenon in presence – as it seems plausible – of different level of wages. Yet, this is plain wrong, when – as we have made clear – exchange rate is fixed through PPP mechanism. The factor that generates the phenomenon is instead the complex interplay between demand (consumption) and technology (labour productivity).

This is not the central topic of this paper, but to show its existence an example will suffice. Let us start from the *ad hoc* case of absence of trade explained in section 3.6.1, with the same structure of sectoral productivities . Now, before of calculating the average productivity in each country, let us suppose that in country A exists an overall demand completely shifted towards the sector with lowest productivity. Conversely, let us suppose that for country B exists an overall demand completely shifted towards the sector with highest productivity. This is just a hypothetical case, but it allows to highlight the consequences. Now the ratio between the two average productivities will not be equal to the constant factor τ . It will be lower than that factor. Hence, country A has a comparative advantage in all individual

sectors. Country B, on the contrary, faces for each commodity a level of international price that would make its production uncompetitive, indistinctly in all sectors.

The example is extreme, but it is made for analytical purposes so to bring forward the precise outcome of unilateral trade. The same result can be obtained by changing labour productivities. This latter case is even more interesting because – in our knowledge – it is not touched in the literature. Yet we shall not deal with it in this paper. Instead, let us jump immediately to the consequences.

The case of unilateral trade may emerge as real possibility when the structure of demand and technology do not well-behave in the comparison between two (or more) countries. Applying the double principle of comparative advantages and the principle of PPP, is not therefore a sufficient condition that guarantees a multilateral trade for all open economies.

Hence, to sum up there are three possible cases when two (or more) countries open up their economies and the principle of comparative advantage is applied.

Case 1: no comparative advantages, which is just a very hypothetical case;

Case 2: existence of comparative advantages for both country A and B, that induces a process of sectoral specialization in both countries;

Case 3: existence of comparative advantages for just one country, that induces such country to produce all the range of goods, forcing the other country (or countries) to produce nothing –a genuine case of trouble from trade.

Indeed, in this latter case, international trade will not bring advantages for all trading parties. Though may not be obvious its existence – as the author himself has experienced with its colleagues – are quite obvious its disruptive consequences. Therefore, our analysis will not concentrate on it.

3.7. Quantities and employment after trade

3.7.1. The new settings

Let us suppose that Case 2 (shared comparative advantages) is in place. Then, country A will specialize its economy in those sectors that show, compare to country B, a relative level of sectoral productivities above the ratio of average productivities. Country B will specialize to the other sectors.

The new level of quantities produced will be[11]:

$$\begin{cases} Q_i = c_i N + c_i^* N^* \\ \vdots \\ Q_k = c_k N + c_k^* N^* \end{cases} \qquad 33$$

[11] Equations 33-36 has been part of the formalization of Araujo et al. (2004a), from which I tried to adopt the same symbols, to increase compatibility. Their work show analytically the Pasinetti's (1981, 1993) argument and highlight elegantly its important implications. Here, I shall take a different route, with a different approach (comparative analysis), method (real systems instead of natural systems), and by concentrating on variables, such as income, average level of productivity, average level of prices, ecc. that are not part, or are not central, to the original model.

With some further elaborations and by fixing the ratio between the population N of country A and population N* of country B, in terms of

$$\xi(t) = \frac{N(t)}{N^*(t)} \tag{34}$$

we are able to write the level of the new effective demand:

$$\begin{cases} Q_i = \left(c_i + \frac{c_i^*}{\xi} \right) N \\ \vdots \\ Q_k = \left(c_k + \frac{c_k^*}{\xi} \right) N \end{cases} \tag{35}$$

Accordingly, the level of employment at sectoral level will be:

$$\begin{cases} L_i = \left(c_i + \frac{c_i^*}{\xi} \right) l_i N \\ \vdots \\ L_k = \left(c_k + \frac{c_k^*}{\xi} \right) l_k N \end{cases} \tag{36}$$

while the total employment will be equal to:

$$L_{Tot} = N \sum_{i=1}^{k} \left(c_i + \frac{c_i^*}{\xi} \right) l_i \tag{37}$$

By dividing both sides for the population *N* we obtain the rate of employment, ϕ, in the open economy. It can be both higher or lower as compared to the closed economy. Formally, the following inequality holds:

$$\phi = \sum_{i=1}^{k} \left(c_i + \frac{c_i^*}{\xi} \right) l_i \gtreqless \sum_{i=1}^{m} c_i l_i \leq 1 \tag{38}$$

3.7.2. Statics: Employment at time T

Our comparative analysis, between the open economy and the same economy in a closed situation of no-trade (NT), can begin by detecting the overall movement of employment at time T. How will the static gains be?

To offer an answer we may re-formulate equation 38, by spitting it in two parts: on the one hand the level of employment and employment rate with no-trade, respectively L_{Tot}^{NT} and ϕ^{NT}, and on the other hand the additional factors (due to specialization) that made up the final result. The following two equations aim precisely at this:

$$L_{Tot}(T) = L_{Tot}^{NT} + N\left[\sum_{i=1}^{k}\left(\frac{c_i^*}{\xi}\right)l_i - \sum_{i=k+1}^{m} c_i l_i\right]$$
$$\phi\,(T) = \phi^{NT} + \left[\sum_{i=1}^{k}\left(\frac{c_i^*}{\xi}\right)l_i - \sum_{i=k+1}^{m} c_i l_i\right] \qquad 39$$

As equation 39 shows, the consequence in terms of employment in opening an economy to trade is ambiguous. Compared to the situation of NT, the economy can be better off as well as worse off. The rate of employment AT is made up of three addenda.

a) The rate of employment with no trade,

b) the gains of employment due to the exports in the first k sectors,

c) the loss of employment due to imports in the sectors above k.

The size of the last two addenda will decide whether at time T, the RPCA and the consequent specialization has brought a gain in terms of employment levels or not. With equation 39, we may face three possible different outcomes. Namely:

$$\sum_{i=k+1}^{m} c_i\, l_i \;>\; \sum_{i=1}^{k} \frac{c_i^*}{\xi} l_i \quad \text{a) additional unemployment}$$
$$\sum_{i=k+1}^{m} c_i\, l_i \;=\; \sum_{i=1}^{k} \frac{c_i^*}{\xi} l_i \quad \text{b) level of employment unchanged} \qquad 40$$
$$\sum_{i=k+1}^{m} c_i\, l_i \;<\; \sum_{i=1}^{k} \frac{c_i^*}{\xi} l_i \quad \text{c) additional employment}$$

The RPCA does not imply any precise condition in this respect. It is indeed compatible with both condition a), b) and c), and there are no evident drawbacks in the principle, whether it drives a country towards one position or another. The principle could be still fulfilled, paradoxically, even in presence of mass unemployment. It is simply silent on this point. Therefore the RPCA is inconclusive in assuring some static gains in terms of employment.

3.7.3. Dynamics: Employment from time T to time T+1

But trade have also long run consequences -it does not just have a once-for-all effect as it has been traditionally – and less traditionally, Pasinetti 1993 – argued.

To analyze the dynamic implications, one needs to take into consideration time, and any change of those variables related to the time dimension. More precisely, the employment rate of the economy will change according to:

$$\eta(T) = \eta^{NT}\,\frac{\phi^{NT}}{\phi} + \frac{\sum_{i=1}^{k}\frac{c_i^*}{\xi} l_i (r_i - \rho_i)}{\phi} - \frac{\sum_{i=k+1}^{m}\phi_i (r_i - \rho_i)}{\phi^{NT}}\,\frac{\phi^{NT}}{\phi} \qquad 41$$

Equation 41 can be made more manageable by conceiving the ratio between ϕ^{NT}/ϕ at time T as due only to the static changes of the rate of employment, so that $\phi^{NT} = \phi - \Delta\phi$, in which $\Delta\phi$ is the static gain. In this case we could write the rate of change η between time T and T+1, just as:

$$\eta(T) = \eta^{NT} + \sum_{i=1}^{k} \lambda_{EX\,i}(r_i - \rho_i) - \sum_{i=k+1}^{m} \lambda_{IM\,i}(r_i - \rho_i) - \frac{\Delta\phi}{\phi}\left[\eta^{NT} - \sum_{i=k+1}^{m} \lambda_{IM\,i}(r_i - \rho_i)\right] \qquad 42$$

This is a rather complicated formulae. To make it simpler, without losses of generality we may adopt the following conventions.

First, let us split up the economy in two broad sectors, when it is in situation of NT as well as AT. In the first case (NT) call (ex-post) the two broad sectors as domestic sector (DO) and import sector (IM). In the second case (AT) call them domestic sector again (DO) and export sector (EX). Let us use the appropriate pedix to denote these four broad sectors.[12] In this case, the relation between the shares of employed labour in the export sector λ_{EX} and that of the import sector λ_{IM} are equal to $\lambda_{EX} = \lambda_{IM} + \frac{\Delta\phi}{\phi}\lambda_{DO}^{NT}$.

Second, the difference in the rate of change of per capita consumption and technical progress in the export end import sector can be defined, for short, with the terms Δr_{EX} and $\Delta\rho_{EX}$. After substituting in the last addenda the factors on which η^{NT} is dependent, and after replacing λ_{EX} with the relationship just written, equation 42 turns in a very simple form, namely:

$$\eta(T) = \eta^{NT} + \lambda_{IM}(\Delta r_{EX} - \Delta\rho_{EX})$$

with $\Delta r_{EX} = r_{EX} - r_{IM}$, $\Delta\rho_{EX} = \rho_{EX} - \rho_{IM}$ 43

The dynamic outcome in terms of employment will be simply dependent on the weighted interplay between the dynamics of demand and the dynamics of technical progress in the export sector and in (what it would have been with NT the sectors that were producing in place of) the import sector.

Will the dynamic gains of employment that emerge from RPCA be positive or negative? Equation 43 gives an answer that is open to both possibilities. If the changes of demand in the export sector are faster than the changes of demand in the import sector, and at the same time the technical progress is lower in the former than in the latter sector, then the level of employment will definitely increase. But if the opposite is true, it will definitely increase the unemployment rate. . The final outcome is therefore inconclusive in the dynamic situation as it was in the static one. The answer about dynamic gains of unemployment does not rest on the RPCA, but on the differential within the round parenthesis of equation 43. All the three cases outlined below are perfectly compatible with the application of the RPCA, and they could alternatively emerge from its application:

$r_{EX} - r_{IM} < \rho_{EX} - \rho_{IM}$ a) dynamic losses of employment 44

[12] The Domestic sector with NT may not coincide in terms of labour shares with the domestic sector after trade, since trade may cause a variation of employment as we shall see.

$r_{EX} - r_{IM} = \rho_{EX} - \rho_{IM}$ b) no dynamic changes of employment

$r_{EX} - r_{IM} > \rho_{EX} - \rho_{IM}$ c) dynamic gains of employment

To sum up the consequences on employment, there may exist static gains or losses as well as dynamic gains or losses. Between static and dynamic considerations there is no direct connection: static gains may turn in dynamic losses and vice versa. International trade may fulfil immediately the condition of full employment (system 38 equal to unity), and in the subsequent periods, the same international trade may cause a level of unemployment worst than it would have been without trade.

[Figure 2 around here]

Figure 2 substantiates in four panels what have been just said. The country represented is supposed to be in a condition of unemployment before trade (equation 4 is a strict inequality, so to allow the increase of the employment rate). The vertical axis measures the differences (negative or positive) of the employment rate, as a consequence of trade. The horizontal axis is the time dimension. International trade is supposed to take place at time t=T, as we said. There is an immediate static effect, then as time goes by there is a dynamic effect. The *b* cases of 40 and 44 are not explicitly considered for being trivial. But nevertheless they are part of the possible outcomes.

3.8. Average Productivity after trade

Other variable that needs to be re-examined after international trade is the average productivity of the country. The average productivity, according to equation 14, is a weighted average of sectoral productivities. Since, international trade drives an economy towards specialization, this will result in a change of weights, and hence in a change of average productivity too.

Any consideration, also in this case, can be made for the static potential gains as well as for the dynamic potential gains. The former emerge immediately at the open up of the economy, at time *T*. The latter affect the rate of change of the average productivity through time: in our essential framework from time *T* to *T+1*.

3.8.1. Statics: Productivity at time T

When country A holds a comparative advantage and specializes from sector 1 to k, its new average labour productivity AT will be:

$$\pi_{\#}(T) = \frac{\sum_{i=1}^{k} \phi_i \cdot \pi_i}{\sum_{i=1}^{k} \phi_i} = \frac{\sum_{i=1}^{k} \left(c_i + \frac{c_i^*}{\xi} \right)}{\sum_{i=1}^{k} \left(c_i + \frac{c_i^*}{\xi} \right) l_i} \qquad 45$$

Let us examine the above equation by connecting it with the level of productivity with no-trade (NT). As first approximation, it is helpful to discuss a simplified version of it, when there exists full employment both before and after international trade. In such a case the normalization is not required ($\phi_i = \lambda_i$), and equation 45 will turn simply in the following:

$$\pi_{\#}(T) = \pi_{\#}^{BT} + \sum_{i=1}^{k} \frac{c_i^*}{\xi} - \sum_{i=k+1}^{m} c_i = \pi_{\#}^{BT} + \sum_{i=1}^{k} \pi_i \lambda_{EXi} - \sum_{i=k+1}^{m} \pi_i \lambda_{IMi} \qquad 46$$

Its interpretation is quite simple: in the ideal case of full employment both before and AT the new level of average productivity could be both above or below the level in place under no trade. The crucial factor to determine the sign of the variation is whether the weighted productivity in the export sector is higher or lower than the weighted productivity (that was in place) in what is now the import sector.

In very simple terms, we could just compare the levels of per capita consumption from sector *i=1* to *k* and i=*k+1* to *m* and see (with the same population) which one is higher. If it is the former the average productivity AT will be increased, otherwise it would decrease.

However, there is no need to make the strong assumptions of full employment. Equation 45 can be made manageable with any level of employment by considering explicitly its changes.

As we have already did, the new level of employment rate can be written as the sum of the old level plus the level of additional employment rate (or additional unemployment rate, in the case specialization drops the level of occupation): $\sum_{i=1}^{k}\left(c_i(T) + \frac{c_i^*}{\xi}(T)\right) l_i(T) = \sum_{i=1}^{m} c_i(T) l_i(T) + \Delta\phi$, which is equal to $\phi = \phi^{NT} + \Delta\phi$ (see section 3.7.1).

Again our attempt will be that of connecting the new level of productivity with the old level of average productivity that was in place without trade (see equation 14). By recalling this latter formulae, and by introducing the above notation, it is possible to write the following meaningful result:

$$\pi_{\#}(T) = \pi_{\#}^{NT} + \left(\sum_{i=1}^{k} \pi_i \lambda_{EXi} - \sum_{i=k+1}^{m} \pi_i \lambda_{IMi}\right) - \frac{\Delta\phi}{\phi} \sum_{i=1}^{k} \pi_i \lambda_{DOi}^{NT} \qquad 47$$

$$\text{with } \frac{\phi^{NT}}{\phi} = \frac{\phi - \Delta\phi}{\phi} = 1 - \frac{\Delta\phi}{\phi}$$

In terms of comparative statics, the level of productivity AT without full employment is made up of an additional factor as compared to equation 46. Namely the impact played by the variation of employment on the weight of the domestic sector. In fact the term $\frac{\Delta\phi}{\phi}\lambda_{DOi}^{NT}$ re-modules the weight of each domestic sector – the only kind of sector that it is not been directly affected by the international trade – according to the changes in the overall level of employment occurred AT.

Equation 47 can be further simplified by splitting – as we already did – the economy in four broad sectors: the export and the domestic sector AT. The import sector (i.e. what would have been produced without imports) and again the domestic sector with NT.

The connection between the four broad sectors in terms of employment shares is the following $\lambda_{EX} = \lambda_{IM} - (\lambda_{DO} - \lambda_{DO}^{NT})$ or equivalently as we have just noticed $\lambda_{EX} = \lambda_{IM} + \frac{\Delta\phi}{\phi}\lambda_{DO}^{NT}$. If we introduce these changes in equation 47 we are able to express the new level of productivity as the sum of three meaningful addenda:

$$\pi_{\#}(T) = \pi_{\#}^{NT} + \lambda_{EX}\left(\pi_{EX} - \pi_{IM}\right) - \frac{\Delta\phi}{\phi}\lambda_{DO}^{NT}\left(\pi_{EX} - \pi_{IM}\right) \qquad 48a$$

and since $\lambda_{EX} = 1 - \lambda_{DO} = 1 - \left(\lambda_{DO}^{NT} - \frac{\Delta\phi}{\phi}\lambda_{DO}^{NT}\right)$ we could write more simply:

$$\pi_{\#}(T) = \pi_{\#}^{NT} + \lambda_{IM}\left(\pi_{EX} - \pi_{IM}\right) \qquad 48b$$

Just after specialization, will the comparative advantages assure a higher level of productivity as compared to the situation in place before trade? The answer in this case too is inconclusive. It rests on the sign of the second addenda of equation 48b, which is dependent on two factors

1) the level of labour productivity of the export sector,

2) the level of labour productivity of the import sector that was in activity before trade, but no longer AT.

The normalized share λ_{IM}, in case of NT, weights the impact of the differences of factor 1) and 2). If international trade is neutral to employment (not a necessary condition), this share of labour is precisely the same share employed at time T in the export sector.

The consequence is that the level of labour productivity AT could be higher, equal or lower than it was with NT, according to the following rule:

$$\begin{aligned} &\pi_{EX} > \pi_{IM} && \text{a) level of average productivity AT higher than with NT,} \\ &\pi_{EX} = \pi_{IM} && \text{b) no changes in the average level of productivity,} \\ &\pi_{EX} < \pi_{IM} && \text{c) level of average productivity AT lower than with NT} \end{aligned} \qquad 49$$

The principle of comparative advantages asks that $\frac{\pi_{EX}}{\pi_{IM}} > \frac{\pi_{EX}^{*}}{\pi_{IM}^{*}}$ but it does not require that such ratio should be higher than one. It follows that just AT the level of average productivity of labour can move both upwards or downwards. This may indeed be surprising, since one aspects that the specialization pattern, made according to the RPCA, would produce some productivity gains for the individual country. According to our analysis, however, this is not necessary the case.

3.8.2. Dynamics: Productivity from time T to time T+1

Other important consequence of international trade to examine is the dynamic effect on productivity? The rate of change of the average productivity, according to equation 19, will be:

$$\rho_{\#}(T) = \frac{\sum_{i=1}^{k}\phi_i \frac{\partial}{\partial t}\left[\frac{\sum_{i=1}^{k}\left(c_i + \frac{c_i^*}{\xi}\right)}{\sum_{i=1}^{k}\phi_i}\right]}{\sum_{i=1}^{k}\left(c_i + \frac{c_i^*}{\xi}\right)} \qquad 50$$

As it was done before, to make the analysis tractable, without significant losses of meaning, we could make the following simplifications and notational conventions.

a) We may aggregate the sectors, in four broad sectors as we did in the case of employment: domestic (with NT and AT), export, and import sector.

b) we shall assume few minor things to make the analysis more focused, namely that the dynamics of demand in each sector, abroad and in the domestic economy, is subject to the same rate of change, so as $r_i = r_i^*$, though it remains $r_i \neq r_j; r_i^* \neq r_j^*$.

Taking the original equation 21 with these three conventions we have that the rate of change of productivity AT, is simply equal to $\rho_{\#}(T) = \rho_{EX}$ being the sum of the share of labour and the share of production in the export sector and in the domestic sector equal to 1, due to specialization, while the level and the rate of change of productivity in these two broad sectors is exactly the same. Under these conditions, a comparison between the dynamics of productivity before and AT is straightforward:

$$\rho_{\#}(T) = \rho_{\#}^{NT} + \lambda_{IM}(\rho_{EX} - \rho_{IM}) \qquad 51$$

This is the rate of change of labour productivity AT. It measures just the dynamic effect of the specialization process, without including the static effect. In other words, equation 51 measure just the Schumpeterian effect, having taken into account already in the static considerations just the Keynesian effect. The equation is simple enough. The change of productivity AT is dependent on

a) the level of the productivity with NT;

b) the differences in the Schumpeterian components between the rate of change in the export sector and the (sectors replaced by the) import sector. The difference between the two rates is weighted with the normalized shared of labour in the import sector, if this was in activity AT.

Again, does the comparative advantages assure a rate of $\rho_{\#}$ higher than before trade? The answer is still negative, since the comparative advantages are concerned with the ratios between levels of productivity, not with rates of changes of productivity. Therefore there is uncertainty whether the rate of change of productivity AT will increase. It could well decrease. And given the irreversible effects of specialization, there is no even the possibility to re-establish new patterns of specialization after that thee different rates of change have modified the comparative advantages, i.e. the relative levels of productivity. Here too we may consider three different cases:

$\rho_{EX} > \rho_{IM}$ a) dynamic gains in productivity, 52

$\rho_{EX} = \rho_{IM}$ b) no changes in $\rho_{\#}$ AT as compared with NT,

$\rho_{EX} < \rho_{IM}$ c) dynamic losses in productivity.

If we sum up graphically as we did for employment the comparative static effect and the dynamic effect of the rate of change of average productivity, the qualitative result will be very similar, if not identical with what we have already analyzed for the employment case.

Also in this case the static and the dynamic effects are dependent on different factors, which may cause changes with the same sign or with the opposite sign of the productivity results. Figure 3 shows precisely the combination of the possible outcomes.

[Figure 3 around here]

3.9. The level of prices

Up to now the principle of comparative advantages did not bring any definitive result, and if one adds, the cost of the transition from the export to the import sectors, there is sufficient room of scepticism. From an individual country point of view the gains from trade (up to now in employment and productivity) seems a matter of chance not of certainty.

However, there is no doubt that the RPCA brings a for-sure result. This consists of a drop of the level of prices, for the goods that are imported. Since the level of prices of goods 1 to k, that AT are still produced for domestic consumption, is unchanged (being the technology for them exactly the same of the NT scenario), this will produce an overall drop of the level of prices, and hence an increase of real wages.

3.9.1. Statics: Prices at time T

Formally we can analyze the comparative static effect by starting from equations 11-12 and by adopting the usual division between export sectors and import sectors.

$$p_{\#}(T) = \frac{\sum_{i=1}^{k} p_i c_i + \sum_{i=k+1}^{m} \frac{p_i^*}{\varepsilon} c_i}{\sum_{i=1}^{k} c_i + \sum_{i=k+1}^{m} c_i} \qquad 53$$

with $\varepsilon = p_{\#}^* / p_{\#}^{BT}$

A comparison is possible with the level of prices before trade by adding and subtracting $\sum_{k+1}^{m} c_i p_i$ and by defining $\frac{\Delta l_i}{l_i} = 1 - \frac{l_i^*}{l_i}\frac{l_{\#}}{l_{\#}^*}$, which is the labour ratio that the principle of comparative advantages allows to save (per unit of

product of good i)[13] through imports. At this point the inequality emerged from specialization in the k+1 to m sectors, $\frac{l_i^*}{l_i} < \frac{l_\#^*}{l_\#}$, can be expressed in terms of equality, $\frac{l_i^*}{l_i - \Delta l_i} = \frac{l_\#^*}{l_\#}$. After some re-arrangements and further simplifications, it is possible to formulate the new level of average prices in relation to the old level of average prices:

$$p_\#(t) = p_\#^{NT}(t) - \sum_{i=1+k}^{m} \frac{\Delta l_i(t)}{l_i(t)} \alpha_i p_i(t) = p_\#(t) = p_\#^{NT}(t) - \sum_{i=1+k}^{m} \frac{\Delta l_i(t)}{l_i(t)} \alpha_i w l_i \qquad 54$$

being the level of wage before trade equal to $\pi_\# p_\#$, we can collect the term, simplify and turn out with the following meaningful equation:

$$p_\#(t) = p_\#^{NT}(t)\left(1 - \sum_{i=1+k}^{m} \frac{\Delta l_i(t)}{l_i(t)} \lambda_i\right) \qquad 55$$

Equation 55 tells exactly the channel through which the RPCA serves its gains from trade to a country. And this channel is the level of prices. There is no possibility that the average prices before trade are multiplied by a value higher than one. If this were so, the term Δl_i would have been negative – an outcome that would violate the RPCA. Therefore, as long as there exist a comparative advantage for imports in sector *k+1* to *m*, the term under parenthesis will be lower than one and the average level of prices AT will be lower than in a situation of NT. This static gain from trade, will turn (everything equal) in an increase of real wages. The size of the gain is completely dependent from the weighted average of Δl_i across the imported sectors: if they show high differentials, the gains from trade will be high, other wise it will be low.

3.9.2. Dynamics: Prices from time T to time T+1

The beneficial immediate static effect that trade plays on prices has been just discussed. It is interesting to ascertain whether in terms of prices there is also a beneficial dynamic effect. In other words, the question we are asking is still made in comparative terms: will inflation in a open economy accelerate, remain constant, or decelerate as compared to a situation without international trade?

Equation 22 gives the rate of inflation in a closed economic system. This equation must be modified in an open economy. The measurement of the internal level of prices now must include the import-goods, and exclude the export-ones. The import-goods are the factor, that in the present context, makes a difference: the other goods – produced internally by the sector 1 to k as before – remained, through time, with the same prices they would have found in a situation of NT.

The relevant equation to consider will be equation 55. We could make two minor simplifications. The first is the usual aggregation between, import, export and domestic sectors (AT and with NT), each of which is considered as internally homogenous. The second is the approximation of $-b/(1-b)$ to $-b$ for b close to zero, where b is the weighted ratio of saved labour due to RPCA, $\sum_{i=1+k}^{m} \frac{\Delta l_i(t)}{l_i(t)} \lambda_i$.

[13]It would have been an addition if the ratios were expressed in terms of labour productivity.

In this case the rate of inflation AT will be connected with the rate of inflation with NT according to this relationship:

$$\iota_{\#}(T) = \iota_{\#}^{NT} - \left[\chi + \left(\eta_{IM} - \eta^{NT}\right)\right] \qquad 56$$

with $\chi = \frac{l}{\Delta l}\frac{d}{dt}\frac{\Delta l}{l}$, i.e. the rate of change of saved labour.

The inflation rate AT may be different from the inflation rate with NT as a result of three rates of change:

a) the rate of change of saved labour $\frac{\Delta l}{l}$, due to RPCA at time T;

b) the rate of change of the employment level in those sectors that would have produced domestically the import goods if no specialization would have occurred;

c) the rate of change of the overall employment level with NT.

If the above rates of change are nil, the rate of inflation with NT and AT will be equal. If χ and the difference $(\eta_{IM} - \eta^{NT})$ are both positive, this means that there are further gains through time due to either improved comparative advantages or an expansion of the "weight" of the import sector (or both). If one of both of them are negative, there is the risk that inflation rate will be AT higher than it was in a scenario with NT.

The outcome of a different inflation rate AT as compared with NT rests therefore on the determinants of the three rates of changes we have just discussed. They will be dependent respectively:

$$\chi = \frac{-d}{1-d}\left[\left(\rho_{\#}^{*} - \rho_{IM}^{*}\right) + \left(\rho_{IM} - \rho_{\#}\right)\right] \qquad 57$$

with $d = 1 - \frac{\Delta l}{l} = \frac{l_i^*}{l_i}\frac{l_{\#}}{l_{\#}^*}$

while the rate of change of the difference of the employment rate in the import sector and the employment rate in the whole economy with NT, will be:

$$\eta_{IM} - \eta^{NT} = \left(r_{IM} - r_{\#}^{NT}\right) - \left(\rho_{IM} - \rho_{\#}^{NT}\right) \qquad 58$$

with $r_{\#}$ that represents the average rate of change of overall per capita demand in the economy.

Putting together equation 57 and 58 and substituting the contents in 56 allows to study the conditions in which a variation of average prices AT is different from the level in place with NT. To study the sign of the variation, we shall detect the term under square brackets, as it emerges after some simplifications:

$$\iota_{\#}(T) = \iota_{\#}^{NT} - \left[\frac{d}{1-d}\left(\rho_{IM}^{*} - \rho_{\#}^{*}\right) + \left(r_{IM} - r_{\#}^{NT}\right) + \left(\rho_{\#}^{NT} - \rho_{IM}\right)\right] \qquad 59$$

As long the comparative advantages persist, d<1, the term $\frac{d}{1-d}$ will be positive. In this case $\iota_{\#}$ will differ from $\iota_{\#}^{NT}$ according to the sign and size taken by three addenda 1) the difference between the rate of change of labour

productivity of the imported goods in the foreign country as compared the average change of productivity in the same country, 2) the difference of the rate of change of demand of the import goods as compared to the average change of the overall demand in the domestic country, 3) the difference between the rate of change of average productivity in the sectors k+1 to m that the domestic country would have experienced with NT. According to the trend taken by these three addenda, there are three possible cases to examine:

a. The inflation $\iota_{\#}$ will be certainly lower than $\iota_{\#}^{NT}$ if addenda, 1), 2) and 3), are all positive, or at least their sum is positive.

b. The inflation $\iota_{\#}$ will be equal to $\iota_{\#}^{NT}$ if all of the three addenda, 1), 2), and 3), are zero or their sum is zero.

c. The inflation $\iota_{\#}$ will be certainly higher than $\iota_{\#}^{NT}$ if all three differences are negative or their sum is negative.

The dynamic equation 61, as it does equation 56, does not bring the same conclusion of equation 55 which emerged from the static analysis. From a dynamic point of view there is no guarantee that the drop of prices (the gain from trade) will be persistently maintained in the future. The size of the drop of price may decrease driving in this way the economy towards a higher level of inflation than the level experienced with NT. International specialization is usually irreversible (in our model) or very little reversible (in reality), so there is also the possibility at some point that a higher rate of inflation AT nullifies the lower level of prices, and the gain embodied in it. So if situation c) occurs also the static price gain discovered so far (the static drop of prices) will turn at some point into a loss from trade. To sum up, Figure 4 will give an impressionistic representation of the two possible outcomes: static and dynamic gains vs. static gains and dynamic losses.

[Figure 4 around here]

3.10. Level of income after trade

The final and the most important variable to scrutinize is the level of per capita income and its rate of change. As in the previous cases we shall attempt to make a comparison by dividing the effects in two: static effects that follows immediately international specialization, and the dynamic effects on the rate of growth of per capita income that could emerge from specialization thereafter.

3.10.1. Statics: Income at time T

The starting point in assessing the static potential gains from trade in terms of income, is equation 16. The difference with the situation of NT consists on the pattern of specialization: now production is concentrated only in the first k sectors. The average productivity, that refers to these first k sectors, has been already determined. Also the average level of prices of the new specialized economy has been determined. But this is not the average level of prices needed to calculate the level of income. In equation 16, the average level of prices should refer only to the bundle of goods (i.e. the sectors) in which the country has specialized. This does not coincide to the average level of prices that we have just calculated, since they include the imported goods, that should not be part of the calculation of income, while they exclude the export prices, which instead are necessary for the calculation of income. Therefore equation 16

requires a different average level of prices. A way of obtaining it is to consider equation 15, which is obviously still in place, for the first k sectors. Consequently the nominal level of per capita income AT will be:

$$y_{\#}(T) = \sum_{i=1}^{k}(c_i + \frac{c_i^*}{\xi})p_i = w\sum_{i=1}^{m}(c_i + \frac{c_i^*}{\xi})l_i = w[1-u] \qquad 60$$

This equation tells that, AT, there will be no change of nominal per capita income if the process of specialization does not bring any change of the level of unemployment. If it does bring changes of unemployment the level of per capita income will change too in the opposite direction. Formally:

$$y_{\#}(T) = y_{\#}^{NT}\left[1 - \frac{\Delta u}{\phi^{NT}}\right] \qquad 61$$

with $\Delta u = u - u^{NT}$ and $\phi^{NT} = 1 - u^{NT}$

It may be argued that equations 60 and 61 determine the nominal level of per-capita income, while it would be interesting to inquiry the *real* level of per-capita income. In this case it is necessary to know the average level of prices of the first k sectors, which represent AT the whole economy from the production side. Re-considering equation 12, the formulae that connects the new level of average price to the old level of average price is the following (the new average level of price AT has been denoted with the apex EX to not be confused with the "internal" level of prices AT given by equation 55):

$$p_{\#}^{EX}(T) = p_{\#}^{NT}\left[\frac{\sum_{i=1}^{k}c_i + \sum_{i=k+1}^{m}c_i}{\sum_{i=1}^{k}c_i + \sum_{i=1}^{k}\frac{c_i^*}{\xi}}\right] + \frac{w\left[\sum_{i=1}^{k}\frac{c_i^*}{\xi}l_i - \sum_{i=k+1}^{m}c_i l_i\right]}{\sum_{i=1}^{k}c_i + \sum_{i=1}^{k}\frac{c_i^*}{\xi}} \qquad 62$$

The average level of prices for the calculation of income AT will be equal to the level before trade if two conditions occur: a) there is a balance between the size of the export demand and the size of the import demand of goods (the first square-bracket addenda turns to 1), b) there is no losses or gains of employment (the second square-bracket addenda goes to zero). In all other cases the level of prices before and AT for the calculation of income will be different. In the still general case in which the level of *nominal* wages is left unchanged in the open up of the economy (as it is perfectly logical in theory and it is a common behaviour in practice), equation 62 turns simply in:

$$p_{\#}^{EX}(T) = p_{\#}^{NT}\frac{\pi_{\#}^{NT}}{\pi_{\#}} \qquad 63$$

By knowing the new level of average prices, it is possible to rewrite equation 60 in terms of average productivity:

$$y_{\#}(T) = \pi_{\#}p_{\#}^{EX}[1-u] \qquad 64$$

Moreover, from the above equation and from the definition of unemployment differentials, given in equation 61, the real level of per capita income $y_{\#r}(T) = y_{\#} / p_{\#}^{EX}$, identified with the pedix *r,* turns out to be related, after some manipulations, with the real per capita income of NT as follows:

$$y_{\#r}(T) = y_{\#r}^{NT} \frac{\pi_{\#}}{\pi_{\#}^{NT}} \left[1 - \frac{\Delta u}{\phi^{BT}} \right] \qquad 65$$

This seems a significant result. It tells that ultimately, in real terms, the per capita income AT is related to the real per capita income with NT, by two ratios:

1) The ratio between average productivities: in absence of additional unemployment, only if the average productivity AT is higher than the average productivity before trade, the real per capita income will increase.

2) The ratio between the employment level AT and the employment with NT: in absence of differences of average productivities only an increase of employment AT can assure, in real terms, a higher per capita income.

Are these two circumstances possible? As it has been discussed in the previous sections, both the new level of average labour productivity and the new level of employment are not univocally determined in their sign by the RPCA. Therefore, also the real income AT may result both higher or lower as compared to the real income before trade. Here, again, the RPCA does not assure a positive gain after all.

3.10.2. Dynamics: Income from time T to time T+1

Dynamically, we are left with the calculation of the rate of change of per-capita income. The equation to be confronted with is equation 24. In a closed economy, considerations on the inflation rate were less important, since the price system left us open two degrees of freedom. Either the choice of the numéraire or – alternatively – the choice of a wage path. An appropriate choice could maintain a stable level of prices overtime. We have seen, in the first part of the paper, the conditions that make this possible, in particular by fixing the level and the rate of change of the wage unit (see equations 26-27). Now, to make a comparison between AT and NT, there is the need of not modifying the latter path, so that the new average level of prices should respond only to the new basket of goods produced and the new level of productivity. The consequence is that the degrees of freedom, that were available under NT, are not longer there, and inflation (or deflation) should be considered explicitly in assessing gains and losses of income.

Considering, first, the nominal rate of growth of per capita income, the starting point is equation 24. Wages continue AT with the same nominal path that they would have been under NT. Taking equation 63 in a dynamic setting, allow us to determine AT the rate of inflation (for the calculation of income), which is equal to

inflation for $$\iota^{EX} = \iota^{NT} + \rho_{\#}^{NT} - \rho_{\#} \qquad 66$$

Also the inflation to be used for the gross domestic product is AT undetermined as compared the situation of NT. The crucial role in determining the sign of the variation is played by the difference between the rates of change of average productivities: if $\rho_{\#}^{NT} > \rho_{\#}$ also the inflation ι^{EX} will be higher than it was under NT. Otherwise it will be lower.

Let us examine first, from equation 61, the nominal rate of growth. It will be equal to:

$$\gamma\,(t) = \gamma^{NT} + \left(\Delta r_{\#} - \Delta\rho_{\#}\right) \qquad 67$$

again with $\Delta r_{\#} = r_{\#} - r_{\#}^{NT}$, and $\Delta\rho_{\#} = \rho_{\#} - \rho_{\#}^{NT}$

The meaning of equation 67 is immediate. The rate of growth of per capita income AT will be equal in nominal terms to the rate of growth in place before trade, if the open up of the economy does not produce changes of the rate of employment as compared to the situation of NT. If the rate of employment drops (unemployment goes up), the rate of growth will be slower. If the rate of employment rises up, the rate of growth of per capita income will be higher.

The point that emerges from equation 67 is interesting, but rests on the fact that it is expressed in nominal terms. A comparison between the real rates of growth, purged from the monetary effect, becomes therefore necessary. It will be the comparison of these real values (with NT and AT), which will indicate whether the RPCA assures some gains from trade in terms of per capita income or not.

Taking equation 64 the real rate of growth will be defined, in general, equal to $\gamma_r = \gamma - \iota = \rho_{\#} + (r_{\#} - \rho_{\#})$. After some elaborations to introduce γ_r^{NT}, or by re-starting from the dynamic aspect of equation 65, the real rate of growth of per capita income will be:

$$\gamma_r(T) = \gamma_r^{NT} + \Delta\rho_{\#} + \Delta\eta \qquad 68$$

with $\Delta\eta = \Delta r_{\#} - \Delta\rho_{\#}$

The real rate of growth of per capita income may diverge from the rate experienced under NT, only if there are differences in:

1) the rate of change of the average productivity,

2) in the rate of change of employment.

If we are already with NT in a situation of full employment, obviously only the Schumpeterian component of the rate of variation of the average productivity can make a contribution to the real growth of per capita income. If, instead, the situation of NT experienced a positive rate of unemployment, trade could contribute to the growth of income also through the absorption of a higher level of employment.

Equation 68 focus the attention on the supply side. But we could emphasize in the same equation the demand side, if it is this side that constitutes the real constrain in the growing of income. A basic simplification turns equation 68 simply in:

$$\gamma_r(T) = \gamma_r^{NT} + \left(r_{\#} - r_{\#}^{NT}\right) \qquad 69$$

It tells us that with specialization, as well as without it, an acceleration of growth of per capita income is *only* possible if there is an acceleration in the dynamics of demand. As in all other dynamic aspects, also in this case we may observe that an increment of $r_{\#}$ is not related to the RPCA. By decomposing the average rate of change of per-capita demand and by applying the results obtain to simplify equation 42, the rate of change of per capita income can be rewritten as follows

$$\gamma_r(T) = \gamma_r^{NT} + \lambda_{IM}\left(r_{EX} - r_{IM}\right) \qquad 70$$

The question of whether the rate of growth AT will be systematically higher than it would have been under NT receives here too an undetermined answer. The two rates of change of per capita demand that are added to γ_r^{NT} may bring a negative as well as a positive result. As we have seen in Section 3.7.1 and in Section 3.8, the final result rests on

the sign of the difference between the growth of per capita demand of export and import sectors. The consequence is that the real rate of growth of per capita income, after international specialization is left to the following open scenario:

$r_{EX} < r_{IM}$ a) dynamic losses of per capita income

$r_{EX} = r_{IM}$ b) no dynamic changes of per capita income (71)

$r_{EX} > r_{IM}$ c) dynamic gains of per capita income

This means that the principle of comparative advantages is necessary an engine of growth from an individual country. It may sometimes turn in a brake of growth. To summarize this conclusion, Figure 5 gives also in this case the possible combination of outcomes, both for the static as well as for the dynamic case.

[Figure 5 around here]

4. Discussion of the results

The connection between trade and growth in a pure labour economy subject to structural change and specialization is not a simple matter. The patterns of trade, in our model, are made according to the RPCA. The model adopted some usual simplifications (full specialization, two trading partners, no joint production), it has rejected many other assumptions, in particular it assumed the presence of many goods, with different productivity and demand levels and different rate of changes. It allowed, discussed, and inquired about the threat of unemployment, the potential static gains as well as the potential dynamic ones, if any. The following observations are derived from the above analysis, and are quite complementary to it.

1. When the rate of exchange between currencies reflects the actual (average) productive capacity of an economy, or to put it shortly, it is fixed in terms of PPP, it is possible to deal with comparative advantages in terms of international prices, that is by turning all prices of one country in that of another country, and then compare what is cheaper. If this is done, the international market works efficiently in the sense that it reveals the comparative advantages, by simply looking at international prices.

2. However, when we allow for many sectors, different technology, different demand, there is no guarantee that the comparative advantages exists for both countries. There is the possibility, which is not just hypothetical, of *unilateral* comparative advantages, rather than *bilateral* (or with many countries *multilateral*) comparative advantages. The case of *unilateral* comparative advantage means that one country will possess an international advantage (in terms of costs) to sell – and to produce, before to sell – every possible good, and the other country (or countries) will have on the international market no advantage at all.

3. If comparative advantages exist for all trading parties (in our case for both parties), a pattern of international specialization made according to this principle will produce some gains from trade. These gains to be positive need the support of strong hypotheses, the main of which is that each country that takes "advantage" of the Ricardian principle, and specializes accordingly, should not drop its level of employment. In an industrial world where the under-utilization of productive capacity is a constant issue, this assumption so crucial in the model, becomes nonsensical in reality.

4. Even if – a big "if" - the above assumption of full employment (or no drop of employment) is fulfilled, a system of international relations based on prices equal costs and exchange rates equal Purchasing power, as described above, does not guarantee that the *global* "gains from trade" are shared by all the parties involved. There is instead the constant

possibility that some countries (or a country in our case) capture all the global "gains from trade", and the other countries do not have any positive gain from trade. In other words, it is not true that the international market makes available positive gains from trade for all participants. International trade, for some of them, may systematically result in a "negative game".

5. The gains from trade are typical global gains "once for all", in the sense that they appear just in the switch from no-trade to trade situations. However the pattern of specialization involved according to the comparative advantage principle, do have long term consequences on the *dynamics* of each economy. Therefore, the gains – again if any – are as much as "once for all" as they are dynamical. However, there is no guarantee either that each country that opens its economy to trade will enjoy a higher long run rate of growth than in a situation of no-trade. It is indeed possible that the opposite occurs, i.e. that an open economy grows slowly than it would have been in an autarky system. It also emerges a clause of no correlation between the possible gains "once for all" and the possible "dynamic" gains from international trade specialization. All possible combinations are possible. A country may gain in the short run and pay the bill with a lower growth in the long run, it may not gain (and even lose) in the short run and gain a faster growth in the long run, it may have both in the short and in the long run a win-win situation, but it cannot exclude also a lose-lose situation as compared to the case of no-trade.

6. The RPCA has appeared inconclusive in its gains on seven over eight variables that we have examined. For employment (static and dynamic), productivity (static and dynamic), per capita income (static and dynamic) and prices (dynamic) the principle does not assure, from an individual country point of view, any certain gain. There is however a for-sure-result in the RPCA, and this consists in the drop of prices "once-for-all" at the open up of the frontiers to trade. This gain allows (temporarily) a better welfare, since the real wages, everything equal, do increase. However, contrary to the common sense, this is not enough to conclude that also the gross per capita income will (temporarily) rise. We have shown that this too cannot be assured. If an average consumer is so unfortunate to live in an economy with sticky prices, then there is the real – unfortunate - possibility that he or she could not catch any advantage from international trade: with eight variables over eight variables that may not move in the right direction. This could help to explain why professional people have always been much more sceptical than theoretical economists about the effectiveness of this principle, took alone, in guiding the trade policies of their own country.

5. Limitations and possible extensions

In summing up the main results, or at least those that appear from my point of view more relevant, there is the impression that the message which emerges is not precisely in line with the mainstream one. This may not surprise completely practical people, which deal daily with the problem of international competition, but it may appear in some respect puzzling for theoreticians.

Did the model here proposed impose *ad hoc* assumptions such as to drive specifically these results? Or to put it in another way, what are the limitations of the model? The abstraction of a "pure labour economy" has been already made clear in the title and discussed at the beginning, so here I shall focus on other possible limitations.

First, the model does not touch the chapter of international finance. It simply focuses on the real aspects of an economy. It discusses (briefly) prices but mainly with the purpose of finding (or avoiding) inflationary or deflationary effects in a world where the price of each good changes overtime and there is no way to keep individual prices constant. Leaving aside finance means that some of our variables that in reality are affected by it (think at the exchange rate), in our model are not. And this does not count as a plus for the model.

Second, if the intermediate goods do not present an insurmountable problem for this model, natural resources and more in general non reproducible goods do present a problem. The latter follow a theory of scarcity, and not a theory of labour value – their price is proportional not with the "effort" spent in producing them, but with the degree of "raretè", to use Walras expression, with which they appear in the market. Since a share of international exchange is made by these commodities, it would be advisable to include them in the picture when discussing of economic international relations. But they did not in the present model.

Third, the model has assumed many sectors, but within each sector it has assumed a unique way of producing it. If this may be acceptable as a first approximation, it is less acceptable, when the real life shows sectors with thousand or at least hundreds of industrial units, each of which with its own "production function". What this variety of industrial units will allow to explain is the intra-industrial trade, and not just as we did the inter-industrial trade, with full specialization.

Forth, having focused almost exclusively on the real aspects of the matter, the model also leaves out any discussion of practices of "strategic" trade, in which countries tweak the price of goods, or the exchange rate in the view of gaining market shares. Some of these practices are very interesting to discuss, because highlights the problems of "free riding" but also the problems of different internal (and legitimate) institutional arrangements, which drive very important international consequences.

Fifth, the model does not discuss what happens either in those markets that are not the commodity market. For instance the labour market, and the issue of possible migration from one economy to another has not been dealt with. The knowledge "market" and the problem of international learning have been only touched briefly. Yet, these are issues that in a globalize world appear of major interest, because people and ideas move from one country to another as much as goods do.

And finally the model has overlooked, by paying little or no attention, to the institutional problem that the economic international relations arise. In our discussion we have pint-pointed how troublesome can the gains from trade be, but we did not turn our analyses in discussing if -and what – international institutions would be able to make all trading participants happy, without leaving any of them bruised.

As much these limitations should be kept in mind when discussing the results, as they appear interesting topics when looking at the possibility of extending the model. With the possible exclusion of the first limitation, which earthquakes the foundation of the production paradigm that our model implies, all other limitations are indeed integrable in the theory here proposed.

6. Conclusions

The paper presented a model on economic international relations. Focus has been given to the issue of short and long term effects on economic growth. A multisectoral, pure labour, economy has been examined *before* and *after* trade and the consequences of international specialization discussed. Under scrutiny was the principle of comparative advantages – still the main theoretical driving force in international economics.

Ricardo (1817) formulated this principle, just when the British crown was starting to establish its commercial and military power at global level. Yet the principle passed through with an anti-mercantilist attitude (as it was) and with a message of hope. No matter how bad or poor countries were, international trade could do something good for each of them. Trade is a positive-sum game. If the principle of comparative advantages is not violated, and few other

conditions are fulfilled, there are gains from trade which lead to an increase in welfare for all parties involved —a typical case of what economists call (sometimes reluctantly) a free lunch.

What emerges from our analysis is that the "free lunch" may not be for all, and it may not certainly last for ever –at least when one considers strictly the gains from trade of the commodity market. There is in the principle of comparative advantages, as we have examined it, a sort of "Trilussa's paradox".[14] On average it may be that international trade offers an additional lunch (if international demand does not bind and other conditions are fulfilled), but this is just an average of different potential situations. It may be that some trading countries count a substantial surplus, others count almost nothing, and still some other countries lose their two spoons of sup that they were previously making (and eating) by themselves.

When the unit of analysis is the individual economy, trade may promote as well as may endanger growth. This is not a call for protectionism, but it is not a call for an unquestionable liberalization either. It seems to create room for a political economy at international level. Practical people probably know the problems discussed in this paper much better of what theoretical economist are used to.

When, some forty years ago, Max Corden (1965) surveyed the field of international trade, he already perceived an unsatisfactory atmosphere surrounding the discipline:

> 'It must be confessed, in conclusion, that the pure theory of international trade has suffered from bad public relations. Some of its main conclusions are often misunderstood, and, even when understood, very often disagreed with. There are two reasons for this. Firstly, the models of the pure theory usually make a large number of assumptions, some of which when stated explicitly sound so unrealistic as to discredit the whole model from the start, while others tend to be forgotten.(...) The second reason for the poor image in some countries of trade theory is the commitment to free-trade liberalism of many of the leading theorists.'

Since then, some important progress has been made, and a new flock of models took the fashionable name of "new trade theory". However these "new" models largely belong to the same basic paradigm with which Corden (1965) confronted to. What we have attempted here is to tackle the problem of international relations from a different paradigm: the one that belongs to Classical-Keynesian economics, where production - not exchange - is central, and where the demand side - not the supply side - sets the level of activity of each economic system. Our concern was Corden's concern: to be relevant by avoiding unrealistic assumptions. We attempted to confront in fact with most of the important questions on which people care about: employment, level of productivity, rate of change of productivity, income and economic growth. In all these fronts, the RPCA appears inconclusive. The only benefit it brings for-certain is a static gain from a drop of internal prices. The drop may be important. Yet, probably not so important to consider – in theory as well as in real life – the RPCA an unconditional truth.

7. References

Araujo, Ricardo Azevedo, Joanilio Teixeira. 2004a. A Pasinettian Approach to International Economic Relations: the Pure Labour Case, *Review of Political Economy*, 16:1:117-29.

Araujo, Ricardo Azevedo, Joanilio Teixeira. 2003. An extension of the structural change model to international economic relations, *Metroeconomica* 54(4): 458-73.

[14] See his tales in the English translation by Salustri (1990).

Araujo, Ricardo Azevedo, Joanilio Teixeira. 2004b. Structural economic dynamics: an alternative approach to uneven development, *Cambridge Journal of Economics*, 28: 5: 705-717.

Corden, Max W. 1965. Recent Developments in the Theory of International Trade, *Princeton Studies in International Economics*, 14, Princeton: Princeton University.

Dosi, Giovanni, Keith Pavitt, Luc Soete. 1990. *The Economics of Technical Change and International Trade*, Brighton: Wheatsheaf.

Fagerberg, Jan. 1988. International Competitiveness. *The Economic Journal* 98, 391 (June): 355-374.

Francisco, Louca. 1997. Turbulence in economics: an evolutionary appraisal of cycles and complexity in historical processes, Cheltenham: Edward Elgar.

Freeman, Christopher, Luc Soete. 1997. *The Economics of Industrial Innovation*, London: Pinter Books. 3nd ed.

Freeman, Christopher. 1982, *The Economics of Industrial Innovation*, London: Pinter Books. 2nd ed.

Heckscher, Eli. 1919. The Effect of Foreign Trade on the Distribution of Income, Engl. ed., in Ellis H. and Metzler L. eds., 1949. *Readings in the Theory of International Trade*, Irwin, Homewood.

Krugman, Paul. 1979. A Model of Innovation, Technology Transfer and the World Distribution of Income, *Journal of Political Economy,* 87: 253-266.

Krugman, Paul. 1991. *Geography and Trade*, Cambridge Mass.: The MIT Press.

Leamer, Edward E. 1985. Sources of International Comparative Advantage: Theory and Evidence. Cambridge Mass.: The MIT Press.

Leontief, Wassily. 1953. Domestic Production and Foreign Trade: The American Capital Position Re-examined, *Proceedings of the American Philosophical Society*, 97: 332-349.

List, Friedrich. 1848. *The National System of Political Economy,* English. ed. 1950, London: Macmillan.

Magnusson, Lars. 2004. *The Tradition of Free Trade*. London: Routledge.

Maneschi, Andrea. 1998. Comparative Advantage in International Trade: A Historical Perspective. Cheltenham, U.K.: Edward Elgar.

Mccombie, John, Anthony Thirlwall. 1997. Economic growth and balance-of-payments constraint revisited, in Arestis, P., Palma, G. & Sawyer, M. eds. *Markets, Unemployment and Economic Policy*, Vol. 2, London: Routledge.

Mccombie, John, Anthony Thirlwall. 2004. *Essays on balance of payments constrained growth – theory and evidence*, London: Routledge.

Mill, John S. 1848. *Principles of Political Economy*, ed. 1964, London: Ashley Longmans.

Ohlin, Bertil. 1933. *Interregional and International Trade*, Cambridge, Mass., Harvard University Press.

Pasinetti, Luigi L. 1993. Structural Economic Dynamics. Cambridge: Cambridge University Press.

Pasinetti, Luigi L. 1981. Structural Change and Economic Growth: A Theoretical Essay on the Dynamics of the Wealth of Nations. Cambridge: Cambridge University Press.

Ricardo, David. 1817. *Principles of Political Economy and Taxation,* in Sraffa P. (ed.) (1952), *Works and Correspondence of David Ricardo*, vol. I, Cambridge, Cambridge University Press.

Ruffin, Roy J. 2005. Debunking a Myth: Torrens on Comparative Advantage. *History of Political Economy* 37(4), (Winter): 711-722.

Salustri, Carlo Alberto. 1990. *Tales of Trilussa*. University of Arkansas Press.

Samuelson, Paul, A. 1969. The Way of an Economist. In International Economic Relations: Proceedings of the Third Congress of the International Economic Association, ed. Paul, A. Samuelson, 1-11. London: Macmillan.

Samuelson, Paul. 1948. International Trade and the Equalisation of Factor Prices, *Economic Journal*, 58: 163-84.

Smith, Adam. 1776. *An Inquiry into the Nature and Causes of the Wealth of Nations*, ed. by Cannan, (2 vols.), London: Methuen.

Torrens, Robert. 1808. *The Economists Refuted*. London: S.A. Oddy.

Torrens, Robert. 1815.*Essay on the External Corn Trade*. London: J. Hatchard.

Figures

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– A theory of international relations"
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FIGURE 1: Timing in modelling international trade

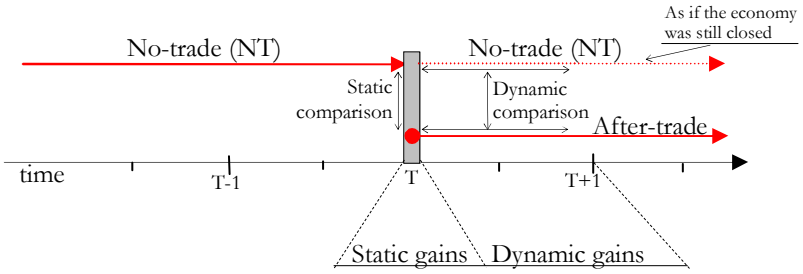

FIGURE 2. Employment: statics and dynamics after trade with comparative advantages

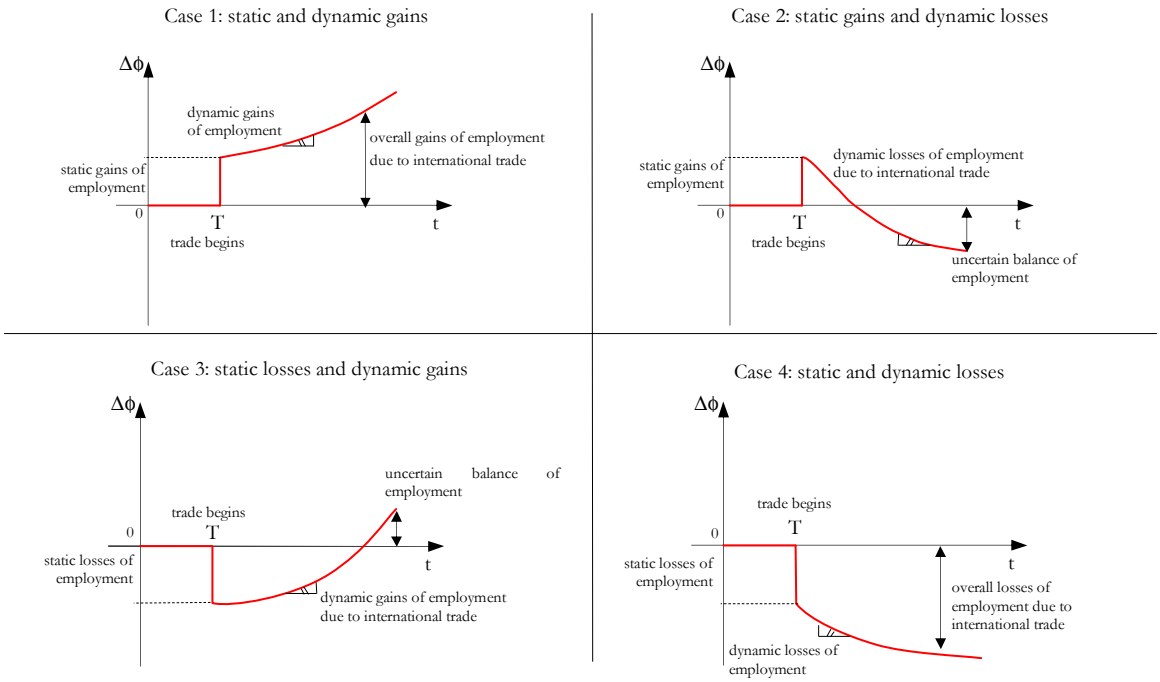

FIGURE 3. Productivity: statics and dynamics after trade with comparative advantages

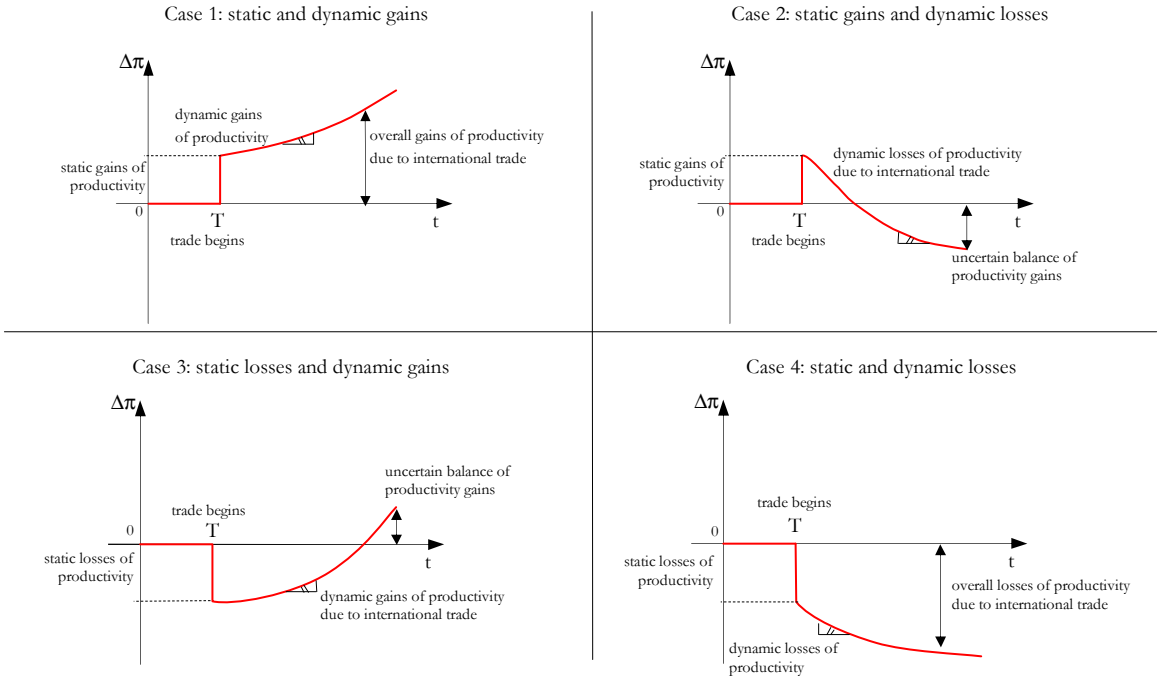

FIGURE 4. Average prices: statics and dynamics after trade with comparative advantages

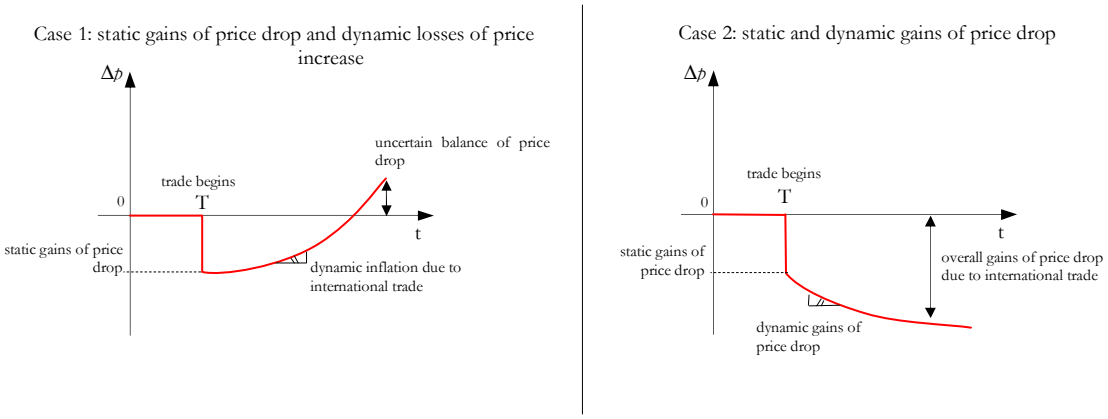

FIGURE 5. Per capita income: statics and dynamics after trade with comparative advantages