



Working Paper Series  
Department of Economics  
University of Verona

# Money illusion and the long-run Phillips curve in staggered wage-setting models

Andrea Vaona

WP Number: 14

September 2010

ISSN: 2036-2919 (paper), 2036-4679 (online)

# Money illusion and the long-run Phillips curve in staggered wage-setting models

Andrea Vaona<sup>1</sup>

University of Verona (Department of Economic Sciences), Palazzina 32

Scienze Economiche - ex Caserma Passalacqua, Viale dell'Università 4,

37129 Verona, Italy. E-mail: andrea.vaona@univr.it. Phone:

+390458028537

Kiel Institute for the World Economy

---

<sup>1</sup>The author would like to thank Dennis Snower for helpful conversations. The usual disclaimer applies.

# Money illusion and the long-run Phillips curve in staggered wage-setting models

## Abstract

We consider the effect of money illusion - defined referring to Stevens' ratio estimation function - on the long-run Phillips curve in an otherwise standard New Keynesian model of sticky wages. We show that if agents under-perceive real economic variables, negative money non-superneutralities will become more severe. On the contrary, if agents over-perceive real variables, positive money non-superneutralities will arise. We also provide a welfare analysis of our results and we show that they are robust to the inclusion of varying capital into the model. In an appendix, we investigate how money illusion affects the short-run effects of a monetary shock.

**Keywords:** Phillips curve, inflation, nominal inertia, monetary policy, dynamic general equilibrium, money illusion, Stevens' ratio estimation function.

**JEL classification code:** E3, E20, E40, E50.

# 1 Introduction

Money illusion has recently attracted renewed attention by the literature - see for instance Fehr and Tyran (2001, 2007) and the literature quoted there.

The way we model money illusion here builds on Stevens (1946, 1951), which has been at the centre of an extensive literature surveyed for instance in Graham (1958), Anderson (1970), Schepard (1981), Luce and Krumhansl (1988) and Michell (1999).

We consider money illusion a biased subjective way economic agents have to evaluate ratios of nominal variables. Suppose an individual receives two pieces of information: his/her nominal wage,  $W$ , and the general level of prices,  $P$ . S/he will have to estimate her/his real wage as a ratio of the two above on the basis of her/his ratio estimation function  $F = f(W, P)$ . Stevens conjectured that subjective values are powers of real values, so that Stevens' ratio estimation function  $f(W, P)$  can be written as:

$$f(W, P) = \left(\frac{W}{P}\right)^\xi$$

Such a distortion could arise for at least two reasons. Economic agents are often found to evaluate economic magnitudes combining real and nominal assessments, given that they have a nominal anchor and, at the same time, they are aware that nominal and real values differ (Shafir et al., 1997). Furthermore, following Pelham et al. (1994), there might be a numerosity effect, whereby people sometimes judge quantity on the basis of the number of units

into which a stimulus is divided without fully considering other important variables - on this point see, for instance, Wertenbroch et al. (2007) and the literature quoted therein<sup>2</sup>.

We nest Stevens' original idea in an otherwise standard New Keynesian model of sticky wages with trend inflation. The effect of trend inflation on output in New-Keynesian models has been the subject of a number of studies by now. Pioneering contributions on this issue were King and Wolman (1996) and Ascari (1998). The former study considers a model with a shopping time technology and it obtains a number of different results, among which there is that long-run inflation reduces firms' markup, boosting the level of output. Ascari (1998), instead, shows that in wage-staggering models money can have considerable negative non-superneutralities once not considering restrictively simple utility and production functions. Devereaux and Yetman (2002) focused on a menu cost model. An analysis of dynamic general equilibrium models under different contract schemes in presence of trend inflation was offered in Ascari (2004). Graham and Snower (2004), instead, examined the microeconomic mechanisms underlying this class of models. In presence of Taylor wage staggering, in a monopolistically competitive labour market, they highlight three channels through which inflation affects output: employment cycling, labour supply smoothing and time discounting. The first one consists in firms continuously shifting labour demand from one cohort

---

<sup>2</sup>Both these mechanisms tend to rule out the possibility for economic agents to *learn* the actual value of real variables.

to the other according to their real wage. Given that different labour kinds are imperfect substitutes, this generates inefficiencies and tends to create a negative inflation-output nexus. The second one is that households demand a higher wage in presence of employment cycling given that they would prefer a smoother working time, decreasing labor supply and aggregate output. Finally under time discounting the contract wage depends more on the current (lower) level of prices than on the future (higher) level of prices and, therefore - over the contract period - the real wage will be lower the greater is the inflation rate, spurring labour demand and aggregate output. The time discounting effect dominates at lower inflation rates, while the other two effects at higher inflation rates, producing a hump-shaped long-run Phillips curve.

Graham and Snower (2004) was extended in a number of different directions. Graham and Snower (2008) showed that under hyperbolic time discounting positive money non-superneutrality is more sizeable than under exponential discounting. Vaona and Snower (2007, 2008) showed how the shape of the long-run Phillips curve depends on the shape of the production function. Finally, Vaona (2010) extended the model by Graham and Snower (2004) from the inflation-output domain to the inflation-real growth one.

The present contribution shows that the shape of the long-run Phillips curve changes for different values of  $\xi$ . The rest of this paper is structured as follows. Section 2 illustrates our baseline model. We then move to its calibration and solution. Section 4 sets out our results and their underlying intuition, also providing a welfare analysis of theirs. Section 5 extends the

model by considering varying capital. The last section summarizes our findings and concludes. In the appendix we consider how money illusion affects the impact of a temporary monetary shock on the model.

## 2 The baseline model

### 2.1 Firms' cost minimization problem and the government

The model here presented only marginally departs from the one by Graham and Snower (2004). Firms populating the final perfectly competitive product market produce an homogeneous output and they minimize their total real cost subject to their production function

$$\begin{aligned} \min_{n_t(h)} \int_{h=0}^1 \frac{W_t(h)}{P_t} n_t(h) dh \\ \text{s.t. } y_t = n_t = \left[ \int_0^1 n_t(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}} \end{aligned}$$

where  $W_t(h)$  and  $n_t(h)$  are respectively the nominal wage and the working time of household  $h$ ,  $y_t$  is output,  $\theta_n$  is the elasticity of substitution among different labour types and  $P_t$  is the general level of prices, which, given that we assume perfect competition in the final product market, equals the price of the homogenous final product.

Taking first order conditions one can obtain the following equations

$$n_t(h) = \left[ \frac{W_t(h)/P_t}{\lambda_n} \right]^{-\theta_n} n_t \quad (1)$$

$$\lambda_n = \left\{ \int_0^1 \left( \frac{W_t(h)}{P_t} \right)^{1-\theta_n} dh \right\}^{\frac{1}{1-\theta_n}} \quad (2)$$

where (1) is the demand for labour services of household  $h$  and (2) is an aggregate real wage index, whereby  $\lambda_n$  - the Lagrangian multiplier - can be renamed  $W_t/P_t$ .

The government rebates its seigniorage proceeds to households by means of lump-sum transfers,  $T_t(h)$ . Therefore:

$$\int_0^1 \frac{T_t(h)}{P_t} dh = \int_0^1 \frac{M_t(h)}{P_t} dh - \int_0^1 \frac{M_{t-1}(h)}{P_t} dh$$

where  $M_t(h)$  is money holdings of household  $h$  at time  $t$ .

## 2.2 Households' maximization problem

Households maximize their discounted expected utility

$$\max_{\{c_{t+i}(h), W_{t+N_i}(h), B_{t+i}(h), M_{t+i}(h)\}} \sum_{j=0}^{\infty} \sum_{i=jN}^{(j+1)N-1} E \left( \beta^i U \left\{ c_{t+i}(h), n_{t+i}(h), \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi \right\} \right)$$



subject to a series of income constraints perceived as follows

$$c_{t+i}(h) = \left[ \frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi n_{t+i}(h) + \left[ \frac{T_{t+i}(h)}{P_{t+i}} \right]^\xi - \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[ \frac{M_{t+i-1}(h)}{P_{t+i}} \right]^\xi - \left[ \frac{B_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[ \frac{B_{t+i-1}(h)}{P_{t+i}} \right]^\xi \iota_{t+i}$$

where  $\beta$  is the discount factor,  $E$  is the expectation operator,  $U$  is the utility function,  $c$  is consumption,  $B$  are private bond holdings and  $\iota$  is the nominal interest rate. The presence of  $\xi$  descends from the fact that economic agents are not able to properly assess ratios and, therefore, real variables. The actual real budget constraint can be obtained setting  $\xi = 1$ . Further constraints derive from the demand for labour services (1).

The first order condition with respect to  $W_{t+N_i}(h)$  turns out to be particularly important for the solution of the model

$$\sum_{i=0}^{N-1} E \left\{ \beta^i |U_n(\cdot)| (-1) (-\theta_n) \left[ \frac{W_{t+i}(h)}{W_{t+i}} \right]^{-\theta_n-1} \frac{1}{W_{t+i}} y_{t+i} \right\} = \sum_{i=0}^{N-1} E \left\{ \lambda_{t+i}(h) \beta^i (\theta_n - \xi) \left( \frac{1}{P_{t+i}} \right)^\xi y_{t+i} \left( \frac{1}{W_{t+i}} \right)^{-\theta_n} W_{t+i}(h)^{(\xi-\theta_n-1)} \right\} \quad (3)$$

where  $U_n(\cdot)$  is the first derivative of  $U(\cdot)$  with respect to  $n_{t+i}(h)$ .

Keeping in mind that  $W_{t+i}(h)$  is constant through the contract period,

one can substitute (2) into (3) to obtain

$$\sum_{i=0}^{N-1} E \left[ \beta^i |U_n(\cdot)| (-1) (-\theta_n) n_{t+i}(h) \right] = \sum_{i=0}^{N-1} E \left\{ \lambda_{t+i}(h) \beta^i (\theta_n - \xi) \left[ \frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi n_{t+i}(h) \right\} \quad (4)$$

Given that households are symmetrical and have the same first order condition for consumption -  $U_c [c_{t+i}(h), \cdot, \cdot] = \lambda_{t+i}(h)$  -, they will all consume the same quantity of the final good. So aggregating one has:

$$\int_0^1 c_t(h) dh = c_t$$

However, in each period, households belonging to different cohorts have different income levels, this implies that, following Ascari (1998), households exchange bonds to keep their consumption constant. Therefore, considering that the total number of households is normalized to 1 and that there exist  $\frac{1}{N}$  households in each cohort  $i$ , one has that

$$c_t = \sum_{i=0}^{N-1} \frac{W_t(i)}{P_t} \frac{n_t(i)}{N}$$

Therefore (4) can be rewritten as

$$\sum_{i=0}^{N-1} E \left[ \beta^i |U_n(\cdot)| \theta_n n_{t+i}(h) \right] = (\theta_n - \xi) N \sum_{i=0}^{N-1} E \left[ \frac{\beta^i \left( \frac{W_{t+i}(h)}{P_{t+i}} \right)^\xi n_{t+i}(h)}{\sum_{i=0}^{N-1} \left( \frac{W_{t+i}(h)}{P_{t+i}} \right)^\xi n_{t+i}(h)} \right] \quad (5)$$

### 3 Calibration and solution of the baseline model

We specify the utility function as follows

$$U(\cdot) = \log c_{t+i}(h) + \varsigma \frac{[1 - n_t(h)]^{1-\eta}}{1-\eta} + V \left\{ \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi \right\} \quad (6)$$

Suppose  $\eta > 0$ , (5) will become

$$\sum_{i=0}^{N-1} E \left\{ \beta^i [1 - n_{t+i}(h)]^{-\eta} \theta_n n_{t+i}(h) \varsigma \right\} = (\theta_n - \xi) N \sum_{i=0}^{N-1} E \left[ \frac{\beta^i \left( \frac{W_{t+i}(h)}{P_{t+i}} \right)^\xi n_{t+i}(h)}{\sum_{i=0}^{N-1} \left( \frac{W_{t+i}(h)}{P_{t+i}} \right)^\xi n_{t+i}(h)} \right] \quad (7)$$

The equilibrium for this model is a sequence  $\{c_t, W_t(h), \mu_t, n_t(h), n_t, y_t, \iota_t\}$  satisfying households' utility maximization and firms' profit maximization.

However, we show below for the steady state that equations (1), (2) and (7) constitute an autonomous system in  $\frac{W^*}{W}, n_0$  and  $y$ , where  $\frac{W^*}{W}$  and  $n_0$  are the relative real wage of cohort zero and its labour supply respectively.

Consider, first, that the inflation rate (equal to the growth rate of money) is  $\mu$ ; second, that, being the final product market perfectly competitive,  $W_t = P_t$  and, finally, that, after (1), the labour supply of cohorts  $j$  and  $j+1$  are connected in the following way:

$$\frac{n_j}{n_{j+1}} = \mu^{-\theta_w}$$

On these grounds, it is possible to solve the sums in (7) and (2) to obtain

$$\sum_{i=0}^{N-1} \beta^i [1 - n_0 \mu^{\theta_n i}]^{-\eta} n_0 \mu^{\theta_n i} = \frac{\theta_n - \xi}{\theta_n \varsigma} N \frac{1 - \beta^N \mu^{(\theta_n - \xi)N}}{1 - \beta \mu^{(\theta_n - \xi)}} \frac{1 - \mu^{(\theta_n - 1)}}{1 - \mu^{(\theta_n - 1)N}} \left( \frac{W^*}{W} \right)^{(\xi - 1)} \quad (8)$$

with

$$\frac{W^*}{W} = \left[ \frac{1}{N} \frac{1 - \mu^{N(\theta_n - 1)}}{1 - \mu^{\theta_n - 1}} \right]^{\frac{1}{\theta_n - 1}} \quad (9)$$

(8) can be solved numerically for  $n_0$ . Aggregate output can be easily obtained by substituting  $n_0$  and (9) into (1).

Further notice that the first order condition with respect to bonds implies that in steady state

$$\beta = \left[ \frac{P_{t+i}}{P_{t+i+1}} \iota_{t+i} \right]^{-\xi} \approx [R]^{-\xi} \quad (10)$$

where  $R$  equals 1 plus the real interest rate.

We calibrate parameters as follows:  $N = 2$ ,  $\eta = 2$ ,  $\theta_n = 5$  and the real interest rate to be 4% in steady state, which means that the discount factor is pinned down by (10). Sensitivity analyses regarding these parameters were extensively discussed and performed by Graham and Snower (2004, 2008). It is offered there also a discussion of the reasons why Taylor contracts can be preferable to other kinds of wage contracts and why inserting wage indexation may not be interesting. Furthermore, we stick to Taylor contracts because Ascari (2004) showed that they are less sensitive to trend inflation than Calvo contracts. These issues will not receive further attention here.

Instead we focus on  $\xi$ . Notice that  $\xi < \theta_n$  not to have a negative mark-up

of the reset wage over the ratio between the weighted marginal disutilities from labour and marginal utilities from consumption over the contract period - see equation (33). The two theoretical arguments advanced at the beginning of the present article for introducing  $\xi$  in our model cannot offer much guidance regarding its value. Resorting to the literature on pay satisfaction and the subjective value of money will not help either, given that there are studies - like Giles and Barrett (1971) and Heneman et al. (1997) - that would favour a value of  $\xi$  greater than one, but there are also studies - like Brandstätter and Brandstätter (1996) and Worley et al. (1992) - which would favour a value of  $\xi$  smaller than one. Therefore, we offer illustrative results for  $\xi = 0.7$  and  $\xi = 4.9$  - which is very close to  $\theta_n$ .

## 4 Results, intuition and welfare analysis of the baseline model

Figure 1 compares the long-run Phillips curves for  $\xi = 0.7$  and for  $\xi = 1$ , while Figure 2 those for  $\xi = 4.9$  and for  $\xi = 1$ . In the first case the long-run Phillips curve in presence of money illusion lies below the one without money illusion, while in the second case the contrary happens. To appreciate the intuition underlying this result the reader should consider equations (10) and (7). For a given real interest rate,  $\xi < 1$  rises the discount factor dampening the time discounting effect. Furthermore, it lowers agents' perception of their labour income reducing labour supply.  $\xi > 1$  instead produces opposite

effects. However, this does not imply that output will grow even in presence of hyperinflation, given that, as it is possible to see in Figure 2, for high inflation rates positive non-superneutralities fade away (turning in the end into negative non-superneutralities).

Building on Woodford (1998) among others, we can specify agents' welfare,  $\mathcal{W}$ , assuming the weight of money holdings to be so small to be negligible:

$$\mathcal{W} = \sum_{j=0}^{\infty} \beta^j \left\{ \log c_{t+j} - \varsigma \frac{[1 - n_{t+j}(h)]^{1-\eta}}{1-\eta} \right\} \quad (11)$$

which, after Graham and Snower (2004, p.23), in steady state can be rewritten as

$$\mathcal{W} = \frac{1}{1-\beta} \log y - \varsigma \frac{1}{1-\beta^N} \sum_{l=0}^{N-1} \beta^l \frac{[1 - n_0 \mu^{\theta_n l}]^{1-\eta}}{1-\eta}$$

Table 1 shows that, varying  $\mu$ ,  $\mathcal{W}$  behaves in a similar way to output. Under wage-staggering and  $\xi = 4.9$  welfare increases with the money growth rate, levelling off at high inflation rates. For  $\xi = 0.7$  welfare decreases as the inflation rate increases. The next section introduces varying capital in the model.

## 5 The model with varying capital

### 5.1 Households' maximization problem

Households maximize their discounted expected utility

$$\max_{\{c_{t+i}(h), W_{t+N_i}(h), B_{t+i}(h), M_{t+i}(h), K_{t+i}(h)\}} \sum_{j=0}^{\infty} \sum_{i=jN}^{(j+1)N-1} \beta^i U \left\{ c_{t+i}(h), n_{t+i}(h), \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi \right\}$$

subject to a series of income constraints perceived as follows

$$\begin{aligned} c_{t+i}(h) + i_{t+i}(h) = & \left[ \frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi n_{t+i}(h) + \left[ \frac{T_{t+i}(h)}{P_{t+i}} \right]^\xi - \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[ \frac{M_{t+i-1}(h)}{P_{t+i}} \right]^\xi - \\ & - \left[ \frac{B_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[ \frac{B_{t+i-1}(h)}{P_{t+i}} \right]^\xi \iota_{t+i} + \left[ \frac{\iota_{t+i}^k(h)}{P_{t+i}} \right]^\xi K_{t+i}(h) \end{aligned}$$

where  $K$  is capital,  $i$  investment and  $\iota_{t+i}^k$  is the nominal rental rate of capital.

Further constraints derive from the demand for labour services (1). Defining the capital depreciation rate as  $\delta$ , the law of motion of capital is

$$K_{t+i}(h) = i_{t+i}(h) + (1 - \delta) K_{t+i-1}(h)$$

which can be substituted in the budget constraint to obtain

$$\begin{aligned}
c_{t+i}(h) + K_{t+i}(h) &= \left[ \frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi n_{t+i}(h) + \left[ \frac{T_{t+i}(h)}{P_{t+i}} \right]^\xi - \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[ \frac{M_{t+i-1}(h)}{P_{t+i}} \right]^\xi - \\
&\quad - \left[ \frac{B_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[ \frac{B_{t+i-1}(h)}{P_{t+i}} \right]^\xi \iota_{t+i} + \left[ \frac{\iota_{t+i}^k(h)}{P_{t+i}} \right]^\xi K_{t+i-1}(h) + \\
&\quad + (1 - \delta) K_{t+i-1}(h)
\end{aligned}$$

Taking the first-order derivative with respect to  $K_{t+i}(h)$  one has

$$\lambda_{t+i}(h) - E \left[ \beta \left( \frac{\iota_{t+i}^k}{P_{t+i}} \right)^\xi \lambda_{t+i+1}(h) \right] - E [\beta (1 - \delta) \lambda_{t+i+1}(h)] = 0 \quad (12)$$

## 5.2 Firms' cost minimization problem and the government

We adopt a two-stage budgeting approach after Chambers (1988, pp. 112-113) and Heijdra and Van der Ploeg (2002, pp. 360-363). In the first stage firms choose the amount of each labour kind as in section 2.1. In the second stage, they minimize their total costs by choosing their preferred amounts of capital and of the aggregate labour input as follows<sup>3</sup>

$$\begin{aligned}
\min_{K_t, n_t} & \frac{\iota_t^k}{P_t} K_t + \frac{W_t}{P_t} n_t \\
s.t. & y_t = K_t^\alpha n_t^{1-\alpha}
\end{aligned} \quad (13)$$

---

<sup>3</sup>We did not insert an index for firms as they are symmetrical and their number is normalized to one.



Solving this problem one has that

$$K_t = MC_t \alpha \frac{y_t}{\frac{\iota_t^k}{P_t}} \quad (14)$$

$$n_t = MC_t (1 - \alpha) \frac{y_t}{\frac{W_t}{P_t}} \quad (15)$$

$$MC_t = \left( \frac{\frac{\iota_t^k}{P_t}}{\alpha} \right)^\alpha \left( \frac{\frac{W_t}{P_t}}{1 - \alpha} \right)^{1 - \alpha} \quad (16)$$

where  $MC_t$  is the real marginal cost at time  $t$ . Profit maximization further implies the price of the final product to be equal to the firms' nominal marginal cost, which leads us to the following equation

$$P_t = \left( \frac{\frac{\iota_t^k}{P_t}}{\alpha} \right)^\alpha \left( \frac{\frac{W_t}{P_t}}{1 - \alpha} \right)^{1 - \alpha} \quad (17)$$

### 5.3 Model solution and results

The equilibrium for this model is a sequence  $\left\{ \mu_t, c_t, n_t(h), n_t, \Pi_t, \iota_t, K_t, \frac{W_t}{P_t}, \frac{W_t(h)}{P_t}, y_t, \frac{\iota_t^k}{P_t} \right\}$  - where  $\Pi_t = \frac{P_t}{P_{t-1}}$  - satisfying households' utility maximization and firms' profit maximization. For  $K$  and  $c$ , we could drop the  $h$  index because households are symmetrical with respect to their consumption and investment choices.

In steady state (12) implies

$$(R^\xi - 1 + \delta) = \left( \frac{\iota^k}{P} \right)^\xi \quad (18)$$

In words, agents equate their perception of the real interest rate to their perception of the gross real rental rate of capital.  $P$  can be normalized to 1 and (18) together with (17) can be used to find  $W$ . Having the ratio between the aggregate wage index and the gross rental rate of capital, (15) and (14) can be used to find  $K/n$ , and, by means of the production function (13),  $y/n$ . Further consider that in steady state  $i = \delta K$ , therefore

$$\frac{c}{n} = \left(\frac{K}{n}\right)^\alpha - \delta \left(\frac{K}{n}\right)$$

In the right-hand side of equation (4) divide and multiply by  $n_{t+i}$  to obtain

$$\sum_{i=0}^{N-1} E \left[ \beta^i |U_n(\cdot)| (-1) (-\theta_n) n_{t+i}(h) \right] = \sum_{i=0}^{N-1} E \left\{ \frac{\beta^i (\theta_n - \xi) \left[ \frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi \frac{n_{t+i}(h)}{n_{t+i}}}{\frac{c_{t+i}}{n_{t+i}}} \right\} \quad (19)$$

Specifying the utility function as in (6), one can find that, considering varying capital, (8) turns out to be

$$\sum_{i=0}^{N-1} \beta^i [1 - n_0 \mu^{\theta_n i}]^{-\eta} n_0 \mu^{\theta_n i} = \frac{\theta_n - \xi}{\theta_n \varsigma} \frac{1 - \beta^N \mu^{(\theta_n - \xi)N}}{c} \frac{1 - \beta^N \mu^{(\theta_n - \xi)N}}{1 - \beta \mu^{(\theta_n - \xi)}} \left(\frac{W^*}{W}\right)^{-\theta_w} W^{*\xi} \quad (20)$$

which can be solved numerically for  $n_0$ . With  $n_0$  at hand, one can easily recover first  $n$  by using (1) and then all the steady state values of the other variables of model.

Figure (3) shows that our results are robust to the inclusion of variable capital.  $\xi > 1$  reinforces positive money non-superneutrality, while for  $\xi < 1$  the opposite holds true.

## 6 Conclusions

To conclude this paper shows how the long-run inflation-output nexus behaves in presence of agents mis-perceiving real economic variables. Under-perception reduces labour supply and time discounting leading to more negative money non-superneutrality. Over-perception, instead, boosts labour supply and time discounting leading to more positive money non-superneutrality. These results hold true even inserting varying capital in our model. In the Appendix, we treat how a temporary shock in money growth affects our model for different values of  $\xi$ .

## References

- [1] Anderson, N.H. (1970). Functional measurement and psychophysical judgment. *Psychological Review*, 77, 153-170.
- [2] Ascari, Guido (1998). Superneutrality of money in staggered wage-setting models. *Macroeconomic Dynamics*, 2, 383-400.
- [3] Ascari, G., 2000. Optimizing agents, staggered wages and persistence in the real effects of monetary shocks. *Economic Journal* 110, 664-686.

- [4] Ascari, G. 2004. Staggered prices and trend inflation: some nuisances. *Review of Economic Dynamics* 7, 642-647.
- [5] Brandstätter Eduard, Hermann Brandstätter (1996), What's money worth? Determinants of the subjective value of money, *Journal of Economic Psychology*, Volume 17, pp. 443-464.
- [6] Chambers, Robert G. (1988) *Applied Production Analysis*. Cambridge, UK: Cambridge University Press.
- [7] Devereux, M.P., Yetman, J., 2002. Menu costs and the long-run output inflation trade-off. *Economics Letters* 76, 95-100.
- [8] Edge, Rochelle M. 2001. "Technical Appendix to \The Equivalence of Wage and Price Staggering in Monetary Business Cycle Models", <http://www.EconomicDynamics.org/appendix/edge01.pdf>.
- [9] Edge, Rochelle M. 2002. "The Equivalence of Wage and Price Staggering in Monetary Business Cycle Models," *Review of Economic Dynamics*, Elsevier for the Society for Economic Dynamics, vol. 5(3), pages 559-585, July.
- [10] Fehr, Ernst and Jean-Robert Tyran (2001), Does money illusion matter?, *American Economic Review*, 91, pp. 1239-1262.
- [11] Fehr, Ernst and Jean-Robert Tyran (2007), Money illusion and coordination failure, *Games and Economic Behavior*, 58, 246-268.

- [12] Giles, B. A. and G. V. Barrett (1971) Utility of Merit Increases. *Journal of Applied Psychology* 55, pp. 103-109
- [13] Graham, C.H. (1958). Sensation and perception in an objective psychology. *Psychological Review*, 65, 65-76.
- [14] Graham, L. and D. J. Snower (2004), "The real effect of Money Growth in Dynamic General Equilibrium", European Central Bank Working Paper n. 412.
- [15] Graham, L. and D. J. Snower (2008). Hyperbolic discounting and the Phillips Curve. *Journal of Money, Credit and Banking*, 40, 3, pp. 428-448.
- [16] Heijdra, Ben J. and Frederick Van der Ploeg (2002) *The Foundations of Modern Macroeconomics*. Oxford University Press: Oxford.
- [17] Heneman Robert L., Gayle Porter, David B. Greenberger, Stephen Strasser (1997) Modeling the Relationship between Pay Level and Pay Satisfaction, *Journal of Business and Psychology*, Vol. 12, pp. 147-158.
- [18] Huang, K.X.D and Liu, Z. (2002) Staggered price-setting, staggered wage-setting, and business cycle persistence, *Journal of Monetary Economics* 49 405-433
- [19] King, R.G., Wolman, A.L., 1996. Inflation targeting in a St. Louis model of the 21st century. *Proceedings. Federal Reserve Bank of St. Louis*, pp. 83-107.

- [20] Luce, R.D. and Krumhansl (1988). Measurement, scaling, and psychophysics. In R.C. Atkinson, R.J. Herrnstein, L. Gardner, and R.D. Luce (Eds.), *Stevens' handbook of experimental psychology: Vol. 1 Perception and Motivation*. New York: Wiley.
- [21] Michell, J. (1999). *Measurement in Psychology: a Critical History of a Methodological Concept*. Cambridge, England: Cambridge University Press.
- [22] Pelham, Brett W., Sumarta Tin Tin and Myaskovsky, Laura. (1994) "The Easy Path from Many to Much: The Numerosity Heuristic." *Cognitive Psychology*, 26, 103-133.
- [23] Shafir, E., Diamond, P., & Tversky, A. (1997). Money illusion. *The Quarterly Journal of Economics*, 112, 341-374.
- [24] Shepard, R.N. (1981). On the status of "Direct" psychological measurement. In C.W. Savage (Ed.), *Minnesota studies in the philosophy of science* (Vol. 9). Minneapolis: University of Minnesota Press.
- [25] Stevens, S.S. (1946). On the theory of scales of measurement. *Science*, 103, 677-680.
- [26] Stevens, S.S. (1951). *Handbook of Experimental Psychology*. New York: Wiley.
- [27] Vaona, A., Snower, D., 2007. Increasing returns to scale and the long-run Phillips curve, *Quaderni di Dipartimento n. 203 (09-07)*. Dipartimento

di Economia Politica e Metodi Quantitativi, Università degli studi di Pavia. <http://economia.unipv.it/eco-pol/abs/abs203.html>.

- [28] Vaona, Andrea and Dennis Snower (2008), Increasing returns to scale and the long-run Phillips curve, *Economics Letters* 100, 83-86.
- [29] Vaona, Andrea (2010), Inflation and growth in the long-run: A New-Keynesian Theory and further Semiparametric Evidence, *Macroeconomic Dynamics*, forthcoming.
- [30] Wertenbroch, Klaus, Soman Dilip and Chattopadhyay, Amitava (2007) "On the Perceived Value of Money: The Reference Dependence of Currency Numerosity Effects". *Journal of Consumer Research*, 34, p1-10.
- [31] Woodford, Michael (1998) Doing without money: controlling inflation in a post-monetary world. *Review of Economic Dynamics* 1, 173-219.
- [32] Worley Christopher G., David E. Bowen, Edward E. Lawler (1992) "On the Relationship between Objective Increases in Pay and Employees' Subjective Reactions", *Journal of Organizational Behavior*, Vol. 13, pp. 559-571.

## 7 Appendix

In this Appendix we consider the impact of a temporary monetary shock on the economy. We stick to the varying capital model. Following Edge (2002) money growth is assumed to evolve according to the process

$$\mu_t = (\mu)^{1-\zeta} (\mu_{t-1})^\zeta \exp(\epsilon_t)$$

where  $\epsilon_t$  is a random shock and  $\zeta$  an autoregressive parameter, that we set equal to 0.57 after Ascari (2004). We also set the steady state money growth rate,  $\mu$ , to 1%. Similarly to Edge (2001), the other equations of the system are

$$1 = \left( \frac{l_t^k}{P_t} \right)^\alpha \left( \frac{W_t}{1-\alpha} \right)^{1-\alpha} \quad (21)$$

$$n_t = \left( \frac{1-\alpha}{\alpha} \right)^\alpha Y_t \left( \frac{W_t}{l_t^k} \right)^{-\alpha} \quad (22)$$

$$K_t = \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} Y_t \left( \frac{W_t}{l_t^k} \right)^{1-\alpha} \quad (23)$$

$$n_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-\theta_n} n_t \text{ for } h \in \left[ 0, \frac{1}{2} \right] \quad (24)$$

$$n_t(h) = \left[ \frac{W_{t-1}(h)}{P_{t-1}} \right]^{-\theta_n} n_t \text{ for } h \in \left[ 0, \frac{1}{2} \right] \quad (25)$$

$$\frac{W_t}{P_t} = \left\{ \frac{1}{2} \left[ \frac{W_t(h)}{P_t} \right]^{1-\theta_n} + \frac{1}{2} \left[ \frac{W_{t-1}(h)}{P_t \Pi_t} \right]^{1-\theta_n} \right\}^{\frac{1}{1-\theta_n}} \quad (26)$$



$$\frac{c_{t-1}}{c_t} \left(1 - \frac{1}{l_t^\xi}\right) / \left(1 - \frac{1}{l_{t-1}^\xi}\right) = \left(\frac{\mu_t}{\Pi_t}\right)^{-\xi} \quad (27)$$

$$\frac{1}{c_t} = \beta E \left\{ \frac{1}{c_{t+1}} \left[ \frac{1}{\Pi_{t+1}} l_t \right]^\xi \right\} \quad (28)$$

$$E(c_{t+1}) = \beta \left[ \frac{l_t^k}{P_t} \right]^\xi c_t + \beta(1 - \delta) c_t \quad (29)$$

$$y_t = c_t + K_t(h) - (1 - \delta) K_{t-1}(h) \quad (30)$$

$$[1 - n_t(h)]^{-\eta} n_t(h) + \beta E \{ [1 - n_{t+1}(h)]^{-\eta} n_{t+1}(h) \} = \quad (31)$$

$$= \frac{(\theta_n - \xi)}{\theta_n \varsigma} \left[ \frac{W_t(h)}{P_t} \right]^\xi \frac{n_t(h)}{c_t} + \frac{(\theta_n - \xi)}{\theta_n \varsigma} \beta E \left\{ \left[ \frac{W_t(h)}{P_t \Pi_{t+1}} \right]^\xi \frac{n_{t+1}(h)}{c_{t+1}} \right\} \quad (32)$$

where (27) comes from the ratio of the first order condition for money holdings at time  $t$  and at time  $t - 1$ .

Figure 4 shows the impulse responses of output to a monetary shock for different values of  $\xi$ . Similarly to Edge (2002) we normalized them on the basis of the impact value of output for  $\xi = 1$ . As it is possible to see, money illusion does not alter output persistence. It only changes the impact value of output as a greater value of  $\xi$  reduces it. To appreciate an intuition for this result consider the equation for the nominal reset wage, that can be obtained from the equation (31)-(32)

$$W_t^*(h) = \left\{ \frac{\sum_{i=0}^{N-1} E [\beta^i |U_n(\cdot)| n_{t+i}(h)]}{\sum_{i=0}^{N-1} E \left[ U_c(\cdot) \beta^i \frac{n_{t+i}(h)}{P_{t+i}^\xi} \right]} \frac{\theta_n}{(\theta_n - \xi)} \right\}^{\frac{1}{\xi}} \quad (33)$$

As noted by Ascari (2000) among others, the nominal reset wage is a mark-up over the ratio between the weighted marginal disutilities from labour and marginal utilities from consumption over the contract period. Overperception of real variables reduces the impact of a monetary shock because agents believe their real wage is higher than what it is in reality and so they change their nominal wage by a lesser amount. This is the meaning of the exponent of  $\frac{1}{\xi}$  in the right hand side of equation (33). The opposite holds true for underperception of real variables, which implies an over-reaction by agents to monetary shocks. Agents' over(under)-reaction produces also over(under)-reaction of output. This intuition is confirmed by the behavior of the real wage index in our model. Similarly to Huang and Liu (2002), after a monetary shock the real wage index declines, however for  $\xi = 1.1$  it does so by 14% less than for  $\xi = 1$ , while for  $\xi = 0.9$  by 21% more than in absence of money illusion.

Table 1 - Percentage deviation from the welfare level with flexible wages

Money growth rate	$\xi = 4.9$	$\xi = 0.7$
2.5	0.04%	-0.24%
5	0.08%	-0.96%
10	0.12%	-3.60%
15	0.13%	-7.44%

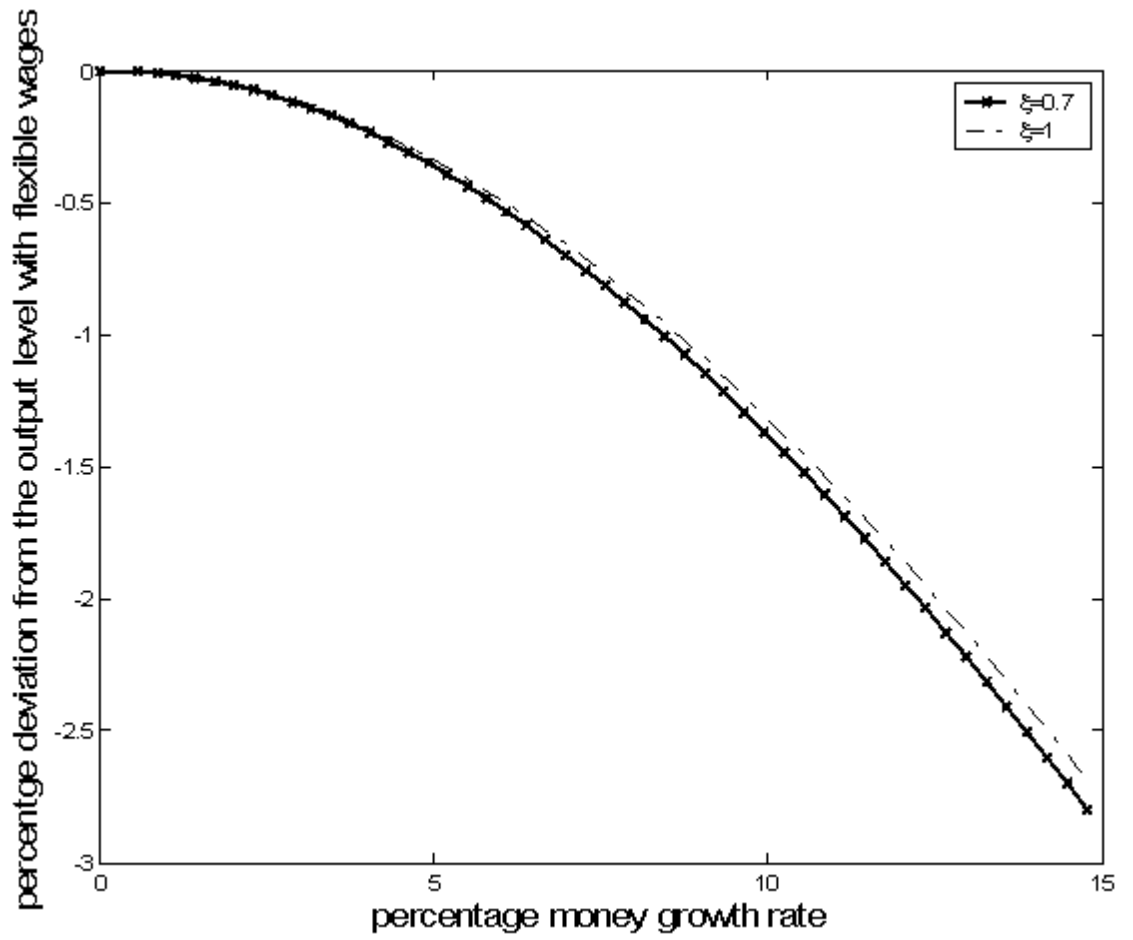


Figure 1: The long-run Phillips curve for  $\xi = 0.7$  and  $\xi = 1$ .

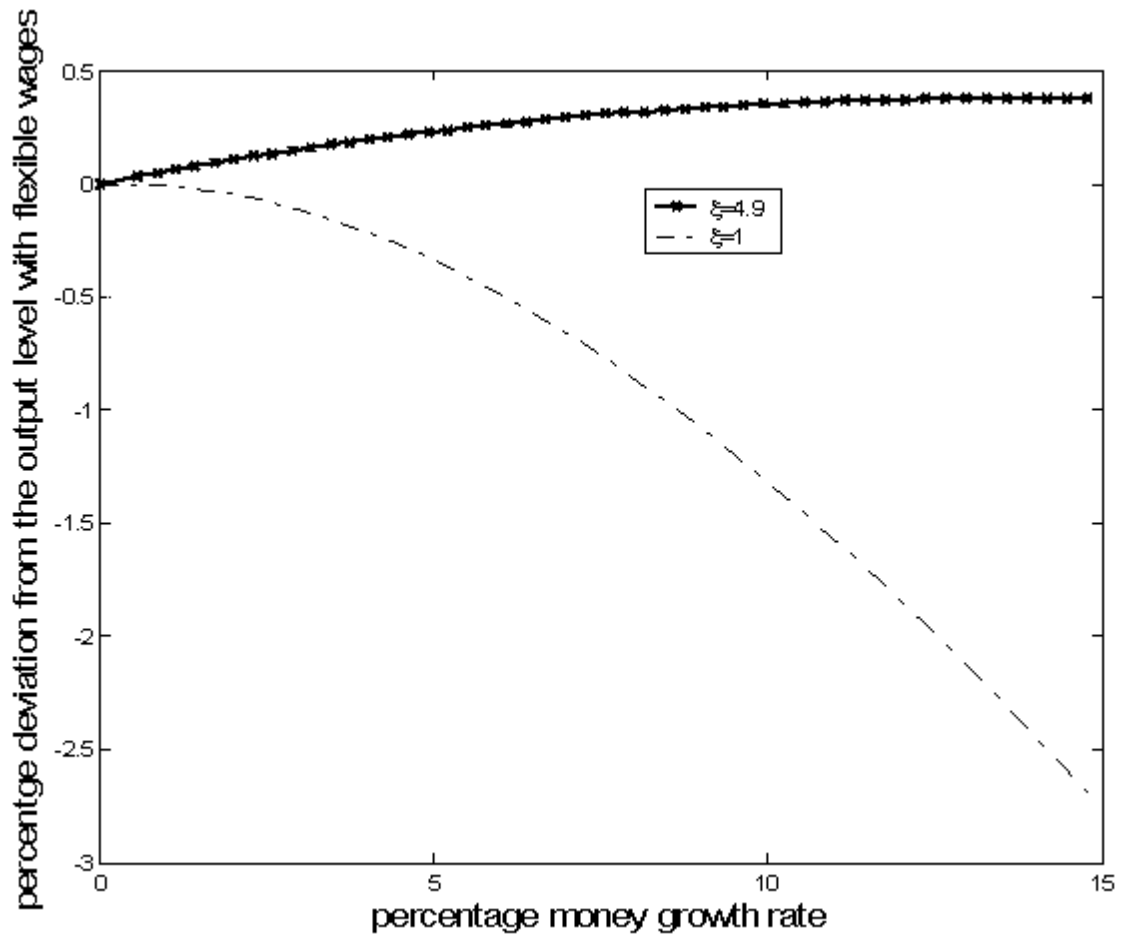


Figure 2: The long-run Phillips curve for  $\xi = 4.9$  and  $\xi = 1$ .

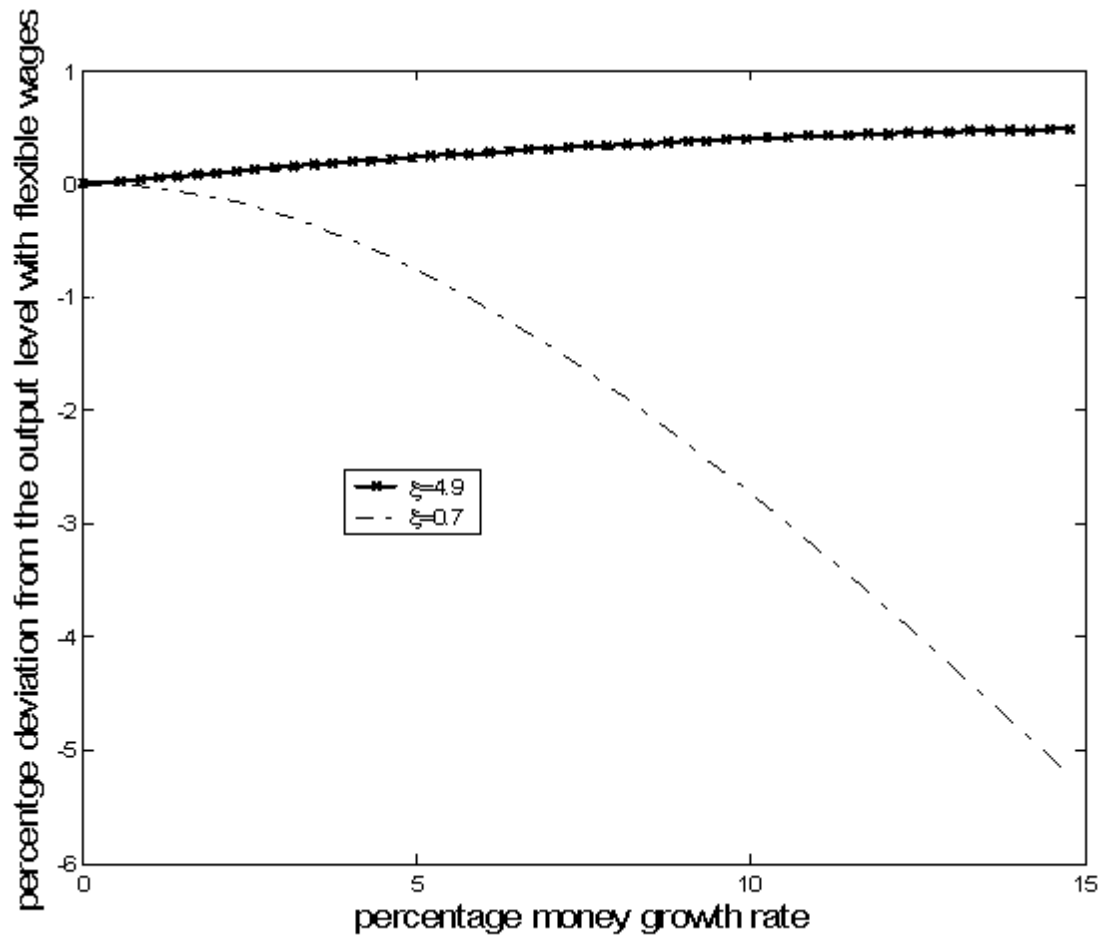


Figure 3: The long-run Phillips curve for different values of  $\xi$  and varying capital.

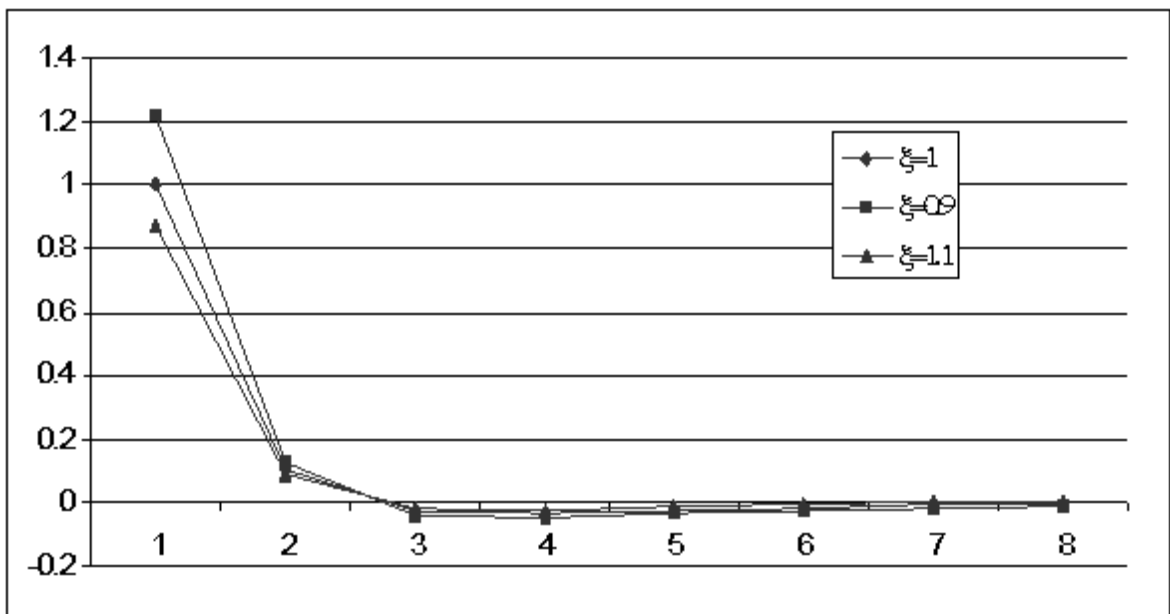


Figure 4: Normalized impulse response functions of output after a monetary shock for different values of  $\xi$ . (The normalization base is the impact value for  $\xi = 1$ ).