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The Sharing Rule: Where Is It?*

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Abstract

The search for a robust and stable sharing rule has led us to novel results about identification of the rule governing the intra-household allocation of resources. We introduce an income proportionality property that directly connects distribution factors with individual incomes and implement this identifying condition to obtain an exact correspondence between the structural and reduced form of the adopted collective demand equations. The paper first reexamines the collective model of household consumption and shows that a) the collective model as traditionally developed is not identified, b) illustrates the novel income proportionality condition, and c) derives a restriction allowing full identification. In the tradition of collective theory, we exploit information about private consumption of an assignable good, clothing for adults and children in our application, prices and distribution factor variation to estimate how resources are shared between adults and children in a sample of Italian households. Previous estimations of collective models where only limited to either the direct or indirect estimation of the structure. Our results open up the possibility of a direct structural estimation of a collective system of demand equations, and associated individual Engel curves, as easily as estimating a demand system based on a unitary framework.

JEL codes: C30, D11, D12, D13.
Keywords: Collective model, Sharing rule, Consumer demand.

1 Introduction

Economists have recognized that modeling decisions of households using an aggregate unitary utility function subject to the household budget constraint is not adequate both under a theoretical and empirical perspective. Considering that decisions are made at the individual level, then decisions should be modeled at the level of each member of the

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household especially if theory is to be empirically tested. The seminal works by Chiappori (1988, 1992) introduce a theoretical household model in which the family is composed of a collection of individuals each of whom is characterized by her own preferences over market goods consumed within the family. Assuming that the decision process results in Pareto efficient outcomes, Chiappori (1988, 1992) shows that when agents are *egoistic* and consumption is purely private the collective model generates testable restrictions and, from observed behaviour, it is possible to recover individual preferences and the rule governing the allocation of resources within the family provided that private consumption of exclusive or assignable goods is observed. The identification of the sharing rule is necessary to derive individual welfare functions and to assess the impact of a policy change in terms of prices, incomes or other exogenous factors of interest on the welfare level of each family member. The identification of individual utility functions is direct only when we assume that the household is formed of two individuals. The identification of the sharing rule and of welfare functions of groups within the household, such as the group of adults and children, requires additional results (Chiappori and Ekeland 2009).

Judging by the numerous theory refinements (Browning, Chiappori and Lechene 2006, Donni 2003, 2006, 2011, Chiappori, Donni, and Komunjer 2012, Vermuelen 2002, Browning, Chiappori, and Weiss 2012) and applications (Browning *et al.* 1994, Chiappori, Fortin, and Lacroix 2002, Lise and Seitz 2011, Bargain and Donni 2009) in the areas of both consumption and labor choices, we may be prone to think that the theory developments and the accuracy of the applications are at a mature stage. Identification issues, though, are traditionally difficult because they involve a simultaneous blend of theory, structural assumptions about functional forms, and data quality considerations to be combined in different doses depending on the study case. This research renews the investigation of identification issues within a collective framework by taking into consideration all these three aspects to show that the search for our object of interest, the sharing rule, is not over yet as often claimed.

The design of our experiment roots on the postulate stating that, for a given functional form of an identifiable collective structure, the estimates obtained from a direct estimation of the structure as, for example, in Browning *et al.* (1994) or Lise and Seitz (2011) should be the same as the estimates derived implementing an indirect least square procedure consisting in a first stage estimation of an unrestricted reduced form and a second stage that uses the collective theory restrictions to recover the structure as, for example, in Chiappori, Fortin, and Lacroix (2002) or in Rapoport, Sofer, and Solaz (2011) and Donni and Moreau (2007). Considering that the identifiability restrictions are determined exploiting the correspondence between a chosen structure and the associated unrestricted reduced form, then a direct or an indirect estimation technique should produce the same structural and sharing rule parameters.

The econometric execution of a demand system for clothing of adults and children in order to estimate the sharing rule between adults and children using Italian data is preceded by a theoretical analysis of the collective model of household consumption. This section derives the restrictions imposed by the collective theory to implement the indirect estimation. In the first place, we recast the conditions for nonparametric identifiability of collective models and repeat the theory exercise given specific assumptions about the functional form of the demand system and the sharing rule in sequence. We find that the functional form assumption about the sharing rule, which is an income scaling function originally characterized by Lewbel (1985), is analogous to the parametric specification of the sharing rule estimated in Browning *et al.* (1994) and is also implicitly adopted in the study by Chiappori, Fortin, and Lacroix (2002). The analysis unveils a new testable restriction that extends the collective proportionality property to what we term the income proportionality property. This restriction proves to be crucial in achieving the postulated equality between the directly estimated structural form parameters and the indirectly estimated reduced form parameters.

This achievement, though novel in the collective literature, is not conclusive. Contrary to what traditionally supposed, we show that the identification of the parameters of the income scaling function does not imply that we are actually estimating the rule governing the allocation of resources within the household. A proper specification, effectively capable to correctly interpret the information about exclusive consumption reported in the data, should scale the best proxy for unobserved individual incomes, or total individual expenditures, rather than household incomes as done in the generality of the previous estimations of the sharing rule. We show how to construct such best proxies for unobserved individual incomes. Failing to do this would be like trying to estimate the sharing rule without exploiting the necessary information to be observed, that is the information about exclusive consumption. This definite result opens up the possibility of a direct structural estimation of a collective demand equation, and associated individual Engel curves resulting from the decomposition of the income term of the Slutsky equation of a collective decentralized program, as easily as estimating a demand system based on a unitary framework.

The paper first reexamines the collective model of household consumption and shows that a) the collective model as traditionally developed is not identified, b) illustrates the novel income proportionality condition, and c) derives a restriction allowing full identification. The subsequent section extends the results to a functional specification, used in the applied section, that closely resembles the specification used in Chiappori, Fortin, and Lacroix (2002) in order to facilitate comparisons across identification strategies and empirical implementations. Concluding remarks close the analysis.

2 The Collective Model of Household Consumption

We describe the consumption decisions of a family of two members or groups such as adults and children, denoted by 1 and 2, within a collective model. Each family member privately consumes an exclusive market good c^i and an ordinary composite market good c_o^i,[1] of which we only observe the whole family consumption $c_o^h = c_o^1 + c_o^2$.[2] Each member faces a market price equal to p_i to buy the exclusive good c^i and a market price of p_o for the ordinary good c_o^i that is assumed equal for both members. Without loss of

[1]Exclusivity and ordinariness are two polar characteristics of goods consumed by a multiple person household. A good is ordinary when a private good is consumed without rivalry in unobserved proportions by all or some non identifiable household members. An example of ordinary good may be the consumption of food. Conversely, a good is exclusive when a private good is consumed by only one identifiable member and its price is different from the price of the other exclusive goods consumed within the family. Examples of exclusive goods are leisure and clothing, when we can observe female and male clothing expenditures and the corresponding prices.

[2]In the paper superscripts indexes denote endogenous variables, subscripts denote exogenous variables.

generality, we exclude the consumption of public goods or the presence of externalities within families. We also abstract from the consumption of domestically produced goods (Apps and Rees 1997, Chiappori 1997).

The collective household model relies on a set of general assumptions, referred to as *collective rationality*, that assures the identifiability of the structure of the household model.[3] Each family member has egoistic preferences over the consumption of exclusive and ordinary goods represented by an *egoistic* quasi-concave utility function $u^i = U^i\left(c^i, c_o^i\right)$, twice differentiable and strictly increasing in its elements.[4] Family outcomes are Pareto-efficient. The rationale of this assumption is that efficient allocations are likely to emerge when agents are able to make binding commitments and have full information as it is often the case in a household setting.

Pareto-efficiency implies that the consumption equilibrium will be on the Pareto frontier of the family. Further, when all goods are privately consumed and there are no consumption externalities, family behaviour can be described by a two-stage process: first the family agrees on a rule to share resources among its members and then each of them maximizes her individual utility function subject to her household income share. We assume that the rule governing the resource allocation is a function of the following set of exogenous variables: individual market prices, p_1 and p_2, household income x,[5] and distribution factors s that do not affect either individual preferences or the budget constraint but do influence the balance of power of the family members and their optimal choices. Distribution factors are helpful in recovering the structure of the collective model and play an important role in empirical applications. Browning, Chiappori, and Lechene (2006), and Chiappori and Ekeland (2009) show the key role played by distribution factors to identify the sharing rule and present an exhaustive review of distribution factors used in the collective household literature.

Under the assumptions of collective rationality, household behaviour can be represented within a two stage model with income transfers. The division of income within the household is governed by the sharing function $\phi_1 = \phi\left(p_1, p_2, x, s\right)$ corresponding to member's 1 level of income, and $\phi_2 = x - \phi\left(p_1, p_2, x, s\right)$ that is the part of household income allocated to member 2 such that $\phi_1 + \phi_2 = x$ with $0 < \phi_i < x \; \forall i = 1, 2$. Each family member then independently chooses her private consumption subject to her budget constraint according to the following decentralized program

$$\max_{c^i, c_o^i} \quad U^i(c^i, c_o^i) \tag{1}$$

$$\text{subject to} \quad p_i c^i + c_o^i = \phi_i,$$

where we assume that the price of the ordinary good is fixed to 1 for both members.

[3]The structure of the household model is defined by preferences of the family members and the household decision process described by the sharing rule.

[4]It is important to remark that *egoistic* preferences are not necessary to recover the structure of the collective model. The collective set-up may be extended to a caring utility function $W^i\left[U^1\left(c^1, c_o^1\right), U^2\left(c^2, c_o^2\right)\right]$ without altering the conclusions of the household model (Chiappori 1992, Chiappori and Ekeland 2009).

[5]In this introductory description of the collective household model we are consistent with the traditional set of assumptions used in the collective literature. As the analysis develops, we will show that including household income x as argument of the sharing rule is a critical assumption for its correct identification.

Solving the individual program (1) for interior solutions only, we obtain the Marshallian demands of member i

$$Reduced\ Form\ \ Structural\ Form$$

$$c^i\left(p_1,p_2,x,s\right)=C^i\left(p_i,\phi_i\left(p_1,p_2,x,s\right)\right), \tag{2}$$

$$c_o^i\left(p_1,p_2,x,s\right)=C_o^i\left(p_i,\phi_i\left(p_1,p_2,x,s\right)\right), \tag{3}$$

with $i=1,2$. For $i=2$, $C^2\left(p_2,x-\phi_1\left(p_1,p_2,x,s\right)\right)$. Note that we can observe only the left-hand side of equations (2) and (3) because the sharing rule function $\phi_i\left(p_1,p_2,x,s\right)$ is not observable. Further, for non assignable goods we can observe the household demand $c_o=c_o^1+c_o^2$, but not its components.

Throughout the paper we refer to the left-hand side of equations (2) and (3), c^i and c_o^i, as the reduced demand functions and to the right-hand side, C^i and C_o^i, as the structural demand functions of member i. We use the term "reduced" form to indicate a functional relationship between the endogenous variable and the set of all observable exogenous variables of the model that are unrestricted in the sense that the collective rationality restrictions are not imposed. Note that in our framework both structure and reduced form are expressed in explicit form. The structure depends on the unobservable sharing rule and incorporates the restrictions underlying the collective model.

The Decentralized Slutsky Equation Let us assume that there exists a unique differentiable solution $C_H^i\left(p_i,\bar{U}_i\right)$ to the dual optimization problem $\min_{c^i,o^i}\phi_i=p_ic^i+c_o^i\mid U^i\left(c^i,c_o^i\right)\geq\bar{U}_i$. We define the individual expenditure function of member i as $\phi_i^*\left(p_i,\bar{U}_i\right)=\min_{c^i,c_o^i}p_ic^i+c_o^i\mid U^i\left(c^i,c_o^i\right)\geq\bar{U}_i$, where $\phi_{p_i}^*=c^i$ by the envelope theorem.

In equilibrium we have

$$C_H^i\left(p_i,\bar{U}_i\right)\equiv C^i\left(p_i,\phi_i^*\left(p_i,\bar{U}_i\right)\right), \tag{4}$$

where $C^i\left(p_i,\phi_i^*\left(p_i,\bar{U}_i\right)\right)$ is the Marshallian demand (2). Note that the the sharing function, which assigns the level of unobserved individual income, plays the same role as the observable household income in the unitary model.

Differentiating (4) with respect to p_i and using the envelope theorem, we obtain the symmetric Slutsky equation of the individual demand of member i

$$C_{H_{p_i}}^i\mid_{d\bar{U}_i=0}=C_{p_i}^i+c^iC_{\phi_i}^i, \tag{5}$$

where $C_{p_i}^i$ is the substitution effect and $C_{\phi_i}^i$ the income effect. Both these effects are unobservable. Using equations (8)-(10) we can express these effects in terms of observable variables so that $C_\phi^1=\frac{c_x^1}{\phi_x}$, and $C_{p_1}^1=c_{p_1}^1-\frac{c_x^1}{\phi_x}\phi_{p_1}$ for member 1; $C_\phi^2=\frac{c_x^2}{1-\phi_x}$, and $C_{p_2}^2=c_{p_2}^2-\frac{c_x^2}{1-\phi_x}\phi_{p_2}$ for member 2. Hence, substituting these expressions into equation (5) we obtain the symmetric[6] Slutsky equation associated with the decentralized program (1)

[6]The Slutsky conditions satisfied by the centralized and decentralized demand systems are different. Because the Pareto weight is a function of both prices and income, the household utility function is price-dependent and, thus, the centralized compensated term is not symmetric (Browning and Chiappori 1998). In a centralized framework, a price change affects both the budget constraint and the objective function giving rise to the "Symmetric Negative plus Rank One" (SNR1) condition.

for the exclusive good c^i for $i = 1, 2$ using duality theory

$$C^i_{H_{p_i}} \mid_{d\bar{U}_1=0} = c^1_{p_1} - \frac{c^1_x}{\phi_x} \left(\phi_{p_1} - c^1\right) \leq 0, \tag{6}$$

and, similarly, for member 2

$$C^i_{H_{p_i}} \mid_{d\bar{U}_2=0} = c^2_{p_2} - \frac{c^2_x}{1-\phi_x} \left(\phi_{p_2} - c^2\right) \leq 0 \tag{7}$$

with $c^1_{p_1} \leq 0, c^2_{p_2} \leq 0$ and $c^1_x \geq 0, c^2_x \geq 0$ for normal goods. This Slutsky equation for the consumption case is expressed in a form analogous to equation (2g) and (2h) in Chiappori, Fortin, and Lacroix (2002) for the labor supply case. We now provide original results about the identification of the partial effects of the unknown sharing rule $\phi(p_1, p_2, x, s)$ following the seminal proof by Chiappori (1992) revisited within consumption theory.

Identification of the Sharing Rule with Two Exclusive Goods and Distribution Factors Chiappori (1988, 1992) shows that the partial effects of the sharing rule $\phi(\cdot)$ are identifiable up to an unknown additive constant provided that at least an exclusive good is observed. The knowledge of individual prices associated with exclusive goods and distribution factors improves the quality of identification (Chiappori and Ekeland 2009, Bourguignon, Browning, and Chiappori 2009). For the purpose of our theory investigation, we assume to observe an information set comprising two exclusive goods, associated individual prices and at least one distribution factor as in Chiappori, Fortin, and Lacroix (2002).

We now show that this result is incomplete.[7] The line of the identification proof that we introduce here is similar to the one developed by Chiappori, Fortin, and Lacroix (2002), but for the fact that we place special attention to the identification of the demand parameters that interact with the parameters of the sharing rule. This different perspective allows us to identify new relations between the structural and the reduced form parameters that are responsible for the lack of identification given the conditions used in Chiappori, Fortin, and Lacroix (2002).

To identify the partial effects of the sharing rule $\{\phi_{p_1}, \phi_{p_2}, \phi_x, \phi_s\}$, there must be a one-to-one correspondence between the elements of the Jacobian and Hessian matrices[8] of the reduced (left-hand side of equation 2) and structural (right-hand side of equation 2) demands of the exclusive good $c^i(p_1, p_2, x, s)$ ensuring that every parameter of the collective structure can be recovered as a function of reduced form parameters only.

Proposition 1 (Non Identification)**.** *For a general reduced $c^i(p_1, p_2, x, s)$ and structural $C^i(p_i, \phi_i(p_1, p_2, x, s))$ specification of collective demand differentiable at least once, the partial effects of the sharing rule $\{\phi_{p_1}, \phi_{p_2}, \phi_x, \phi_s\}$ and the individual demand functions are not identified because for each element of the Jacobian of the reduced form it is not possible to establish a one-to-one correspondence with the elements of the Jacobian of the structural form. In the sense that for each element of the set of structural parameters*

[7]Our presentation is in the context of consumption analysis. The result can be extended to labor supply without loss of generality.

[8]For the Hessian matrix to be non empty the reduced form must be at least twice differentiable in the variables.

$S=\left\{C^i_{p_1},C^i_{p_2},C^i_x,C^i_s\right\}$ *there is not a unique relationship* $S=f(R)$ *for* $f:\mathbb{R}\rightarrow\mathbb{R}$ *being a bijection with the elements of the set of reduced form parameters* $R=\left\{c^i_{p_1},c^i_{p_2},c^i_x,c^i_s\right\}.$

Proof. Recall first that $c^i(p_1,p_2,x,s)=C^i(p_i,\phi_i(p_1,p_2,x,s))$ for both $i=1,2$. Note that this equality drives the equality of the corresponding element of the reduced and structural matrices of first derivatives. Then, the Jacobians of the demand equations of the two members are

Member 1		*Member 2*		
Reduced	*Structural*	*Reduced*	*Structural*	
$c^1_{p_1}$	$C^1_{p_1}+C^1_\phi\phi_{p_1},$	$c^2_{p_1}$	$-C^2_\phi\phi_{p_1},$	(8)
$c^1_{p_2}$	$C^1_\phi\phi_{p_2},$	$c^2_{p_2}$	$C^2_{p_2}-C^2_\phi\phi_{p_2},$	(9)
c^1_x	$C^1_\phi\phi_x,$	c^2_x	$C^2_\phi(1-\phi_x),$	(10)
c^1_s	$C^1_\phi\phi_s,$	c^2_s	$-C^2_\phi\phi_s,$	(11)

where c^i_j represents the first derivative with respect to the $j-th$ variable of the left-hand side of equation (2) and C^i_j the first derivative of the right-hand side for $i=1,2$ and $j=p_1,p_2,x,s.$

Using equations (8), (10), and (11) of member 1, we obtain

$$A=\frac{c^1_{p_2}}{c^1_x}=\frac{\phi_{p_2}}{\phi_x},\qquad C=\frac{c^1_s}{c^1_x}=\frac{\phi_s}{\phi_x},\tag{12}$$

for $c^1_x\neq 0$.[9]

From equation (9) to (11) of member 2, we have

$$B=\frac{c^2_{p_1}}{c^2_x}=-\frac{\phi_{p_1}}{(1-\phi_x)},\qquad D=\frac{c^2_s}{c^2_x}=-\frac{\phi_s}{(1-\phi_x)},\tag{13}$$

for $c^2_x\neq 0$.

From the terms C and D we derive the partial effects ϕ_x and ϕ_s

$$\phi_x=\frac{c^1_xc^2_s}{c^1_xc^2_s-c^1_sc^2_x}=\frac{D}{D-C},\tag{14}$$

$$\phi_s=\frac{c^1_sc^2_s}{c^1_xc^2_s-c^1_sc^2_x}=\frac{CD}{D-C},\tag{15}$$

assuming that $D\neq C$.

Using the definitions of A, D and C, we recover the partial effect ϕ_{p_2}

$$\phi_{p_2}=\frac{c^1_{p_2}c^2_s}{c^1_xc^2_s-c^1_sc^2_x}=\frac{AD}{D-C},\tag{16}$$

[9]We adopt the notation in capital letters as in Chiappori, Fortin, and Lacroix's (2002) to facilitate comparisons.

and from B, C and D, we get the partial effect ϕ_{p_1}

$$\phi_{p_1} = \frac{c^2_{p_1} c^1_s}{c^1_x c^2_s - c^1_s c^2_x} = \frac{BC}{D - C}, \tag{17}$$

for any point such that $D \neq C$ where the unobservable parameters of the sharing rule are all expressed in terms of observable reduced form information.

The identifying relations (14)-(17) are not complete. Consider that

$$\phi_x = \frac{c^1_x c^2_s}{c^1_x c^2_s - c^1_s c^2_x} = \frac{c^1_x}{C^1_\phi} = 1 - \frac{c^2_x}{C^2_\phi}, \tag{18}$$

and

$$\phi_s = \frac{c^1_s c^2_s}{c^1_x c^2_s - c^1_s c^2_x} = \frac{c^1_s}{C^1_\phi} = -\frac{c^2_s}{C^2_\phi}, \tag{19}$$

$$\phi_x = C^{-1} \phi_s = \frac{c^1_x}{c^1_s} \phi_s, \tag{20}$$

$$C^1_{p_1} = c^1_{p_1} - c^2_{p_1} \frac{c^1_s}{c^2_s} = c^1_{p_1} - C^1_\phi \phi_{p_1}, \tag{21}$$

$$C^2_{p_2} = c^2_{p_2} - c^1_{p_2} \frac{c^2_s}{c^1_s} = c^2_{p_2} + C^2_\phi \phi_{p_2}. \tag{22}$$

For the preference parameters not included in the sharing function, we have

$$C^1_\phi = \frac{c^1_x}{1 - \frac{c^2_x}{C^2_\phi}} = -C^2_\phi \frac{c^1_s}{c^2_s} = c^1_x - c^2_x \frac{c^1_s}{c^2_s}, \tag{23}$$

so that

$$C^1_\phi = c - c^2_x \frac{c^1_s}{c^2_s} = \frac{c^1_x}{C^1_\phi} = -\frac{c^1_x}{(1 - c^2_x)} \frac{c^1_s}{c^2_s} C^2_\phi = -\frac{c^1_x}{c^2_x} \frac{(1 - \phi_x)}{\phi_x} C^2_\phi, \tag{24}$$

$$C^2_\phi = c^2_x - c^1_x \frac{c^2_s}{c^1_s} = \frac{c^2_x}{(1 - \phi_x)}, \tag{25}$$

implying that identification is not achieved because it is not possible to establish a one-to-one correspondence. All parameters of the sharing rule can be written as a function of other sharing rule parameters. □

All unobservable sharing rule parameters $\phi_{\{x,p_1,p_2,s\}}$ can be expressed both in terms of reduced form parameters, thus leading to identification as in the first part of the proof, and in terms of a mixed expression with at least one sharing rule parameter as in the second part of the proof, thus contradicting identification. In general, if A is any logical statement, then $A \& \neg A$ implies that the statement is false.

Suppose now that the specification of the reduced form is nonlinear in the variables so that it can be differentiated more than once. The second derivatives do not add useful information for identification. To see this, we concentrate our attention without loss of relevant information on equations (14)-(17) describing the parameters of the sharing rule.

The one-to-one correspondence condition applies to the elements of both the Hessian matrix of the reduced form H_R and the Hessian matrix of the structural form H_S as follows

$$H^R = \begin{bmatrix} \frac{\partial \frac{D}{D-C}}{\partial x} & \frac{\partial \frac{D}{D-C}}{\partial s} & \frac{\partial \frac{D}{D-C}}{\partial p_1} & \frac{\partial \frac{D}{D-C}}{\partial p_2} \\ \frac{\partial \frac{CD}{D-C}}{\partial x} & \frac{\partial \frac{CD}{D-C}}{\partial s} & \frac{\partial \frac{CD}{D-C}}{\partial p_1} & \frac{\partial \frac{CD}{D-C}}{\partial p_2} \\ \frac{\partial \frac{AD}{D-C}}{\partial x} & \frac{\partial \frac{AD}{D-C}}{\partial s} & \frac{\partial \frac{AD}{D-C}}{\partial p_1} & \frac{\partial \frac{AD}{D-C}}{\partial p_2} \\ \frac{\partial \frac{BC}{D-C}}{\partial x} & \frac{\partial \frac{BC}{D-C}}{\partial s} & \frac{\partial \frac{BC}{D-C}}{\partial p_1} & \frac{\partial \frac{BC}{D-C}}{\partial p_2} \end{bmatrix}, \quad \text{and } H^S = \begin{bmatrix} \frac{\partial \phi_x}{\partial x} & \frac{\partial \phi_x}{\partial s} & \frac{\partial \phi_x}{\partial p_1} & \frac{\partial \phi_x}{\partial p_2} \\ \frac{\partial \phi_s}{\partial x} & \frac{\partial \phi_s}{\partial s} & \frac{\partial \phi_s}{\partial p_1} & \frac{\partial \phi_s}{\partial p_2} \\ \frac{\partial \phi_{p_1}}{\partial x} & \frac{\partial \phi_{p_1}}{\partial s} & \frac{\partial \phi_{p_1}}{\partial p_1} & \frac{\partial \phi_{p_1}}{\partial p_2} \\ \frac{\partial \phi_{p_2}}{\partial x} & \frac{\partial \phi_{p_2}}{\partial s} & \frac{\partial \phi_{p_2}}{\partial p_1} & \frac{\partial \phi_{p_2}}{\partial p_2} \end{bmatrix}. \tag{26}$$

Consider for example the $(1,1)$ element

$$\frac{\partial \frac{D}{D-C}}{\partial x} = \frac{\partial \phi_x}{\partial x} \Longrightarrow \frac{DC_x - CD_x}{(D-C)^2} = -\frac{c_{xx}^2 C_\phi^2 + c_x^2 C_{\phi x}^2}{\left(C_\phi^2\right)^2}.$$

Similarly, for all other elements. It follows that the correspondence condition is not satisfied. Each element of H^R cannot be uniquely associated with the corresponding element of H^S because the unobservable parameters of the structural form are expressed as function of both structural and reduced form parameters. We may conclude that there does not exist either local or global identification.

Note that the equality of the Hessian's cross-derivative restrictions

$$H_{ij}^R = H_{ji}^R \text{ and } H_{ij}^S = H_{ji}^S \quad \nabla i, j, \tag{27}$$

along with the negative semi-definiteness of the symmetric Slutsky matrix for member 1

$$c_{p_1}^1 - c_x^1 \left(\frac{BC}{D-C} - c^1 \right) \left(\frac{D-C}{D} \right) \leq 0, \tag{28}$$

and, similarly, for member 2

$$c_{p_2}^2 - c_x^2 \left(\frac{AD}{D-C} - c^2 \right) \left(-\frac{D-C}{C} \right) \leq 0, \tag{29}$$

corresponding to equation (6) and (7). This system of differential equations would provide the necessary and sufficient conditions for demand equations to be consistent with the collective model by guaranteeing integrability if we are able to establish a correspondence between H_{ij}^R and H_{ij}^S $\nabla i, j$.

Let us examine the implications of this indeterminateness. The proportionality condition, that is fundamental because solves the over-identification problem generated by the presence of more than one distribution factor, still holds (Bourguignon, Browning, Chiappori 1995, Chiappori, Fortin, and Lacroix 2002, Bourguignon, Browning, and Chiappori 2009). We now reproduce the proof using the frame developed in Proposition 1

Proposition 2 (Proportionality Condition. Bourguignon, Browning, and Chiappori (1995))**.** *With several distribution factors $r = 1, ..., R$ the ratio of the parameters associated with each distribution factor of equations of member 1 and 2 is the same*

$$\frac{C_1}{D_1} = \frac{C_r}{D_r} \quad \forall r = 2, ..., R. \tag{30}$$

Proof. Consider the case with $r = 2$. From the relationships shown in 11 we have for member 1

$$c^1_{s_1} = C^1_\phi \phi_{s_1} \text{ and } c^1_{s_2} = C^1_\phi \phi_{s_2} \tag{31}$$

and for member 2

$$c^2_{s_1} = -C^2_\phi \phi_{s_1} \text{ and } c^2_{s_2} = -C^2_\phi \phi_{s_2}. \tag{32}$$

Note that the parameters associated with the distribution factors is the same for both members. Their ratio is

$$\frac{\phi_{s_1}}{\phi_{s_2}} = \frac{c^1_{s_1}}{c^2_{s_1}} \text{ and } \frac{\phi_{s_1}}{\phi_{s_2}} = \frac{c^1_{s_2}}{c^2_{s_2}}. \tag{33}$$

Therefore

$$\frac{c^1_{s_1}}{c^2_{s_1}} = \frac{c^1_{s_2}}{c^2_{s_2}} \tag{34}$$

implying that

$$\frac{c^1_{s_1}}{c^2_{s_1}} \frac{c^2_x}{c^1_x} = \frac{C_1}{D_1} = \frac{C_r}{D_r} = \frac{c^1_{s_r}}{c^2_{s_r}} \frac{c^2_x}{c^1_x} \quad \forall r = 2, ..., R. \tag{35}$$

□

Proposition 2 is not complete. If we further exploit the relationships implied by the correspondence conditions described in Proposition 1, we can see that the proportionality condition constraining the ratio of the distribution factors' partial effects of each family member to be the same is also proportional to the ratio of the income effects specific to each family member. This is shown in the following proposition.

Proposition 3 (Income Proportionality Condition)**.** *With several distribution factors $r = 1, ..., R$ the ratio of the parameters associated with each distribution factor of equations of member 1 and 2 is the same and is also equal to the ratio of the structural income's partial effects of each family member,*

$$\frac{C_1}{D_1} = \frac{C_r}{D_r} = \frac{C^1_\phi}{C^2_\phi} = -\frac{c^1_x}{c^2_x} \frac{1 - \phi_x}{\phi_x}, \quad \forall r = 2, ..., R. \tag{36}$$

Proof. Recall that

$$C_1 = \frac{c^1_{s_1}}{c^1_x} = \frac{\phi_{s_1}}{\phi_x}, \quad \text{and} \quad D_1 = \frac{c^2_{s_1}}{c^2_x} = -\frac{\phi_{s_1}}{1 - \phi_x}, \tag{37}$$

with

$$\frac{C_1}{D_1} = \frac{c^1_{s_1}}{c^2_{s_1}} = -\frac{c^1_x}{c^2_x} \frac{1 - \phi_x}{\phi_x}, \tag{38}$$

noting also that $\frac{c^1_{s_1}}{c^2_{s_1}} = \frac{C^1_\phi}{C^2_\phi}$, and extending to the case of many distribution factors, we have

$$\frac{C_1}{D_1} = \frac{C_r}{D_r} = \frac{C^1_\phi}{C^2_\phi} = -\frac{c^1_x}{c^2_x}\frac{1-\phi_x}{\phi_x} \quad \forall r = 2, ...R, \tag{39}$$

for $\phi_x \neq 0$ and $\phi_x \neq 1$. □

The restriction derived in Proposition 3 says that distribution factors are informative about the rule governing the intra-household distribution of resources only if we can implement an exact relationship with income. This is intuitive if we consider that distribution factors affect optimal choices only through the sharing function that describes across-households variations in individual income levels.[10] Note also that while the restrictions described in Proposition 2 can be imposed in the estimation because it involves only reduced from parameters, the ratio restriction derived in Proposition 3 cannot be imposed in the estimation because it involves both reduced and structural parameters. This can be interpreted as a further evidence of the identification difficulties.

Closer inspection of the restrictions associated with the sharing rule parameters described in Proposition 1 reveals that it would be sufficient to know *a priori* at least one sharing rule parameter to identify all other parameters exactly. An interesting candidate is ϕ_x because of the natural interpretation of the parameter as a share. Suppose that we do not have sufficient variation in both prices and distribution factors. Therefore, we cannot longer assume that the reduced form parameters associated with prices and distribution factors are statistically significantly different from zero. Then, the sharing function can be reasonably described as

$$\begin{cases} \phi_1 = \phi_x x \\ \phi_2 = (1-\phi_x)\, x \end{cases} \quad \text{for } \phi_x \neq 0;\ \phi_x \neq 1, \tag{40}$$

showing that ϕ_x is well-defined within the following range as required by the constraint $\phi_1 + \phi_2 = x$

$$0 < \phi_x < 1, \tag{41}$$

thus, the parameter ϕ_x acts as a proportion that scales the income parameter of both the reduced form $-\frac{c^1_x}{c^2_x}\frac{1-\phi_x}{\phi_x}$ and the structural form $\frac{C^1_\phi}{C^2_\phi}\left(\frac{\phi_x}{1-\phi_x}\right)$ as shown in equation (39). The minimal representation of the sharing rule in equation (40) tell us that information about total income is not sufficient for identification, unless something is known *a priori* about how income is shared within the household. Part of this information is provided by the knowledge of at least one privately consumed good, in line with the minimal informational requirement of the collective theory, so that it is possible to construct the best approximation of individual incomes before estimation.[11] Therefore, ϕ_x is a resource

[10] It is interesting to note that the relationship of Proposition 3 is present also in Chiappori, Fortin, and Lacroix (2002), though not explicitly. The equalities across ratios of distribution factors in equation (8) summarizing the collective restrictions on labor supply are also equal to the negative of the ratio of the demand parameters α_2 and β_2 in equation (11) and (12).

[11] This interpretation explains why Bourguignon *et al.* (1994) and Lise and Seitz (2011) use a logistic transformation to ensure that the estimated sharing function has to be maintained within the zero-one range.

share that, if known *a priori* as best as possible, permits the derivation of an approximate measure of individual incomes.

Our objective is now to investigate whether it is possible to achieve exact identification taking the interpretation of ϕ_x derived above into account. To do so, we suppose that ϕ_x is a known constant $\xi \in (0,1)$. As a consequence, the sharing rule function is separable in a function that scales individual incomes as

$$\phi_1(x, p_1, p_2, s) = \vartheta(x) m_1(p_1, p_2, s) \text{ with } \vartheta(x) = \xi x \approxeq x_1 \tag{42}$$

where $m_1(p_1, p_2, s) \in \mathbb{R}_+$. Note that when $0 < m_1(\cdot) < \frac{x}{\xi x}$ approximates 1 the function reduces to equation 40. The separability property states that is the resource share $\vartheta(x) = \xi x \approxeq x_1$ that may depend on income, not the scaling function $m_1(p_1, p_2, s)$ that only describes how resources are distributed across household members.

We can now show that this is sufficient to obtain the exact identification of all the remaining sharing rule parameters belonging to the scaling function $\{\phi_{p_1}, \phi_{p_2}, \phi_s\}$.

Proposition 4 (Identification)**.** *The sharing rule parameters $\{\phi_{p_1}, \phi_{p_2}, \phi_s\}$ are exactly identified.*

Proof. Recall equation (2) for $i = 1, 2$ and use $\phi_x = \xi$. Then, the elements of the Jacobian matrices become

Member 1		*Member 2*		
Reduced	*Structural*	*Reduced*	*Structural*	
$c^1_{p_1}$	$C^1_{p_1} + C^1_\phi \phi_{p_1},$	$c^2_{p_1}$	$-C^2_\phi \phi_{p_1},$	(43)
$c^1_{p_2}$	$C^1_\phi \phi_{p_2},$	$c^2_{p_2}$	$C^2_{p_2} - C^2_\phi \phi_{p_2},$	(44)
c^1_x	$C^1_\phi \xi,$	c^2_x	$C^2_\phi (1-\xi),$	(45)
c^1_s	$C^1_\phi \phi_s,$	c^2_s	$-C^2_\phi \phi_s,$	(46)

as a consequence, all the sharing rule parameters are exactly identified because there exists a one-to-one correspondence ensuring that all the structural parameters are function of reduced form parameters alone

$$\phi_{p_1} = -\frac{c^2_{p_1}}{C^2_\phi} = \frac{c^2_{p_1}}{c^2_s}\xi, \tag{47}$$

$$\phi_{p_2} = \frac{c^1_{p_2}}{C^1_\phi} = \frac{c^1_{p_2}}{c^1_s}(1-\xi), \tag{48}$$

$$\phi_{s_r} = \frac{c^1_s}{C^1_\phi} = -\frac{c^2_s}{C^2_\phi} = \frac{c^1_s}{c^1_x}\xi = -\frac{c^2_s}{c^2_x}(1-\xi). \tag{49}$$

□

The restrictions in the right-hand side of conditions (47), (48), and (49) can be used to recover the structure only from the estimates of the reduced form parameters. For this reason, the restrictions hold globally. No functional form assumptions are necessary to identify the sharing rule. Note also that the knowledge of $\phi_x = \xi \in (0,1)$ permits rewriting the income proportionality condition described in Proposition 3 in a form that can be imposed in the estimation because it involves only reduced from parameters

$$\phi_{s_r} = \frac{c^1_{s_1}}{c^2_{s_1}} = \frac{c^1_{s_r}}{c^2_{s_r}} = -\frac{C^1_\phi}{C^2_\phi} = -\frac{c^1_x}{c^2_x}\frac{(1-\xi)}{\xi}, \quad \forall r = 2, ..., R. \tag{50}$$

The ratio of the reduced form parameters associated with the distribution factors must be proportional to the negative ratio of the reduced form parameters associated with income scaled by the ratio between member 2's and member 1's share. Now distribution factors are directly related to income by a restriction that can be enforced in the reduced form estimation. The structure, which embeds the reduced form restrictions by construction, is also estimable.

Remark 1. If resources are shared equally between the two household's members assuming that $\xi = \frac{1}{2}$, then the income proportionality condition does not depend on the share parameter ξ

$$\frac{c^1_{s_r}}{c^2_{s_r}} = -\frac{c^1_x}{c^2_x} = -\frac{C^1_\phi}{C^2_\phi}. \tag{51}$$

In this special case, the collective household model reduces to the unitary model where it is assumed that resources are shared equally among the household's members.

We now apply these general results to the functional form of the demand equations used in the empirical analysis.

3 Parametric Specification of the Collective Consumption Model

We present a collective model where the sharing rule is defined by an income modifying function scaling total expenditure. Although this structural form is often used in empirical applications (Browning *et al.* 1994, Lewbel and Pendakur 2008, Lize and Seitz 2010), we argue that scaling total expenditure at the household level captures demographic differences across households but this specification does not identify the decision process of allocating resources between household members. We show that the specification of the structural sharing rule should be defined using individuals' total expenditure because total household expenditures provide no information about the distribution of resources within the household.

We choose a double-log functional form for the collective demand equations (2)-(3) leaving unspecified the functional form of the sharing rule. The specification of the demand equation does not include cross price effects to facilitate comparisons with the labor supply application developed by Chiappori, Fortin, and Lacroix (2002).

In order to recover the underlying structure of the collective model, we use the available information about the private consumption of clothing for adults and children. In line with Rothbarth's observation (Rothbarth 1941, 1943), the presence of a child induces a reallocation of consumption within the household. Assuming that income remains unchanged before and after a child arrival, the associated new demand for children goods is met by reducing the expenses on adult goods. At the same level of total expenditure, larger families spend less on adult goods. As shown in Perali (2003:183, Figure 6.3), food and clothing for children are positively associated with the presence of a child. Greater expenditure allocated on children goods is matched by a reduction in the budget share devoted to clothing for adults and, to a less extent, all other goods. These income reallocation effects,[12] present also in our data, are what our empirical identification strategy intends to capture.

The reduced and structural equations of the exclusive good $c^i(p_1, p_2, x, s)$ are specified according to the following differentiable equations,[13] where parameters in greek style are associated with the structural form while the latin style notation is associated with the reduced form parameters,

Reduced Form	**Structural Form**

Member 1

$$\ln c^1 = a_0 + \sum_{i=1}^{D} a_{d_i} \ln d_{1i} + a_{p_1} \ln p_1 + a_{p_2} \ln p_2 + a_x \ln x + \sum_{r=1}^{R} a_{s_r} \ln s_r, \qquad (1.R)$$

$$\ln C^1 = \alpha_0 + \sum_{i=1}^{D} \alpha_{d_1} \ln d_{1i} + \alpha_{p_1} \ln p_1 + \alpha_x \phi(\ln p_1, \ln p_2, \ln x, \ln s), \qquad (1.S)$$

Member 2

$$\ln c^2 = b_0 + \sum_{i=1}^{D} b_{d_i} \ln d_{2i} + b_{p_1} \ln p_1 + b_{p_2} \ln p_2 + b_x \ln x + \sum_{r=1}^{R} b_{s_r} \ln s_r, \qquad (2.R)$$

$$\ln C^2 = \beta_0 + \sum_{i=1}^{D} \beta_{d_i} \ln d_{2i} + \beta_{p_2} \ln p_2 + \beta_x (\ln x - \phi(\ln p_1, \ln p_2, \ln x, \ln s)), \qquad (2.S)$$

where d represents a set of exogenous socio-demographic characteristics capturing observable heterogeneity, p_1 and p_2 are the prices of the two exclusive goods, x is total family expenditure, s_r is the rth element of the $r = 1, ..., R$ set of distribution factors. As defined in Section 2, $\phi(\cdot)$ is the sharing rule in levels not in share and $\phi_2(\cdot) = \ln x - \phi(\cdot)$ is the amount of family resources allocated to member 2 given by the difference between overall family resources and the sharing rule. We choose a log-linear specification for the

[12] Clothing for children is also a public good when the same clothes are used for more than one child. The sharing of children clothes mitigates the income reallocation effect and can be modeled using a Barten-like price substitution effect as done in Dunbar, Lewbel, and Pendakur (2012). The modeling of the public nature of children goods, which is fully compatible with the present approach, would improve the quality of the identification strategy, but would imply the estimation of both a price and an income scaling function that, while not adding estimation difficulties, may distract attention from the core identification issues discussed in the present work. For this reason, the empirical application of our work is limited to families with only one child thus making public or joint consumption effects not relevant.

[13] Note that Chiappori, Fortin, and Lacroix (2002) use a semi-log specification for the spouses' labour equations.

unrestricted equations (1.R) and (2.R) as well. There are no interaction or quadratic terms because our intention is to have a parsimonious specification associated with the minimum amount of information sufficient to uniquely recover the structural equations (1.S)-(2.S) and ensure a global identification of the partial effects.

The objective is identification of the structural demand parameters $(\alpha_{p_1}, \alpha_x, \beta_{p_2}, \beta_x)$ and of the unobservable function $\phi(\cdot)$ of equations (1.S) and (2.S). We use the same technique described in Proposition 1. The identifying information set comprises two exclusive goods and R distribution factors as in the initial theory section. Our purpose is now to uncover the identifying conditions underlying the structural equations (1.S) and (2.S).

Following Proposition 1, we obtain the following Jacobian matrices for member 1

Reduced Form	Structural Form	
$\ln c^1_{d_i} = a_{d_i}$	$\ln C^1_{d_i} = \alpha_{d_i}, \quad i = 1, ..., D,$	(52)
$\ln c^1_{p_1} = a_{p_1}$	$\ln C^1_{p_1} = \alpha_{p_1} + \alpha_x \phi_{p_1},$	(53)
$\ln c^1_{p_2} = a_{p_2}$	$\ln C^1_{p_2} = \alpha_x \phi_{p_2},$	(54)
$\ln c^1_x = a_x$	$\ln C^1_x = \alpha_x \xi,$	(55)
$\ln c^1_{s_r} = a_{s_r}$	$\ln C^1_{s_r} = \alpha_x \phi_{s_r}, \quad r = 1, ..., R,$	(56)

and, similarly, for member 2

Reduced Form	Structural Form	
$\ln c^2_{d_i} = b_{d_i}$	$\ln C^2_{d_i} = \beta_{d_i}, \quad i = 1, ..., D,$	(57)
$\ln c^2_{p_1} = b_{p_1}$	$\ln C^2_{p_1} = -\beta_x \phi_{p_1},$	(58)
$\ln c^2_{p_2} = b_{p_2}$	$\ln C^2_{p_2} = \beta_{p_2} - \beta_x \phi_{p_2},$	(59)
$\ln c^2_x = b_x$	$\ln C^2_x = \beta_x (1 - \xi),$	(60)
$\ln c^2_{s_r} = b_{s_r}$	$\ln C^2_{s_r} = -\beta_x \phi_{s_r}, \quad r = 1, ..., R.$	(61)

Note that all the sharing rule parameters are function of α and β as we should expect from Proposition 4. Given the restriction $\phi_x = \xi$ for $\xi \in (0, 1)$, the demand parameters are identified as follows

$$\alpha_x = \frac{1}{\xi} a_x, \tag{62}$$

$$\beta_x = \frac{1}{(1-\xi)} b_x, \tag{63}$$

and

$$\alpha_{p_1} = a_{p_1} + b_{p_1} \frac{a_x}{b_x}, \tag{64}$$

$$\beta_{p_2} = b_{p_2} + a_{p_2} \frac{b_x}{a_x}. \tag{65}$$

The identifying conditions of the parameters of the sharing rule simplify to

$$\phi_{s_r} = -\xi \frac{a_{s_r}}{a_x} = -(1-\xi) \frac{b_{s_r}}{b_x}, \tag{66}$$

$$\phi_{p_1} = -\xi \frac{b_{p_1}}{b_x} = (1-\xi)\frac{b_{p_1} a_{s_r}}{a_x b_{s_r}}, \tag{67}$$

$$\phi_{p_2} = -\xi \frac{a_{p_2}}{a_x} = (1-\xi)\frac{a_{p_2} b_{s_r}}{b_x a_{s_r}}. \tag{68}$$

The simplicity of this set of restrictions is evidence of the informational value of knowing ϕ_x. The income proportionality condition then becomes

$$-\frac{a_{s_r}}{b_{s_r}} = \frac{a_x}{b_x} = \frac{\alpha_x}{\beta_x}\frac{(1-\xi)}{\xi}. \tag{69}$$

Solving the system of differential equations (66) to (68) of the partial effects $(\phi_{p_1}, \phi_{p_2}, \phi_{s_r})$, we obtain a logarithmic specification of the sharing rule function analogous to the one obtained by Chiappori, Fortin, and Lacroix (2002) in equation (10)

$$\phi_1 = \xi \ln x + (\phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2) = \ln x_1 + \ln m(p_1, p_2, s) \tag{70}$$

that can be aggregated as

$$\phi_1 = \ln x_1 + \ln m(p_1, p_2, s), \tag{71}$$

and

$$\phi_2 = \ln x - \phi_1 = \ln x_2 - \ln m(p_1, p_2, s), \tag{72}$$

where $\ln x_1 = \xi \ln x$, $\ln x_2 = (1-\xi)\ln x$. The value of the scaling function $m(p_1, p_2, s)$ must be $0 < \ln m(p_1, p_2, s) < \ln \frac{x}{x_i}$.[14] This functional assumption shows that the sharing rule specified as in 70 is an income scaling function in the style described by Lewbel (1985, Theorem 8) and in the tradition of the demographic functions modifying either prices or income (Pollak and Wales 1981, Lewbel 1985, Bollino, Perali, and Rossi 2000, Perali 2003). The sharing rule and demographic functions have in common the fact that both are unobservable functions.

In line with the interpretation of ξ as a resource share, it is appropriate to redefine the sharing function as the product between the best approximation for individual income and an income scaling function that includes all the exogenous variables in the model. It is worth remarking that the scaling function $\ln m(p_1, p_2, s)$, which must be different from zero, depends on individual prices p_1, p_2 and distribution factors s only, not on total expenditure.[15]

Note that if $\xi = 1$, which would assign all household resources to member 1, the sharing rule specification becomes

$$\ln \phi_1(p_1, p_2, x, s) = \ln(x m(p_1, p_2, s)). \tag{73}$$

[14]For the scaling function $m(p_1, p_2, s)$ to be bounded between 0 and 1, and thus assuring that members do not receive either negative expenditure or income share that exceeds the household resource endowments, in general it is adopted a logistic functional form (Browning *et al.* 1994, Lise and Seitz 2011, Browning, Chiappori, and Lewbel 2010, Lewbel and Pendakur 2008, and Donni and Bargain 2009). In the present work, the scaling function $m(p_1, p_2, s)$ is estimated without imposing this functional form.

[15]This separability property is analogous to the properties of independence of income of the sharing rule adopted by Dunbar, Lewbel and Pendakur (2012).

This specification has been adopted in previous papers (Browning et al. 1994, Lise and Seitz 2011, Browning, Chiappori, and Lechene 2006, and Chiappori, Donni, and Komunjer 2012) either in consumption or labor supply frameworks. However, our analysis shows that rather than learning something about the rule governing the intra-household allocation of resources, we in fact explain only variations in income across households. The assumption $\xi = 1$ is therefore observationally equivalent to the unitary model.

Given the log-linear specification of the sharing function as in (70) and that the resource share ξ is assumed to be known we obtain the following directly estimable structural specification of the demand function and the sharing rule is[16]

$$\ln C^1 = \alpha_0 + \sum_{i=1}^{D} \alpha_i \ln d_i + \alpha_{p_1} \ln p_1 + \alpha_x \ln x_1 + \alpha_x \left(\phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2 \right), \tag{74}$$

$$\ln C^2 = \beta_0 + \sum_{i=1}^{D} \beta_i \ln d_i + \beta_{p_2} \ln p_2 + \beta_x \ln x_2 - \beta_x \left(\phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2 \right). \tag{75}$$

Individual shares of total household expenditure $\xi_i \in (0,1)$ are constructed using all the observable information about private consumption in the following way
2011

$$\xi_i = \frac{x_i}{x} = \frac{\mathbf{0.5}\, \mathbf{p}_\mathbf{o}' \mathbf{c}_\mathbf{o} + (\mathbf{p}_i \mathbf{C}^i)}{x}, \tag{76}$$

for $i = 1, 2$ and $\xi_1 + \xi_2 = 1$. We allocate the expenditure of non-assignable goods to each household member in equal proportion and add the assignable expenditure of each individual. This approximation of the real individual shares is the best that can be done given the available information about the consumption of assignable goods. The modifying function $m(p_1, p_2, s)$ can be interpreted as a correction factor. Individual incomes obtained by multiplying the resource share ξ_i by $\ln x$ vary across households in relation to the levels of non-assignable expenditure and the levels of individual consumption. The latter variability permits explaining the variation in the intra-household resource allocation across households provided that the assumption of a uniform distribution of non-assignable consumption does not influence how the income scaling function captures the inter-household deviations from the uniform distribution.

Proposition 5 (Neutrality of non-assignable expenditure)**.** *The assumption of fair division of the non-assignable goods between household members is neutral because it does not affect the relative magnitude of the parameters of the scaling function* $m^\star(p_1, p_2, s)$.

Proof. See appendix. □

The assumption of a division in equal shares is innocent because both the parameters associated with the approximation of individual incomes and the parameters of the scaling function capture the difference between the observed individual allocations as it is

[16] If the resource share ξ were not known the specification of the income term $\alpha_x \ln \phi_1$ would be $\alpha_x \ln \phi_1 (p_1, p_2, x, s) = \alpha_x (\phi_x \ln x + \phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2)$ thus making it apparent the source of the parametric identification problem. It would be in fact possible to identify the parameters' product $A_x = \alpha_x \phi_x$, but not either α_x or ϕ_x thus precluding the direct estimation of the structural model.

required for the identification of the sharing rule that properly accounts for the exclusive information. As it is natural to anticipate, if assignable expenditure is also distributed uniformly across household members, the information about private consumption is not informative about the intra-household distribution of resources.

Remark 2. *The exclusive good must be "good."* If the objective is the estimation of the rule governing the allocation of resources within the household, then the difference between the approximation of individual expenditures $(x_1 - x_2) > \theta$ where θ is a sufficiently large value. If not and $(x_1 \sim x_2)$, then the estimates of the parameters of interest are the same up to a neutral normalization and nothing is learned about the intra-household allocation of resources.

Therefore, to obtain identification of the structure of the collective model the individual demand equations of the assignable or exclusive goods must be significantly different across members of the same household.

The consumption collective model can be empirically implemented using a) a two stage indirect estimation where the first stage consists in a GMM estimation of the reduced form followed by a minimum distance recovery of the structural form as in Chiappori, Fortin, and Lacroix (2002), or b) a direct estimation of the structural form using maximum likelihood. Because of the correspondence theorem described in Proposition 4, the indirect and direct method lead to the same estimates. By implementing both methods, we control our identification experiment ensuring that the theory results are matched in the empirical application. In our experiment design, we also measure the precision of the parameters of the sharing rule in capturing the inter-household variation in assignable expenditure by comparing the estimation with $\xi = 1$, which presumes a uniform distribution of resources within the household as in the unitary model, and ξ is the resource share. The following section is devoted to the data description.

4 Data

We work in the context of a consumption model based on the observation of the private consumption of two exclusive goods, adult and children clothing. The household data used in the empirical application are drawn from the 2007 Italian household budget survey (Italian Statistics 2007). For the purposes of our study, we select families with two parents and a young dependent child. The sample size thus reduces to 1,795 households.

In the Italian household survey, expenditures of highly detailed goods are often censored in a non negligible size. The realization of zero expenditures can be explained by the short duration of the recall period of the survey design or by a stringent budget constraint (Pudney 1989).[17] Censored expenditure categories, including clothing, have been corrected with the specification of a bivariate model as suggested in Blundell and Meghir (1987). We compute total expenditure from the imputed expenditures of the censored goods and correct for remaining measurement errors and possible simultaneity bias by instrumenting total expenditure using, among other instruments, total household income

[17]The choice for the recall period for the expenditure of semi-durable goods, such as clothing, is one of the most important problems encountered by researchers when designing an expenditure survey (Grosh and Glewwe 2000). An excessively long recall period can lead to underestimation of the effective expenditure.

as shown in the next section.

A limit of the Italian household budgets is the absence of information on quantities consumed by each household necessary for the computation of household specific unit values. We approximate unit values with a method suggested by Lewbel (1985) and empirically developed by Atella, Menon, and Perali (2003) and Hoderlein and Mihaleva (2008). This technique captures the spatial and quality variability typical of unit values as it can be deduced from household socioeconomic characteristics. We then add this variability to the price indexes published monthly at the province level and construct nominal unit values from prices in 2007 levels.

Table 2 reports the definition of the variables used in the empirical analysis and the descriptive statistics for our sample. The dependent variables are the log of clothing expenditures of parents and children divided by the respective prices. Distribution factors are the age and education ratio of the parents. In the estimation we control for geographic location (North, Center and South Italy), seasonality effects using a dummy taking the value of one if the family has been interviewed in December, the presence of double-earner families and for parents' and child's age expressed in classes. We also introduced a dummy whether the child is in the 15-17 age class to account for the fact that more grown-up children can participate, to some degree, to the allocation of family resources (Dauphin *et al.* 2011). Note that some demographic attributes, such as age, level of education and working condition, are individual specific, while other characteristics, such as location and working condition of the couple, are the same across household members.

5 Empirical Strategy and Results

In this section, we present the empirical methods for clothing expenditure estimation within a collective framework and their corresponding results. We consider couples with a young dependent child. In these families the presence of a child generates a budget reallocation effect from the parents to the child. Economies of scale in clothing use can be reasonably considered absent. Our main interest is to estimate the sharing rule within a collective consumption model whose specification resembles as closely as possible the collective labor supply model implemented in Chiappori, Fortin, and Lacroix (2002) in order to maintain the set of theory restrictions as comparable as possible. Differently from Chiappori, Fortin, and Lacroix (2002), we use the finding shown in Proposition 1 that the parameter associated with the income term ϕ_x cannot be identified and, as a best estimate of the information content of this parameter, we construct a resource share from the available data on private clothing consumption, in order to implement the income proportionality property and the correspondence described in Proposition 2. The collective model is specified in quantity space in an *ad hoc* fashion in the sense that it is not directly derived from a known utility or expenditure function. This functional choice is similar to the one adopted in the seminal work of Browning *et al.* (1994) of collective consumption with the difference that we do not add a quadratic income term. In general, the simplicity of the demand specification permits us to concentrate on the identification issues associated with estimation.[18]

[18] If the main interest were behavioral and of applied welfare, then a theoretically plausible flexible collective demand system would be required (Menon, Perali, and Piccoli 2012).

We now describe the indirect and direct methods to estimate the system of collective demand equations (74)-(75) for clothing for adults and children in Italy.

Empirical Strategy

Indirect Estimation of the Collective Structure

The indirect estimation uses a Minimum Distance method that is implemented in two stages as it is done in Chiappori, Fortin, and Lacroix (2002).

Stage I - Estimation of the reduced form

$$\ln c^1 = a_0 + \sum_{i=1}^{D} a_{d_i} \ln d_{1i} + a_{p_1} \ln p_1 + a_{p_2} \ln p_2 + a_x \ln x + \sum_{r=1}^{2} a_{s_r} \ln s_r + \epsilon_1, \tag{77}$$

$$\ln c^2 = b_0 + \sum_{i=1}^{D} b_{d_i} \ln d_{2i} + b_{p_1} \ln p_1 + b_{p_2} \ln p_2 + b_x \ln x + \sum_{r=1}^{R} b_{s_r} \ln s_r + \epsilon_2, \tag{78}$$

where we have introduced additive unobserved individual heterogeneity ϵ_i which is assumed to be independent and identically distributed. The notation $k = 1$ represents the parents and $k = 2$ the child. The system is estimated with the income proportionality restriction imposed because the system is otherwise not identified as shown in Proposition 1

$$-\frac{a_{s_r}}{b_{s_r}} = \frac{a_x}{b_x} \quad \nabla r. \tag{79}$$

The parameters of interest $\{\phi_{p_1}, \phi_{p_2}, \phi_s\}$ and $\{\alpha_x, \alpha_{p_1}, \beta_x, \beta_{p_2}\}$ are recovered in the second stage.

Stage II - Recovery of the collective structure The collective structure can be fully recovered by imposing the cross-equation restrictions summarized in Table 1 using the minimum chi square (MCS) estimator. Since the estimates of the unrestricted $\{a, b\}$ parameter vectors are consistent, the MCS method yields consistent asymptotically normal efficient estimates (Berkson 1949 and 1980, Ferguson 1958, Rothenberg 1973). The minimum distance technique is used to impose the collective theory restrictions only. In the spirit of Malinvaud (1970), minimum distance is utilized to recover the demand structure and the sharing rule parameters.

We denote the vector of restricted parameters to be estimated by $\gamma = \{\alpha \mid \beta\}$ where the sub-vectors α and β are vertically concatenated. The collective theory restrictions can be summarized via the mapping $\gamma = F(g)$ relating the restricted parameters $g = \{a \mid b\}$ to the unrestricted structural parameters. The function $F(g)$ incorporates the prior information provided by the collective demand theory as well as the restrictions that identify the sharing rule parameters. Let the Jacobian of this function be denoted by $G\{g\} = \partial F(g) / \partial g$. The Jacobian G has full column rank. The MCS method chooses the estimator g that minimizes the following quadratic form (Rothenberg 1973)

$$\left\{ \hat{a, b} \right\} = \operatorname{argmin}_{\{a,b\}} \left\{ [\gamma - F(g)]' \, \Sigma_g^{-1} [\gamma - F(g)] \right\}, \tag{80}$$

where Σ_g is a consistent estimate of the variance-covariance matrix of g obtained from the jackknife technique. Note that the computational algorithm that estimates the unrestricted parameters g along with its covariance matrix Σ_g in the first stage, and then recovers the structure in the second stage, is an indirect feasible generalized least square procedure. To the extent that it generates consistent and asymptotically efficient estimates, it provides a computationally convenient alternative to the full-information maximum likelihood estimation method. This approach has the unpleasant feature that the partial effects of the sharing rule are specific to each structural specification of the collective model and corresponding reduced functional form making the practical derivation of the partial effects cumbersome in case of demand systems with many equations. Table 1 reports the set of identifying restrictions described both in Proposition 1 and Proposition 4 in the context of the parametric specification of the collective consumption model illustrated in Section 3.

Direct Estimation of the Collective Structure

We directly estimate the structural demand equations using Full Information Maximum Likelihood

$$\ln C^1 = \alpha_0 + \sum_{i=1}^{D} \alpha_{d_1} \ln d_{1i} + \alpha_{p_1} \ln p_1 + \alpha_x \left(\xi_i \ln x + \ln m\left(p_1, p_2, s\right)\right) + \varepsilon_1, \tag{81}$$

$$\ln C^2 = \beta_0 + \sum_{i=1}^{D} \beta_{d_i} \ln d_{2i} + \beta_{p_2} \ln p_2 + \beta_x \left(\xi_i \ln x - \ln m\left(p_1, p_2, s\right)\right) + \varepsilon_2, \tag{82}$$

where ε_i is the error term assumed to be independent and identically distributed. The restrictions required by the collective household model are implicitly embodied in the functional form. The direct estimation of the scaling income function $m\left(p_1, p_2, s\right)$ is akin to the problem of estimating a regression containing unobservable independent variables (Goldberger 1972). We estimate the above sharing rule both in terms of total household expenditure $\ln x$, as in Browning *et al.* (1994) and Lise and Seitz (2011), and in terms of individual expenditure $\ln x_k = \xi \ln x$ where $\xi \in [0, 1]$ is the resource share defined in equation (76). We do so in order to appreciate the contribution of private consumption of clothing in the construction of the resource share which is at the core of the empirical identification strategy.

The log of total expenditure is likely to be endogenous because it depends on economic behaviour and due to measurement error, either because of infrequency of purchases or because of recall errors. To adjust for the possibility of endogeneity of the log of total expenditure, we apply the control function method (Blundell and Robin 1999). According to this method, we regress the log of total expenditure against the exogenous explanatory variables and the log of net household income. The residual r of the auxiliary regression is then included into the clothing equations (81)-(82). The log of total expenditure is exogenous conditional on the additional regressor. Thus, the error term ε_i can be written as the orthogonal decomposition $\varepsilon_i = \nu_i r + \upsilon_i$, where ν_i is a parameter to be estimated and υ_i is an i.i.d error term. Testing if ν_i is significantly different from zero for $i = 1, 2$ corresponds to testing the exogeneity of the log of total expenditure.

Estimation Results

The estimation results for the collective consumption models specified in terms of total household income $\ln x$, that we term model (a),[19] and total individual income $\ln x_k = \xi \ln x$, that we term model (b), are reported in Table 3. We estimate model (a) and (b) both using the indirect and direct method as illustrated above. We choose not to report the parameter estimates with the income proportionality restriction imposed of both the first and second stage of the indirect method, because the results of the second stage are the same as the parameters obtained form the direct structural estimation. Further, we do not report the parameter estimates of the indirect estimation of the model with only the proportionality restriction imposed because the parameters of the sharing rule are not identified as explained in Proposition 1.

Most parameters are statistically significant at conventional levels. The parameters ν_i $\forall i$ associated with the control factor r in both models are significantly different from zero showing that the log of total expenditure without correction would be endogenous. Interestingly, even though restricted to be equal to the ratio of distribution factors, the coefficients associated with the log of household income are statistically significantly different from zero. The comparison of the parameters associated with the income term of model (a) and (b) reveals that they differ significantly both statistically and economically. The income effect is much higher for children's clothing, especially in model (b) that estimates Engel effects at the individual level.

The parameters of the control variables, such as region, month of interview, number of labour income recipients, age of the child and presence of an adult-child, are equal in sign and magnitude in both model (a) and model (b) because the parameters associated with the control variables enter the structural specification linearly. The parameters associated with the own-price is statistically significantly different from zero at conventional levels in both specifications. The sign is correct.

The coefficients of the income scaling function $m(.)$ have the same sign in both model (a) and (b), but differ significantly in magnitude with the exception of the age and education ratio that are not significantly different from zero in both specifications. This is evidence that the estimation expressed in terms of individual expenditures is effectively capturing inter-household variations in the intra-household Pareto allocation of resources as described by the clothing consumption habit of the adult and child component of each household. Model (a) is in fact describing inter-household variations in total household expenditure and we learn nothing about the allocation of resources within the household. The modifying function $m(.)$ in model (a) is simply an income scaling function, not a sharing rule.

Therefore, we comment the impact on the sharing rule only referring to the parameters of the scaling function presented in model (b). A change in children's clothing prices affect the allocation rule more than a change in adults clothing price. The sign of the price effect affecting directly clothing consumption is the opposite of the price effect exerted indirectly through the sharing rule. As the price of both clothing for adults and children increase, adults hold more resources for themselves. An increase in age ratio, that is in the age gap between father and mother, is often associated with higher wages due to a

[19] Because of the income proportionality restriction, model (a), that is expressed in terms of $\ln x$, implies that $\xi_i = 0.5$ as it is explained in Remark 1.

higher position of the father in the life cycle, and higher bargaining power leading to more resources available to children as the sign shows. The effect associated with the education ratio, though not precisely estimated, is positive. This lack of significance does not allow us to verify whether more educated mothers are more or less "altruistic" towards children in the context of our experiment.

Figure 1 shows the sharing rule's pattern as total household expenditure rises. The rule governing the intra-household allocation of resources gives more resources to children at low income levels and less at higher levels, though the function stays in general constant across the income distribution.

6 Conclusions

In this paper we show how to achieve full identification of the collective consumption model identify by restricting the sharing rule parameter associated with total expenditure to a known resource share, effectively capable to correctly interpret the information about exclusive consumption reported in the data, and implementing an additional property that guarantees the proportionality of the ratio of distribution factors with the ratio of the demand parameters associated with individual total expenditure levels. This property was not spelled out in previous collective work, probably because the property was not implementable though embedded in the collective structure. It is however a fundamental property of a collective approach because, as it is reasonable to expect, the effect of distribution factors is directly linked to the resource levels. Failing to do this would be like trying to estimate the sharing rule without exploiting the necessary information to be observed to implement a collective model, that is the information about exclusive consumption. Previous work applying both a direct estimation of the sharing rule, as in Browning *et al.* (1994), and an indirect estimation, such as in Chiappori, Fortin, and Lacroix (2002), did not identify the rule governing the allocation of resources within the household. Conditional on the functional restrictions introduced in this work, we claim that the search for an identifiable sharing rule led us to a robust proposal.

We implement the collective consumption model using Italian household budget data limited to young couples with one child to estimate the rule governing the allocation of resources between adults and children. We perform both an indirect and direct estimation of the structural model to show that there is full correspondence between the structure and the associated reduced form in line with the set of restrictions derived in the theory section. We also compare the estimates of the sharing rule supposing that the resource share is equal to $1/2$, as implied by the unitary model, or to a positive fraction of 1 using as best we can the available information about private consumption as required by the collective theory. This exercise illustrates the relevance of assignable information in the quality of the estimation of the sharing rule. We may then conclude that our results open up the possibility of a direct structural estimation of a collective system of demand equations (Arias *et al.* 2001, Caiumi and Perali 2010, Dunbar, Lewbel, and Pendakur 2012, and Menon, Perali, and Piccoli 2012) as easily as estimating a demand system based on a unitary framework. Further, it permits a neat estimation of individual Engel curves resulting from the decomposition of the income term of the Slutsky equation of a collective decentralized program. According to our estimates, the intra-household distribution of resources is relatively pro-child at low level of total household expenditure.

It should be stressed that the quality of the estimation can improve with the quantity and quality of the information available about the consumption of private goods and with our ability to describe economies of scale derived from the sharing of household public goods by properly modeling household technologies. This is the objective of our future research.

Table 1: Identification Conditions of the Engel Curves and the Sharing Rule

RESTRICTIONS	$\frac{a_{s_1}}{b_{s_1}} = \frac{a_{s_2}}{b_{s_2}}$	$\frac{a_{s_1}}{b_{s_1}} = \frac{a_{s_2}}{b_{s_2}} = -\frac{a_x}{b_x}$
PARAMETERS		
ϕ_x	$\frac{a_x}{a_x - b_x \frac{a_{s_1}}{b_{s_1}}}$	ξ
ϕ_{p_1}	$\frac{b_{p_1}}{a_x \frac{b_{s_1}}{a_{s_1}} - b_x}$	$-\xi \frac{b_{p_1}}{b_x}$
ϕ_{p_2}	$\frac{a_{p_2}}{a_x - b_x \frac{a_{s_1}}{b_{s_1}}}$	$\xi \frac{a_{p_2}}{a_x}$
ϕ_{s_1}	$\frac{a_{s_1}}{a_x - b_x \frac{a_{s_1}}{b_{s_1}}}$	$\xi \frac{a_{s_1}}{a_x}$
ϕ_{s_2}	$\frac{a_{s_2}}{a_x - b_x \frac{a_{s2}}{b_{s_2}}}$	$\xi \frac{a_{s_2}}{a_x}$
α_x	$a_x - b_x \frac{a_{s_r}}{b_{s_r}}$	ξa_x
β_x	$b_x - a_x \frac{b_{s_r}}{a_{s_r}}$	ξb_x
α_{p_1}	$a_{p_1} - b_{p_1} \frac{a_{s_r}}{b_{s_r}}$	$a_{p_1} + b_{p_1} \frac{a_x}{b_x}$
β_{p_2}	$b_{p_2} - a_{p_2} \frac{b_{s_r}}{a_{s_r}}$	$b_{p_2} + a_{p_2} \frac{b_x}{a_x}$

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Appendices

A: Irrelevance of Equal Access Assumption

We show that the assumption of a fair division by one half of non-assignable goods **C** guaranteeing that each household member has equal access to the good that is not assignable, such as food, is an innocent assumption as stated in Proposition 5. We do so by describing how non-assignable and exclusive consumption can be separated so that we can learn how resources are distributed within the households from exclusive consumption alone.

Proof. Consider a two-person household with $i = 1, 2$. Individual total expenditure x_i can be decomposed into two components: an assignable and observable expenditure x_i^a and an unobservable, therefore non-assignable, individual expenditure x_i^{no}

$$x_i = x_i^a + x_i^{no} \tag{83}$$

Note that total household expenditure x can be written as

$$x = \sum_{i=1,2} \left(x_i^a + x_i^{no}\right). \tag{84}$$

Let us define x_i^{no} as an unknown proportion of the non-assignable household expenditure $(x - x^a)$

$$x_i^{no} = \lambda_i \left(x - x^a\right) \text{ with } 0 \leq \lambda_i \leq 1, \tag{85}$$

with $x^a = \sum_{i=1,2} x_i^a$. Individual total expenditure can then be rewritten as

$$\begin{aligned} x_i &= x_i^a + x_i^{no} = x_i^a + \lambda_i \left(x - \sum_{i=1,2} x_i^a \right) = \\ &= x_i^a + \lambda_i x - \lambda_i x_1^a - \lambda_i x_2^a. \end{aligned} \tag{86}$$

If we assume a fair division rule $\lambda_1 = \lambda_2 = \frac{1}{2}$ in case of a two-person household we can rewrite the above expression for $i = 1$ as

$$x_1 = x_1^a + \frac{1}{2}x - \frac{1}{2}x_1^a - \frac{1}{2}x_2^a = \frac{1}{2}x + \frac{1}{2}\left(x_1^a - x_2^a\right). \tag{87}$$

Let us now recall the structural equation for clothing consumption of member i where for simplicity we omit the demographic terms

$$\ln C^i = \alpha_0 + \alpha_x \left[\ln x_i + \ln m^\star \left(p_1, p_2, s\right)\right],$$

with

$$\ln x_i = \frac{x_i^a + \lambda_i x^{no}}{x} \ln x = w_i \ln x,$$

where $x^{no} = x - x_1^a - x_2^a$, and w_i measures the household total expenditure share allocated to member i. Then for $\lambda_1 = \lambda_2 = \frac{1}{2}$

$$\begin{aligned}
\ln C^i &= \alpha_0 + \alpha_x \frac{x_1^a + 0.5\left(x - x_1^a - x_2^a\right)}{x} \ln x + \alpha_x \ln m^{\star}\left(p_1, p_2, s\right) = \\
&= \alpha_0 + \alpha_x \frac{0.5\left(x + x_1^a - x_2^a\right)}{x} \ln x + \alpha_x \ln m^{\star}\left(p_1, p_2, s\right) = \\
&= \alpha_0 + \alpha_x \left(0.5 + \frac{0.5\left(x_1^a - x_2^a\right)}{x}\right) \ln x + \alpha_x \ln m^{\star}\left(p_1, p_2, s\right) = \\
&= \alpha_0 + \alpha_x 0.5 \ln x + \alpha_x \left(0.5\left(x_1^a - x_2^a\right)\right) \frac{\ln x}{x} + \alpha_x \ln m^{\star}\left(p_1, p_2, s\right),
\end{aligned}$$

where total household income and the term that includes individual specific expenditures are separate and the assumption of a fair division of total household income $\lambda_i = \frac{1}{2}$ is neutral because it does not affect the relative magnitude of the parameters of the scaling function. On the other hand, if the second term describing intra-household allocations were close to zero, then the scaling function would describe only inter-household rather than intra-household allocation differences. □

The assumption of a fair division is innocent because both α_x and the parameters of the scaling function do capture the difference between the observed individual allocations $\left(x_1^a - x_2^a\right)$ as is required for the identification of the sharing rule that properly accounts for the exclusive information. Note further that the fair division of income of the first term is corrected by the scaling function $m\left(p_1, p_2, s\right)$.

Remark. The exclusive good must be "good." If the objective is the estimation of the rule governing the allocation of resources within the household, then the difference $\left(\left(x_1^a - x_2^a\right)\right) > \theta$ where θ is a sufficiently large value. If not, that is $\left(x_1^a \sim x_2^a\right)$, then the estimates of both α_x and the parameters associated with the scaling function $m\left(p_1, p_2, s\right)$ are not affected by any individual specific information and nothing is learned about the intra-household allocation of resources.

We now consider the multi-persons household case confining our interest to a single sharing rule. The classical example is the division between the two household components of adults and children. We focus on a fair or a needs-based division rule. Suppose that the family is composed by n adults and m children.

Remark. The multi-persons household case for a fair division rule. Let resources be shared in equal parts and $\lambda_1 \neq \lambda_2$. Specifically, $\lambda_1 = \frac{n}{n+m}$ and $\lambda_2 = \frac{m}{n+m}$. The separability property of Proposition 5 between total household income and exclusive consumption is maintained as shown in the following decomposition:

$$x_1 = x_1^a + \lambda_1 x - \lambda_1 x_1^a - \lambda_2 x_2^a = \lambda_1 \left[x + \frac{1-\lambda_1}{\lambda_1} x_1^a\right] - \lambda_2 x_2^a =$$

$$= \frac{n}{n+m}\left[x + \frac{m}{n} x_1^a\right] - \frac{m}{n+m} x_2^a = \frac{n}{n+m} x + \frac{m}{n+m}\left(x_1^a - x_2^a\right),$$

and for x_2 we have

$$x_2 = \frac{m}{n+m} x + \frac{n}{n+m}\left(x_1^a - x_2^a\right).$$

Remark. The multi-person household case for a needs-based division rule. Let resources be shared according to needs as described by an equivalence scales and $\lambda_1 \neq \lambda_2$. Let $n^* = n\,\omega_a$ and $m^* = m\,\omega_c$ where ω_a, ω_c are respectively the male and female's weight. Then

$$x_1 = \frac{n^*}{n^* + m^*}x + \frac{m^*}{n^* + m^*}\left(x_1^a - x_2^a\right).$$

B: Sample Statistics and Estimation Results

Table 2: Descriptive Statistics and Variable Definition (1,795 Households)

Variable	Definition	Mean	Std. Dev.
$\ln q_p$	Log of parents' clothing demand	5.752	0.492
$\ln q_c$	Log of child's clothing demand	5.941	0.716
$\ln p_p$	Log of parent clothing price	-0.831	0.773
$\ln p_c$	Log of child clothing price	-1.708	1.019
$\ln x_p$	Log of parent individual expenditure	5.016	0.326
$\ln x_c$	Log of child individual expenditure	2.512	0.157
$\ln x$	Log of household total expenditure	7.527	0.479
North	= 1 if family located in Northwest regions	0.257	
South	= 1 if family located in South or Islands regions	0.337	
December	= 1 if family interviewed in December	0.087	
DE - Double Earner	= 1 if double-earner family	0.588	
Parent age	Parents' age in classes	7.505	1.427
Child age	Child's age in classes	1.684	0.706
Child 1517	= 1 if dependent child 15-17 years old	0.141	
Age ratio	Age ratio: age wife/(age wife + age husband)	0.478	0.034
Educ. ratio	Education ratio: educ. wife/(educ. wife + educ. husband)	0.488	0.084

Table 3: Estimation Results of the Collective Consumption Models

	Model (a) $\xi = 0.5$		Model (b) $\xi \in [0,1]$	
	$\ln x$		$\ln x_i = \xi \ln x$	
	Adults	Child	Adults	Child
Intercept	1.641***	-0.044	1.630***	−0.071
	0.356	*0.149*	*0.378*	*0.169*
North	−0.035*	0.044**	−0.035*	0.044**
	0.016	*0.015*	*0.016*	*0.015*
South	0.051***	0.089***	0.051***	0.089***
	0.016	*0.015*	*0.015*	*0.015*
December	-0.006	0.125***	−0.006	0.125***
	0.023	*0.020*	*0.019*	*0.019*
DE - Double Earner	0.004	0.034**	0.003	0.033**
	0.023	*0.019*	*0.018*	*0.016*
Age	−0.032***	-0.004	−0.033***	−0.004
	0.010	*0.003*	*0.010*	*0.003*
Child 1517	−0.071**	−0.053**	−0.071**	−0.053**
	0.023	*0.018*	*0.023*	*0.017*
$\ln p_{cloth}$	−1.101***	−0.648***	−1.074***	−0.502***
	0.047	*0.040*	*0.033*	*0.066*
$\ln x$ - model (a)	0.920***	1.224***		
	0.095	*0.032*		
$\ln x_i$ - model (b)			0.693***	1.845***
			0.076	*0.080*
$\hat{\upsilon}$	−0.121***	0.187***	−0.122**	0.185***
	0.035	*0.050*	*0.042*	*0.052*
ϕ_{p_p}	0.059**		0.039**	
	0.024		*0.015*	
ϕ_{p_c}	0.120***		0.159***	
	0.022		*0.030*	
ϕ_{s_1} (age ratio)	−0.053		−0.061	
	0.061		*0.053*	
ϕ_{s_2} (educ. ratio)	0.029		0.027	
	0.028		*0.021*	

Note: * denotes significance at the 10% level, ** at the 5% level, and *** at the 1% level. Standard errors are in italics.

Figure 1: Child's Sharing Rule