Working Paper Series
Department of Economics
University of Verona

A Characterization of Collective Individual Expenditure Functions

Martina Menon, Elisa Pagani, Federico Perali

WP Number: 20 June 2012

ISSN: 2036-2919 (paper), 2036-4679 (online)

A Characterization of Collective Individual Expenditure Functions*

Martina Menon,† Elisa Pagani,‡ and Federico Perali§

This draft: June 2012

Abstract. We examine the properties ensuring that individual expenditure functions, derived within the context of the collective household model, are legitimate individual cost functions. Our curvature results are important for the characterization of collective demand functions as well and for the measurement of inequality within the household.

Keywords: Individual expenditure function, Collective household model, Concavity, Homogeneity.

JEL Classification: C30, D11, D12, D13.

*The authors wish to thank Letizia Pellegrini and Alberto Peretti for helpful comments and suggestions. Any errors and omissions are the sole responsibility of the authors.

†University of Verona (Italy), Department of Economics and CHILD, martina.menon@univr.it.

‡University of Verona (Italy), Department of Economics, elisa.pagani@univr.it.

§University of Verona (Italy), Department of Economics and CHILD, federico.perali@univr.it.

1 Introduction

Although neoclassical theory of demand explains constrained optimal choices of individuals, for long time the behavior of multi-person households has been described within the fiction of a unitary utility function. The unitary approach has been extensively applied in theoretical and empirical works because it allows testing restrictions on household behavior. However, a number of empirical evidences find that the unitary model fails in representing the behavior of multi-person households and, moreover, is unable to measure welfare of individuals. For these reasons, new approaches to household behavior have been studied either assuming cooperation or noncooperation between members of the household [16].

In this work, we model household optimal choices using the collective approach [8, 5]. The collective approach relies on two main assumptions: preferences of each individual are described by a utility function and household outcomes are Pareto-efficient. These two assumptions together with the assignability of some expenditures to specific household members permit identifying the rule governing resource allocation within the family and recovering individual preferences. Further research has extended the identification of the structure underlying the collective model to corner solutions [4, 15] and, recently, to account for the presence of children in the family [7, 17, 23]. The sharing function is in general expressed in terms of individual levels of income and defines expenditure functions at the individual level.

The regularity properties of individual cost functions have not been characterized yet. Our aim is then to study the properties that must be satisfied by individual cost functions in order to be theoretically plausible cost functions. We also discuss the implications of our curvature results for the measurement of inequality within the household.

Section 2 adapts the theory of legitimate cost functions [20] to the individual cost function defined in the collective framework. In Section 3 the conditions for individual cost functions to be a legitimate function are specialized to precise functional forms for the sharing rule. The conclusive section discusses some implications of our novel results.

2 Modified Individual Expenditure Function in a Collective Context

Consider a family of two persons $k = 1, 2$.[1] Each member k privately consumes a bundle of market goods $c_k \in I\!R^\ell$ and faces a vector of prices $p_k \in I\!R^\ell$. Note that purchases of individuals may include also leisure time and thus the vector of prices would include the corresponding wages. In the analysis, we omit the consumption of public goods or the presence of externalities within families. We also abstract from the consumption of domestically produced goods [2, 9].

The collective model relies on two main assumptions. Firstly, preferences of each family member over the consumption of c_k are represented by an individual quasi-concave utility function $U_k(c_k, d)$ twice differentiable and strictly increasing in c_k, where $d \in I\!R^n$ describes observable heterogeneity both at the individual level, such as age or education of individual k, and at the family level, such as quality of the living area. Secondly, outcomes of the family decision problem are assumed to be Pareto-efficient. Pareto-efficiency implies that the consumption equilibrium will be on the

[1] Our results can be straightforwardly extended to larger family units.

Pareto frontier of the family. Further, when all goods are privately consumed and there are no consumption externalities, these two assumptions allow describing family behavior using a two-stage process: first, the family agrees on a rule to share resources among its members, then, each member maximizes her individual utility function subject to her household income share.[2]

In the dual representation of individual consumption choices, each individual minimizes her share of family resources to achieve a given level of utility. Formally, we define the collective individual expenditure function of member k as $e_k^*(p_k, u_k, d) = \min_{c_k} p_k c_k \mid U_k(c_k, d) \geq u_k$, where $e_k^*(p_k, u_k, d) : I\!R^\ell \times I\!R^r \times I\!R^n \rightarrow I\!R$ is defined on the consumption set $\Gamma = I\!R_+^\ell \times I\!R_+^r \times I\!R^n$ and represents the minimum level of expenditure needed to individual k to achieve the level of utility u_k at given prices p_k. Note that the expenditure function of household members add to the collective household expenditure function, that is $e^*(p_1, p_2, u_1, u_2, d) = \sum_{k=1}^{2} e_k^*(p_k, u_k, d)$. The function $e_k^*(p_k, u_k, d)$ is a legitimate expenditure function if it is: a) homogeneous of degree 1 in p_k; b) positive, strictly increasing in u_k and non-decreasing in p_{ki}, $i = 1, \ldots, \ell$; c) concave in p_k; d) continuous in p_k and u_k.

We specify the expenditure function e_k as follows

$$e_k(p_k, u_k, d, z) = f_k\left[e_k^*(p_k, u_k, d), p_k, z\right], \tag{1}$$

where $e_k^*(p_k, u_k, d)$ is a legitimate cost function and $z \in I\!R^q$ is a vector of distribution factors.[3] Note that demographic attributes d affect individual expenditure e_k indirectly, while distribution factors z affect individual expenditures directly. The function f_k, which is continuous, permits interaction of distribution factors and prices with the cost function. Our interest is to derive restrictions on f_k guaranteeing that e_k is also a theoretically plausible cost function for which properties a)-d) hold. We derive the following propositions.

Proposition 1 *(Homogeneity of degree 1 in p_k). Let e_k^* be a legitimate cost function and let $e_k(p_k, u_k, d, z) = f_k\left[e_k^*(p_k, u_k, d), p_k, z\right]$. If f_k is homogeneous of degree 1 in (e_k^*, p_k), then e_k is homogeneous of degree 1 in p_k.*

Proof: By definition of homogeneous function $f_k\left[e_k^*(tp_k, u_k, d), tp_k, z\right] = f_k\left[te_k^*(p_k, u_k, d), tp_k, z\right] = tf_k\left[e_k^*(p_k, u_k, d), p_k, z\right]$. □

Proposition 2 *(Positive, strictly increasing in u_k and non-decreasing in p_k). Let $e_k(p_k, u_k, d, z) = f_k\left[e_k^*(p_k, u_k, d), p_k, z\right]$, with e_k^* a legitimate cost function. If $f_k(e_k^*(p_k, u_k, d), p_k, z) > 0$, f_k increasing in e_k^* and non-decreasing in p_{ki}, $\forall i = 1, \ldots, \ell$, then $e_k(p_k, u_k, d, z)$ is positive, strictly increasing in u_k and non-decreasing in p_{ki}.*

Proof: The condition $f_k(e_k^*(p_k, u_k, d), p_k, z) > 0$ implies that $e_k > 0$. Then, we have that e_k^* is a legitimate cost function, hence, it is increasing in u_k and non-decreasing in p_{ki}, $\forall i = 1, \ldots, \ell$. The former condition implies that, taking two different values of u_k, $u_{k1} < u_{k2}$, then $e_k^*(p_k, u_{k1}, d) < e_k^*(p_k, u_{k2}, d)$, and becuase f_k is increasing in e_k^*, we have $f_k[e_k^*(p_k, u_{k1}, d)] < f_k[e_k^*(p_k, u_{k2}, d)]$, that

[2] For a complete appraisal of the collective household theory see for instance [5].

[3] Distribution factors are variables that affect the household decision process without influencing either individual preferences or the budget constraint. Distribution factors are helpful in recovering the structure of the collective model and play an important role in empirical applications [10, 23].

is the strict monotonicity of f_k with respect to u_k. Then, the latter condition implies that, taking two different vectors of prices $p_k^1 = \left(p_{k1}^1, \ldots, p_{kl}^1\right)$ and $p_k^2 = \left(p_{k1}^2, \ldots, p_{kl}^2\right)$, $p_{ki}^1 \leq p_{ki}^2$, $e_k^*\left(p_{ki}^1, u_k, d\right) \leq e_k^*\left(p_{ki}^2, u_k, d\right)$, with $i = 1, \ldots, \ell$ and we have $f_k[e_k^*\left(p_{ki}^1, u_k, d\right)] < f_k[e_k^*\left(p_{ki}^2, u_k, d\right)]$, that is the monotonicity of f_k with respect to p_{ki}.

□

Proposition 3 *(Concave in p_k). Let e_k^* be a legitimate cost function and let $e_k\left(p_k, u_k, d, z\right) = f_k\left[e_k^*\left(p_k, u_k, d\right), p_k, z\right]$. Assume that f_k is concave in (e_k^*, p_k) and increasing in e_k^*. Then $e_k\left(p_k, u_k, d, z\right)$ is concave in p_k.*

Proof: In order to have $e_k\left(p_k, u_k, d, z\right)$ concave in p_k, it has to satisfy, $\forall \alpha \in [0,1]$ and $\forall p_k^1, p_k^2 \in \mathbb{R}_+^\ell$, $e_k\left(\alpha p_k^1 + (1-\alpha)p_k^2, u_k, d, z\right) \geq \alpha e_k\left(p_k^1, u_k, d, z\right) + (1-\alpha)e_k\left(p_k^2, u_k, d, z\right)$. We start from the hypothesis of the concavity of e_k^* and, hence, we have: $e_k^*\left(\alpha p_k^1 + (1-\alpha)p_k^2, u_k, d\right) \geq \alpha e_k^*\left(p_k^1, u_k, d\right) + (1-\alpha)e_k^*\left(p_k^2, u_k, d\right)$. Then, from the monotonicity of $f_k(\cdot)$ we obtain

$$\begin{gathered} f_k[e_k^*\left(\alpha p_k^1 + (1-\alpha)p_k^2, u_k, d\right), \alpha p_k^1 + (1-\alpha)p_k^2, z] \geq \\ f[\alpha e_k^*\left(p_k^1, u_k, d\right) + (1-\alpha)e_k^*\left(p_k^2, u_k, d\right), \alpha p_k^1 + (1-\alpha)p_k^2, z] \end{gathered} \tag{2}$$

and, because f_k is concave in (e_k^*, p_k), then

$$\begin{gathered} f_k[\alpha e_k^*\left(p_k^1, u_k, d\right) + (1-\alpha)e_k^*\left(p_k^2, u_k, d\right), \alpha p_k^1 + (1-\alpha)p_k^2, z] \geq \\ \alpha f_k[e_k^*\left(p_k^1, u_k, d\right), p_k^1, z] + (1-\alpha)f_k[e_k^*\left(p_k^2, u_k, d\right), p_k^2, z]. \end{gathered} \tag{3}$$

□

The above propositions are general results that reduce to Lewbel's characterizations [20] if we assume also differentiability. They describe the properties that f_k must have to guarantee that the function $e_k\left(p_k, u_k, d, z\right) = f_k\left[e_k^*\left(p_k, u_k, d\right), p_k, z\right]$ is also legitimate for any legitimate cost function e_k^*.

In the following section, we characterize the function f_k as a product between a legitimate cost function and a scaling function that depends on prices and distribution factors.

3 Scaled Individual Expenditure Function

We now specify the modifying function f_k describing how prices and distribution factors interact with $e_k^*\left(p_k, u_k, d\right)$ as follows

$$\begin{aligned} e_k\left(p_k, u_k, d, z\right) = f_k\left[e_k^*\left(p_k, u_k, d\right), p_k, z\right] = \\ = y_k^*\left(p_k, u_k, d\right) m(p_k, z) \;\text{ with }\; 0 < m(\cdot) < \frac{y^*}{y_k^*}, \end{aligned} \tag{4}$$

where $y_k^*\left(p_k, u_k, d\right)$ is a legitimate individual cost function and y^* is the cost function at the household level.[4] Note that the function f_k corrects an individual income $y_k^*\left(p_k, u_k, d\right)$ measured with error according to the index $0 < m(p_k, z) < \frac{y^*}{y_k^*}$ that may correct towards the bottom or the top depending on whether $m(p_k, z) \lesseqgtr 1$.

[4] A similar scaling transformation of a plausible cost function is described also in [20], Theorem 8. This specification has been adopted both in [17, 23] to estimate the sharing rule.

The aim of this section is to investigate the properties of the scaling function $m(p_k, z)$ needed to preserve a regular and theoretically plausible e_k.

Proposition 4 *(Homogeneity of degree 1 in p_k). If the scaling function $m(p_k, z)$ is homogeneous of degree zero in p_k and $y(p_k, u_k, d)$ is a legitimate cost function, then $e_k(p_k, u_k, d, z) = y(p_k, u_k, d)m(p_k, z)$ is homogeneous of degree 1 in p_k.*

Proof: By the definition of homogeneity

$$\begin{aligned} e_k(tp_k, u_k, d, z) &= y_k^*(tp_k, u_k, d)m(tp_k, z) = ty_k^*(p_k, u_k, d)t^\alpha m(p_k, z) \\ &= t^{\alpha+1}y_k^*(p_k, u_k, d)m(p, z) = t^{\alpha+1}e_k(p_k, u_k, d, z), \end{aligned} \tag{5}$$

thus e_k is homogeneous of degree 1 in p_k if and only if $m(p_k, z)$ is homogeneous of degree 0 in p_k.□

Proposition 5 *(Positive, strictly increasing in u_k and non-decreasing in p_{ki}) Let $e_k(p_k, u_k, d, z) = y_k^*(p_k, u_k, d)m(p_k, z)$. If $m(p_k, z)$ is positive and $y_k^*(p_k, u_k, d)$ is a legitimate cost function, then $e_k(p_k, u_k, d, z)$ is positive and strictly increasing in u_k. If, moreover, $m(p_k, z)$ is non-decreasing in p_{ki}, $\forall i = 1, \ldots, \ell$, then $e_k(p_k, u_k, d, z)$ is also non-decreasing in $p_{ki}, \forall i = 1, \ldots, \ell$.*

Proof: The positivity of $e_k(p_k, u_k, d, z)$ follows immediately by the definition. Then, taking two different values of u_k, $u_{k1} < u_{k2}$, we have $y_k^*(p_k, u_{k1}, d)m(p_k, z) < y_k^*(p_k, u_{k2}, d)m(p_k, z)$, because y_k^* is a cost function and $m(p_k, z)$ is positive, hence, $e_k(p_k, u_k, d, z)$ is increasing in u_k. Finally, taking two different vectors of prices $p_k^1 = (p_{k1}^1, \ldots, p_{kl}^1)$ and $p_k^2 = (p_{k1}^2, \ldots, p_{kl}^2)$, with $p_{ki}^1 \leq p_{ki}^2$, we have $y_k^*(p_{ki}^1, u_k, d)m(p_{ki}^1, z) \leq y_k^*(p_{ki}^2, u_{k2}, d)m(p_{ki}^2, z)$, because both $y_k^*(p_k, u_k, d)$ and $m(p_k, z)$ are non-decreasing in $p_{ki}, \forall i = 1, \ldots, \ell$. □

If e_k must be non-decreasing in p_{ki}, $\forall i = 1, \ldots, \ell$, and we suppose, in addiction, that $y_k^*(p_k, u_k, d)$ and $m(p_k, z)$ are differentiable with respect to $p_{ki}, \forall i = 1, \ldots, \ell$, we must satisfy

$$\begin{cases} \frac{\partial e_k(p_k,u_k,d)}{\partial p_{k1}} = \frac{\partial y_k^*}{\partial p_{k1}}(p_k, u_k, d)m(p_k, z) + y_k^*(p_k, u_k, d)\frac{\partial m}{\partial p_{k1}}(p_k, z) \geq 0, \\ \vdots \\ \frac{\partial e_k(p_k,u_k,d)}{\partial p_{k\ell}} = \frac{\partial y_k^*}{\partial p_{k\ell}}(p_k, u_k, d)m(p_k, z) + y_k^*(p_k, u_k, d)\frac{\partial m}{\partial p_{k\ell}}(p_k, z) \geq 0. \end{cases} \tag{6}$$

This follows from the assumptions on $m(p_k, z)$ and recalling that $y_k^*(p_k, u_k, d)$ is a legitimate cost function.

Remark 1 *We note that if $\frac{\partial m}{\partial p_{ki}} \geq 0, \forall i = 1, \ldots, \ell$, then it follows that the ℓ inequalities of the system are satisfied. Note that system (6) is satisfied even if we assume*

$$\frac{\partial y_k^*}{\partial p_{ki}} \frac{1}{y_k^*} \geq -\frac{\partial m}{\partial p_{ki}} \frac{1}{m}. \tag{7}$$

If we multiply by p_{ki} each member, we obtain

$$\frac{\partial y_k^*}{\partial p_{ki}} \frac{p_{ki}}{y_k^*} \geq -\frac{\partial m}{\partial p_{ki}} \frac{p_{ki}}{m}, \tag{8}$$

that is the relationship between the elasticity of the expenditure $y_k^*(p_k, u_k, d)$ and the elasticity of the scaling function $m(p_k, z)$, with respect to the i-th price.

Remark 2 *(Continuity) In order to maintain property d) of $e_k(p_k, u_k, d, z)$, $m(p_k, z)$ has to be continuous with respect to p_k.*

The concavity requirement for the expenditure $e_k(p_k, u_k, d, z)$ is given by what follows. Propositions 6-7 and Theorem 1 correspond to Proposition 3.

Proposition 6 *(Corollary 5.18 of [3]) Let $g_k : X \subseteq I\!R^s \to I\!R$ be a non-negative and concave function and $h_k : X \subseteq I\!R^s \to I\!R$ be a positive and concave function. Hence, the function $f_k(x) = g_k(x)h_k(x)$ is semi-strictly quasiconcave on X, with respect to x.*

We may apply this Proposition to the functions $g_k = y_k^*$ and $h_k = m$. We observe that we need the same domain for the functions y_k^* and $m(p_k, z)$, and this implies that we suppose that these functions vary only in prices.

Proposition 7 *(Proposition 3.30 of [3]) If $f_k : X \subseteq I\!R^s \to I\!R$ is an upper semicontinuous semi-strictly quasi-concave function, defined on a convex set X, then it is also quasi-concave.*

Moreover, the following theorem holds (Theorem 21.15 of [25]).

Theorem 1 *Suppose that $f_k : X \subset I\!R^s \to I\!R$ be positive and X is a convex cone. If f_k is homogeneous of degree one and quasi-concave on X, then it is concave on X.*

We note that the expenditure function $e_k(p_k, u_k, d, z) : I\!R^\ell_+ \times I\!R^k \times I\!R^n \times I\!R^q \to I\!R$ satisfies the hypotheses of the previous theorem in p_k, hence it is also concave in p_k.

Our results have important implications both for providing a testable hypothesis about the curvature of the sharing rule [11, 24] and for the measurement of inequality within the household. We would like to show that as the concavity of $m(p_k, z)$ increases also the level of inequality in the distribution of household resources that are increasingly concentrated in the hands of one household member increases. To this aim, suppose that $m(p_k, z)$ is twice continuously differentiable with respect to prices. Consider further the definition of an absolute and a relative inequality aversion coefficient:

$$\rho_a(m) = -\frac{\dfrac{\partial^2 m}{\partial p_{ki}^2}}{\dfrac{\partial m}{\partial p_{ki}}} \text{ and } \rho_r(m) = -\frac{\dfrac{\partial^2 m}{\partial p_{ki}\partial p_{kj}}}{\dfrac{\partial m}{\partial p_{ki}}}. \tag{9}$$

This inequality aversion coefficient $\rho(m)$, akin to a risk aversion coefficient, can be interpreted as the coefficient describing how a change in the scaling function $m(p_k, z)$ in response to a change in price of one good ($\rho_a(m)$), or more ($\rho_r(m)$), affects the degree of equity adopted by a family when sharing goods in favor of one household member and against another.

It should be noted that, $m(p_k, z)$ being a concave function in p_k, does not mean that $\frac{\partial^2 m}{\partial p_{ki}\partial p_{kj}} \leq 0$, but $v^T H_{wm} v \leq 0$.[5] For example, the definition of ultramodularity and supermodularity [22] for a twice continuously differentiable function $m : I\!R^\ell \times I\!R^q \to I\!R$ is, respectively,

$$\begin{aligned} \tfrac{\partial^2 m}{\partial p_{ki}\partial p_{kj}} \geq 0, \text{ for any } 1 \leq i \leq j \leq \ell \text{ and} \\ \tfrac{\partial^2 m}{\partial p_{ki}\partial p_{kj}} \geq 0, \text{ for any } 1 \leq i < j \leq \ell. \end{aligned} \tag{10}$$

[5]For the analysis, we suppose that $w : I\!R^s \to I\!R$ is a twice differentiable function of s variables. The Hessian of w

One can easily see that if m is non-decreasing in p_{ki} and $-m$ is also ultramodular with respect to prices, then $\rho_a(m) \geq 0$ and also $\rho_r(m) \geq 0$, while if $-m$ is supermodular, then $\rho_r(m) \geq 0$. Ultramodular and supermodular functions are used in social sciences to analyze how one agent's decision affect the incentives of others.

It is worth noting that, if the scaling function $m(p_k, z)$ is not concave in p_k, under some hypotheses, it can be transformed in a concave function. In fact, we may use the class of concave transformable (or transconcave) functions [3, 6, 14, 18, 19] by adopting a one to one transformation of their domain, and then, into concave functions by an increasing function. The function $m : A \subseteq I\!R^\ell \times I\!R^q \to C \subseteq I\!R$ is said to be G-concave if there exists a continuous real-valued increasing function G defined on C such that $G(m(p_k, z))$ is concave over A. Alternatively, letting G^{-1} denote the inverse of G, if

$$m\left(\alpha p_k^1 + (1-\alpha)\, p_k^2, z\right) \geq G^{-1}\left[\alpha G\left(m\left(p_k^1, z\right)\right) + (1-\alpha)\, G\left(m\left(p_k^2, z\right)\right)\right] \tag{11}$$

holds $\forall p_k^1, p_k^2 \in A$ and $\alpha \in [0, 1]$.

To be concavifiable $m(p_k, z)$ must be at least quasiconcave. Note that semistrict quasiconcavity is not sufficient to support transconcavity. We observe also that in the twice continuously differentiable case, necessary and sufficient conditions lie between the properties of pseudoconcavity and strong pseudoconcavity.[6] It could also be interesting to consider additional assumptions on G required to obtain a scaling function $G\left(m(p_k, z)\right)$ that satisfies the requirement of Propositions 4 and 5.

4 Conclusions

This study describes the properties that must be satisfied by collective individual cost functions in order to be theoretically plausible cost functions. We also describe the implications of our curvature results for the measurement of inequality within the household.

---

at x is

$$H_w(x) = \begin{pmatrix} w''_{11}(x) & \cdots & w''_{1s} \\ \vdots & \ddots & \vdots \\ w''_{s1}(x) & \cdots & w''_{ss} \end{pmatrix}$$

where, w''_{ij} is the second derivative of w, with respect to x_i and x_j. If w has continuous partial derivatives of first and second order on an open convex set X, and let $H_w(x)$ is its Hessian, then the following assertions hold: 1) w is concave if and only if $H_w(x)$ is negative semidefinite ($v^T H_w v \geq 0, \forall v \in I\!R^s$), and 2) if $H_w(x)$ is negative definite $\forall x \in X$ ($v^T H_w v > 0, \forall v \in I\!R^s$), then w is strictly concave.

[6]To establish conditions under which a twice continuously differentiable function is transformable concave for some transformation see [3]. For results referring to functions not necessarily twice differentiable see, for instance, [12, 13, 14, 18, 19].

References

[1] Apps, P. F., and R. Rees (1988): "Taxation and the Household," *Journal of Public Economics*, 35(3): 355-369.

[2] Apps, P. F., and R. Rees (1997): "Collective Labor Supply and Household Production," *Journal of Political Economy*, 105(1): 178-190.

[3] Avriel, M., W.E. Diewert, S. Shaible, and I. Zang (1988): *Generalized Concavity*, Plenum Press, New York.

[4] Blundell, R., P.A. Chiappori, T. Magnac, and C. Meghir (2007): "Collective Labour Supply: Heterogeneity and Non-Participation," *Review of Economic Studies*, 74(2): 417-445.

[5] Browning, M., P.A. Chiappori, and Y. Weiss (2011): *The Economics of the Family*, Tel Aviv University, unpublished textbook manuscript, forthcoming.

[6] Caplin, A., and B. Nalebuff (1991): "Aggregation and Social Choice: A Mean Voter Theorem," *Econometrica*, 59(1): 1-23.

[7] Cherchye, L., B. De Rock, and F. Vermeulen (2012): "Married with Children: A Collective Labor Supply Model with Detailed Time Use and Intrahousehold Expenditure Information," *American Economic Review*, forthcoming.

[8] Chiappori, P.A. (1988): "Rational Household Labor Supply," *Econometrica*, 56(1): 63-90.

[9] Chiappori, P.A. (1997): "Introducing Household Production in Collective Models of Labor Supply," *Journal of Political Economy*, 105(1): 191-209.

[10] Chiappori, P.A., and I. Ekeland (2009): "The Microeconomics of Efficient Group Behavior: Identification," *Econometrica*, 77(3): 763-799.

[11] Couprie, H., E. Peluso, and A. Trannoy (2010): "Is Intrahousehold Power more Balanced in Poor Households?," *Journal of Public Economics*, 94(7-8): 493-507.

[12] Crouzeix, J.P. (1977): *Contributions à l'ètude des Fonctions Quasi-convexes*, Thése de Doctorat, U.E.R. des Sciences Exactes et Naturelles, Universitè de Clermont-Ferrand II, France.

[13] Debreu, G. (1976): "Least Concave Utility Functions," *Journal of Mathematical Economics*, 3(2): 121-129.

[14] De Finetti, B. (1949): "Sulle Stratificazioni Convesse," *Annali di Matematica Pura ed Applicata*, 30(1): 123-183.

[15] Donni, O. (2003): "Collective Household Labor Supply: Nonparticipation and Income Taxation," *Journal of Public Economics*, 87(5-6): 1179-1198.

[16] Donni, O., and P.A. Chiappori (2011): "Nonunitary Models of Household Behavior: A Survey of the Literature," *International Series on Consumer Science*, edited by J. A. Molina, New York: Springer.

[17] Dunbar, G., A. Lewbel, and K. Pendakur (2012): "Children's Resources in Collective Households: Identification, Estimation and an Application to Child Poverty in Malawi," *American Economic Review*, forthcoming.

[18] Fenchel, W. (1953): *Convex Cone, Sets and Functions*, Princeton University Press, Princeton, New Jersey.

[19] Fenchel, W. (1956): "Über Konvexe Funktionen mit Voreschriebenen Niveaumannigfaltigkeiten," *Mathematische Zeitschrift*, 63: 496-506.

[20] Lewbel, A. (1985): "A Unified Approach to Incorporating Demographic or Other Effects into Demand Systems," *Review of Economic Studies*, 52(1): 1-18.

[21] Mangasarian, O.L. (1969): *Nonlinear Programming*, Mc Graw Hill Company, New York.

[22] Marinacci, M., and L. Montrucchio (2005): "Ultramodular Functions," *Mathematics of Operation Research*, 30(2): 311-332.

[23] Menon, M., and F. Perali (2012): "The Sharing Rule: Where Is It?" Department of Economics, University of Verona, Working Paper Series 16/2012.

[24] Peluso, E., and A. Trannoy (2007): "Does Less Inequality among Households Mean Less Inequality among Individuals?," *Journal of Economic Theory*, 133(1): 568-578.

[25] Simon, C.P., and L. Blume (1994): *Mathematics for Economists*, Norton & Company, New York, London.