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Abstract

We investigate the link between inflation, growth and unemployment nesting a model of fair wages into one of endogenous growth of learning by doing and assuming that firms protect wages' purchasing power against inflation in exchange of worker's effort. Unemployment decreases with higher inflation and real growth rates. These effects tends to vanish as inflation and growth increase. Depending on the assumptions on learning-by-doing mechanisms, the effect of inflation on growth can be either nil or positive, but tiny. The Appendix shows that the short run effects of a monetary shocks mirror the long-run effects of inflation.

Keywords: efficiency wages, money growth, long-run Phillips curve, trend inflation

JEL classification codes: E3, E2, E4, E5,

1 Introduction

Bewley (1999, 160-161, 164-165, 208-209) documents that firms are concerned by the effects of inflation on the purchasing power of wages. Though they do not tend to favour wage indexation, they will be often ready to defend workers' standard of living against inflation if they perform well. This exchange of gifts - namely effort vis à vis a shield against inflation for wages' purchasing power - is what we term *inflation gifts.*

Vaona (2010, 2012b) formalized this concept resorting to a fair wages model similar to the one by Danthine and Kurmann (2004). The effects of inflation on output and unemployment are investigated in a number of different varieties of this model and it is showed that in this context both a short and a long-run Phillips curve can emerge even under flexible prices and nominal wages.

The aim of this paper is to extend this framework to an endogenous growth of learning-by-doing. Further in an appendix we investigate the short-run properties of the model, after calibrating it on the basis of our empirical results.

This research question is interesting for many different reasons. One of the most long-standing debates in economics is whether inflation can have real economic effects both in the short and in the long-run. The political relevance of the existence of a non-vertical Phillips curve hardly needs to be mentioned. Since Phillips (1958) modern macroeconomics was animated by debates on this issue. Regarding the specific streams of literature involved in our research question, this paper extends results obtained for the link between inflation and the level of output to the inflation-growth domain.

Under this respect, our objective here is similar to the one achieved by Vaona (2012a) with respect to sticky wages/prices models of the inflation-output trade-off set out in King and Wolman (1996), Ascari (1998) and Graham and Snower (2004, 2008) among others[1]. The originality of the present paper is in considering a different structure of the labour market with respect to the bespoken contributions.

In so doing, we give continuity to the research strategy inaugurated by Vaona (2012a), that shifted the focus of the literature on the inflation-growth nexus from transmission channels passing through asset accumulation, credit and product markets[2], to ones passing through the labour market. Finally, this study takes part to the recent renaissance of efficiency wages models in the explanation of macroeconomic trends, which involved both fairness and shirking theories (Danthine and Kurmann, 2004, 2008, 2010; Alexopoulos, 2004, 2006, 2007).

The rest of this paper is structured as follows. A model nesting inflation gifts into an endogenous growth theory through learning-by-doing is set out, starting from the households' problem and the government budget constraint

[1] A similar research strategy was pursued also by Annicchiarico et al. (2011), where the link between monetary volatility and growth was investigated.

[2] For surveys of theoretical and empirical studies see Temple (2000) and Gillman and Kejak (2005).

and moving to the firm side of the economy before giving the long-run solution. In this first model, knowledge spillovers are assumed to depend on capital per worker. Next we consider a model where knowledge spillovers depend instead on the aggregate stock of capital. Once again we provide the long-run solution after illustrating the model structure. In the Appendix we investigate the short-run properties of the two models.

2 A model of spillovers depending on the average level of capital per worker

2.1 The problem of the household and the budget constraint of the government

In our model, a continuum of households populates the economy. Within each household there exists a continuum of individuals. Both the number of households and individuals are normalized to 1. We share these assumptions with the models presented in Danthine and Kurmann (2004, 2008, 2010). Furthermore, similarly to the trend inflation literature we resort to a money-in-the utility function setup (Ascari, 2004; Graham and Snower, 2004, 2008), also because this kind of models was showed to be functionally equivalent to liquidity costs ones (Feenstra, 1986).

The households' maximization problem is

$$\max_{\{C_{t+j}(h),B_{t+j}(h),M_{t+j}(h),e_{t+j}(h),K_{t+j}(h)\}} \sum_{j=0}^{\infty} \beta^{t+j} E\left(U\left\{C_{t+j}(h), N_{t+j}(h)\, G\left[e_{t+j}(h)\right], V\left[\frac{M_{t+j}(h)}{P_{t+j}}\right]\right\}\right) \tag{1}$$

subject to a series of income constraints

$$\begin{aligned} C_{t+j}(h) + K_{t+j}(h) &= \frac{W_{t+j}(h)}{P_{t+j}} N_{t+j}(h) + \frac{T_{t+j}(h)}{P_{t+j}} - \frac{M_{t+j}(h)}{P_{t+j}} + \frac{M_{t+j-1}(h)}{P_{t+j}} - \frac{B_{t+j}(h)}{P_{t+j}} \\ &\quad + \frac{B_{t+j-1}(h)}{P_{t+j}} i_{t+j-1} + \frac{R_{t+j}}{P_{t+j}} K_{t+j-1}(h) + (1-\delta) K_{t+j-1}(h) + Q_{t+j}(h) \end{aligned}$$

where β is the discount factor, E is the expectation operator, U is the utility function, $C_{t+j}(h)$ is consumption by household h at time $t+j$, $B_{t+j}(h)$ are the household's bond holdings, i_{t+j} is the nominal interest rate, $N_{t+j}(h)$ is the fraction of employed individuals within the household, $G\left[e_{t+j}(h)\right]$ is the disutility of effort - $e_{t+j}(h)$ - of the typical working family member, $V\left[\frac{M_{t+j}(h)}{P_{t+j}}\right]$ is the utility arising from nominal money balances - $M_{t+j}(h)$ - over the price level - P_{t+j}. $W_{t+j}(h)$ and $T_{t+j}(h)$ are the household's nominal wage income and government transfers respectively. Finally, $K_{t+j}(h)$ is the capital held by household h, δ is the capital depreciation rate, R_{t+j} is the capital rental rate, and $Q_{t+j}(h)$ are profits accruing to households from firms.

As it appears from the problem above, in our framework all decisions pertain to households and not to individuals. Similar assumptions were taken not only in Danthine and Kurmann (2004, 2008, 2010), but also in Merz (1995), Blanchard and Galì (2010) and Alexopoulos (2004) among others.

Furthermore, though individuals are identical ex-ante, they are not so ex-post, being some of them employed and some other unemployed. Households are instead all symmetric both ex-ante and ex-post, because the fraction of employed people is the same across all households. Matching between firms and households is assumed to be random and costless. Finally, leisure does not provide any utility to agents, so their labour supply is inelastic and it is normalized to one unit of time. The same applies to unemployment related activities.

On the footsteps of Akerlof (1982), in our model workers would not prefer to exert effort. However, they are ready to do so in exchange for some gift, as a real wage above some reference level. Building on Danthine and Kurmann (2004), Vaona (2012b) specified the disutility of effort as

$$G\left[e_{t+j}(h)\right]=\left\{e_{t+j}(h)-\left[\begin{array}{c}\phi_0+\phi_1\log\frac{W_{t+j}(h)}{P_{t+j}}+\\+\phi_2\log u_{t+j}(h)+\phi_3\log\frac{W_{t+j}}{P_{t+j}}+\phi_4\log\frac{W_{t+j-1}}{P_{t+j}}\end{array}\right]\right\}^2 \quad (2)$$

where $u_{t+j}(h)=1-N_{t+j}(h)$ and W_{t+j} is the aggregate nominal wage. The novelty of this specification consists in the fact that in the last term, the nominal wage at time $t+j-1$ is assessed at the prices of time $t+j$. This modelling device allows to formalize *inflation gifts*. More in detail, inflation can challenge households' living standards. Therefore, they perceive firms' pay policies preserving their purchasing power as a gift and they are ready to exert effort in exchange. In other terms, the reference wage falls with a higher inflation rate[3].

As customary in the relevant literature, $\phi_1, \phi_2 > 0$ and $\phi_3, \phi_4 < 0$. This means that households exert greater effort when they receive a higher real wage and when the unemployment rate is higher. On the contrary, a higher reference wage, captured by the level of the aggregate real wage and the real value of the past aggregate nominal wage, reduces effort. In (2) the reference wage only depends on aggregate variables, as in the social norm case. We do not explore the possibility that it may depend on households' variables here. This is because Vaona (2010) showed that the personal norm case can produce implausible results in presence of trend inflation.

We detrend nominal variables for nominal growth (π) and real variable for real growth (γ). The resulting maximization problem is

$$\max_{\{c_{t+j}(h),b_{t+j}(h),m_{t+j}(h),e_{t+j}(h),k_{t+j}(h)\}} \sum_{j=0}^{\infty} \beta^{t+j} E\left(U\left\{c_{t+j}\left(h\right), N_{t+j}\left(h\right) G\left[e_{t+j}(h)\right], V\left[\frac{m_{t+j}(h)}{p_{t+j}}\right]\right\}\right) \tag{3}$$

[3]This specification does not entail any money illusion. For an in-depth discussion of this issue see Anonymous and Vaona (2012).

subject to a series of constraints

$$\begin{aligned} c_{t+j}\left(h\right)+k_{t+j}(h) &= \frac{w_{t+j}\left(h\right)}{p_{t+j}}N_{t+j}(h)+\frac{t_{t+j}\left(h\right)}{p_{t+j}}-\frac{m_{t+j}\left(h\right)}{p_{t+j}}+\frac{m_{t+j-1}\left(h\right)}{p_{t+j}}\frac{1}{\pi\gamma}-\frac{b_{t+j}(h)}{p_{t+j}} \\ &+\frac{b_{t+j-1}(h)i_{t+j-1}}{p_{t+j}}\frac{1}{\pi\gamma}+\frac{(1-\delta)}{\gamma}k_{t+j-1}(h)+\frac{r_{t+j}}{p_{t+j}}\frac{k_{t+j-1}(h)}{\gamma}+q_{t+j}(h) \\ G\left[e_{t+j}(h)\right] &= \left\{e_{t+j}(h)-\left[\begin{array}{c}\phi_0+\phi_1\log\frac{w_{t+j}(h)}{p_{t+j}}+\phi_2\log u_{t+j}(h)+ \\ +\phi_3\log\frac{w_{t+j}}{p_{t+j}}+\phi_4\log\left(\frac{w_{t+j-1}}{p_{t+j}}\frac{1}{\pi\gamma}\right)\end{array}\right]\right\}^2 \end{aligned}$$

where lower case letters are the detrended counterparts of the upper case ones. Note that in order to avoid either the difference $e_{t+j}(h)-\left[\begin{array}{c}\phi_0+\phi_1\log\frac{w_{t+j}(h)}{p_{t+j}}+ \\ +\phi_2\log u_{t+j}(h)+ \\ +\phi_3\log\frac{w_{t+j}}{p_{t+j}}+\phi_4\log\left(\frac{w_{t+j-1}}{p_{t+j}}\frac{1}{\pi\gamma}\right)\end{array}\right]$ or $e_{t+j}(h)$ to be trended, we have to assume that $\phi_1+\phi_3+\phi_4=0$.

On the footsteps of Danthine and Kurmann (2004), we adopt the following specification for the utility function

$$U\left(\cdot\right)=\log c_{t+j}(h)-N_{t+j}\left(h\right)G\left[e_{t+j}\left(h\right)\right]+b\log\left[\frac{m_{t+j}\left(h\right)}{p_{t+j}}\right]$$

The first order conditions with respect to capital, effort, consumption, bond and money holdings imply

$$\frac{1}{c_{t+j}(h)} = \left[\frac{r_{t+j}}{p_{t+j}}\frac{\beta}{\gamma}\frac{1}{c_{t+j+1}(h)} + \frac{1}{c_{t+j+1}(h)}\beta\left(1-\delta\right)\frac{1}{\gamma}\right] \tag{4}$$

$$e_{t+j}(h) = \left[\begin{array}{c} \phi_0 + \phi_1 \log\frac{w_{t+j}(h)}{p_{t+j}} + \phi_2 \log u_{t+j}(h) + \\ +\phi_3 \log\frac{w_{t+j}}{p_{t+j}} + \phi_4 \log\left(\frac{w_{t+j-1}}{p_{t+j}}\frac{1}{\pi\gamma}\right) \end{array}\right] \tag{5}$$

$$\frac{1}{c_{t+j}(h)} = E\left[\frac{p_{t+j}}{p_{t+j+1}}\frac{i_{t+j}}{c_{t+j+1}(h)}\beta\frac{1}{\pi\gamma}\right] \tag{6}$$

$$\left(\frac{\mu_{t+j}}{\tilde{\pi}_{t+j}}\right)^{-1} = \frac{c_{t+j-1}\left(h\right)}{c_{t+j}\left(h\right)}\left(1-\frac{1}{i_{t+j}}\right)/\left(1-\frac{1}{i_{t+j-1}}\right) \tag{7}$$

where μ is the money growth rate and $\tilde{\pi}_{t+j}$ is the off-trend portion of the inflation rate. Finally the government budget constraint is

$$\int_0^1 \frac{T_{t+j}\left(h\right)}{P_{t+j}}dh = \int_0^1 \frac{M_{t+j}\left(h\right)}{P_{t+j}}dh - \int_0^1 \frac{M_{t+j-1}\left(h\right)}{P_{t+j}}dh \tag{8}$$

2.2 The firm side of the model

Similarly to many studies in the New Keynesian tradition, we assume the existence of an intermediate labour market and of a final product market. There is neither price nor wage stickiness both in the intermediate labour market and in the final one. An alternative, but equivalent model set up would be to use two-stage budgeting (Chambers, 1988, 112-113; Heijdra and Van der Ploeg, 360-363).

2.2.1 The intermediate labour market

In the intermediate labour market, households sell their labour force for their wage to labour intermediaries. The different labour kinds of each household are assumed to be imperfectly substitutes. They are assembled into an homogeneous labour input to be sold to firms on the final product market. The maximization problem of the representative labour intermediary is

$$\max_{\{N_{t+j}(h),W_{t+j}(h)\}} W_{t+j}N_{t+j} - \int_0^1 W_{t+j}(h)N_{t+j}(h)dh \qquad (9)$$

$$s.t.\ N_{t+j} = \left[\int_0^1 e_{t+j}(h)^{\frac{\theta_n-1}{\theta_n}} N_{t+j}(h)^{\frac{\theta_n-1}{\theta_n}} dh\right]^{\frac{\theta_n}{\theta_n-1}} \qquad (10)$$

We drop the index of labour intermediaries to simplify notation. Given that the number of labour intermediaries is normalized to one and given that they are all symmetric, W_{t+j} and $N_{t+j} \in [0,1]$ can be directly considered the aggregate wage and employment (rate), respectively. θ_n is the elasticity of substitution between different labour kinds.

Taking the ratio of the first order conditions with respect to $N_{t+j}(h)$ and $W_{t+j}(h)$ one has

$$e_{t+j}(h) = \phi_1 \qquad (11)$$

Households' symmetry and (10) imply $\phi_1 = 1$ and $N_{t+j}(h) = N_{t+j}$

2.2.2 The final product market

In the final product market, perfectly competitive firms hire homogenous capital and labour inputs to produce an homogeneous output. Their maximization problem is

$$\max_{\{N_{t+j}(f),K_{t+j-1}(f)\}} P_{t+j}Y_{t+j}(f) - W_{t+j}N_{t+j}(f) - R_{t+j}K_{t+j-1}(f)$$

$$s.t. \;\; Y_{t+j}(f) = A\left[N_{t+j}(f)\frac{K_{t+j-1}}{N_{t+j}}\right]^{1-\alpha}[K_{t+j-1}(f)]^{\alpha} \tag{12}$$

where $Y_{t+j}(f)$ is output of the firm f at time $t+j$, $N_{t+j}(f)$ and $K_{t+j-1}(f)$ are labour and capital of firm f respectively. A is a productivity index and α is a parameter. $\frac{K_{t+j-1}}{N_{t+j}}$ is aggregate capital per worker. In (12) we assume the existence of learning-by-doing effects. More specifically, we assume the existence of knowledge spillovers from one worker to the other, depending on the average availability of capital for each worker in the aggregate economy (Lucas, 1988; Barro and Sala-i-Martin, 1995, 152).

The first order conditions with respect to $N_{t+j}(f)$ and $K_{t+j-1}(f)$ imply

$$(1-\alpha)\frac{Y_{t+j}(f)}{\frac{W_{t+j}}{P_{t+j}}} = N_{t+j}(f)$$

$$\alpha\frac{Y_{t+j}(f)}{\frac{R_{t+j}}{P_{t+j}}} = K_{t+j-1}(f)$$

Under symmetry

$$Y_{t+j} = AK_{t+j-1}$$

therefore, after detrending,

$$\frac{r_{t+j}}{p_{t+j}} = \alpha A \tag{13}$$

$$\frac{w_{t+j}}{p_{t+j}} \frac{N_{t+j}}{y_{t+j}} = (1-\alpha) \tag{14}$$

2.3 The long-run solution

To obtain γ, consider (4). In steady state one has

$$\gamma = \beta \left(\frac{r}{p} + 1 - \delta \right)$$

where we drop time subscripts to denote steady state variables. Considering (13) one has

$$\gamma = \beta \left(\alpha A + 1 - \delta \right) \tag{15}$$

Combine (5) and (11) to obtain

$$\log u_{t+j}(h) = \frac{\phi_1 - \phi_0}{\phi_2} - \frac{\phi_1}{\phi_2} \log \frac{w_{t+j}(h)}{p_{t+j}} - \frac{\phi_3}{\phi_2} \log \frac{w_{t+j}}{p_{t+j}} - \frac{\phi_4}{\phi_2} \log \left(\frac{w_{t+j-1}}{p_{t+j}} \frac{1}{\pi\gamma} \right)$$

Under symmetry and in steady state, recalling that $\phi_1 + \phi_3 + \phi_4 = 0$, one has

$$\log u = \frac{\phi_0 - \phi_1}{\phi_2} + \frac{\phi_4}{\phi_2}\log \pi + \frac{\phi_4}{\phi_2}\log\left[\beta\left(\alpha A + 1 - \delta\right)\right] \tag{16}$$

Our model has therefore two implications regarding the relationship between long-run growth, unemployment and inflation. Long-run growth only depends on deep parameters and there exists a long-run link between inflation and unemployment. After Karanassou et al. (2005, 2008a, 2008b) one may calibrate $\frac{\phi_4}{\phi_2} \approx -0.29$. Note that (16) does not imply that hyperinflation reduces unemployment, given that $\lim_{\pi\to\infty} \frac{d\log u}{d\pi} = 0$. Finally a higher growth rate decreases the unemployment rate, though at a declining extent given that $\lim_{\gamma\to\infty} \frac{d\log u}{d\gamma} = 0$.

3 A model of spillovers depending on the aggregate capital stock

When assuming that knowledge spillovers depend on aggregate capital instead of average capital per worker, the problem of firms on the final product market changes to

$$\begin{aligned} & \max_{\{N_{t+j}(f), K_{t+j-1}(f)\}} P_{t+j}Y_{t+j}(f) - W_{t+j}N_{t+j}(f) - R_{t+j}K_{t+j-1}(f) \\ s.t.\ \ Y_{t+j}(f) &= A\left[N_{t+j}(f)K_{t+j-1}\right]^{1-\alpha}\left[K_{t+j-1}(f)\right]^{\alpha} \end{aligned} \tag{17}$$

As a matter of consequence (13) changes to

$$\frac{r_{t+j}}{p_{t+j}} = \alpha A N_{t+j}^{1-\alpha} \tag{18}$$

Bearing in mind that

$$\log u = \frac{\phi_0 - \phi_1}{\phi_2} + \frac{\phi_4}{\phi_2} \log \pi + \frac{\phi_4}{\phi_2} \log \gamma \tag{19}$$

one has that

$$\gamma = \beta \left\{ \alpha A \left[1 - (\pi\gamma)^{\frac{\phi_4}{\phi_2}} \exp\left(\frac{\phi_1 - \phi_0}{\phi_2} \right) \right]^{1-\alpha} + 1 - \delta \right\} \tag{20}$$

Exploiting the implicit function theorem on can obtain the absolute change in the growth rate induced by an absolute change in the inflation rate

$$\frac{d\gamma}{d\pi} = \frac{-\beta\alpha A (1-\alpha) (N)^{-\alpha} u \frac{\phi_4}{\phi_2} \frac{1}{\pi}}{1 + \beta\alpha A (1-\alpha) [N]^{-\alpha} u \frac{\phi_4}{\phi_2} \frac{1}{\gamma}} \tag{21}$$

Both to assess γ and the sign of $\frac{d\gamma}{d\pi}$, it is necessary to calibrate our model. Furthermore, (20) needs a numerical solution. We set $\alpha = 0.33, \beta = 0.99$ and $\delta = 0.08$ as customary in the literature. Considering US data from 1956Q1 to 2010Q4, the average real growth rate was equal to 3.08%. Therefore, we attach a value to A to have $\gamma = 1.0308$, namely $A = 0.3688$. Again considering the bespoken US data, the inflation rate was equal to 3.92% in the whole sample and to 3.88% focusing on years after 1978. Therefore we have $\mu = \pi = 1.0392$. $\frac{\phi_0 - \phi_1}{\phi_2}$ was normalized to obtain through (16) a value of u_{ss} equal to the US average of 0.06. $N = 1 - u$. Finally $\frac{\phi_1 + \phi_3}{\phi_2} = -\frac{\phi_4}{\phi_2} \approx 0.29$ as mentioned above.

Before exposing our results, note that in our model there could potentially arise the complementarities mentioned in Aghion and Howitt (1998, 139) leading to multiple equilibria in learning by doing models. In other words, an increase in the growth rate induces a decline in the unemployment rate - $\frac{du}{d\gamma} < 0$ from (19) - which in its turn can generate an increase in the growth rate of the economy - $\frac{d\gamma}{du} < 0$ from (20). This chance is strengthened by the non-linearity of the left hand side of (20). Therefore, we double checked that, once setting $A = 0.3688$, we do not have more than one value of γ satisfying (20). We can confirm that in the present model no multiple equilibria arises for plausible parameter values.

Figure 1 sets out our results, concerning the inflation-growth nexus. $\frac{d\gamma}{d\pi} > 0$, however its magnitude is small and it declines the higher is trend inflation. This implies that, on the basis of our model, by increasing the trend inflation rate to 80% the US growth rate would have reached 3.15% per year, instead of its actual value of 3.08. Therefore, in this model, money is very close to being superneutral regarding the growth rate of the economy.

Regarding the inflation-unemployment nexus, our results do not change with respect to those obtained in our previous model.

4 Conclusions

In the present paper we analysed two models of inflation gifts and endogenous growth through learning by doing. In the former model, learning by doing depends on the average capital per worker in the whole economy, while, in the latter model, on the aggregate capital stock. Inflation is showed to have an impact on inflation, which can be calibrated on the basis of the empirical literature. The impact of inflation on growth, instead, is nil in the former model and positive and tiny in the latter one. Real growth reduces the unemployment rate in both models. Finally, in the Appendix we show that the short-run effects of inflation on the unemployment rate and real growth mirror the long-run ones.

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5 Appendix: short run dynamics

.1 Spillovers depending on the average level of capital per worker

In this Appendix we assess the short-run implications of our model, calibrating it on the basis of parameter values customarily adopted in the literature.

Our system of equations is (4), (5), (6), (7), (14), $N_{t+j} = 1 - u_{t+j}$, the aggregate budget constraint, $c_{t+j} + k_{t+j} = y_{t+j} + \frac{(1-\delta)}{\gamma} k_{t+j-1}(h)$, the detrended production function, $y_{t+j} = \frac{A}{\gamma} k_{t+j-1}$, and the money growth generating process $\mu_{t+j} = \mu_{t+j-1}^{\rho} \epsilon_{t+j}$. The transversality conditions for capital, money and bond holdings are customary, namely that for i going to infinity $\beta^{t+j} \frac{k_{t+j}}{c_{t+j}}$, $\beta^{t+j} \frac{m_{t+j}}{c_{t+j}}$ and $\beta^{t+j} \frac{b_{t+j}}{c_{t+j}}$ all go to zero. Our unknowns are $\left\{ \mu_{t+j}, \tilde{\pi}_{t+j}, k_{t+j}, u_{t+j}, \frac{w_{t+j}}{p_{t+j}}, i_{t+j}, N_{t+j}, c_{t+j}, y_{t+j} \right\}$.

It is a well-known fact that AK models do not have transitional dynamics (Barro and Sala-i-Marin, 1995, 142). Our model partly inherits this trait. To show that consumption is completely smoothed also off the steady state, consider (4), (13) and (15) to obtain

$$\frac{c_{t+j+1}(h)}{c_{t+j}(h)} = \frac{\beta \left(A\alpha + 1 - \delta \right)}{\gamma} = 1$$

As a consequence we can re-write the system as follows

$$
\begin{aligned}
\log u_{t+j} &= \begin{bmatrix} -\frac{\phi_0-\phi_1}{\phi_2} - \frac{\phi_1+\phi_3}{\phi_2}\log \frac{w}{p}\Big|_{t+j} - \frac{\phi_4}{\phi_2}\log\frac{w}{p}\Big|_{t+j-1} - \\ -\frac{\phi_4}{\phi_2}\log\left(\frac{1}{\tilde{\pi}_{t+j}}\right) - \frac{\phi_4}{\phi_2}\log\left(\frac{1}{\pi\gamma}\right) \end{bmatrix} && (22)\\
\frac{1}{i_{t+j}} &= \frac{1}{E\left(\tilde{\pi}_{t+j+1}\right)}\beta\frac{1}{\pi\gamma} && (23)\\
\left(\frac{\mu_{t+j}}{\tilde{\pi}_{t+j}}\right)^{-1} &= \left(1-\frac{1}{i_{t+j}}\right)/\left(1-\frac{1}{i_{t+j-1}}\right) && (24)\\
\frac{w_{t+j}}{p_{t+j}}\frac{N_{t+j}}{k_{t+j-1}} &= (1-\alpha)\frac{\gamma}{A} && (25)\\
N_{t+j} &= 1-u_{t+j} && (26)\\
\bar{c}+\gamma k_{t+j} &= \frac{A}{\gamma}k_{t+j-1}+\frac{(1-\delta)}{\gamma}k_{t+j-1} &&\\
\mu_{t+j} &= \mu_{t+j-1}^{\rho}\epsilon_{t+j} && (27)
\end{aligned}
$$

and, after log-linearization and some minor manipulations

$$
\begin{aligned}
\hat{u}_{t+j} &= -\frac{\phi_1+\phi_3}{\phi_2}\left.\frac{\hat{w}}{p}\right|_{t+j} - \frac{\phi_4}{\phi_2}\left.\frac{\hat{w}}{p}\right|_{t+j-1} + \frac{\phi_4}{\phi_2}\left(\hat{\pi}_{t+j}\right) && (28)\\
\hat{\pi}_{t+j+1} &= i\hat{\pi}_{t+j} - (i-1)\,\hat{\mu}_{t+j} && (29)\\
\hat{k}_{t+j} &= \left.\frac{\hat{w}}{p}\right|_{t+j} - \frac{u}{N}\hat{u}_{t+j} && (30)\\
\hat{k}_{t+j+1} &= (A+1-\delta)\,\frac{1}{\gamma}\hat{k}_{t+j} && (31)\\
\hat{\mu}_{t+j} &= \rho\hat{\mu}_{t+j-1}+\hat{\epsilon}_{t+j} && (32)
\end{aligned}
$$

where hats denote deviations from steady sate values and variables without time subscripts are taken at their steady state values. Our unknown variables are the following ones $\left\{\hat{\mu}_{t+j}, \hat{\pi}_{t+j}, \hat{k}_{t+j}, \hat{u}_{t+j}, \left.\frac{\hat{w}}{p}\right|_{t+j}\right\}$. $\hat{\epsilon}_{t+j}$ is assumed to have zero mean and a constant unit variance. With respect to the calibration mentioned above, also recall that in steady state $i = \frac{\gamma\pi}{\beta}$. As in Ascari (2004), $\rho = 0.57$.

Note that equation (31) would imply a constant rate of change of the deviations of the capital stock from steady state, unless $\hat{k}_{t+j} = 0$ for all i. We impose this condition, which implies that the model has no transitional dynamics for capital too. Next we focus on the other equations of the system above.

Figure A1 shows the impulse response functions under our calibration. In presence of a monetary shock, unemployment decreases, inflation increases and the real wage is nearly unaffected. So it is possible to conclude that our model generates a Phillips curve both in the short and in the long-run, as well as short run real wage stickiness. Capital displays no reaction to a monetary shock.

.2 Spillovers depending on the aggregate capital stock

When knowledge spillovers depend on the aggregate capital stock, the system of equations changes into

$$
\begin{aligned}
\log u_{t+j} &= -\frac{\phi_0-\phi_1}{\phi_2}-\frac{\phi_1+\phi_3}{\phi_2}\log\frac{w_{t+j}}{p_{t+j}}- \qquad (33)\\
&\quad -\frac{\phi_4}{\phi_2}\log\left(\frac{w_{t+j-1}}{p_{t+j-1}}\frac{1}{\tilde{\pi}_{t+j}}\frac{1}{\pi\gamma}\right)\\
\frac{1}{c_{t+j}(h)i_{t+j}} &= \frac{1}{\tilde{\pi}_{t+j+1}}\frac{1}{c_{t+j+1}(h)}\beta\frac{1}{\pi\gamma} \qquad (34)\\
\left(\frac{\mu_{t+j}}{\tilde{\pi}_{t+j}}\right)^{-1} &= \frac{c_{t+j-1}(h)}{c_{t+j}(h)}\left(1-\frac{1}{i_{t+j}}\right)/\left(1-\frac{1}{i_{t+j-1}}\right) \qquad (35)\\
\frac{1}{c_{t+j}(h)} &= \frac{r_{t+j}}{p_{t+j}}\frac{\beta}{\gamma}\frac{1}{c_{t+j+1}(h)}+\frac{1}{c_{t+j+1}(h)}\beta(1-\delta)\frac{1}{\gamma} \qquad (36)\\
\frac{r_{t+j}}{p_{t+j}} &= \alpha A N_{t+j}^{1-\alpha} \qquad (37)\\
(1-\alpha) &= \frac{N_{t+j}}{y_{t+j}}\frac{w_{t+j}}{p_{t+j}} \qquad (38)\\
N_{t+j} &= 1-u_{t+j} \qquad (39)\\
c_{t+j}+k_{t+j} &= y_{t+j}+\frac{(1-\delta)}{\gamma}k_{t+j-1}(h)\\
y_{t+j} &= \frac{A}{\gamma}k_{t+j-1}N_{t+j}^{1-\alpha} \qquad (40)\\
\mu_{t+j} &= \mu_{t+j-1}^{\rho}\epsilon_{t+j} \qquad (41)
\end{aligned}
$$

and the unknowns are the sequence $\left\{\mu_{t+j},\tilde{\pi}_{t+j},k_{t+j},u_{t+j},\frac{w_{t+j}}{p_{t+j}},\frac{r_{t+j}}{p_{t+j}},i_{t+j},N_{t+j},c_{t+j},y_{t+j}\right\}$.

Note that the steady state of c is indeterminate. However, one can study the dynamics of the system by computing the steady state values of $\frac{c}{k}$ and $\frac{y}{c}$.

The system of log-linearized equations is

$$
\begin{aligned}
\hat{u}_{t+j} &= -\frac{\phi_1+\phi_3}{\phi_2}\frac{\hat{w}_{t+j}}{p_{t+j}} - \frac{\phi_4}{\phi_2}\left(\frac{\hat{w}_{t+j-1}}{p_{t+j-1}}\right) + \qquad (42)\\
&\quad +\frac{\phi_4}{\phi_2}\left(\hat{\pi}_{t+j}\right)\\
\hat{c}_{t+j}+\hat{\imath}_{t+j} &= \hat{c}_{t+j+1}+\hat{\pi}_{t+j+1} \qquad (43)\\
\left(\hat{\pi}_{t+j}-\hat{\mu}_{t+j}\right) &= \frac{1}{i-1}\left(\hat{\imath}_{t+j}-\hat{\imath}_{t+j-1}\right)+\hat{c}_{t+j-1}-\hat{c}_{t+j} \qquad (44)\\
\left(\hat{c}_{t+j+1}-\hat{c}_{t+j}\right) &= \frac{r}{p}\frac{\beta}{\gamma}\left(\frac{\hat{r}_{t+j}}{p_{t+j}}\right) \qquad (45)\\
\left(\frac{\hat{r}_{t+j}}{p_{t+j}}\right) &= (1-\alpha)\hat{N}_{t+j} \qquad (46)\\
\left[(\hat{w}/p)_{t+j}+\hat{N}_{t+j}-\hat{y}_{t+j}\right] &= 0 \qquad (47)\\
\hat{N}_{t+j} &= -\frac{u}{N}\hat{u}_{t+j} \qquad (48)\\
\hat{c}_{t+j}+\frac{k}{c}\hat{k}_{t+j} &= \frac{y}{c}\hat{y}_{t+j}+\frac{(1-\delta)}{\gamma}\frac{k}{c}\hat{k}_{t+j-1}(h)\\
\hat{y}_{t+j} &= \left[\hat{k}_{t+j-1}+(1-\alpha)\hat{N}_{t+j}\right] \qquad (49)\\
\hat{\mu}_{t+j} &= \rho\hat{\mu}_{t+j-1}+\hat{\epsilon}_{t+j} \qquad (50)
\end{aligned}
$$

The calibration of the system does not change with respect to the above one. Figure A2 projects the impulse-response function of the deviation of the unemployment rate from steady sate against the one pertaining the inflation rate. A 1% monetary shock produces a 0.6% decrease in the unemployment rate and a 2% increase in inflation. Being of the order of 10^{-2}, the percentage deviations from steady state of the other variables are negligible. Therefore, money effects on off-steady state output dynamics are small. Results would not change much by considering higher trend inflation rates.

Figure 1 - Absolute change in the real growth rate after a unit increase in trend inflation

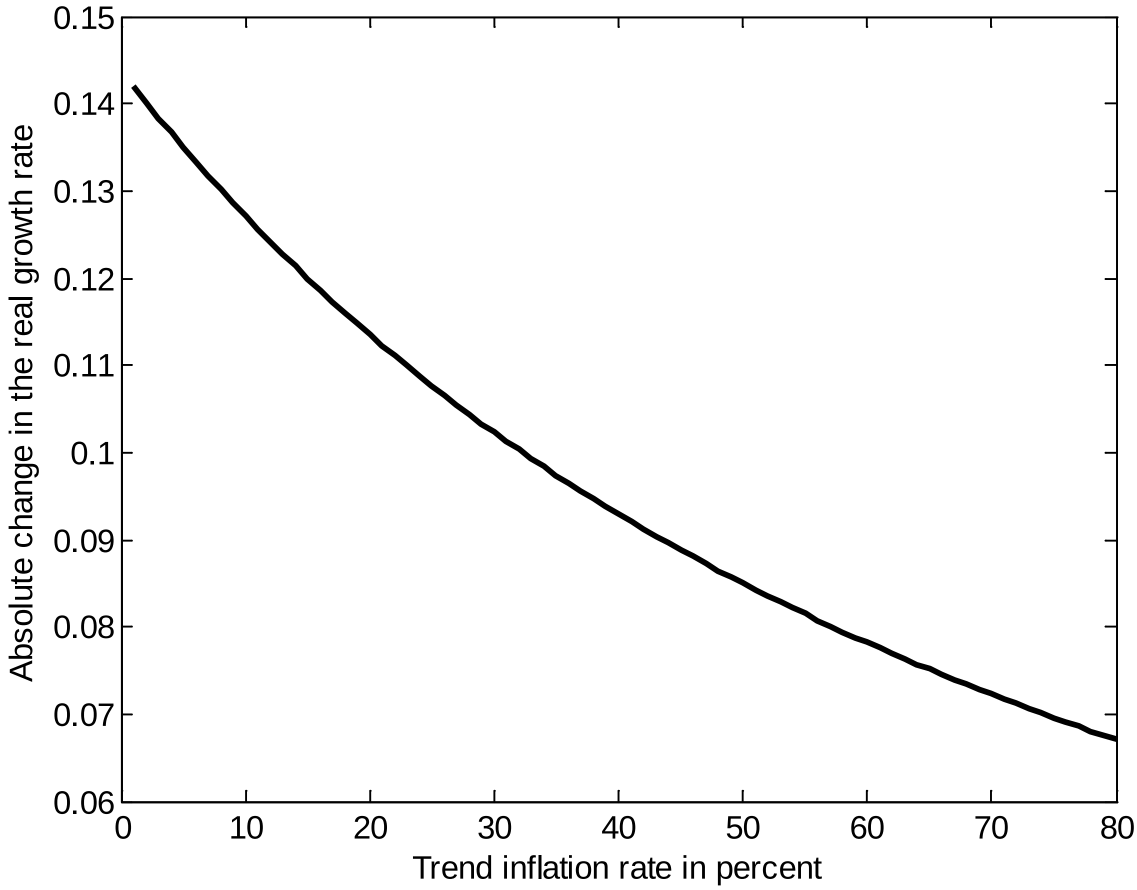

Figure A1 - Impulse response functions after a one percent monetary shock under learning by doing depending on capital per worker

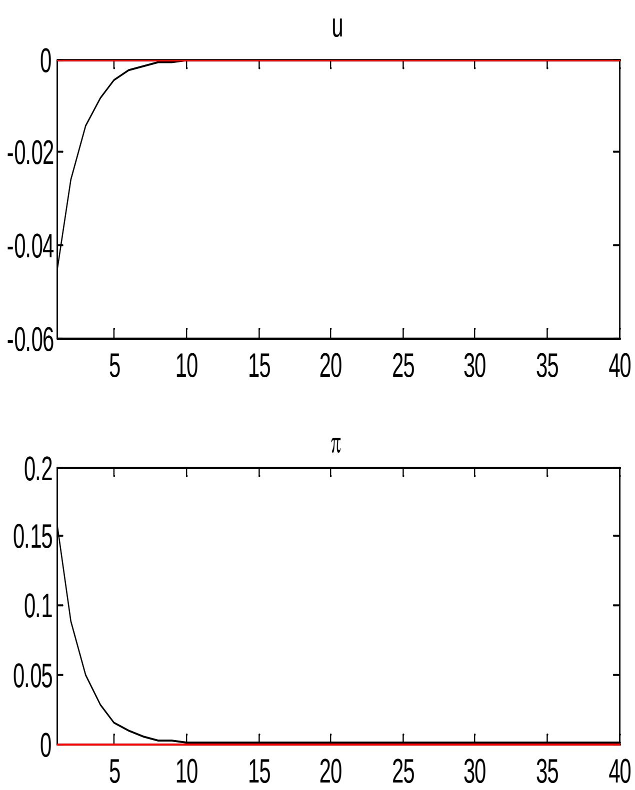

Figure A2 - Impulse response functions after a one percent monetary shock under learning by doing depending on the aggregate capital stock