Working Paper Series
Department of Economics
University of Verona

A remark on 'Decomposition of bivariate inequality indices by attributes' by Abul Naga and Geoffard, Economics Letters 90 (2006), pp. 362-367

Marco Giovanni Brambilla, Eugenio Peluso

WP Number: 5 March 2010

ISSN: 2036-2919 (paper), 2036-4679 (online)

A remark on "Decomposition of bivariate inequality indices by attributes" by Abul Naga and Geoffard, Economics Letters 90 (2006), pp. 362-367.

Marco Giovanni Brambilla*

DEFAP - Catholic University of Milan.

Eugenio Peluso†

Department of Economics, University of Verona.

February, 2010

Abstract

We correct the generalized version of the decomposition of multivariate inequality indices by attributes proposed by Abul Naga and Geoffard "Abul Naga, R. H. and Geoffard, P. Y., 2006. Decomposition of bivariate inequality indices by attributes. Economic Letters 90, pp. 362-367."

Key Words: Multidimensional inequality; Correlation increasing transfers; Copulas

JEL codes: D31, D63.

Abul Naga and Geoffard (2006) showed an elegant decomposition of a bidimensional inequality index of the Atkinson-Kolm-Sen class into level of inequality in each dimension, assessed through Atkinson–Kolm–Sen indices for each attribute, and a residual statistic that depends on the degree of association among the distribution of attributes. This is an important result both for the axiomatic characterization provided in the paper and for its potential interest for applied works (see Croci Angelini and Michelangeli 2008 for a first attempt).

*Piazza Buonarroti 30, 20145 Milano (Italy). E-mail: marco.brambilla@unicatt.it.

†Corresponding author. Via dell'Università 3, 37129 Verona (Italy). Tel +390458028104, fax +398028529. E-mail: eugenio.peluso@univr.it.

According to Abul Naga and Geoffard, when the number of dimensions is set equal to $p \geq 2$, the multidimensional extension of the model accounts for the association among the attributes through the following term (eq. 16, p. 366):

$$\kappa = \frac{n \sum_{i=1}^{n} x_{i1}^{\alpha_1} ... x_{ip}^{\alpha_p}}{\sum_{i=1}^{n} x_{i1}^{\alpha_1} ..... \sum_{i=1}^{n} x_{ip}^{\alpha_p}}.$$

This formula, which is extremely relevant for (suitable) empirical applications, unfortunatly contains a mistake. Below we prove that the correct measure of association is:

$$\kappa = \frac{n^{p-1} \sum_{i=1}^{n} x_{i1}^{\alpha_1} ... x_{ip}^{\alpha_p}}{\sum_{i=1}^{n} x_{i1}^{\alpha_1} ..... \sum_{i=1}^{n} x_{ip}^{\alpha_p}}$$

The proper coefficient at the numerator is n^{p-1} and not just n.

Proof. We study the general case with n individuals (indexed by i) and p attributes (indexed by j). Retaining the definitions of the paper for θ, μ_j and ρ_j, and starting from

$$(\theta \mu_1)^{\alpha_1} (\theta \mu_2)^{\alpha_2} ... (\theta \mu_k)^{\alpha_p} = (\gamma_1 \mu_1)^{\alpha_1} \prod_{j=2}^{p} \rho_j^{\alpha_j},$$

after simple manipulations we get:

$$\sum_{j=1}^{p} \alpha_j \ln \theta = \alpha_1 \ln \gamma_1 + \alpha_2 \ln \gamma_2 ... + \alpha_p \ln \gamma_p + \ln \left(\frac{\prod_{j=2}^{p} \rho_j^{\alpha_j}}{\prod_{j=2}^{p} \gamma_j^{\alpha_j}} \right).$$

The term we are looking for is:

$$\kappa = \left(\frac{\prod_{j=2}^{p} \rho_j^{\alpha_j}}{\prod_{j=2}^{p} \gamma_j^{\alpha_j}} \right). \tag{1}$$

From

$$w_0 = (\gamma_h \mu_h)^{\alpha_h} \prod_{j \neq h} \rho_j^{\alpha_j} \mu_j^j = \frac{1}{n} \sum_i x_{ih}^{\alpha} \prod_{j \neq h} \rho_j^{\alpha_j} \mu_j^j$$

we get:

$$\gamma_h = \frac{1}{\mu_h} \left[\frac{1}{n} \sum_i x_{ih}^{\alpha_h} \right]^{\frac{1}{\alpha_h}} \quad \text{for any } h = 1, ..p. \tag{2}$$

We also know that:

$$w_0 = \frac{1}{n} \sum_{i=1}^{n} x_{i1}^{\alpha_1} ... x_{ip}^{\alpha_p} = \frac{1}{n} \sum_{i=1}^{n} x_{i1}^{\alpha_1} \prod_{j=2}^{p} \rho_j^{\alpha_j} \mu_j^j$$

that gives:

$$\prod_{j=2}^{p} \rho_j^{\alpha_j} = \frac{\sum\limits_{i=1}^{n} x_{i1}^{\alpha_1} ... x_{ip}^{\alpha p}}{\prod_{j=2}^{p} \mu_j^{\alpha_j} \sum\limits_{i=1}^{n} x_{i1}^{\alpha_1}}. \tag{3}$$

Replacing (3) and (2) into (1), we get the result:

$$\kappa = \frac{\frac{\sum\limits_{i=1}^{n} x_{i1}^{\alpha_1} ... x_{ip}^{\alpha_p}}{\prod_{j=2}^{p} \mu_j^{\alpha_j} \sum\limits_{i=1}^{n} x_{i1}^{\alpha_1}}}{\prod_{j=2}^{p} \left[\frac{1}{\mu_j} \left(\frac{1}{n} \sum\limits_{i=1}^{n} x_{ij}^{\alpha_j} \right)^{\frac{1}{\alpha_j}} \right]^{\alpha_j}} = \frac{n^{p-1} \sum\limits_{i=1}^{n} x_{i1}^{\alpha_1} ... x_{ip}^{\alpha_p}}{\prod_{j=1}^{p} \sum\limits_{i=1}^{n} x_{i1}^{\alpha_1}}.$$

■

References

[1] Abul Naga, R. H. and Geoffard, P. Y., 2006. Decomposition of bivariate inequality indices by attributes. Economics Letters 90, pp. 362-367.

[2] Croci Angelini, E. and Michelangeli, A., 2008. Measuring Well-Being differences across EU Countries. A Multidimensional Analysis of Income, Housing, Health, and Education. Working Papers 15-2008, Macerata University, http://www.unimc.it/sviluppoeconomico/wpaper/wpaper00015