On the Definition and Measurement of Transient and Chronic Poverty

Magne Mogstad*
Research Department, Statistics Norway

Abstract:
Moving from a static to a dynamic perspective of poverty is not simply a matter of having access to repeated income observations of the same units, which makes it possible to measure the persistence of poverty and study poverty patterns based on information about changes in income from one year to the next. Access to longitudinal data should draw poverty scholars to revisit life-cycle theories of consumption as a framework to reflect on what kind of behavioural assumptions are underlying their chosen definition of poverty. In this paper, we will evaluate various approaches for measuring transient and chronic poverty by critically drawing upon theories of life-cycle theories of consumption. On this basis, we deduce that it is necessary to extend the accounting period of income in order to reduce the measurement problem of fluctuating annual income, and to obtain a reliable estimate of the economic resources available for consumption and saving to distinguish the chronically poor from the non-poor. Furthermore, we argue that the present methods for measuring chronic and transient poverty lack a theoretically coherent foundation. As an attempt to overcome the weaknesses of the present methods, we suggest an alternative method for measuring chronic and transient poverty. In fact, the method proposed in this paper encompasses the present methods for measuring chronic and transient poverty based on an extended accounting period of income.

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*Corresponding address:
Magne Mogstad
Research Department, Statistics Norway
Postboks 8131 Dep, 0033 Oslo, Norway
E-mail: magne.mogstad@ssb.no
1. Introduction

Over the last decades, increasing discontent with poverty snapshots based on information about annual incomes provided by the national statistical bureaus in most OECD-countries has been expressed. Thus, poverty scholars have worked on broadening the perspective of distributional analysis to study the dynamics of poverty. In particular, the measurement of chronic rather than transient poverty has become an important issue. Although it is commonly agreed that the distinguishing feature of chronic poverty compared to transient poverty is its extended duration, the approaches taken to account for the persistence of poverty differ substantially. In fact, within the uniformity of the term ‘chronic poverty’ it exists some distinct notions of the concept of chronic poverty and how it should be measured. Specifically, the methodological choices necessary for measuring chronic poverty are related to whether or not, and to what degree, the approaches taken are, implicitly or explicitly, based on a model of intertemporal optimisation. In this regard, a key choice is to determine whether poverty is connected to the time-patterns or the streams of annual income. Furthermore, when deciding on a focal variable for analysis of poverty it is critical how incomes accumulated over different periods are made comparable.

In this paper, we evaluate various methods for measuring transient and chronic poverty by critically drawing upon theories of life-cycle theories of consumption. In particular, we will argue that although some aspects of the present methods may be explicitly justified, they lack a theoretically coherent foundation. The standard method for measuring chronic and transient poverty employs a sequence of income periods of one year, and distinguishes the chronically from the transient poor in terms of how long time they have endured low annual income. An underlying assumption for the validity of this approach is that income is perfectly transferable within the year that it is earned, but not transferable between years. This implies that the method pays excessively attention to the time-patterns of income, but is rather insensitive to the actual income streams. In fact, a non-poor may have an income stream that is lower than the income stream of a person defined as chronically poor over a given period of time.

As a response to the fact that the standard approach fails to recognise that just as one bad year does not make an otherwise wealth person poor, one good year does not move an otherwise impoverished person out of poverty, alternative methods for measuring chronic and transient poverty have been proposed where the accounting period of income is extended. Although these methods may be considered as important improvements compared to the standard approach by moving the focus away from the time-patterns of annual income, each suffer from its own weaknesses. An approach followed in certain analyses is to measure chronic poverty by simply adding up data on real income over a set of consecutive calendar years, and identify the chronically poor as those with aggregate real income below the aggregate poverty threshold. However, by relying on the consumer price index rather than real interest rates when making incomes accumulated over time comparable, one only account for part of the benefits of receiving income early rather than late in the sequence of years. Alternatively, one may follow the method suggested in Rodgers and Rodgers (1993) where real interest rates are used to make real incomes added up over several years comparable. Specifically, their focal variable for the measurement of chronic poverty is the annual income level equal to the maximum sustainable annual consumption expenditure he can achieve given the actual income stream over the period provided that he may transfer income by borrowing and saving at the prevailing interest rates. That is, each individual is assumed to strictly prefer to carry out intertemporal income transfers to ensure an exactly constant consumption level over time. Such a behavioural assumption requires preferences structures for intertemporal choice of consumption where either the rates of time preferences are equal to the real interest rates or the intertemporal elasticities of substitution are zero. Thus, one is forced to take the


2 See Duncan and Rodgers (1991) and Aaberge et al. (2000).
stand that it is reasonable that each individual either views the periods as perfect complements or has a rate of time preferences, which is supposed to be a deep parameter, equal to fluctuating real interest rates. In particular, it seems dubious that individuals facing relatively high interest rates on borrowing, or even borrowing constraints, should have a similarly high rate of time preferences and therefore prefer constant consumption. As an attempt to overcome the weaknesses of the present methods, we suggest an alternative method for measuring chronic poverty that is compatible with a more general preference structure. Moreover, it is flexible enough to handle relatively high and diverse interest rates on borrowing and savings as well as liquidity constraints in a sound way. Specifically, our choice of focal variable is the minimum expenditure level that the individual would need in each period to be as well off as he could be by making intertemporal transfers of income at the prevailing interest rates on borrowing and saving to achieve his preferred consumption profile. The method proposed in this paper encompasses the present methods for measuring chronic and transient poverty based on an extended accounting period of income.

The paper is organized as follows. In Section 2, we critically discuss the issue of identifying the chronically poor from the perspective of life cycle theories of consumption. This discussion provides the structure and framework to consider the material that follows. In Section 3, we provide a review of alternative methods for measuring chronic and transient poverty. As an attempt to overcome the weaknesses of the proposed methods, we present in section 4 an alternative method for measuring chronic and transient poverty, for which we attempt to provide a coherent justification for the set of underlying assumptions. Acknowledging that, in end effect, the importance of using one method instead of others is an empirical matter, we will in Section 5 quantify the extent of chronic and transient poverty and the shape of the demographic and geographic poverty profiles following from various approaches. The informational basis for this empirical analysis is a longitudinal data set covering the entire resident population of Norway for the period 1993-2001, and will be supplemented with information from Statistics Norway's detailed income register for the respective years. Policy implications are discussed in section 6.

2. Identification of the chronically poor: Life-cycle theories of consumption revisited

"The identification of low measured income with 'poor' and high measured income with 'rich' is justified only if measured income can be regarded as an estimate of expected income over a lifetime or a large fraction thereof" (Friedman, 1957; p38).

When evaluating distributional issues, such as poverty, advocates of life-cycle theories of consumption would argue that relying on information about income for a relatively short time period can give a misleading picture of poverty in a society. The principal proposition is that identification of an individual with low income over the time period observed with 'poor' is only justifiable if measured income can be regarded as an empirically sound estimate of the economic resources available for the individual to fulfil his or her specific end. The underlying behavioural assumption in life-cycle theories is that rational individuals optimise their consumption intertemporally. Instead of basing the analysis on observed temporary income, it is argued that consumption is a function of expected income over a lifetime, or at least a large fraction thereof, and that individuals may smooth out the effect of transitory shocks on consumption by borrowing and saving. Specifically, income is decomposed into permanent and transitory components, where the former is interpreted as reflecting the effect of factors influencing future labour and capital income such as endowment of wealth.

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3 During the 1950s and 1960s, a new framework for consumption theory was developed. Unlike the Keynesian consumption function, attempts were made to derive models for consumer behaviour explicitly from individuals' utility maximising behaviour in a dynamic framework. In the seminal work of Modigliani and Brumberg (1954), consumption decisions were integrated in an intertemporal optimisation program along the lines of Fisher's (1907, 1930) seminal two-period model, from which they derived a life-cycle hypothesis implying that the individual's consumption in a given period is determined by his or her expected lifetime income, rather than the income in that period as Keynes postulated. In Friedman and Kuznets (1954) and Friedman (1957, 1963), a conceptually somewhat analogous theory to that of Modigliani and Brumberg was proposed, namely the permanent income hypothesis. For a discussion of similarities and discrepancies between the permanent income hypothesis and the life-cycle hypothesis see Jappelli (2005).
education and skills, and occupation. The transitory components on the other hand are interpreted as factors that are likely to be treated by the unit affected as accidental or chance occurrences, for example short-term illness and cyclic fluctuations in economic activity.

The rationale for the view that consumption in a given period is supposed to reflect some unobserved permanent income rather than the time-pattern of income, is that by the law of large numbers the transitory components, which are assumed to have expectation equal zero, should average out. If this is the case, the exercise of identifying those with insufficient economic resources to achieve an acceptable living standard may have little to do with the distribution of annual incomes or incomes accumulated over a few years, which may very well fluctuate heavily do to transitory shocks. Furthermore, one has acknowledged that, in addition to transitory exogenous shocks, individuals may themselves behave in certain ways causing annual incomes to vary substantially over time without reflecting (long-term) changes in economic resources available for consumption. In particular, accounting rules and standards of taxation for income from self-employment and financial assets seems to have strong impact on distributional analysis based on information about annual income (Aaberge et al., 2000). As a way of reducing the measurement problem of fluctuating annual income, and to obtain a reliable estimate of the economic resources available for consumption and saving (i.e. future consumption) to determine whether a person is poor or non-poor, it seems plausible that the accounting period of income should be extended.

It is worth noticing that the objection against basing distributional analysis on information about annual income is not only critical for the relevance of poverty snapshots based on cross-section data, but also existing studies of poverty spells, which so far has been treated as equivalent to studying changes in the lower part of distribution of annual incomes. In particular, an underlying assumption for the meaningfulness of studying poverty patterns based on information about changes in income from one year to the next is that it makes sense to define poverty in terms of annual incomes. Otherwise, we are studying the dynamics of objects that have no well-defined meaning. If low annual income is unsuitable as an indicator of individuals inability to pursue well-being because of the lack of economic means, as advocates of life cycle theories claim, it is necessary to take a step back, and reconsider how we should identify the poor from longitudinal data before continuing modelling and evaluating poverty spells and transitions.

2.1 Are lifetimes the ideal accounting periods of income for analysis of poverty?

Arguably, analysis of poverty should not be based on observed annual income, rather the income over a longer time period. This raises the question: how long should the accounting period of income be? Are lifetimes the ideal accounting period of income for analysis of poverty?

In order to connect poverty strictly to lifetime income, without paying attention to the time-pattern of income, it is necessary that individuals may smooth out the effect of transitory shocks on consumption by borrowing and saving at the prevailing interest rates. The rationale for such a viewpoint is that by the law of large numbers the transitory components, which are assumed to have expectation equal zero, should average out. Appealing to the law of large numbers to deduce that the effects of transitory shocks average out will, of course, only make sense if the numbers really are sufficiently large. The justification proposed in Friedman (1957; p22) is that data indicates "For any considerable group of consumer units, the resulting transitory components tend to average out." It might very well be true that average consumption in a population reflects average permanent income, since the transitory components tend to average out across the population over a relatively short period of time. This does not, however, imply that transitory components should average out for every single individual over the same time period. As Friedman (1957; p23) himself acknowledges, "The experience of one unit is

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4 For example, in the first sentence of the Presidential Address to the European Society of Population Economics Twelfth Annual Congress, Jenkins (2000; p1) states that "This lecture is about the longitudinal dynamics of personal economic well-being, i.e. the patterns of change, from one year to the next, of needs-adjusted household net income for each person in the population".
itself but a small sample of a more extensive hypothetical universe, so there is no reason to suppose that transitory components average out to zero over the unit's lifetime." Since the ultimate objective of any theory of distributional justice is equality along some dimension (Sen, 1992) - the distinguishing feature being what one claim equality should be judged by, i.e. the choice of focal variable - arguing that the effects of transitory shocks may average out across a population has no bearing for analysis of poverty. Consequently, advocates of the life-cycle theories for exercises of social evaluation, such as poverty analysis, are forced, if their reference to the law of large numbers is to hold, to take the stand that a reasonable approximation is that each individual has an infinite planning horizon. Otherwise, it is no reason to believe that the effects of transitory shocks should, from the viewpoint of each individual, average out. In fact, if individuals have finite planning horizon, it may be no reason to believe that an individual who are unlucky and hit by adverse shocks should be either able to borrow in the capital market to smooth consumption as he pleases, since a borrower would limit the credit of the unlucky one as he realises that it is a fair chance that the transitory shocks may not average out over a finite time period. Therefore, the unfortunate ones may find that adverse transitory shocks, in end effect, result in a life in poverty despite that they, ex ante the shocks, had a reasonable level of expected lifetime resources.

A pressing issue is, therefore, whether or not it is a reasonable approximation for an exercise of social evaluation, such as poverty analysis, to assume that each individual maximises utility over an infinite planning horizon. On one hand, the assumption of costless optimisation over an infinite planning horizon may be useful for deriving analytical solutions to problems concerning the choices between consumption and saving among households over time, and as Barro (1974) argued an infinite horizon may be justifiable if individuals have completely altruistic bequest motives. On the other hand, one may argue that the assumption suppresses the complexity of the calculations involved and the great deal of uncertainty that is necessary to quantify to make perfectly optimising decisions over long time periods. In fact, the findings of Tversky and Kahneman (1974) and Loewenstein and Thaler (1989) indicate that individuals appear to depart consistently and systematically from the predictions of utility maximizing models over an infinite horizon. Instead, individuals seem to follow rules of thumb in choosing their long-term consumption plan, which may very well be a rational response to factors such as computation costs and the great deal of uncertainty about future income flow. In fact, Friedman (1957; p23), unlike some of his Chicago-school followers, acknowledges this himself asserting that permanent income should be interpreted as something like an average of the expected level of income over a fraction of a lifetime, and that although "It is tempting to interpret the permanent components as corresponding to the average lifetime values and the transitory components as the difference between such lifetime averages and the measured values in a specific period. It would, however, be a serious mistake to accept such an interpretation." Instead, he argues that households, in practice, would adopt a much shorter horizon than the remainder of their lifetimes.5 If individuals have shorter horizons than their lifetimes when making complex saving and consumption decisions then not only permanent components but also transitory components matter from their viewpoint. Consequently, distribution of incomes observed over a time period less than a lifetime may indeed be of relevance for analysis of poverty.

Even if one accepts that each individual, at least as an approximation, maximises utility over an infinite planning horizon and the transitory shocks therefore tend to average out for each person, it is nevertheless far from trivial to assume that he may smooth consumption by borrowing and saving without facing any constraints. Specifically, an underlying assumption in the benchmark version of the permanent income hypothesis is that individuals may borrow and save at the same interest rate provided that they at some point repay their debts. In contrast, is seems clear that the interest rates differ between borrowing and saving, partly due to transaction costs. Furthermore, it can be difficult to borrow, at least unconstrained, against future labour income since judgements about income prospects

5 Specifically, Friedman (1957, 1963) presents some evidence that makes him infer that the empirically appropriate definition is to regard the permanent components as reflecting the influence of factors affecting income for a period of three or more years.
involves not only great uncertainty but also asymmetric information about skills and propensities to induce effort - especially for individuals outside the labour force such as unemployed, students, and disabled.⁶ An implication of binding liquidity constraints is that consumption may not exceed income early in life, and so consumption, at least to some degree, has to follow income. Furthermore, even if constraints are not binding today, the fact that they may bind in the future can reduce current consumption. Therefore, connecting poverty entirely to low stream of income over a lifetime may conceal the picture of poverty in a population, since individuals may in some periods have significantly restricted consumption possibilities due to insufficient current income, such as early in life, despite reasonable lifetime income.

To sum up, above we have argued that it is more to evaluations of distributional issues, such as poverty, than comparing lifetime incomes, due to liquidity constraints and/or individuals with shorter horizon than their lifetimes when making complex saving and consumption decisions implying that not only permanent components but also transitory components matters from their viewpoint. Therefore, poverty analysis also involves identifying those who lack the economic resources to achieve a reasonable level of material well-being at different stages over their lifetime. Social justice requires a system that provides individuals with a fair opportunity to achieve a reasonable living standard, relative to the conventional standards in the society to which they belong, at each stage in life. This does not, however, imply that the length of the accounting period of income to be used for identification of the chronically poor should be a year. Conceptually, the choice of accounting period of income on depends on how long one horizon that is reasonable to assume that individuals have when making complex saving and consumption decision. In this regard, it is of interest that consumer expenditure analyses show that the poor spend more than their annual income (Slesnick, 1992, Mayer and Jencks, 1989), which may be interpreted as evidence for borrowing and saving behaviour among the poor.⁷ Furthermore, survey information about the ability to save and borrow among the poor support that individuals can borrow (Mayer and Jencks, 1989). At the end of the day it is likely, however, that the choice of account period of income is, as the exact specification of the poverty line, somewhat arbitrary. Consequently, we might be forced to use various accounting periods of income, and evaluate whether and to what degree the results are robust to this methodological choice.

3. A critical review of proposed methods for measuring chronic and transient poverty
Although it is commonly agreed that the distinguishing feature of chronic poverty compared to transient poverty is its extended duration, the approaches taken to account for the persistence of poverty differ substantially. In fact, within the uniformity of the term ‘chronic poverty’ it exists some distinct notions of the concept of chronic poverty and how it should be measured. Specifically, previous studies of chronic and transient poverty differ in terms of whether or not it is the time-patterns or the streams of annual income that matters when identifying the poor, and how incomes accumulated over different periods are made comparable.

3.1 Is it the time-patterns or streams of annual income that matters when measuring chronic and transient poverty?
The standard method for measuring chronic and transient poverty employs a sequence of income periods of one year, and distinguishes the chronically from the transient poor in terms of how long time they have endured low annual income. This could be done can by tabulating the proportion of the population with annual incomes below the period-specific poverty line in x out of T time periods, where 0 ≤ x ≤ T.⁸ In this case, the chronically poor are those who had annual income below the period-specific poverty line in most or all periods, while the transient poor are those who had annual income above the period-specific poverty line for not all but a sufficiently number of periods. If the cut-off line between the chronically poor and transient poor is set equal to T, it is necessary to have annual income below the period-specific poverty line at each step in the sequence of periods to be chronically

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⁶ Flavin (1981) and Blinder and Deaton (1985) reject the permanent income hypothesis with perfect capital markets.
⁷ Alternatively, annual expenditures exceeding annual incomes may be because the poor underreports their incomes.
poor. The crucial issue is therefore to decide on how many periods with annual income below the period-specific poverty line it takes to be defined as chronically poor. Alternatively, one may distinguish the chronically poor from the transient poor in terms of how long time they have been poor by using hazard regression models and/or first-order Markov models to estimate transition rates and the lengths of spells with annual income below or above the period-specific poverty line. In this case, chronic poverty corresponds to having relatively long spells (or somewhat shorter but multiple spells).

The important thing to notice is that the standard method for identifying the chronically poor - independently of whether one relies on the tabulation or spell approach- implicitly assumes, like the cross-section studies of poverty, that income is perfectly transferable within the period that it is earned, but not transferable between periods (Rodgers and Rodgers, 1993). Consequently, the method pays excessively attention to the time-patterns of annual income, and is rather insensitive to the actual income streams. The implications of this strong dependency on the time-patterns of annual income are illustrated in the table 1. Let us first assume that the chronically poor are defined as those experiencing a poverty spell of at least three years in length out of the sequence of six years, or alternatively as those being tabulated as having annual income below the period-specific poverty line in at least three out of the six years. In either case, all 4 individuals presented in table 1 will be defined as poor. However, defining individual D as chronically poor is clearly unreasonable, since he by saving could achieve an annual income level in each period that is well above the respective period-specific poverty line. In comparison, one may argue that individual C is correctly defined as chronically poor since he could not achieve an annual income level above the period-specific poverty line in each period by intertemporal income transfers.

Suppose instead that we take a more conservative approach and define the chronically poor as those experiencing a complete poverty spell over the period, or alternatively as those being tabulated as having annual income below the period-specific poverty line in every year. In this case, individual B is the only chronically poor, despite his relatively high income stream compared to individual A. This illustrates that if one follows the standard approach, a non-poor person may have an income stream that is lower than the income stream of a person defined as chronically poor. The reason is that the former individual happens to have, in a single year, annual income above the period-specific poverty line, which excludes him from the poor population regardless of the relative size of his income stream. On this basis, one may argue that it is meaningless to view individual B, unlike individual A, as chronically poor; individual B could, by saving, sustain a higher consumption level than A in each period. Furthermore, the fact that the standard approach pays excessively attention to the time-patterns of income, and is rather insensitive to the actual income streams, is particularly worrying if one acknowledges that factors such as accounting rules and standards of taxation for income from self-employment and financial assets may all cause annual income to vary substantially over time without reflecting (long-term) changes in economic resources available for consumption.

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Table 1. Annual incomes and period-specific poverty lines of four illustrative individuals for six consecutive years

<table>
<thead>
<tr>
<th></th>
<th>Year 1: Income</th>
<th>Year 2: Income</th>
<th>Year 3: Income</th>
<th>Year 4: Income</th>
<th>Year 5: Income</th>
<th>Year 6: Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual A</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>101</td>
<td>10</td>
</tr>
<tr>
<td>Individual B</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>99</td>
</tr>
<tr>
<td>Individual C</td>
<td>101</td>
<td>101</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>101</td>
</tr>
<tr>
<td>Individual D</td>
<td>300</td>
<td>300</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>300</td>
</tr>
<tr>
<td>Period-specific poverty line</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

3.2 Extending the accounting period of income: How should incomes accumulated over different periods be made comparable for the purpose of poverty analysis?

Moving from a static to a dynamic perspective of poverty is not simply a matter of having access to repeated income observations of the same units, which makes it possible to measure the persistence of poverty and study poverty patterns based on information about changes in income from one year to the next. Access to longitudinal data should draw poverty scholars to revisit life-cycle theories of consumption as a framework to reflect on what kind of behavioural assumptions are underlying their chosen definition of chronic poverty. If one finds the underlying assumption of the standard approach for identifying the chronically poor unreasonable - namely that income is perfectly transferable within the year that it is earned, but not transferable between years - it appears sensible to extend the accounting period of income. When extending the accounting period of income, and consequently incorporating that rational individuals may optimise their consumption intertemporally (although not to the extent that pure life-cycle theorists would advocates provided that the accounting period of income is shorter than the a lifetime), it is necessary to make incomes from different time periods comparable.

In order to identify the chronically poor in an environment of inflation based on information about incomes accumulated over time it is necessary to acknowledge that simply comparing nominal incomes across periods with different price levels does not suffice, since we do not consume dollars; we consume goods. Otherwise, one risks overrating the level of economic resources available for consumption for individuals receiving a relatively large fraction of their income late in life, and vice versa. A possible response, used in Duncan and Rodgers (1991) and Aaberge (2000), is to let the individual's long-term income be determined by average income level, \( I \), over the given time period after adjusting for yearly inflation (measured by the consumer price index\(^\text{10}\)). In the simple case where the indicators of poverty relies on incomes accumulated over two years, this corresponds to:

\[
I = \frac{1}{2}(y_1 + \frac{y_2}{1 + \pi})
\]

where \( y_t \) denotes income in year \( t = 1, 2 \), and \( \pi \) represents the rate of inflation. That is, one attempts to make the incomes added up over time comparable by accounting for the average change in prices for a (supposedly) representative basket of consumer goods. A person may then be defined as chronically poor if he has a mean real income level over a sequence of years that is below a poverty line defined as a percentage of the mean real median income over the same time period.

However, making incomes from different periods comparable is not merely a question of accounting for changes in the price of goods, rather changes in the price of consumption. Should not the price of

\(^{10}\) The consumer price index is used rather than the gross domestic product deflator, since the former measures the change in average prices on goods consumed in an economy while the latter measures the average price on final goods produced in an economy. Obviously, the two measures need not be the same because the goods produced in an economy do not necessarily coincide with the goods purchased by the consumer.
consumption today reflect its opportunity cost? The opportunity cost of consumption today has to do with the alternative use of your money, which depends on the real interest rates determining how much future consumption we must give up in exchange for being able to consume more today. Hence, it is the real interest rates rather than the consumer price index should be used to make incomes accumulated over a set of periods comparable. Only if it is costless to carry out intertemporal transfers of income (i.e. the real interest rates are equal to zero) the price of goods coincides with the price of consumption. In fact, relying on the consumer price index rather than real interest rates when comparing incomes accumulated over a set of periods, implies that one fails to account for the benefits from receiving real income in an early stage in life, and vice versa. For example, the real income stream for an individual given by (1, 100) is viewed as equally good as the real income stream of another individual given by (100, 1), despite the fact that the latter individual can achieve greater consumption in each period compared to the former individual simply by utilizing the opportunity to save at a positive real interest rate.

As an alternative to simply adding up data on real income gathered over a set of consecutive calendar years, Rodgers and Rodgers (1993) develop a method where real interest rates are used to make incomes added up over several years comparable. Specifically, their focal variable for the chronic poverty analysis is measured as the annual income level \( A \) equal to the maximum sustainable annual consumption expenditure that he can achieve given the actual income stream over the period provided that he may transfer income by borrowing and saving. The chronically poor are then defined as those individuals with \( A \) below the poverty line \( (z) \) over the given sequence of years. For simplicity, suppose that the real interest rates on saving and borrowing are equal, and denoted \( r \). Then, the focal variable is defined as the annuity that has the same value as the individual’s actual income stream over the given period. To calculate this focal variable one simply replace the variables representing the level of consumption in the different periods of a standard intertemporal budget constraint by \( A \). Consider the simple two-period case as an illustration, where \( A_T \), computed at year \( T=2 \), is the annuity determined by:

\[
y_t(1.0r) + y_{t+1} = A_t(1.0r) + A_{t+1} \iff A_t = \frac{(1.0r \times y_t) + y_{t+1}}{1.0r + 1}
\]

where \( y_t \) denotes real income in year \( t = 1, 2 \).\(^{11}\) If \( y_1 > y_2 \) the annual income level that is equal to the maximum sustainable annual consumption expenditure is greater then his average income, and vice versa, due to prevailing interest rates for borrowing and saving that he may utilise to smooth consumption.\(^{12}\)

Even if one refute the underlying assumption in life-cycle theories of possible smoothing of consumption by borrowing unconstrained, and takes the stand that (certain) individuals are prohibited from borrowing, it is clear that even they can save. Consequently, one must, at least, account for the real interest rates from saving when comparing incomes accumulated over a set of periods across a population. Rodgers and Rodgers (1993) acknowledge that the cost of borrowing may not perfectly mirror that of saving, and therefore let the saving account interest rate represent the saving rate, while the borrowing rate is defined by the much higher credit card interest rate. But what Rodgers and Rodgers (1993) fail to account for is the issue of binding liquidity constraints for certain groups of the population. If chronic poverty essentially is about identifying those with so low resources over time

\(^{11}\) Provided that the interest rates for saving and borrowing are equal and year \( T \) is the basis for the annuity calculations, the annual income level that is equal to the maximum sustainable annual consumption expenditure over \( T \) years can, which is showed in Rodgers and Rodgers (1993), be determined as;

\[
A_T = \frac{\sum_{t=1}^{T} \left[ \Pi (1 + r_t) \right] y_t + y_T}{\sum_{t=1}^{T} \left[ \Pi (1 + r_t) \right] y_t + 1}
\]

\(^{12}\) In reality, the rates of interest vary not only through time, but also differ for saving and borrowing. If the interest rate for saving is allowed to differ from that of borrowing, the maximum sustainable annual consumption expenditure must be computed by an iterative procedure. See Rodgers and Rodgers (1993) for the algorithm of the iterative procedure.
that they are, in effect, excluded from customs and activities expected by the society they belong to, it is problematic to expect that the very same individuals can participate in the capital market as borrowers. Specifically, it can be difficult to borrow against future labour income since judgements about income prospects involve not only great uncertainty but also asymmetric information about skills and propensities to induce effort - especially for individuals outside the labour force, such as unemployed and disabled, who are at risk of falling seriously short of the resources commanded by the average individual or household in the community in which they live.

Translated to the framework of Rodgers and Rodgers (1993), binding borrowing constraints is equivalent to letting the interest rate on borrowing ($r^b$) tend toward infinity. In the two-period case, this implies that $A_2 = y_1$ for individuals with $y_2 > y_1$, i.e. the maximum sustainable annual consumption expenditure is determined by the relatively low income in the first period and, in fact, insensitive to the size of $y_2$. Such an approach would, however, suffer from the same weakness as cross-section studies of poverty in the sense that neither recognise that a bad year does not render an otherwise well-off person poor. Consequently, one may infer that the framework of Rodgers and Rodgers (1993) is not flexible enough to incorporate binding borrowing constraints. But more importantly, we will argue that the extreme insensitivity of the maximum sustainable annual consumption expenditure to the gains and losses from borrowing and saving as the interest rates increases reflects a deeper problem in Rodgers and Rodgers's approach, which they do no address. For an illustration consider table 2, where we in the two-period case, using Rodger and Rodger's own formula, calculate the maximum annual consumption expenditure for individuals with different income streams facing various interest rates on borrowing and saving. This table illustrates that as the interest rate on borrowing increases individual A will continue to transfer income from period 2 to period 1 in order to smooth consumption perfectly, even though the gain in terms of resources available of consumption in period 1 from such equalising transfer declines radically. Similarly, as the interest rate increases and future consumption becomes relative cheap compared to current consumption individual B is assumed to insist on having equal consumption. Finally, as the interest rate increases individual C, who has equal real income in the two periods, will not trade-off some consumption in period 1, which now is relatively expensive, for a substantial increase in consumption in period 2. That is, the use of maximum annual consumption expenditure as the focal variable for analysis of poverty implies that each individual, per assumption, actually would actually prefer to make inter-period transfers at the prevailing interest rates to smooth consumption perfectly. This raises two questions. Firstly, ‘what is the underlying preference structure of the individuals for this focal variable?’. Secondly, ‘is the assumed preference structure a reasonable approximation to reality?’.
Table 2. Real income, preferred consumption profile, and focal variables for analysis of poverty

<table>
<thead>
<tr>
<th>Interest rate</th>
<th>Individual</th>
<th>Real income in year 1</th>
<th>Real income in year 2</th>
<th>A₂</th>
<th>(c₁*, c₂*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>r₂ = 0.00</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>55</td>
<td>(55.5, 54.5)</td>
</tr>
<tr>
<td>r₂ = 0.01</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>54.8</td>
<td>(55.1, 54.4)</td>
</tr>
<tr>
<td>r₂ = 0.03</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>54.3</td>
<td>(54.3, 54.3)</td>
</tr>
<tr>
<td>r₂ = 0.05</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>53.9</td>
<td>(53.6, 54.3)</td>
</tr>
<tr>
<td>r₂ = 0.10</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>52.9</td>
<td>(51.8, 54.1)</td>
</tr>
<tr>
<td>r₂ = 0.20</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>50.8</td>
<td>(48.5, 53.7)</td>
</tr>
<tr>
<td>r₂ = 0.40</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>47.5</td>
<td>(43.4, 53.2)</td>
</tr>
<tr>
<td>r₂ = 0.60</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>44.6</td>
<td>(39.4, 52.9)</td>
</tr>
<tr>
<td>r₂ = 0.80</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>42.1</td>
<td>(36.3, 52.7)</td>
</tr>
<tr>
<td>r₂ = 1.00</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>40.0</td>
<td>(33.7, 52.5)</td>
</tr>
<tr>
<td>r₂ → ∞</td>
<td>A</td>
<td>10</td>
<td>100</td>
<td>10.0</td>
<td>(10.0, ∞)</td>
</tr>
<tr>
<td>r₂ = 0.00</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>55</td>
<td>(55.5, 54.5)</td>
</tr>
<tr>
<td>r₂ = 0.01</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>55.2</td>
<td>(55.6, 54.9)</td>
</tr>
<tr>
<td>r₂ = 0.03</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>55.7</td>
<td>(55.7, 55.7)</td>
</tr>
<tr>
<td>r₂ = 0.05</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>56.1</td>
<td>(55.7, 56.5)</td>
</tr>
<tr>
<td>r₂ = 0.10</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>57.1</td>
<td>(55.9, 58.5)</td>
</tr>
<tr>
<td>r₂ = 0.20</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>59.1</td>
<td>(56.3, 62.4)</td>
</tr>
<tr>
<td>r₂ = 0.40</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>62.5</td>
<td>(57.1, 70.1)</td>
</tr>
<tr>
<td>r₂ = 0.60</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>65.4</td>
<td>(57.8, 77.5)</td>
</tr>
<tr>
<td>r₂ = 0.80</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>67.9</td>
<td>(58.4, 84.8)</td>
</tr>
<tr>
<td>r₂ = 1.00</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>70.0</td>
<td>(59.0, 91.9)</td>
</tr>
<tr>
<td>r₂ → ∞</td>
<td>B</td>
<td>100</td>
<td>10</td>
<td>100.0</td>
<td>(100, ∞)</td>
</tr>
<tr>
<td>r₂ = 0.00</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(55.5, 54.5)</td>
</tr>
<tr>
<td>r₂ = 0.01</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55.5</td>
<td>(55.4, 54.6)</td>
</tr>
<tr>
<td>r₂ = 0.03</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55.5</td>
<td>(55.0, 55.0)</td>
</tr>
<tr>
<td>r₂ = 0.05</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(54.7, 55.4)</td>
</tr>
<tr>
<td>r₂ = 0.10</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(53.9, 56.3)</td>
</tr>
<tr>
<td>r₂ = 0.20</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(52.4, 58.1)</td>
</tr>
<tr>
<td>r₂ = 0.40</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(50.2, 61.7)</td>
</tr>
<tr>
<td>r₂ = 0.60</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(48.6, 65.2)</td>
</tr>
<tr>
<td>r₂ = 0.80</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(47.4, 68.7)</td>
</tr>
<tr>
<td>r₂ = 1.00</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(46.4, 72.2)</td>
</tr>
<tr>
<td>r₂ → ∞</td>
<td>C</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>(55.0, 75.0)</td>
</tr>
</tbody>
</table>

Notation:
A₂ is the maximum annual sustainable consumption expenditure, which is computed at year 2
(c₁*, c₂*) is the preferred consumption profile

Assumptions:
When calculating (c₁*, c₂*): σ = 2/3 and δ = 0.03
When calculating A₂: σ = 0 or δ = r₂

According to Rodgers and Rodgers (1993; p34), "Our concept of chronic poverty is a response to the question: 'What level of deprivation, relative to the poverty threshold, would be experienced if individuals could undertake consumption-equalizing inter-period transfers at reasonable rates of interest, and did so?'. A necessary condition for this to be a sound approach for measuring chronic poverty is that each individual actually would prefer to make inter-period transfers at the prevailing interest rates to smooth consumption perfectly. In order to evaluate whether or not the choice of focal variable in Rodgers and Rodgers is a reasonable, we need to identify the underlying preference structure. In general, there are two kinds of preference structures that are consistent with using maximum annual sustainable consumption expenditure as the focal variable for identifying the chronically poor, and as we shall see neither of them are obviously reasonable.

To show this, consider the benchmark problem of intertemporal choice, outlined in Deaton (1992), where preferences are assumed to be intertemporal additive, in which case the utility function for a given individual takes the form:
\( U(c_1, \ldots, c_T) = u_1(c_1) + u_2(c_2) + \ldots + u_T(c_T) \), where \( c_t \) represents the consumption level in period \( t \) and the individual period subutility functions are increasing and concave in the single arguments, i.e. \( u'_t \geq 0, u''_t \leq 0 \) for all \( t = 1, 2, \ldots, T \) periods.

Utility is maximized subject to the intertemporal budget constraint:

\[
\sum_{t=1}^{T} \frac{c_t}{(1 + \delta)^{-t}} = \sum_{t=1}^{T} \frac{y_t}{(1 + \gamma)^{t-1}}
\]

That is, each individual faces the following maximisation problem:

\[
\begin{align*}
\text{Max } U & = \sum_{t=1}^{T} u(c_t)(1 + \delta)^{-t} \\
\text{s.t. } \sum_{t=1}^{T} \frac{c_t}{(1 + \delta)^{-t}} & = \sum_{t=1}^{T} \frac{y_t}{(1 + \gamma)^{t-1}}
\end{align*}
\]

Applying the Lagrange method, we can derive the first order conditions:

\[
u'_t(c_t)(1 + \delta)^{-t} = \lambda(t(1 + \gamma)^{t-1} \text{ for all } t = 1, \ldots, T
\]

Suppose that the subutility function can be captured by the following function, which is regularly applied in empirical analysis (Rödsseth, 1997):

\[
u(c_t) = \frac{-\sigma}{(1 - \sigma)} e^\left(\frac{(1 - \sigma)}{\sigma} \cdot \frac{c_t}{(1 + \delta)}\right)
\]

For simplicity, we have now ignored the possibility of taste-shifting variables that may affect the desirability of consumption at different points in the life cycle. Furthermore, we have assumed that the intertemporal elasticity of substitution is a deep parameter that is constant over time.

The first order conditions can then be rewritten as:

\[
c_t (1 + \delta)^{-t} = \lambda(1 + \gamma)^{t-1}, \forall t
\]

The intertemporal choice of consumption between, say, period 1 and 2 is then determined by:

\[
c_1 = c_1 \frac{(1 + \delta)}{(1 + \delta)}
\]

That is, the preferred time-pattern of consumption depends on the real interest rate, the rate of time preferences, and the intertemporal elasticity of substitution.

It is instructive to distinguish in terms of whether \( \sigma \) is strictly positive or not. Suppose \( \sigma \) is strictly positive. Then, consumption will be growing or declining over time according to whether the real interest rate is greater or less that the rate of time preferences. For example, growing consumption implies that the impatience of the individual in terms of current consumption is dominated by the effect from the price differential between consumption today and tomorrow. Provided that the interest rates differ from the rate of time consumption, the greater \( \sigma \) is the steeper is the profile of the time-pattern of consumption, i.e. the more willing is the individual to substitute between consumption in different periods. Specifically, when \( \sigma \) exceeds 1 the growth or decline in consumption will exceed the difference between real interest rates and the rate of time preferences. Provided that \( \sigma \) is strictly positive the individual will prefer constant consumption if, and only if, the real interest rates for each period is equal to the rate of time preferences. If the rate of time preferences is supposed to be a deep parameter, as it often is claimed, it seems hard to make such an argument for constant consumption as

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13 Alternatively, we could, for example, use a utility function of the kind \( u_t(c_t) = (1 + \delta)^{1-t}v(c_t, z) \), where \( z \) are taste-shifting variables that may affect the desirability of consumption at different points in the life cycle while \( \delta \) is the rate of time preferences.
real interest rates are fluctuating heavily over time. In particular, it seems dubious that individuals facing relatively high interest rates on borrowing should have a similarly high rate of time preferences, and therefore prefer constant consumption.

Suppose that \( \sigma \) is zero. When \( \sigma \) tends to zero the marginal utility of consumption becomes very sensitive to small changes in consumption, i.e. individuals are becoming unwilling to change their consumption much to take advantage of intertemporal incentives since the profile of the marginal utility can be matched to changed interest rates with little change in consumption. If \( \sigma \) is zero there is no intertemporal substitution of consumption between the periods. Specifically, each individual is assumed to make intertemporal income transfers according to a maximin rule of the kind:

\[
\text{Carry out intertemporal transfers of income by borrowing and saving at the prevailing interest rates to maximise the minimum level of annual consumption that is achievable over the given sequence of years}
\]

For example, if the individual has a dollar more in period 2 compared to 1, he would transfer this dollar even if he only received a fraction of an additional cent in period 1. Notice that this rule is not even of a leximin type; the individual is not concerned with the second-lowest annual consumption level. That is, the individual is assumed to prefer to make intertemporal income transfer independent of the price he has to pay in terms of interest rate payments as long as such transfer do not reduce his maximum annual sustainable consumption. Translated to the notion of utilities, each individual is assumed to have a utility function that is neither continuously differentiable nor strictly quasiconcave, rather a Leontief utility function where the periods are viewed as perfect complements and the utility is equal to the minimum consumption level over the sequence of years. Arguably, it is far from straightforward to see that a meaningful picture of poverty can be derived from using a focal variable with such a theoretical basis.

Above, we have argued that neither of the preference structures that are compatible with using maximum sustainable annual consumption expenditure as the focal variable for identifying the chronically poor is obviously reasonable. Arguably, the focal variable of Rodgers and Rodgers lacks the theoretical basis of intertemporal optimisation that it claims to have. While life-cycle consumption theories suppose that individuals can, if they prefer, make inter-year income transfers, Rodgers and Rodgers insists that they strictly prefer to make such equalising income transfers.

### 4. A method for measuring chronic and transient poverty based on a general preference structure for intertemporal choice of consumption

As a way of reducing the measurement problem of fluctuating annual income, and to obtain a reliable estimate of the economic resources available for consumption and saving to distinguish the chronically poor from the non-poor, it is necessary to extend the accounting period of income. In this regard, it is critical how incomes accumulated over different periods be made comparable for the purpose of poverty analysis. In the previous section, we argued that although the existing methods, especially that of Rodgers and Rodgers (1993), may be considered as important improvements compared to the standard approach by moving the focus away from the time-pattern of annual income, each suffer from its own weakness. Specifically, the proposed methods fail, when comparing incomes accumulated over a sequence of years, to provide a satisfactory way of accounting for the benefits of receiving income early rather than late. As an attempt to overcome the weaknesses of the present methods, we will in this section suggest an alternative approach for identifying the chronically poor, for which we attempt to provide a coherent justification for the set of underlying assumptions. In fact, the method proposed in this paper encompasses the present approaches suggested for identifying the chronically poor based on an extended accounting period of income.

In this section, we propose a method for making incomes accumulated over different periods comparable that, unlike Rodgers and Rodgers (1993), is compatible with a more general preference structure for the intertemporal choice of consumption. Specifically, our choice of focal variable is a response to the question:
What is the minimum expenditure level that the individual would need in each period to be as well off as he could be by making intertemporal transfers of income at the prevailing interest rates on borrowing and saving to achieve his preferred consumption profile?

In order to derive this focal variable, we will outline a two-step procedure:

1. Determine the optimal consumption profile \((c_1^*, c_2^*)\) and the corresponding utility level \(\hat{U}\)
2. Calculate the minimum expenditure level \((V)\) that the individual would need in each period to obtain the utility level \(\hat{U}\)

Consider again the first order conditions derived from the intertemporal optimisation problem outlined above:

\[ c_i (1 + \delta)^{1-\sigma} = \lambda (1 + r_i)^{1-\sigma}, \forall_i \]

From these first order conditions the intertemporal choice of consumption between, say, period 1 and 2 can be determined by:

\[ c_1^* = c_2^* \left(1 + \frac{\delta}{r_2} \right) \]

From the intertemporal budget constrain we can determine the preferred consumption profile \((c_1^*, c_2^*)\):

\[ y_1 (1,0_{r_2}) + y_2 = c_2 (1,0_{r_2}) \left(1 + \frac{\delta}{1 + r_2} \right)^{\sigma}, \] \[ + c_2^* \Leftrightarrow c_2^* = \frac{(1,0_{r_2} \times y_1) + y_2}{\left(1 + \left(1 + \frac{\delta}{1 + r_2} \right) \right) (1,0_{r_2}) + 1} \]

\[ c_1^* = \frac{(1,0_{r_2} \times y_1) + y_2}{(1 + r_2) + \left(1 + \frac{\delta}{1 + r_2} \right)^{\sigma}} \]

Only when \(\sigma = 0\) or \(r_2 = \delta\) will the individual prefer to make intertemporal income transfers to smooth consumption perfectly. Otherwise, the preferred consumption profile will differ from the constant consumption profile underlying the choice of maximum sustainable annual consumption expenditure as the focal variable for analysis of poverty.

After calculating the optimal consumption profile, we derive the corresponding utility level \(\hat{U}\) by inserting the consumption levels into the utility function:

\[ U(c_1^*, c_2^*) = u(c_1^*) + u(c_2^*) = -\frac{\sigma}{(1-\sigma)} c_1^* - \frac{\sigma}{(1-\sigma)(1+\delta)} c_2^* = \hat{U} \]

Then, we are able to determine the \(V\) simply inverting the indirect utility function:\(^{14}\)

\[ \hat{U} = -\frac{\sigma}{(1-\sigma)} V^{(\frac{1-\sigma}{\sigma})} - \frac{\sigma}{(1-\sigma)(1+\delta)} V^{(-\frac{1-\sigma}{\sigma})} \Leftrightarrow V = \left[ -\frac{\sigma}{(1-\sigma)} \frac{\hat{U}}{\left(1 - \frac{\sigma}{1 - (1-\sigma)(1+\delta)} \right)} \right] \]

By following the approach outlined above, we can calculate the minimum expenditure level that the individual would need in each period to be as well off as he could be by making intertemporal transfers of income at the prevailing interest rates on borrowing and saving to achieve his preferred consumption profile.

Notice that the method for identifying the chronically poor outlined above encompasses the present approaches suggested for identifying the chronically poor based on an extended accounting period of

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(^{14}\) In the static consumption theory framework, the inverted of the indirect utility function is often referred to as the money metric utility function (McKenzie, 1957, Samuelson, 1974).
income. Firstly, the analysis of chronic poverty where the real incomes gathered over a set of consecutive calendar years simply are added up is a special case of the method for identifying the chronically poor proposed in this paper, in which both the real interest rate and the time preference rate are zero. Secondly, the method suggested in Rodgers and Rodgers (1993) is a special case in which the real interest rates are equal to the time preference rates. If the real interest rates differ from the time preference rates the minimum expenditure level will exceed the maximum sustainable annual consumption expenditure; how much will depend on the deviation between the rate of time preferences and the real interest rates as well as the elasticity of substitution. The reason why the minimum expenditure level will exceed the maximum sustainable annual consumption expenditure is that the method for deriving the minimum expenditure level is, unlike that of the maximum sustainable annual consumption expenditure, based on a preference structure flexible enough to allow individuals to adjust their consumption profile to take advantage of the intertemporal incentives provided by the real interest rates. For example, consider an individual facing a binding borrowing constraint. In the simple two-period case, this implies that the maximum sustainable annual consumption expenditure is determined solely by the relatively low income in the first period and is, in fact, unaffected by the size of the income in the second period. Therefore, one may argue that the method proposed in Rodgers and Rodgers (1993) will, in the case of binding liquidity constraints, exhibit the same contentious feature as cross-section studies of poverty suffer from, namely that a bad year does not render an otherwise well-off person poor. In comparison, the minimum expenditure level of an individual facing binding a borrowing constraint will not be determined solely by the income level in the first period; although he is not as well off as he would if there were no binding liquidity constraints, he will still receive some utility from the income in the second period.

4.1 Allowing the interest rates of borrowing to differ from those of saving

Above, we made the simplistic but unrealistic assumption that the interest rates on saving and borrowing are equal. Below, we will relax this assumption. To be specific, we will assume that the interest rate on borrowing (rb) is greater than the interest rate on saving (rs). In the simple two-period case, which may straightforwardly be generalised to any finite number of periods, this lead to a piecewise-linear and convex budget set with two segments. That is, the budget set is convex, while the budget constraint is concave, as illustrated in Figure 1.

In the absence of non-convexity and with a piece-wise linear budget constraint, the optimal consumption profile may be derived by computing a conditional consumption profile for each of the linear segments- the term conditional is used because it describes the choice of consumption conditional on having chosen a segment or a kind on which to locate. In Figure 1, the individual has three options: to locate on segment 1, on segment 2, or at the kink. The unconditional consumption profile describes the choice of consumption in the two periods conditional on choice of segment or kink, and the choice of segment or kink itself. The income and substitution effects will then depend on whether individuals stay or remain at the segments or kink. Small changes in interest rates and income may, therefore, affect the consumption profile through adjustments within the given segment and/or by changing the choice of segment and kink, generating consumption functions whose expansion paths for interest rates and incomes themselves have kinks (points of non-differentiability).
Figure 1. Piecewise-linear and convex intertemporal budget set with two segments

Formally, we can apply the Kuhn-Tucker method to derive the individuals’ utilities of the set of consumption profiles for the segments and kink. Whether an individual is a saver or a borrower depends on his utility maximising choice subject to this piecewise-linear constraint, i.e. which segment or kink he chooses to locate on. In table 3, we illustrate this optimisation problem, as a two-step procedure, for an intertemporal additive utility function with subutility functions as previously described. After applying this two-step procedure for determining the preferred consumption profile when the interest rates for saving and borrowing differ, we can calculate $V$ in the same way as outlined above.

Table 3. Two-step procedure for determining the preferred consumption profile in the two-period case

<table>
<thead>
<tr>
<th>Step 1: Determining the conditional consumption profile</th>
<th>Prospective borrower: Optimisation problem for segment 2 and kink</th>
</tr>
</thead>
<tbody>
<tr>
<td>MaxU = $u(c_1) + \frac{u(c_2)}{1 + \delta}$</td>
<td>MaxU = $u(c_1) + \frac{u(c_2)}{1 + \delta}$</td>
</tr>
<tr>
<td>s.t. (i) $y_1 + \frac{y_2}{1 + rs} = c_1 + \frac{c_2}{1 + rs}$</td>
<td>s.t. (i) $y_1 + \frac{y_2}{1 + rb} = c_1 + \frac{c_2}{1 + rb}$</td>
</tr>
<tr>
<td>(iia) $c_1 \leq y_1$</td>
<td>(iia) $c_1 \geq y_1 \Leftrightarrow -c_1 \leq -y_1$</td>
</tr>
<tr>
<td>$L = u(c_1) + \frac{u(c_2)}{1 + rs} - \lambda(c_1 + \frac{c_2}{1 + rs} - y_1 + \frac{y_2}{1 + rs}) - \mu(c_1 - y_1)$</td>
<td>$L = u(c_1) + \frac{u(c_2)}{1 + rb} - \lambda(c_1 + \frac{c_2}{1 + rb} - y_1 + \frac{y_2}{1 + rb}) + \mu(c_1 - y_1)$</td>
</tr>
<tr>
<td>1.O.C.</td>
<td>1.O.C.</td>
</tr>
<tr>
<td>(1) $u'_c - \lambda - \mu = 0$</td>
<td>(1) $u'_c - \lambda + \mu = 0$</td>
</tr>
<tr>
<td>(2) $u'_c - \frac{\lambda}{1 + rs} = 0$</td>
<td>(2) $u'_c - \frac{\lambda}{1 + rb} = 0$</td>
</tr>
<tr>
<td>(3) $c_1 + \frac{c_2}{1 + rs} = y_1 + \frac{y_2}{1 + rs}$</td>
<td>(3) $c_1 + \frac{c_2}{1 + rb} = y_1 + \frac{y_2}{1 + rb}$</td>
</tr>
<tr>
<td>(4) $\mu(c_1 - y_1) = 0, (\mu = 0 \text{ if } c_1 &lt; y_1), \mu \geq 0$</td>
<td>(4) $\mu(y_1 - c_1) = 0, (\mu = 0 \text{ if } c_1 &lt; y_1), \mu \geq 0$</td>
</tr>
<tr>
<td>Subutility function: $u(c_i) = -\frac{\sigma}{(1 - \sigma)} c_i^{(1 - \sigma)} \sigma$, where $\sigma \geq 0$ is the intertemporal elasticity of substitution</td>
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</tr>
</tbody>
</table>

Suppose $\mu = 0$
From the 1.O.C. we get:
\[
\frac{c_1}{1 - \sigma} = \frac{c_2}{1 - \delta} \Rightarrow c_2 = \frac{c_1(1 + rs)}{1 + \delta}, \text{ which together with the intertemporal budget constraint and the complementary slackness condition give the consumption profile:}
\]
\[
c_1^* = \frac{y_1(1 + rs) + y_2}{(1 + rs) + \frac{1 + rs}{1 + \delta}} \leq y_1 \Rightarrow y_2 \leq y_1 \left(1 + \frac{rs}{\delta}\right)^\sigma
\]
\[
c_2^* = \frac{y_1(1 + rs) + y_2}{(1 + rs) \left(1 + \frac{rs}{\delta}\right)} + 1
\]

Suppose \( \mu > 0 \)

Then:
\( c_1^* = y_1 \) and \( c_2^* = y_2 \)

By comparing the utilities of the two candidates for optimum, we can determine the choice of consumption profile conditional on choice of segment 1 or kink.

\[
\frac{c_1}{1 - \sigma} = \frac{c_2}{1 - \delta} \Rightarrow c_2 = \frac{c_1(1 + rb)}{1 + \delta}, \text{ which together with the intertemporal budget constraint and the complementary slackness condition give the consumption profile:}
\]
\[
c_1^* = \frac{y_1(1 + rb) + y_2}{(1 + rb) + \frac{1 + rb}{1 + \delta}} \geq y_1 \Rightarrow y_2 \geq y_1 \left(1 + \frac{rb}{\delta}\right)^\sigma
\]
\[
c_2^* = \frac{y_1(1 + rb) + y_2}{(1 + rb) \left(1 + \frac{rb}{\delta}\right)} + 1
\]

Suppose \( \mu > 0 \)

Then:
\( c_1^* = y_1 \) and \( c_2^* = y_2 \)

By comparing the utilities of the two candidates for optimum, we can determine the choice of consumption profile conditional on choice of segment 2 or kink.

### Step 2: Determining the unconditional consumption profile

By comparing the utilities of the two conditional consumption profiles, we can determine the choice of unconditional consumption profile. Furthermore, we know whether the individual concerned is a saver or a borrower.

**Notation and assumptions:**
- \( c_t \) represents the consumption level in period \( t = 1, 2 \)
- The utility function \( U \) depends on the individual period subutility functions \( u_t \), which are increasing and concave in the single arguments, i.e. \( u'_t \geq 0, u''_t \leq 0 \) for all \( t = 1, 2 \) periods.
- \( y_t \) represents the real income level in period \( t = 1, 2 \)
- \( rb \) represents the interest rate for borrowing.
- \( rs \) represents the interest rate for saving.

### 5. The sensitivity of poverty analysis to the choice of method for measuring chronic and transient poverty

(Forthcoming)

### 6. Policy implications

(Forthcoming)

### 7. References


Friedman, M. (1963): "Windfalls, the horizon, and related concepts in the permanent income hypothesis", in Consumption and Savings, Vol. 2, (eds.) I. Friends and J. Jones


