A Theory of Child Targeting*

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Abstract

Designing policies to target specific individuals in the household, especially children, is an important concern among policy makers. This is illustrated very clearly by the recent orientations of the Labour Government in the UK, aiming at reducing child poverty through a variety of instruments. Despite a large empirical literature on this subject, there is surprisingly little theoretical foundation to analyze the optimality of different instruments for this purpose. In the present paper, we discuss the merit of targeting children through two general types of policies, namely selective commodity taxation and cash transfer to family with children. The Rotten Kid model of Becker (1991) provides a very simple and intuitive framework to study this question. We formally derive the conditions under which a subsidy placed on a particular good ameliorates child welfare, show that the subsidy can dominate cash-transfers for this purpose and characterize the optimal policy mix when both instruments are available. In contrast to traditional results, we also demonstrate that well-chosen subsidies can be more efficient than cash transfers in alleviating child poverty.

Key Words: commodity taxation, universal child benefit, targeting, intra-household distribution, social welfare.

Classification JEL : D13, D31, D63, H21, H31.

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1 Introduction

Many economic policies, in developing and developed countries alike, aim at reducing poverty among children.\footnote{For industrialized countries, see e.g. Bradbury et al. (2001, eds) or UNICEF (2005). The concern is especially acute in the US and the UK, as witnessed by the important empirical literature on the subject (see a survey and a US-UK comparison in Dickens and Ellwood, 2001). The concern is echoed in the recent orientations of the Labour Government in the UK, aiming at "eradicating child poverty" within 20 years (see e.g., Sutherland and Piachaud, 2001, Brewer et al., 2003).} One of the most natural approach to achieve this goal consists in making cash transfers to families with children. This raises issues of agency, however, because cash transfers are not made directly to the intended recipients (i.e., children), but instead, supplement the income of adults with the assumption that the standard of living of children will improve as well.\footnote{See e.g., Cox and Jakubson (1995), Alderman et al. (1997), Phipps (1999), Bingley and Walker (2000), Banks and Brewer (2002), Tabor (2002), Blow et al. (2004), Bhalotra (2004), for a discussion of this issue.} Even if adults and children are altruistically linked, decisions regarding children’s consumption are ultimately made by adults and the impact of cash transfers may be partially or totally neutralized by the intrahousehold redistribution process.

In response to a tightening of government finance in almost all countries, the emphasis on designing more efficient poverty-alleviation programs is central. From this perspective, several studies argue that subsidizing certain goods consumed by children may be more advantageous than cash transfers as far as the objective of reducing child poverty is concerned (see e.g. Barrientos and DeJong, 2004, Bhalotra, 2004). Clearly enough, reducing the price of child-specific goods may incite parents to purchase these goods and improve children’s welfare. Here too, however, the final impact on child welfare is unclear since subsidies on child-specific goods may be offset by a reduction of intrahousehold allocations to children.\footnote{Prominent examples are school lunch subsidies or free milk vouchers (see e.g. Bingley and Walker, 2000, Whitmore, 2002, or Currie, 2005).}

Despite the large literature on child poverty, there is surprisingly little theoretical grounds to assess alternative policies in the light of intra-household mechanisms. The present paper contributes filling the gap.\footnote{See Cigno and Pettini (2002) and Cigno et al. (2003) in a framework with fertility choices. See also Ray (1997) for a survey of optimal commodity taxation in the presence of children.} For this purpose, we consider a household, comprising an egoistic child and a benevolent adult which makes decisions about con-
sumption, and suppose that the Rotten Kid Theorem of Becker (1991) can be applied. The model turns out to be a very simple and intuitive framework to study our question. In particular, it helps understanding how variations in prices may independently affect the well-being of children and adults within the household, which is central to properly assess the potential of subsidies as a targeting device.

We examine in which circumstances price subsidies — simply defined as a reduction in the free market price of one good — will improve the standard of living of the child. We then compare the effectiveness of these subsidies to that of comparable cash transfers.\footnote{In the development literature, both instruments are often refer to as marginal and ‘infra-marginal’ subsidies respectively. The latter is given in the form of a ration of a certain good that is made available below the market price. If the ration can be sold again, infra-marginal subsidies are formally equivalent to cash transfers (equal to the ration quantity multiplied by the difference between market and subsidized prices).}

The child’s utility is maximized — taking into account the parent’s redistribution scheme — subject to the budget constraint of the social planner. The approach is actually inspired from the literature on targeting (Kanbur, 1987; Besley and Kanbur, 1988, 1993; Besley, 1990; Keen, 1992; Kanbur et Keen et Tuomala, 1994) but differs to the extent that the targeted person is located within a household (see Haddad and Kanbur, 1992, for an introduction to these issues).

Our main results can be summarized as follows. First, we formally derive the conditions under which a subsidy placed on a particular good ameliorates child welfare. This result is general in the sense that the subsidized good may be consumed by both the parent and the child. Still, a special case states that subsidizing a child-specific good will improve her situation if this good is superior. Second, we show that in this specific case the subsidy can prove to be better welfare-improving than cash transfers when these instruments are implemented on the same cost basis for the social planner. The optimal level of price subsidies is characterized. Third, we demonstrate that the reduction in child poverty may be greater using well-chosen subsidies rather than cash transfers. This result contrasts with the more traditional view according to which cash transfers are more efficient at alleviating poverty.

The paper is structured as follows. In Section 2, we describe the implications of the Rotten Kid Theorem on household demands and the intrahousehold distribution of resources. In Sections 3 and 4, we present the main theoretical results.
2 The Rotten Kid Model

2.1 Preferences and the Decision Process

In our framework, a household which comprises an adult \((a)\) and a child \((c)\) makes decisions about consumption. The vectors of goods consumed by the adult and the child are denoted by \(x_a\) and \(x_c\), respectively. The child is egoistic and her utility is given by:

\[
U_c = u_c(x_c),
\]

whereas the adult is altruistic in the sense of Becker, so that her utility can be written as:

\[
U_a = W[x_a, u_c(x_c)].
\]

(1)

The separability of the child’s consumption is accepted by the majority of economists, though not without reservations (Gronau, 1988), and is a crucial component of the welfare analysis that follows. To make things easier, we use a slightly more restrictive form and suppose that the utility function (1) is additively separable:\(^6\)

\[
U_a = u_a(x_a) + \rho u_c(x_c),
\]

(2)

where \(u_a(x_a)\) is the adult’s subutility function and \(\rho > 0\) represents the degree of the adult’s altruism.\(^7\) The functions \(u_a(x_a)\) and \(u_c(x_c)\) are strictly concave and differentiable in all their arguments. Under these assumptions, the Rotten Kid Theorem (Becker, 1991) posits that the household behaves as if the adult’s utility was maximized under the household budget constraint. Thus, the optimization program can be written as:

\[
\max_{x_a, x_c} \ u_a(x_a) + \rho u_c(x_c) \quad \text{s.t.} \quad p' (x_a + x_c) \leq Y
\]

(3)

where \(p\) is the vector of prices and \(Y\) is the household income. Then, the first order conditions are:

\[
\frac{\partial u_i}{\partial x_i} = \lambda_i p,
\]

where \(\lambda_i\) is member \(i\)’s marginal utility of income, with \(\lambda_a/\lambda_c = \rho\). Solving first order conditions, with the budget constraint, yields:

\[
x_i = \bar{x}_i (p, Y),
\]

(4)

\(^6\)This simplification does not fundamentally alter the main conclusions of the paper.

\(^7\)In this representation of household decision making, ordinal preferences of children are ‘translated’ into cardinal preferences by the decision maker (the parent) so that the interpretation in terms of altruism is all relative. The problem is not new (see Becker, 1991, and the discussion in Bhalotra, 2001).
where $\bar{x}_i$ denotes the vector of individual demands, and

$$ x = \bar{x}(p, Y) $$

where $\bar{x} = \bar{x}_a + \bar{x}_c$ denotes the vector of household demands. These functions are traditional Marshallian demands and thus satisfy the Slutsky condition. Since the optimization program is additively separable, they have other properties which will be examined in the next subsection.

Strictly speaking, the Rotten Kid Theorem relies on a set of restrictive assumptions which are not necessarily satisfied (Bergstrom, 1989). Nevertheless, the postulate that the household maximizes the utility index of the benevolent adult seems reasonable, since our objective is to investigate economic policies that aim at reducing poverty among young children. More fundamentally, we refer to the Rotten Kid Theorem for conveniency while the present framework is compatible with various representations such as the consensus model (Samuelson, 1956), the Pareto model (Apps and Rees, 1988) or the Rothbarth model (Gronau, 1991). It differs from the collective set-up (Chiappori, 1988), however, in that the weight of the child in (3) is constant, i.e. does not depend on total income or the price system.

### 2.2 Properties of Household Demands

The optimization program (3) is (additively) separable, hence, as is well known, the decision process can be decentralized (two-stage budgeting). Firstly, total income is divided between the adult and the child according to some rule; secondly, each of them independently chooses consumption subject to her own budget constraint. This interpretation will be useful in the following.

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8 The main underlying assumption is naturally that the child cannot influence household decisions. A more minor aspect is that parents behave here as a single decision maker. Alternatively, our analysis could be seen as restricted to lone parents, which is a relevant focus group with respect to child poverty. As a matter of fact, the change of family structure (and the trend away from marriage) seems one of the main responsible for the rise in child poverty in the US, as argued by Lerman (1996) and Bergmann (1994), who recommends increased aid and more appropriate measures for single mother families.

9 For instance, program (3) could be seen as the maximization of a household welfare function where $\rho$ is the bargaining power of the child. In Cigno et al. (2000), this weight corresponds to the number of children, which, in a Rotten Kid model, is interpreted as the parent’s utility increasing with the number of children but, in a bargaining model, could also reflects the bargaining weight of the group of children.
We shall first examine how the sharing rule is determined. To do so, let \( v_i(p, \phi_i) \) be the member \( i \)'s indirect subutility function where \( \phi_i \) is her share of income. The latter is the solution to:

\[
\max_{\phi_a, \phi_c} v_a(p, \phi_a) + \rho v_c(p, \phi_c) \quad \text{s.t.} \quad \phi_a + \phi_c = Y. \tag{6}
\]

The first order condition of this program is:

\[
\frac{\partial v_a}{\partial \phi_a} - \rho \frac{\partial v_c}{\partial \phi_c} = 0, \tag{7}
\]

and the second order condition is:

\[
\frac{\partial^2 v_a}{\partial \phi_a^2} + \rho \frac{\partial^2 v_c}{\partial \phi_c^2} < 0. \tag{8}
\]

Once total income is divided between the child and the adult, each of them maximizes her utility subject to her own budget constraint. That is,

\[
\max_{x_i} u_i(x_i) \quad \text{s.t.} \quad p' x_i \leq \phi_i. \tag{9}
\]

Hence, the individual demands are characterized by the following structure:

\[
x_i = \hat{x}_i(p, \phi_i),
\]

where \( \hat{x}_i \) is a vector of Marshallian demands, expressed as functions of \( p \) and \( \phi_i \). To simplify notation, let \( \phi = \phi_c \) and \( Y - \phi = \phi_a \), where \( \phi(p, Y) \) is the ‘sharing rule’. This function has properties that we examine below.

**The effect of total income on the sharing rule.** We differentiate the first order condition (7) with respect to \( Y \) and obtain:

\[
\frac{\partial^2 v_a}{\partial \phi_a^2} \left( 1 - \frac{\partial \phi}{\partial Y} \right) - \rho \frac{\partial^2 v_c}{\partial \phi_c^2} \frac{\partial \phi}{\partial Y} = 0.
\]

We use the following notation:

\[
\frac{\partial v_i}{\partial \phi_i} = \lambda_i \,(> 0), \quad \frac{\partial^2 v_i}{\partial \phi_i^2} = \lambda'_i \,(< 0),
\]

noticing that \(-\lambda'_i/\lambda_i\) is a measure of concavity of member \( i \)'s subutility. This measure will hereafter be referred to as the (absolute) redistribution aversion of member \( i \).\(^{10}\) Then, after some manipulations, the effect of income on the sharing rule can be written as:

\[
\frac{\partial \phi}{\partial Y} = \frac{\lambda'_a}{\lambda'_a + \rho \lambda'_c} = -\frac{\lambda'_a/\lambda_a}{\theta} \tag{10}
\]

\(^{10}\) This terminology is a little misleading because the value of \(-\lambda'_i/\lambda_i\) is not necessarily a characteristic of the child preferences (see remark in footnote 7).
where:

\[
\theta = - \left( \frac{\lambda'_a}{\lambda_a} + \frac{\lambda'_c}{\lambda_c} \right) > 0.
\]

According to the last term in expression (10), the share of each additional unit of total income accruing to the child is equal to the ratio of the adult’s aversion to the sum of both members’ aversion. That is to say, the increment of income will accrue more importantly to the household member located on the least curve portion of her utility function. Note also that the effect of total income on the sharing rule is clearly comprised between zero and one: the expenditure devoted to the child is a normal good for the adult. This is hardly controversial.11

The sum of both members’ redistribution aversions, denoted \( \theta \) above, has an attractive interpretation in terms of complementarity: it corresponds to the derivative of the benevolent parent’s marginal rate of substitution between the child’s allocation \( \phi \) and the parent’s allocation \( Y - \phi \) with respect to the relative implicit prices of these allocations (i.e. \( \frac{\partial^2 \phi}{\partial x c} \) when total household utility is hold constant). In other words, it measures the convexity of the preferences of the benevolent parent regarding allocations \( \phi \) and \( Y - \phi \). If the latter tend to be perfect substitutes (complements), the indifference curve of the altruistic parent tends to be linear (right-angled) and \( \theta \) tends to zero (\( \theta \) tends to infinity). The term \( \theta \) will thus be referred to as an index of complementarity hereafter.

The effect of prices on the sharing rule. We differentiate the first order condition (7) with respect to \( p_j \) and obtain:

\[
\left( \frac{\partial^2 v_a}{\partial \phi_a \partial p_j} - \frac{\partial^2 v_a}{\partial \phi_a^2} \right) \frac{\partial \phi}{\partial p_j} = \rho \left( \frac{\partial^2 v_c}{\partial \phi_c \partial p_j} + \frac{\partial^2 v_c}{\partial \phi_c^2} \right) = 0.
\]

Now, from Roy’s identity, we have:

\[
\frac{\partial v_i}{\partial p_j} = -\lambda_i x_{ji}, \quad \frac{\partial^2 v_i}{\partial p_j \partial \phi_i} = -\lambda' x_{ji} - \lambda_i \frac{\partial \hat{x}_{ji}}{\partial \phi_i}.
\]

Using these expressions in equation (11) leads to:

\[
\frac{\partial \phi}{\partial p_j} = -\frac{1}{\theta} \left[ \left( \frac{\lambda'_c}{\lambda_c} x_{jc} - \frac{\lambda'_a}{\lambda_a} x_{ja} \right) + \left( \frac{\partial \hat{x}_{jc}}{\partial \phi_c} - \frac{\partial \hat{x}_{ja}}{\partial \phi_a} \right) \right].
\]

11 This property of normality, however, is not general but comes from the additivity property of the adult’s utility function.
This expression may be either positive or negative. Remark that the size of the complementarity index tempers the impact of variations in prices on the sharing rule, and incidentally limits the possibility for the social planner to interfere with the intrahousehold allocation process. Now, using expression (10), we obtain:

\[
\frac{\partial \phi}{\partial p_j} = \left(1 - \frac{\partial \phi}{\partial Y} \right) x_{jc} - \frac{1}{\theta} \left( \frac{\partial \hat{x}_{jc}}{\partial \phi_c} - \frac{\partial \hat{x}_{ja}}{\partial \phi_a} \right).
\]  

(13)

The interpretation of this expression is quite complicated. Also, we consider the simple—and empirically relevant—case where good \( j \) is exclusively consumed by the child. Then, we have:

\[
\frac{\partial \phi}{\partial p_j} = x_j - \frac{1}{\theta} \frac{\partial \hat{x}_{jc}}{\partial \phi_c} - \frac{\partial \phi}{\partial Y} x_j.
\]

(14)

There are three terms in the right hand side of this expression. The last term is a ‘conventional’ income effect: the child endowment decreases because the real income of the household is reduced by the price rise. The sum of the first two terms corresponds to a compensated effect, i.e., the change in the child’s share resulting from a simultaneous variation in the price \( p_j \) and in total income \( Y \) such that the adult’s utility remains constant. The first term corresponds to the direct increase in the share that allows the child to purchase the same bundle of goods; the second effect—close to a substitution effect—is the change in the child expenses due to the price change; differentiating the definition of the child’s share \( \phi = p' \cdot \bar{x}_c \), using the derivative (14) and the Slutsky decomposition, we can indeed show that:

\[
\frac{1}{\theta} \frac{\partial \hat{x}_{jc}}{\partial \phi_c} = -p' \frac{\partial \bar{x}_c}{\partial p_j} \bigg|_{dU_a=0}.
\]

(15)

In the present context, this combination of substitution effects can be written as a function of the slopes of the Engel curves because the adult’s utility function is separable in the child’s and adult’s consumptions (Deaton and Muellbauer, 1980).

The sum of the first and last terms is clearly positive. However, the sign of (15) is generally undetermined so that the derivative \( \partial \phi/\partial p_j \) can be either positive or negative. In the particular case where \( x_j \) is an inferior good, we have \( \partial \phi/\partial p_j > 0 \): an increase in the price of the exclusive good is compensated by an increase in the share of the member who consumes this good. Yet, even in the general case where \( x_j \) is a normal good, the derivative \( \partial \phi/\partial p_j \) will be positive if the second term is small enough due to a large complementarity index \( \theta \) in the household. This may appear fairly natural that the income accruing to
the child does not decrease as the price of toys increases. In that case, a price subsidy on a child-specific good would contribute to decrease the child endowment so that the final effect of such policy on child welfare is a priori ambiguous. But this also suggests that, empirically, the complementarity index may be quite large so that the substitution between $\phi$ and $Y - \phi$ could be limited.

### 2.3 Example: The Complementarity Index

To illustrate the properties of the sharing rule and emphasize the role of the complementarity index, we shall consider the case where the adult’s and the child’s utility functions are of the CARA form: $u_i = -\exp(-\alpha_i(p) \cdot \phi_i)$, where $\alpha_i(p)^{-1}$ is a linearly homogeneous function of prices $p$. To begin with, the adult’s and the child’s Engel curves are easily computed using the Roy’s identity and are written as:

\[
\begin{align*}
    x_{aj} &= -\left(\frac{\partial \alpha_a/\partial p_j}{\alpha_a}\right) \phi_a, \\
    x_{cj} &= -\left(\frac{\partial \alpha_c/\partial p_j}{\alpha_c}\right) \phi_c.
\end{align*}
\]
i.e., the impact of the share of total income is linear. Similarly, the child’s share of income is linear and can be written as:

$$\phi_c = \frac{\alpha_a}{\theta} \cdot Y + \frac{\ln \alpha_c + \ln \rho - \ln \alpha_a}{\theta};$$

thus, the parameter of adult’s altruism influences the intercept of the sharing rule. In the CARA specification, the individual measures of redistribution aversion are constant and corresponds to $\alpha_i$, so that the complementarity index $\theta$ is simply equal to $\alpha_a + \alpha_c$. Computed at the equilibrium, this index coincides with the value of the second order derivative of the adult’s indifference curve. Clearly enough, however, the value of the complementarity index is not invariant to changes in measurement units; hence its interpretation is complicated. To take a numerical example, let us suppose that $Y = \mathcal{L}20,000$, $\alpha_a = \alpha_c$ and $\rho = 0$, so that each member receives $\mathcal{L}10,000$. The indifference curves for various values of the complementarity index ($\theta = 0, \theta = 0.0001, \theta = 0.001, \theta = \infty$) are drawn in Figure 1. We observe that the equilibrium is defined by the point where the first derivative of the indifference curve is equal to $-1$ and, at this very point, the second derivative of the indifference curve is equal to $\theta$. Moreover, a value of $\theta$ greater than 0.001 represents a considerable degree of complementarity.

3 Targeting Child Welfare

In the present context, the targeting problem for the social planner can be formalized as one of maximizing the well-being of the child, represented by $u_c(x_c)$, subject to a budgetary constraint. In doing so, we implicitly suppose that (i) the child utility function incorporated in the benevolent parent’s decision process (3) truly reflects child welfare/preferences and (ii) the social planner’s anticipation of the child welfare coincides with that of the adult, i.e. child consumption is not treated as merit goods.\(^\text{13}\)

\(^{12}\)The fact that the complementarity index is independent of total income, even if convenient in the present example, is not necessarily realistic.

\(^{13}\)For interesting developments on welfare-improving price distortions and the taxation of merit goods, see Geoffard (2002). Note that pro-poor policies actually seem to focus quite often on merit goods. For instance, recent programs in Latin America have made cash transfers to poor families conditional upon their sending their children to school and health clinic (e.g. Janvry et al., 2004). This is not necessarily due to a supposedly lack of altruism from the parents but indeed to dissonance between parents and the State upon what is best for the child (see discussions in Bhalotra, 2004, or Becker, 1999). Similar justifications apply to in-kind benefits such as food stamps (see Whitmore, 2002).
Maximizing simply child welfare is naturally a polar case but it conforms to the objective of targeting child welfare and allows to derive simple results. A social function \( u_a(x_a) + \rho' u_c(x_c) \) with \( \rho' > \rho \) would entail that parents are not enough altruistic from the State’s point of view. Instead, we simply derive rules in a framework where child is the unit of concern and parents’ preferences are the agency problem.

3.1 The Marginal Impact of Subsidies

The first question is then to determine whether a marginal increment of the subsidy actually improves the child’s welfare and if so, how large is the improvement. To do so, we assume that each unity of good \( j \) is subsidized at a constant rate. Hence the price of good \( j \) is written as \( p_j = q_j - s_j \) where \( q_j \) is the price in the no-subsidy situation and \( s_j \) is the subsidy. Thus, if we differentiate \( v_c(p, \phi(p, Y)) \) with respect to \( s_j \), we obtain:

\[
\frac{\partial U_c}{\partial s_j} = -\left( \frac{\partial v_c}{\partial p_j} + \frac{\partial v_c}{\partial \phi} \frac{\partial \phi}{\partial p_j} \right).
\]

Now, substituting expression (13) for the derivative of the sharing rule, using expression (12) and simplifying give the marginal impact of the subsidy on welfare:

\[
\frac{\partial U_c}{\partial s_j} = \lambda_c \left[ x_j \frac{\partial \phi}{\partial Y} + R_j \right]
\]

where \( R_j = \frac{1}{\theta} \left( \frac{\partial \hat{x}_{jc}}{\partial \phi_c} - \frac{\partial \hat{x}_{ja}}{\partial \phi_a} \right) \).

The multiplicative term \( \lambda_c \) is the marginal utility of the household expenditures for the child (it depends on the normalization chosen for the representation of the child’s utility) whereas the whole expression in square brackets is a monetary measure of the variation in utility. The first term in this expression is positive. This is the traditional income effect: if the social planner introduces a subsidy on good \( j \), the household real income will increase by \( x_j \) and the child’s share will increase by \( x_j \) times \( \partial \phi/\partial Y \). The second term, \( R_j \), will be referred to as the redistributive or targeting effect of the subsidy. It is equal to the inverse of the complementarity index multiplied by the difference in the slopes of the Engel curves. Specifically, the subsidy will be very effective if the complementarity index is small – i.e. there is some substitutability between the child’s and the adult’s welfares – and if the slope of the Engel curve of the child is very steep in comparison with that of the adult (and, in particular, if good \( j \) is exclusively consumed by the child). On the contrary, the subsidy may have a negative effect on the child’s welfare if \( \partial \hat{x}_{jc}/\partial \phi_c < \partial \hat{x}_{ja}/\partial \phi_a \).
The effectiveness of the policy also depends on its cost. Therefore, the marginal impact of the subsidy per unit spent by the social planner must be computed as well. If the total cost of the subsidy is noted $B = s_j x_j$, the marginal cost of an increase in the subsidy is given by:

$$\frac{\partial B}{\partial s_j} = x_j - s_j \frac{\partial x_j}{\partial p_j}. \quad (18)$$

This expression is supposed to be positive. Thus, the marginal impact in terms of child’s welfare per unit spent by the social planner is given by:

$$\frac{\partial U_c}{\partial B} = \lambda_c \left[ x_j \frac{\partial \phi}{\partial Y} + R_j \right] \left( x_j - s_j \frac{\partial x_j}{\partial p_j} \right)^{-1}. \quad (19)$$

To gain insight into this rule, however, we compute the marginal increase in the child’s utility per budget unit at the no-subsidy situation, i.e., $s_j = 0$, and obtain:

$$\frac{\partial U_c}{\partial B} \bigg|_{s_j=0} = \lambda_c \left[ \frac{\partial \phi}{\partial Y} + \frac{R_j}{x_j} \right]. \quad (20)$$

Following this expression, the optimal rule for the practitioner who wants to target children’s welfare is simple: the subsidy must be put on the good for which the difference between the slope of the Engel curve of the child and that of the adult divided by the level of consumption – referred to as the targeting effect per unit consumed – is the largest.

To implement these results, the social planner must have information about the slopes of members’ Engel curves. However, the identification of these curves for goods consumed by both the child and the adult – even if not impossible – is quite complicated. Hence, to make things easier, we shall consider subsidizing goods which are exclusively consumed by the child. This is a very desirable situation, since in this case, the subsidy will necessarily have a positive targeting effect (at least if the exclusive goods are normal).

Thus, if good $j$ is specific to children, the effect of total income on demand can be written as:

$$\frac{\partial x_j}{\partial Y} = \frac{\partial \tilde{x}_{jc}}{\partial \phi_c} \frac{\partial \phi}{\partial Y}. \quad (21)$$

and thus:

$$\frac{\partial \tilde{x}_{jc}}{\partial \phi_c} = \frac{\partial \tilde{x}_j}{\partial Y} \left/ \frac{\partial \phi}{\partial Y} \right.. \quad (22)$$

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14 This is also empirically relevant, since some countries use subsidies on child-specific goods to influence the intrahousehold allocation of resources. One of the most prominent examples is the absence of VAT on children’s clothing and footwear in the United-Kingdom (see e.g. Ward-Batts, 2003).
This preliminary information is useful. Indeed, let us assume that goods \( j \) and \( k \) are exclusively consumed by the child. Then, if we substitute the value (22) for the slope of the Engel curves in the marginal impact (20), we demonstrate that a small subsidy, starting from the no-subsidy situation, will be more effective on good \( j \) than on good \( k \) if and only if

\[
\frac{Y \partial \bar{x}_j}{x_j \partial Y} > \frac{Y \partial \bar{x}_k}{x_k \partial Y},
\]

i.e., the income elasticity of good \( j \) is greater than that of good \( k \). The social planner is thus able to select empirically the ‘child-specific’ good for which the subsidy will have the largest effect on child welfare, namely superior/luxury goods.

This simple and intuitive result applies whatever the degree of altruism of the parents. But the nature of the targeted goods may change with the degree of altruism. Altruistic parents for whom ‘basic’ child goods (food, clothes) are necessities should receive subsidies placed on child superior goods (books, toys, computer) if the planner wants to further increase child welfare. In contrast, basic child goods may be superior goods for non-altruistic parents. This is ultimately an empirical question and we suggest a simple rule of thumb for implementation in the concluding section.\(^{15}\)

### 3.2 The Marginal Impact of Transfers: A Comparison

We now suppose that the social planner has the possibility to use lump-sum transfers – such as universal child benefits – as an additional instrument. The advantage of such transfers over price subsidies is that the social planner can target the subsidy to specific goods. For example, if the planner wants to increase child welfare, they can focus on goods that have a higher income elasticity, such as superior/luxury goods.

\(^{15}\)Concerning the recent pro-child reforms in the UK, Gregg et al. (2005) show that low-income families were spending a higher share of their budget on children’s clothing and footwear and a lesser on adults’ clothing before reform. Banks and Brewer (2002) also survey recent evidence on the fact that money spent on children in low-income families is not much lower than in (slightly) higher-income household since parents tend to make sacrifices to provide their children with essentials. In this case, increasing cash transfers toward these families may end up improving the situation of parents more than that of children – a justification in favor of complementing child benefits with in-kind benefits and public services directed to children. In our framework, the sharing rule \( \frac{\partial \phi}{\partial Y} \) applied to additional income would then be small enough to limit the effect of increased child benefits, making the point in favor of price subsidies on child basic goods. As a matter of fact, Blow et al. (2004) find that child benefits would be disproportionately spent on alcohol and women’s clothing and interpret this as evidence that altruistic parents insure children against shocks by spending constant amounts on children goods while unexpected fluctuations in child benefits are used to consume adult goods. Other studies constrast these results: Gregg et al. (2005) find that recent increase in child benefits has lead to faster increase in child-related items (clothes, books) than other expenses, and in particular than vice goods.
transfers, in comparison with subsidies, is that they do not generate distortions: the cost of the policy for the social planner exactly coincides with the benefit for the household (as a whole).\textsuperscript{16} We thus question whether the increment in the price subsidy is dominated by a similar increment in a lump-sum transfer, when the social objective is to improve child welfare. To examine this issue, the household income is written as \( Y = Z + T \) where \( Z \) is the income in the no-transfer situation and \( T \) is the lump-sum transfer. This way, the marginal impact of the direct transfer on the child’s welfare is simply equal to:

\[
\frac{\partial U_c}{\partial T} = \lambda_c \frac{\partial \phi}{\partial Y}.
\]

The effectiveness of the lump-sum transfer is thus reduced by the fact that the derivative of the sharing rule is less than one.\textsuperscript{17} Then, we can establish the difference between the welfare improvement of the subsidy and that of the cash transfer for a comparable marginal increase in public budget:

\[
\Delta = \frac{\partial U_c}{\partial B} - \frac{\partial U_c}{\partial T}.
\]

Using expressions (19) and (23), we can easily show that the marginal impact of the subsidy is larger than that of the cash transfer if the following condition is satisfied:

\[
\frac{\partial \phi}{\partial Y} \frac{\partial \bar{x}_j}{\partial p_j} s_j + R_j > 0.
\]

In this expression, the first term is the dead weight loss of the subsidy (i.e., the difference between the effect of a marginal increase in the subsidy on the household real income, 

\textsuperscript{16}We oppose here two standard instruments, as done for instance in Besley and Kanbur (1988). Naturally, in practice, many other policies are suggested to fight child poverty, either directly (in-kind benefits, means-tested child benefits, tax allowance or credit, increments to social or housing benefits due to the presence of children), through measures improving parents’ employment (in-work policies, increased child-care facilities, etc.) or indirectly through educational and health policies. Studies have often focused on the relative merit of cash transfers and in-kind benefits (see Currie, 2005), including vouchers and food stamps (see e.g. Whitmore, 2002, Bingley and Walker, 2000).

\textsuperscript{17}We ignore specific aspects often related to unconditional child transfers like the potential labeling effect. Empirical results are mixed, some studies finding that child benefits are disproportionately spent on child goods (Kooreman, 2000, Madden, 1999, Grogan, 2004) while other do not (Edmonds, 2002). Another aspect is naturally the question of who receives the transfer in the household. In particular, there is some evidence that “wallet to purse” reforms would improve the situation of children (see Lundberg et al., 1997, and Ward-Batts, 2003; Blow et al. (2004, p.6) also argue that results in Kooreman (2000) are driven by an underlying wallet to purse effect). This finding is not generalized within affluent countries (Bradbury, 2003). Evidences are probably stronger in the developing world (see discussion in Phipps, 1999).
$x_j$, and its effect on the budget of the social planner, $x_j - s_j \frac{\partial \bar{x}_j}{\partial p_j}$ multiplied by the derivative of the sharing rule, i.e. the proportion of the dead weight loss supported by the child. This distortionary effect is negative, except if good $j$ is a Giffen good. It is counteracted by the second term, the targeting effect. As seen before, the sign of the targeting effect depend on the difference between the child’s and the adult’s Engel curves.\(^{18}\) In the special case where the subsidy is equal to zero, the marginal dead weight loss vanishes (Auerbach and Hines, 2000) so that the marginal increase in the subsidy has purely a targeting effect.

The introduction of a subsidy will be more favorable to children than that of a cash transfer up to the point where expression (24) is equal to zero, which defines an upper threshold for the subsidy:

$$s^*_j = -\left( \frac{\partial \phi}{\partial Y} \frac{\partial \bar{x}_j}{\partial p_j} \right)^{-1} R_j.$$  \hspace{1cm} (25)

The subsidy will be more effective than the transfer as long as $s_j < s^*_j$.

### 3.3 Optimal Subsidy Rates

We now suppose that the social planner can subsidize an arbitrary number of goods, which are given \textit{a priori}. The objective, as previously, is to maximize child welfare with a fixed budget constraint. To achieve this goal, the optimal level of each subsidy (or possibly tax in the present case) must be chosen. However, the computation of optimal subsidies does not provide analytical solutions and the interpretation of the first order conditions of the optimization program of the social planner is intricate. Hence, to make things easier, we suppose that cross-derivatives of individual demands are equal to zero. This assumption is usual in optimal taxation problems.\(^{19}\) In that case, the first order condition for each subsidized good $j$ becomes:

$$\lambda_c \left[ x_j \frac{\partial \phi}{\partial Y} + R_j \right] \left( x_j - \frac{\partial \bar{x}_j}{\partial p_j} s_j \right)^{-1} = \mu,$$

\(^{18}\)If the individual subutility functions are of the Gorman Polar form with the same Engel curves, there is no opportunity of targeting: the effect of a subsidy will be, in the best circumstances, equivalent to the effect of a cash transfer. This is reminiscent of a result by Deaton (1979).

\(^{19}\)The assumption that cross-derivatives are equal to zero is not a big impediment since we may only have a limited number of subsidized goods and choose them sufficiently unrelated so that the assumption is plausible.
where $\mu > 0$ is the Lagrange multiplier of the planner’s budget constraint. This condition naturally means that the marginal impact of the subsidy per unit spent by the social planner must be the same for each subsidized good. If this equation is solved with respect to $s_j$, we obtain a simple expression for the subsidy rate:

$$\frac{s_j^0}{p_j} = -\varepsilon_{jj}^{-1} \left[ \frac{\lambda_c}{\mu} \left( \frac{\partial \phi}{\partial Y} + \frac{R_j}{x_j} \right) - 1 \right]$$

(26)

where

$$\varepsilon_{jj} = \frac{p_j \partial \bar{x}_j}{x_j \partial p_j} < 0$$

is the price-elasticity of the demand for good $j$. Therefore, the optimal subsidy rate on good $j$ is inversely related to its own price elasticity (the efficiency motive, i.e., a typical Ramsey principle) and positively related to the targeting effect divided by $x_j$ (the redistributive motive). In this interpretation, the term $\mu^{-1}$ represents the shadow price of the social welfare evaluated by the social planner. If this price increases the weight of the redistributive motive in setting subsidy rates becomes relatively more important.

To conclude, we suppose that the social planner has the possibility to use lump-sum transfers (or taxes). This yields an interesting interpretation of the subsidy rates given by formula (26). In effect, in this case, the first order condition of the planner’s optimization program can simply be written as:

$$\lambda_c \frac{\partial \phi}{\partial Y} = \mu.$$ 

If this expression is introduced in (26), the subsidy rates simplify to:

$$\frac{s_j^*}{p_j} = - \left( \frac{\partial \phi}{\partial Y} \varepsilon_{jj}^{-1} \right) \frac{R_j}{x_j}$$

This expression is simply a reformulation of (25). Thus the subsidy rate is proportional to the average targeting effect and will take its sign. The optimal rule is then simple. The social planner fixes each subsidy rate at the value $s_j^*$ and gives what is left of her budget to the household under the form of a transfer. The optimal level of such transfer can simply be written as:

$$T^* = B - \sum_j s_j^* x_j.$$
4 Targeting Child Poverty

4.1 The Definition of Child Poverty

If we now consider the issue of poverty, subsidies may not be as desirable as other instruments. In effect, the cash transfer is identical for all households while the gain due to a subsidy depends on the consumption level of that good. If the good is normal, the subsidy will benefit more to rich households and miss its target. This simple idea is formally presented by Besley and Kanbur (1988, 1993). However, when tackling child poverty, results may differ to the extent that subsidies on certain goods can have redistributive intra-household effect in favor of children.

To show this, we must specify first of all what we mean by child poverty. Let us assume that households differ only with respect to the income level \( Y \) and denote \( f(Y) \) the density function of household income in the population. We then define a poverty line which cuts off the poor from the non-poor among children. There are many ways of arriving to such a line,\(^{20}\) but for our purposes, it suffices to have an arbitrary cutoff at \( Z_E \) so that children with \( \phi(p, Y) \leq Z_E \) are classified as being in poverty. Since the function \( \phi \) is monotonically increasing in \( Y \), there exists a value \( Z \) such that for any household with \( Y \leq Z \), the child is in poverty. This cut-off is implicitly defined by \( Z_E = \phi(p, Z) \).

The proportion of poor children is given by:

\[
H = \int_{0}^{Z} f(Y) \cdot dY.
\]

We then need a poverty index which aggregates information on units below the poverty line. In what follows, we shall use the income gap measure of poverty, which is defined as:

\[
P = \int_{0}^{Z} \left( \frac{Z_E - \phi(p, Y)}{Z_E} \right) f(Y) \cdot dY.
\]

One criticism of this measure is that it is insensitive to the distribution of income among the poor. However, for the question we are asking, this approach is very simple and

\(^{20}\)While US indices rely on absolute measures, traditional practice in Europe consists of using relative poverty lines (usually a proportion of the median income); this practice has also been adopted among Laeken indicators to monitor social policy in EU-25. However, national reports tend to use multiple criteria. In the UK, for instance, Department for Work and Pensions (2003) recommends a combination of absolute low income measure (£210 per week for a couple with one child), relative measure (60% of the median) and a measure of material deprivation.
convenient.\footnote{Measuring depth of poverty using this poverty gap measure – instead of e.g. headcount ratio – receives much support by administrations and policy makers. See, e.g. Department of Work and Pension (2003).}

An important point is that measures of child poverty usually assume that children in poor households (e.g. households with income below 60% of the median) are children in poverty. Whether this is the case or not depends on how income is shared within the household. Originally, in our framework, we need not to assume equal sharing and focus on resources actually accruing to children to define child poverty. This way, the threshold $Z$ is the level of household income corresponding to the child poverty line, but this does not necessarily give a poverty line for the household or the adult.

\section*{4.2 The Marginal Impact of Subsidies and Transfers}

Before examining the marginal impact on child poverty, let us note that the budgetary cost of a subsidy on good $j$ is simply:

$$B = \int_0^\infty s_j \bar{x}_j(p, Y) f(Y) \cdot dY,$$

and thus,

$$\frac{\partial B}{\partial s_j} = \int_0^\infty \left( x_j - s_j \frac{\partial \bar{x}_j}{\partial p_j} \right) f(Y) \cdot dY = E(x_j) - s_j E\left( \frac{\partial \bar{x}_j}{\partial p_j} \right), \quad (27)$$

where

$$E(x_j) = \int_0^\infty x_j f(Y) \cdot dY,$$

$$E\left( \frac{\partial \bar{x}_j}{\partial p_j} \right) = \int_0^\infty \frac{\partial \bar{x}_j}{\partial p_j} f(Y) \cdot dY,$$

are the demand for good $j$ and its derivative averaged over the population of households. We then investigate the effect of a marginal increment in the budget due to a subsidy on good $j$. That is,

$$\frac{\partial P}{\partial B} = -\frac{H}{Z_E} \times \left[ E_P \left( x_j \frac{\partial \phi}{\partial Y} \right) + E_P \left( R_j \right) \right] \times \left[ E(x_j) - s_j E\left( \frac{\partial \bar{x}_j}{\partial p_j} \right) \right]^{-1}, \quad (28)$$

where

$$E_P \left( x_j \frac{\partial \phi}{\partial Y} \right) = \frac{1}{H} \int_0^Z x_j \frac{\partial \phi}{\partial Y} f(Y) \cdot dY,$$
\[ E_P(R_j) = \frac{1}{H} \int_0^Z R_j f(Y) \cdot dY, \]

are the income effect and the targeting effect of the subsidy, averaged over the population of households with a child in poverty. The interpretation of this expression is analogous to that of (19). In particular, the subsidy will decrease child poverty if the average targeting effect is positive. However, the cost of the policy is evaluated over the whole population.

For the sake of the comparison, we measure now the marginal impact of a lump-sum transfer. That is,

\[
\frac{\partial P}{\partial T} = -\frac{H}{Z_E} E_P \left( \frac{\partial \phi}{\partial Y} \right),
\]

(29)

where

\[ E_P \left( \frac{\partial \phi}{\partial Y} \right) = \frac{1}{H} \int_0^Z \frac{\partial \phi}{\partial Y} f(Y) \cdot dY. \]

The comparison of the marginal impact of the subsidy and the marginal impact of the transfer is given by:

\[ \Delta = \frac{\partial P}{\partial B} - \frac{\partial P}{\partial T}. \]

This expression will be positive if, as easily shown, the following condition is negative:

\[ s_j E_P \left( \frac{\partial \phi}{\partial Y} \right) E \left( \frac{\partial x_j}{\partial p_j} \right) + E_P(R_j) + \left[ E_P \left( x_j \frac{\partial \phi}{\partial Y} \right) - E(x_j) E_P \left( \frac{\partial \phi}{\partial Y} \right) \right]. \]

The last term, in brackets, resembles that in Besley and Kanbur (1988) and shows that if mean consumption in families with poor children is less than the mean consumption in the population, the impact of subsidies is smaller than that of cash transfers. However, the whole expression needs to be negative thanks to the first two terms, reminiscent of (24) hence easily interpretable.

### 4.3 Example: Transfers versus Subsidies

We take the preceding example and suppose that the utility functions are of the CARA form. The individual demands are linear in \( \phi_i \), i.e.,

\[ x_{ij} = a_i \phi_i, \]

(30)

where \( a_i = -\left((\partial \alpha_i / \partial p_j) / \alpha_a \right) \) is computed in the previous discussion of this example, and the sharing rule is an affine function of \( Y \), i.e.,

\[ \phi = \eta_1 + \eta_2 Y \]
so that \( Z_E = \eta_1 + \eta_2 Z \). We then suppose that household incomes are uniformly distributed between zero and an upper bound, \( \tilde{Y} \). In that case, the proportion of households with a child in poverty is denoted by \( H = \frac{Z}{\tilde{Y}} \) and the average demand for good \( j \) is simply equal to

\[
E(x_j) = \frac{1}{\tilde{Y}} \int_{0}^{\tilde{Y}} \left[ (a_c - a_a) \eta_1 + (a_a(1 - \eta_2) + a_c \eta_2) Y \right] \cdot dY \\
= A + BE(Y), \quad \text{with } E(Y) = \tilde{Y}/2
\]

where \( A = (a_c - a_a) \eta_1 \) and \( B = (a_a(1 - \eta_2) + a_c \eta_2) \) are the parameters of the household demand for good \( j \).

We shall now compute the marginal impact of the subsidy (from the no subsidy situation) and compare it to the marginal impact of the transfer. Using expressions (28) and (29) previously derived, we obtain the marginal impact of the subsidy at \( s_j = 0 \):

\[
\frac{\partial P}{\partial B} = -\frac{1}{Z_E} \times H \times \left( A\eta_2 - \frac{B}{2} \eta_1 + \frac{A}{\theta_1} + \frac{B}{2} Z_E \right) \times [A + BE(Y)]^{-1},
\]

and the marginal impact of the transfer:

\[
\frac{\partial P}{\partial T} = -\frac{1}{Z_E} \times H \times \eta_2.
\]
Now, if we make the difference between these two expressions, we demonstrate that (from the no-subsidy situation) the subsidy is more efficient than the transfer if and only if the following condition is satisfied:

$$\theta E(Y) < \frac{A}{B\eta_1\eta_2(1-H)}.$$ 

That means, in particular, that the subsidy is specially desirable when the complementarity index (multiplied by average income as a normalization) is small and the poverty rate is large. When good $j$ is exclusively consumed by the child, this formula reduces to:

$$\theta E(Y) < \frac{1}{\eta_2^2(1-H)}.$$ (31)

In this case, the condition does not depend on the parameters of the demand. To illustrate the relative importance of these variables, we shall suppose that $E(Y) = \mathcal{L}20,000$ and sketch the relation between $H$ and $\theta$ given by (31). In Figure 2, the case where $\eta_2 = 0.25$ ($\eta_2 = 0.50$) is described by the upper (lower) line whereas the dotted horizontal lines correspond to $\theta = 0.001$ and $\theta = 0.0001$ which are, as previously seen, realistic upper and lower bounds for $\theta$. We observe that, if the fraction of $Y$ given to the child is relatively small ($\eta_2 = 0.25$), the subsidy is more efficient than transfers – true in the area below the upper line – even for a large complementarity index and a small poverty of rate. If the fraction of $Y$ given to the child is relatively large ($\eta_2 = 0.50$), the subsidy is more efficient than transfers only if the complementarity index is very small ($\theta \approx 0.0002$).

In particular, for a non-decreasing and very concave Engel curve, the first effect will support the cash transfer while the second effect will be favorable to the subsidy. This simply means that even though poor households receive less transfer – since the subsidy is proportional to expenses while the transfer is uniform – a lot of intra-household distribution in favor of children is taking place. Note that the optimal subsidy rates can be computed as well.

5 Conclusion

Within the framework of a general representation of households with children, we have (i) derived the conditions under which a subsidy placed on a particular good ameliorates child welfare; (ii) shown that the subsidy can prove to dominate a cash transfer; (iii) characterized the optimal level of price subsidies and the optimal policy mix when both...
subsidies and direct transfers are allowed; (iii) demonstrate that the reduction in child poverty may be greater using well-chosen subsidies rather than cash transfers.

The basic concept that we must know to apply these theoretical tools on real data is the difference between the slopes of the adult’s and child’s Engel curves. Indeed, this concept will allow us to extract the set of goods for which the subsidy will be the most effective.

The easiest way to identify the sharing rule is to employ the Rothbarth’s method which is generally used to estimate the cost of children. This method requires that at least one good is not consumed by the child. The standard choice for such goods is men’s and women’s clothing or ‘vice’ goods (alcohol, tobacco, etc.). Note that the household consumption of these items coincides with adult consumption so that there is an identity:

\[
\bar{x}_k(p, Y) = \hat{x}_{ka}(p, Y - \phi(p, Y)),
\]

for any adult-specific good \(x_k\). Then, in Gronau’s (1991) variant of the method, the demands for ‘adult-specific’ goods are estimated from a sample of childless families (i.e., where \(\phi_a = Y\)). This way, the derivatives of identity (32) are:

\[
\frac{\partial \hat{x}_k(p, Y)}{\partial Y} \text{ estimated on couples} = \frac{\partial x_{ka}}{\partial \phi_a} \text{ estimated on singles} \left(1 - \frac{\partial \phi}{\partial Y}\right)
\]

\[
\frac{\partial \hat{x}_{ka}(p, Y)}{\partial p_k} \text{ estimated on couples} = \frac{\partial x_{ka}}{\partial p_k} \text{ estimated on singles} - \frac{\partial \hat{x}_a}{\partial \phi_a} \frac{\partial \phi}{\partial p_k}
\]

and estimations from childless families provide the direct derivatives with respect to income and to the price of the good. Thus partial derivatives of the sharing rule can be identified (as long as \(\hat{x}_k\) is a monotonic function of \(Y - \phi\)) and the total redistribution aversion can be easily computed from equation (13), which gives:

\[
\theta = \frac{\partial \hat{x}_1}{\partial \phi_a} \left(\frac{\partial \phi}{\partial p_1} - \frac{\partial \phi}{\partial Y} x_1\right)^{-1}.
\]

If the social planner wants to introduce a subsidy on child-specific goods, the components of the structural model that are retrieved are sufficient to compute optimal subsidy rates. The usual difficulty with this method is that it assumes that preferences for the exclusive commodity are independent of family composition (the presence of children does not change parents’ preferences for, say, adult clothing).

Finally, two limitations of the present framework deserve attention in future work. First, the production side has been ignored in our partial equilibrium analysis. If the
supply of subsidised goods is very rigid, its price will increase as the subsidy is introduced and exchanged quantities will not change, cancelling distortions but also the targeting effect. Second, our analysis has considered only private goods, while the level of public consumption achieved by the household can be crucial for the welfare of children. Measures of material deprivation often include public durable goods (see Gregg et al., 2005, for instance). Nonetheless, it is possible to extend the present model to public goods, as done in the collective model literature (see Blundell et al., 2006).

Appendix: Measuring the Targeting Effect

References


