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Risk adjustment, investment policy, and valuation for an unlevered firm

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Risk adjustment, investment policy, and valuation for an unlevered firm

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Abstract

We characterize the optimal investment decision and the stock value of an unlevered firm that holds the non-standard option of improving the growth rate of cashflows from its assets in place upon incurring an irreversible cost. The firm's investment policy and equity price are studied as a function of the market price of risk, of cashflow's exposure to systematic risk, and of cashflow volatility.

JEL classification: C6; E2

Keywords: Investment; Equity pricing; Market price of risk

1 Introduction

The analysis of the investment policy of an unlevered firm and of the firm's ensuing value in the presence of risk premia is known to be a delicate issue since the pioneering works of MacDonald and Siegel (1986), Pindyck (1988), and Dixit (1989). The presence of economy-wide risk aversion significantly impacts the firm's investment decision and the firm's fair value. Chiefly, ignoring risk premia hampers the analysis as the risk-adjusted drift of the firm's cashflow process and its volatility remain unnaturally delinked. If the market price of risk is positive, a surge in the systematic component of cashflow volatility does inflate the

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value of the option-like components of firm's equity but has also a general deflating effect on the value of any claim on the firm's cashflows.

In contrast, the link between equity valuation, optimal investment, and risk adjustment is the center of our attention. We focus on a unlevered firm that has the investment option of improving the expected growth rate of cashflows from its assets in place rather than the usual investment option of either scaling them up or buying incremental assets. The investment policy and the stock value are investigated for different levels of the economy-wide risk aversion, of cashflow's correlation with systematic shocks, and of cashflow volatility.

The emphasis on systematic risk places our analysis close to the works of Sarkar (2000), Cappuccio and Moretto (2001), Sarkar (2003), and Lund (2005). These works have a different focus. They are mainly concerned with firm's 'nearness' to invest as assessed by the objective probability of investing within a fixed horizon and by measures of expected investment. Also, they study the firm's classical investment option of acquiring incremental assets. Our explicit treatment of equity pricing for an unlevered firm that holds the non-standard option of improving the growth rate of its assets in place makes a distinct contribution to their analyses.

Our work is organized as follows. Section 2 describes the primitives of our continuous-time model. Equity valuation is carried out in Section 3. Section 4 gives numerical evidence on how risk premia shape investment policy and equity value. Section 5 concludes.

2 Stochastic discount factor and equity risk premium

We assume the existence of an exogenous state-price density process $\{\xi\}$ that prices any traded asset in the economy and its dynamics is

$$\frac{d\xi}{\xi} = -rdt - \lambda dW^{\mathbb{P}},$$

where r is the riskfree rate, λ is the market price of systematic risk, and $\mathbb{P}$ denotes the objective probability measure.

The firm's cashflow level has the following dynamics:

$$\frac{dD}{D} = \mu dt + \sigma dZ^{\mathbb{P}}.$$

The parameter ρ quantifies how much shocks to the firm's cashflows comove with the systematic shocks in the economy:

$$d\left[W^{\mathbb{P}}, Z^{\mathbb{P}}\right] = \rho dt,$$

$$-1 \leq \rho \leq 1,$$

where $[\cdot,\cdot]$ denotes the quadratic covariation of two diffusive processes. The parameter ρ represents the firm's exposure to systematic risk. When the exposure is negative, the firm's stock offers insurance and its risk premium

$$\frac{S_D}{S} D\lambda\rho\sigma$$

becomes negative, where S_D denotes the derivative of the stock price S with respect to the firm's cashflow level D.

As it is natural for cashflows stemming from real business operations, we assume that the process $\{D\}$ cannot be seen as the value process of a self-financing portfolio of traded assets. Hence, the expected growth rate μ bears no *a priori* relation with r, λ and σ apart from the one established by a boundedness condition on the firm's fair value $s(D)$ in the absence of investment options:

$$0 \leq s(D) = \frac{D}{r + \lambda\rho\sigma - \mu} < \infty \quad \text{for bounded levels of } D.$$

Such a condition is equivalent to the well-known expected-return-shortfall condition in the real options literature:

$$r + \lambda\rho\sigma - \mu > 0.$$

The firm is an all-equity corporation and is run by equityholders who maximize the value of equity. The firm has an investment opportunity to improve the growth rate of its activities from μ to μ' ($r+\lambda\rho\sigma > \mu' > \mu$) upon incurring an irreversible cost I.

Since the firm's equity is a traded security that provides an intermediate payout equal to D per unit time, the no-arbitrage dynamics of equity prices in the absence and in the presence of the investment option has the same structure:

$$\begin{aligned}
\frac{ds}{s} + \frac{D}{s}dt &= \left(r + \frac{s_D}{s} D\lambda\rho\sigma\right) dt + \frac{s_D}{s} D\sigma dZ^{\mathbb{P}}, \\
\frac{dS}{S} + \frac{D}{S}dt &= \left(r + \frac{S_D}{S} D\lambda\rho\sigma\right) dt + \frac{S_D}{S} D\sigma dZ^{\mathbb{P}},
\end{aligned}$$

respectively.

3 Equity pricing

At the investment date, equityholders contribute new equity funds for an amount I. These funds are used to exercise the growth option, so that the drift in the cashflow process is upgraded to μ'. Given

$$\beta_1 = \frac{-\left(\mu - \lambda\rho\sigma - \frac{1}{2}\sigma^2\right) - \sqrt{\left(\mu - \lambda\rho\sigma - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} > 1,$$

equity's no-arbitrage value is

$$S(D) = \begin{cases} s(D) + \left(\frac{U}{r+\lambda\rho\sigma-\mu'} - s(U) - I\right)\left(\frac{D}{U}\right)^{\beta_1} & \text{if} \quad 0 \leq D \leq U, \\ \frac{D}{r+\lambda\rho\sigma-\mu'} - I & \text{if} \quad U < D, \end{cases}$$

$$U = I\frac{\beta_1}{\beta_1 - 1}\frac{(r+\lambda\rho\sigma-\mu')(r+\lambda\rho\sigma-\mu)}{\mu'-\mu},$$

where $S(D)$ meets the dynamic no-free-lunch restriction

$$S_D D\mu + \frac{1}{2}S_{DD}D^2\sigma^2 + D = Sr + S_D D\lambda\rho\sigma$$

with the value-matching and smooth-pasting conditions:

$$S(U) = \frac{U}{r+\lambda\rho\sigma-\mu'} - I \quad \text{and} \quad \left.\frac{d}{dD}S(D)\right|_{D=U} = \frac{1}{r+\lambda\rho\sigma-\mu'}.$$

Within this framework, an increase of the non-systematic component of cashflow risk can be manifestly understood as an increase in σ accompanied by a decrease in ρ. The accompanying decrease in the exposure to systematic risk is such that the expected-return shortfalls $r+\lambda\rho\sigma-\mu$ and $r+\lambda\rho\sigma-\mu'$ remain untouched at their original levels. This accommodates the situations envisaged by Cappuccio and Moretto (2001) and Lund (2005). It must be noticed that such situations do modify the firm's investment policy and equity value.

In contrast, the magnitude of the economy-wide attitude towards risk and the exposure to systematic risk can be changed in ways that leave the firm's investment policy and equity value unaffected. This occurs whenever λ and ρ are revised without altering their product $\lambda\rho$. Two polar cases of such an observational equivalence are notable: the case of a risk-neutral economy ($\lambda = 0$) and the case of a firm's zero exposure to systematic risk ($\rho = 0$).

4 Numerical results

The first numerical issue we address is the direct effect on investment policy and on equity value of the market price of systematic risk and of the firm's exposure to systematic risk. We adopt the base-case parameter levels reported in Table 1.

Table 1: Base case parameters

μ	μ'	σ	ρ	λ	r	I
0%	2.5%	15%	0.5	0.5	7%	1

Figures 1A and 1B show how changes in the market price of risk impact the investment threshold U and equity's market value $S(D)$. If the firm is positively exposed to systematic risk ($\rho = 0.5$), an increase in λ implies a downward revision of the risk-adjusted percentage drift $\mu - \lambda\rho\sigma$ of the cashflow process. This lessens the risk-adjusted probability of hitting a given upper barrier, but it also implies a deflation of any security exposed to systematic risk, including a depression of the investment payoff $\frac{U}{r+\lambda\rho\sigma-\mu'} - I$. The second effect prevails so that investment is delayed to improve the cashflow level U at which it is optimally carried with the objective of maximizing the stock price. If the firm's cashflows exhibit no exposure to systematic risk ($\rho = 0$), alterations in the economy-wide attitude towards risk have no impact on the firm's investment policy and on its stock price.

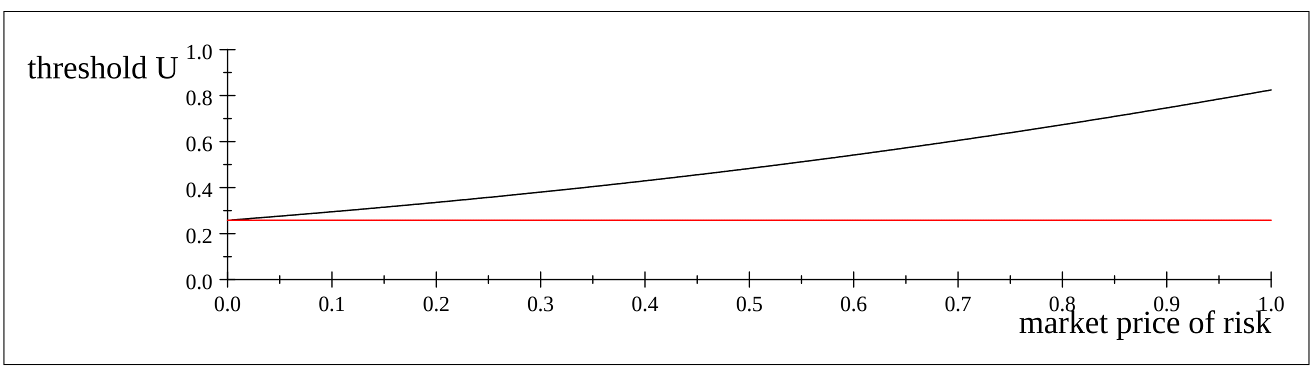

Figure 1A: U versus λ for $\rho = 0.5$ (black) and for $\rho = 0$ (red)

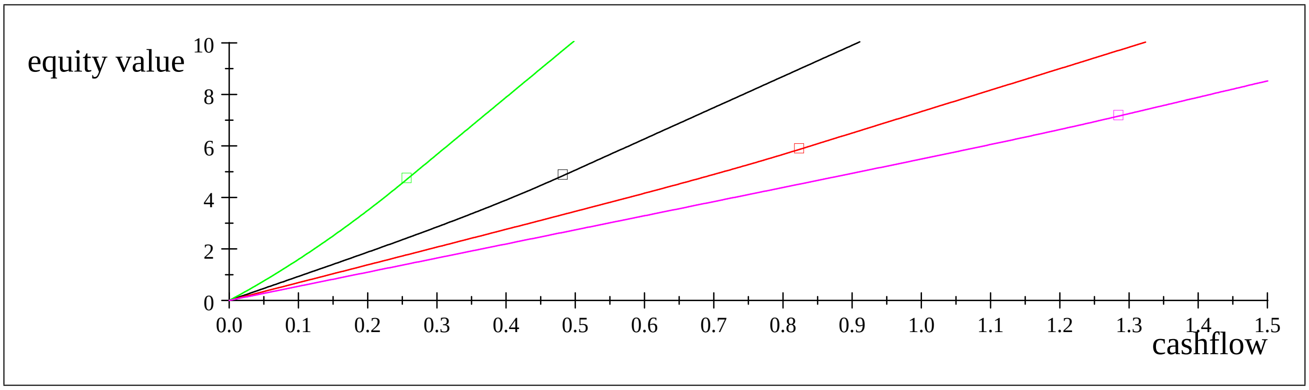

Figure 1B: $S(D)$ versus D given $\rho = 0.5$ for $\lambda = 0$ (green), $\lambda = 0.5$ (black), $\lambda = 1$ (red), and $\lambda = 1.5$ (magenta). The boxes represent the investment threshold U and equity's corresponding value $S(U)$

Figures 2A and 2B provide evidence on what is the effect on U and on $S(D)$ of changes in the firm's exposure to systematic risk. If systematic risk is disliked at the economy-wide level ($\lambda = 0.5$), cashflow's negative correlation ρ with systematic shocks implies an upward revision of its risk-adjusted percentage drift $\mu - \lambda\rho\sigma$, which raises the risk-adjusted probability of hitting a given upper barrier. A negative ρ also implies a value swelling of any claim on the firm's cashflows, counting the investment payoff $\frac{U}{r+\lambda\rho\sigma-\mu'} - I$. Both effects go in the same direction of being conducive to an anticipated optimal investment as U can be confortably lowered to maximize equity value. The opposite arguments hold for a positive ρ. If there is economy-wide risk neutrality ($\lambda = 0$), changes in ρ have no impact on the firm's investment decision and on its equity value.

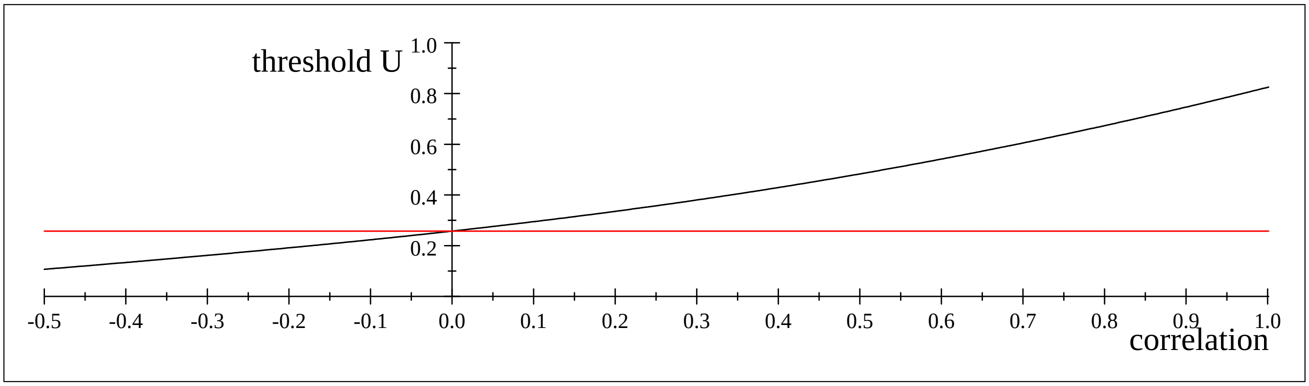

Figure 2A: U versus ρ for $\lambda = 0.5$ (black) and for $\lambda = 0$ (red), given $r + \lambda\rho\sigma > \mu'$

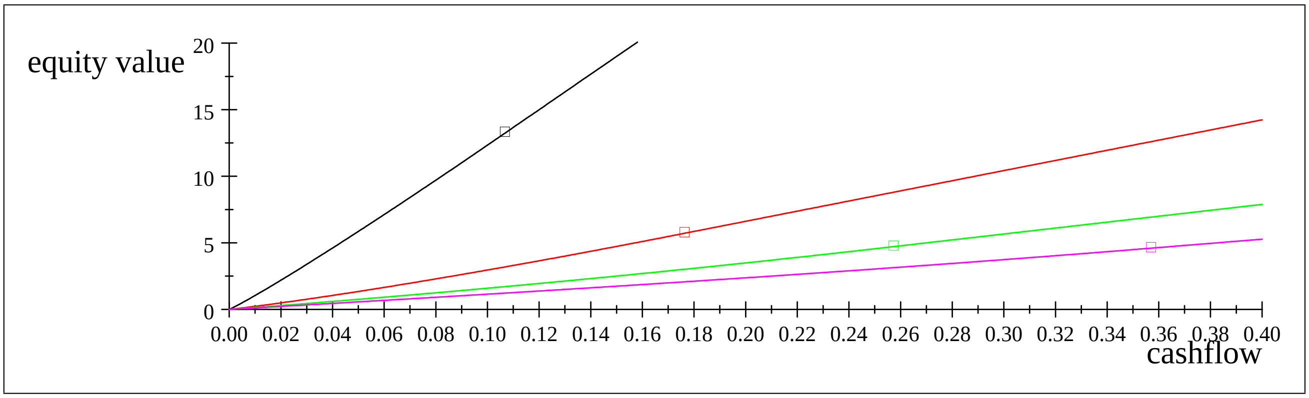

Figure 2B: $S(D)$ versus D given $\lambda = 0.5$ for $\rho = -0.5$ (black), $\rho = -0.25$ (red), and $\rho = 0$ (green), and $\rho = 0.25$ (magenta). The boxes represent the investment threshold U and equity's corresponding value $S(U)$

Changes in cashflow volatility modify risk premia even if λ and ρ are kept constant. Obviously, such changes must be understood as modifications of the systematic component of cashflow volatility. The second issue we address is how the market price of systematic risk and the firm's exposure to systematic risk contribute to the effects of volatility on investment policy and on equity valuation. Figure 3A shows how sensitive is optimal investment to increases in σ. In the absence of any risk premium in the economy ($\lambda = 0$), a swelled volatility implies that any given upper barrier is lesser difficult to be reached, while the investment payoff $\frac{U}{r+\lambda\rho\sigma-\mu'} - I$ remains unaffected. Hence, optimal investment is deferred to maximize the stock price. Investment postponement is more prominent in the presence of economy-wide risk premia

$(\lambda = 0.5)$ and of positive correlation between the firm's cashflows with systematic risk $(\rho = 0.5)$. A swelled volatility downsizes the risk-adjusted percentage drift $\mu - \lambda\rho\sigma$, but it also comes along with a deflation of any claim on the firm's cashflows, including a devaluation of the investment payoff $\frac{U}{r+\lambda\rho\sigma-\mu'} - I$. The second effect wins so that investment is even more postponed to pump up the cashflow level U at which the investment option is optimally exercised. Figure 3B shows that, in the two observationally equivalent cases $\lambda = 0$ and $\rho = 0$, the value of the option of waiting to invest experiences a barely visible increase as σ moves up, leaving the firm's stock price nearly unaffected. Hence, risk premia $(\lambda = 0.5)$ and higher volatility of cashflows will predictably dent the market value of a firm with positive exposure to systematic risk $(\rho = 0.5)$. This is confirmed by Figure 3C.

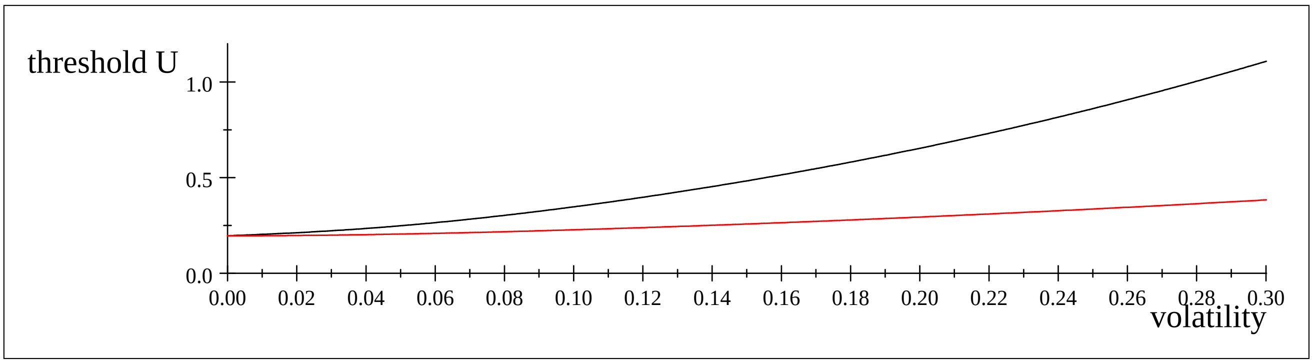

Figure 3A: U versus σ for $\lambda = 0.5$ and $\rho = 0.5$ (black) and for either $\lambda = 0$ or $\rho = 0$ (red)

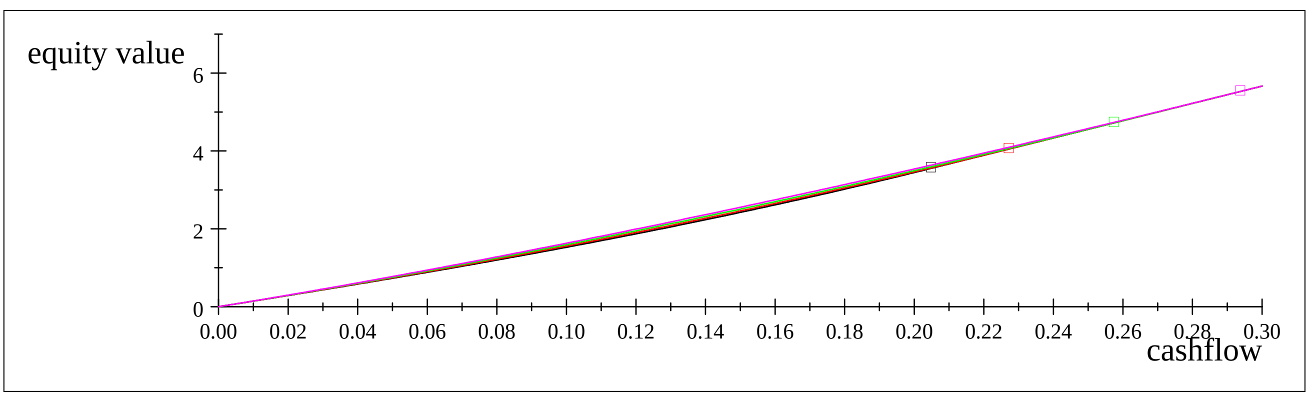

Figure 3C: $S(D)$ versus D given either $\lambda = 0$ or $\rho = 0$ for $\sigma = 0.05$ (black), $\sigma = 0.10$ (red), $\sigma = 0.15$ (green), and $\sigma = 0.20$ (magenta). The boxes represent the investment threshold U and equity's corresponding value $S(U)$

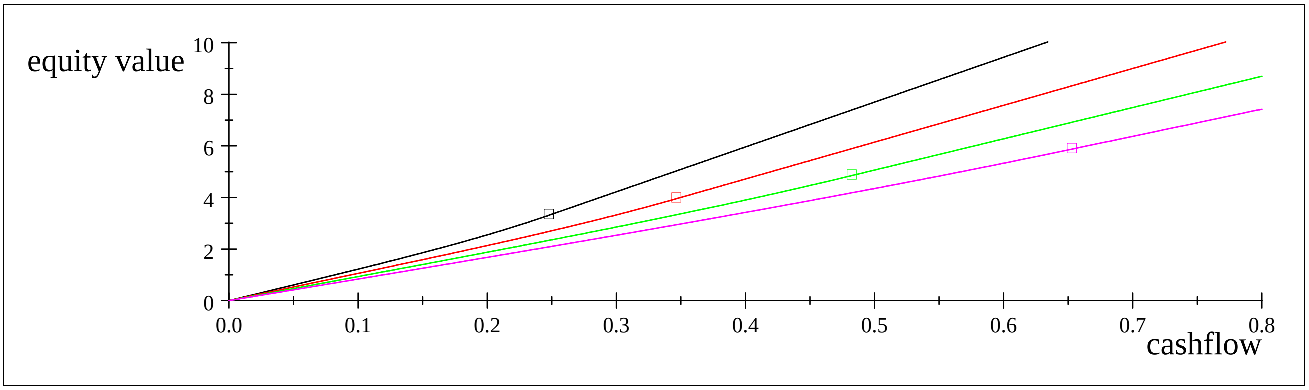

Figure 3B: $S(D)$ versus D given $\lambda = 0.5$ and $\rho = 0.5$ for $\sigma = 0.05$ (black), $\sigma = 0.10$ (red), $\sigma = 0.15$ (green), and $\sigma = 0.20$ (magenta). The boxes represent the investment threshold U and equity's corresponding value $S(U)$

5 Conclusions

Our work builds on the extant literature that gauges the impact of systematic risk on real options. We consider an unlevered firm that, upon incurring an irreversible cost, has the option of lifting the expected growth rate of its existing cashflows rather than the standard option of either scaling them up or buying incremental assets. The investment policy and the stock value of the unlevered firm are studied as a function of economy-wide risk premia, of firm's exposure to systematic risk, and of cashflow volatility. Even in the presence of the investment option, a surge in the systematic component of cashflow volatility is shown to have a net deflating effect on the firm's equity value.

An interesting avenue for future research is the examination, via an extension of the analysis in Mauer and Ott (2000), of the impact of risk premia on the cost of Myers's (1977) debt overhang problem for a levered corporation endowed with growth options.

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