The economic dynamics in Amoroso's contribution

by Nardi Spiller C.* e di Pomini M.**

> rather than following their secular growth path, progressive economies are under pressure to deviate; and we have seen that even if initially such deviations produce different types of movement, in the end they all result in fluctuations, though of a different size and length in each case (Fanno 1956: 168).

Summary: 1. Introduction; 2. The need for a dynamic theory: Amoroso's research programme; 3. Cyclical movements; 4. The phases of economic fluctuations; 5. From a cycle theory to a dynamic theory 6. An energetic approach to the production process; 7. The limits of Amoroso's research; 8. Concluding Remarks. Selected Bibliography.

A study carried out as part of research project funded by the University of Verona and presented at the Ninth Annual Conference ESHET Conference, University of Stirling (Scotland), 9-12 June 2005.

Although this work is the result of joint discussion and analysis, paragraphs 3, 4 and 8 were written by Cristina Nardi Spiller and paragraphs 1,2,5,6 and 7 by Mario Pomini.

*Professor of Macroeconomics and Professor of Cultural Economics, Faculty of Economics, University of Verona. Professor of Economics of Arts, Faculty of Arts, University of Verona.
E-mail: cristina.spiller @univr.it
** Professor of Microeconomics and Macroeconomics, Faculty of Law, University of Padua.
E-mail: mario.pomini@unipd.it

1. **Introduction**

Ramsey's model is unanimously considered the achievement of dynamic macroeconomics, thanks to its ability to extend the analysis of optimising behaviour to an intertemporal context of resource allocation[1]. Despite the considerable emphasis on the analytical properties of models *à la* Ramsey which abound in literature, we should be aware that in the last century Ramsey's approach was not the only neoclassical approach to a genuinely dynamic theory. Other less important currents offered a significant contribution; for various reasons their impact on scientific debate was modest. One such theory was developed in Italy by the mathematical economist Luigi Amoroso. However Amoroso's original vision of economic dynamics never found the following it deserved, despite initially arousing interest at an international level.

This paper has dual aims. Firstly we intend to highlight the principal features of Amoroso's reflections on macroeconomic dynamics, in terms of cycle theory and general equilibrium theory, the two traditional components of that time. The subsequent analysis will further our understanding of why Amoroso's work did not achieve the success it deserved, irrespective of his political inclination towards fascism: Amoroso's work was based on a mechanical, nineteenth century style interpretation of economic dynamics.

2. **The need for a dynamic theory**: **Amoroso's research programme**

Amoroso (1886-1965) was the leading exponent of the Italian School of Mathematical Economics[2]. He began his mathematical studies at the Scuola Normale of Pisa and later continued in Rome, graduating in 1907. He pursued a brilliant academic career in research, lecturing on Geometry and Rational Mechanics, and already by the academic year 1913-1914 had progressed to teach Advanced Analysis. Later he became a professor of Political Economy and Mathematical Physics, and in 1914 he was appointed Professor of Financial Mathematics at the University of Bari. He then took up the Chair of Political Economy at the University of Rome, which he held until 1956.

A mathematician by training, Amoroso's interest in political economy was first aroused during lectures at Rome University delivered by Pantaleoni, who guided him towards Pareto.

---

[1] See Barro and Sala–i–Martin (1995).

[2] The term "Scuola Italiana di Matematica Economica" was coined by Edgeworth (1922).

Amoroso became Pareto's assistant, as numerous documents testify; indeed Amoroso is justly considered[3] one of the most faithful interpreters of Pareto's General Equilibrium Theory.

One of the most significant aspects of Amoroso's work was to become his divulgation of the mathematical aspects of Pareto's theory. What is more Pareto urged Amoroso to further the study of Mathematical Economics in Italy as an academic subject in its own right distinct from Political Economy, with its own field of analysis and rigorous methods inherited from rational mechanics. Amoroso's endeavours led to the publication of *Lezioni di Economia Matematica* [*Lectures on Mathematical Economics*] in 1921, for decades one of the most authoritative texts on the subject.

There is no doubt that Amoroso's work focused on the two central themes (Giva, 1996) which he pursued throughout his career: the birth and development of mathematical economics as a discipline in its own right, and, at a more scientific level, his plan to make Pareto's model dynamic. The second theme, which forms the focus of our analysis, was already clearly defined in Amoroso's first economic paper published in 1912 entitled *Contributo alla teoria matematica della dinamica economica* [*A Contribution to the Mathematical Theory of Economic Dynamics*], in which Amoroso affirmed that

> like rational mechanics, mathematical economics has two parts: a static part and a dynamic part. The mathematical formulation of the principles of economic statics is defined in the work of Vilfredo Pareto. Economic dynamics lags far behind (Amoroso, 1912, citation reproduced in Guerraggio, 1996: 739).

This citation reveals the two factors which are vital to understand the scientific path pursued by Amoroso: Pareto, the acclaimed master, and dynamics. For the second generation of marginalists, economic dynamics constituted a new field of research; Amoroso ventured into this field from different angles, with varying degrees of success. It is well known that the Pareto system was essentially static, and that Pareto himself left merely a trace of his plans to extend the model in a dynamic perspective, in a few paragraphs of the *Corso*. There were two explanations for Pareto's behaviour. Firstly, extending the analytical categories of rational mechanics to dynamic cases was tricky: economic and mechanical phenomena had little in common, while greater analogies existed in static treatments. Secondly Pareto was convinced that social dynamics were best investigated through sociology, rather than by means of the more refined but less

---

[3] Among others see Guerraggio (1996) and Tusset (2004).

productive science of economics. In 1907, having already abandoned economics in favour of sociology, Pareto concluded that

> It is likely that the day will come when mathematical economics adopts similar principles to those of D'Alambert. But it is better not to forestall the events. For the time being only economic statics is scientifically developed and has yielded useful results. (Pareto, 1907: 450).

Pareto left his pupils with a momentous task: the transformation of his system from one of static equations to one of dynamic equations. In Italy it was the young Amoroso who took up the challenge, which became the central theme of his research programme. Indeed almost all Amoroso's writings on economics attempted to offer a dynamic interpretation of some economic phenomenon, from his earliest essays in the 1920s on the supply curve to his energetic interpretation of the economic phenomena of the 1940s just like Fisher before him, with an interlude dedicated to an interpretation of economic cycles.

Amoroso's greatest innovation was his method, more than his single contributions to the field of economics: he was effectively one of the founders of modern dynamic analysis in economics. Anyway he said that it is difficult to formulate right forecast (Amoroso, 1933a: 21). He went well beyond the work of Pareto, who limited dynamic analysis to the simplest case of subsequent adjustments or to comparative statics as we would call it today. Amoroso believed that the achievement of genuinely dynamic analysis made it necessary to abandon comparative statics and explain changes in variables over time. In other words the unknown variables became derivatives over time of the relevant variables; similarly the equations became differential equations. So the need arose to employ more sophisticated mathematical tools: Amoroso was the first to introduce to Italy the calculus of variations, which he used extensively. In the 1930s Amoroso made two important contributions to economic dynamics in Italy: he singled out the field of enquiry of modern economic dynamics, and he identified the appropriate analytical tools. He was nevertheless well aware of the interpretational difficulties, and he used mechanical analogies in an attempt to overcome them:

> The passage from statics to dynamics is extremely arduous. It implies the construction of a theoretical method similar to that developed in mechanics from Galileo to D'Alambert, a method which centres on the inertial force of the system. Within the field of economics there is something comparable to inertial resistance in mechanics. This resistance needs to be measured and represented by adequate algorithms: precisely what has been done recently with the continuation of Pareto's work (Amoroso,1948: 13-14).

The analogy with rational mechanics continued to emerge in the writings of Amoroso. While on the one hand it provided him with a range of well-established analytical tools, on the other it restricted his vision to a nineteenth century perspective and prevented him from taking new trends into account: in the aftermath of the Keynesian revolution, a shift from a Paretian-Walrasian perspective towards one of aggregate quantities enabled modern dynamic macroeconomics to emerge.

Amoroso was fully aware that the economy, with its 'perennial ebbs and flows' (Amoroso, 1938:. 226) could never be completely accommodated within the classical mechanics models. It was however an issue that could not be resolved, and he himself considered the models to be sufficiently explicative.

3. Cyclical Movements

Within the field of economic mechanics, Amoroso devoted special attention to fluctuations in the complex structures of the economy, which he believed were of a cyclical nature due to the existence of a principle analogous to that of action and reaction in mechanics. Opposing forces, which were typical of the commercial and industrial sectors, gave rise to fluctuations in prices:[4] each time prices rose, output increased, albeit with a lag due to the length of production processes. The increase in output was counteracted at an early phase by the actions of speculators, who expected prices to fall.

> In other words acceleration in the rate of price increases *naturally develops forces within the economy* which tend to lower prices; vice versa acceleration in the rate of price decreases develops forces which tend to raise them.
>
> From this dual contrast it follows that prices (*and consequently the volume of output*) do not follow a constantly increasing or decreasing path, but rather alternate waves of expansion or depression. Movements in the economic phenomena linked with the cycle are the result of this (Amoroso, 1933b [author's own italics]: 36).

---

[4] Nevertheless, and particularly with regard to the level of stocks, opinions differed. Fanno (1956) noted that possibile variations were principally due to changes in the level of production, with distinct reactions in terms of direction and magnitude, on prices. The author underlined how the sterility of Amoroso's research (1940) 'which aimed to express in general analytical terms the reactions between stocks and prices, even though the price trend was similar, but not parallel, to the trend of the volume of output' (Fanno, 1956 n. 3: 164). Vinci (1940) agreed with Fanno.

Thus economic fluctuations [5] could not be exclusively attributed to monetary phenomena, since cyclical fluctuations and dynamic equilibriums were jointly determined by a number of forces: live forces in action, inertial forces representing the weight of the past, and guiding forces embodying the future[6]. Reference to variables representing the future was a possible anticipation of the value of expectations, although Amoroso did not offer a formal treatment of their formation. Cyclical movements were produced as the primary forces developed, generating opposing trends. A movement in one direction caused an equivalent movement in the opposite direction, and this in turn produced further primary movements. Hence the principle in mechanics of action and reaction was also confirmed here.

Amoroso[7] returned to Pareto (1906) to underline how economic movements originated from the forces which expressed individual needs or tastes, and how such needs and tastes were in turn restricted by the existence of economic goods, the difficulty of obtaining the desired quantities, and any transformation that such goods needed to undergo. From the imbalance between forces and constraints, and vice versa, equilibrium could in any case be achieved following a static, Pareto approach. After all

> I dissociate entirely from Marshall, Edgeworth and their followers. Walras deserves the merit of having posed the problem in general; Irving Fisher deserves a place alongside Walras […]. I have nothing in common with Marshall, Edgeworth, etc.; my theory is different, and I naturally deem it superior, otherwise I would modify it (Pareto, 1973: 63).

By rejecting the primitive notion of cardinal marginal utility, Pareto and others considered utility to be not measurable but rather comparable and orderable thanks to the map of indifference curves.[8] Although Amoroso was fully aware of the intrinsic difficulty of making economic

---

[5] Vinci (1934) dissociated from Amoroso's views, since he assumed that the volume of output varied in relation to the speed of change of prices and income. Vinci proposed his own model.

[6] The guiding forces, which were the forces of the future, consisted of political actions which dominated the forces centering on production processes; hence politics governed economics in the 20th century (Amoroso, 1932b).

[7] For a concise survey of Amoroso's thought, see Palomba (1966). For an overview of the areas of light and shade in Italian economic thought between the two wars see Faucci (1990).

[8] Pareto used the term ophelimity, from the Greek *óphelos*, utility, to indicate the intensity of individual preference. Pareto defined the index of ophelimity as a number $U(P)$, such that $U(P_1)>=<U(P_2)$, if and only if the basket of goods (available for consumption) P_1 was preferred, indifferent or not preferred to the basket P_2. The index was assumed given unless there was a positive transformation, i.e. if U is an index of ophelimity then so is $V=F(U)$, with $F'>0$. Amoroso had hoped that the term would become popular as a means of expressing independence from considerations on the existence and measurability of the psychological pleasure related to consumption (cardinal utility), however this was not the case and the term ordinal utility or utility came to be used instead.

forecasts in the light of interference between subjective and objective factors, he defended the innovative power of the theory.

Amoroso's approach went as far as to investigate the economic dynamics of the economy perceived as the result of reciprocal actions and reactions[9] in the three fundamental sectors of the system: industry, trade and banking. In reality

> In modern terms we would speak of production, trade and money. It is evident that Amoroso's subdivision cannot not be construed in terms of productive sectors, but rather of phases in a capitalist economy, in which the behaviour of agents is typical (Vinci, 1999, *Introduzione:* XVI).

In his investigation of the phases of the economic cycle Amoroso aimed to

> provide a rational foundation to the economic barometer devised by Pantaleoni around 1906 and reconstructed in 1919 by Harvard University (Amoroso, 1938: 78).

There was however one significant difference: in the Harvard barometer, the market prices of goods were included in the business curve, while in Amoroso's analysis they were grouped together with share price trend in the general price index and therefore assumed to be elements underlying the speculation curve.

From Amoroso's perspective, the index T (volume of production) included both industrial and agricultural output, P denoted the general price index and J the interest rate, which included both the cost and the price of money (the discount rate and the actual interest rate); T, P, and J were functions of time, t. Within this analytical framework two aspects of crucial importance emerged: the performance over time of the indices, i.e. whether the trend was upwards or downwards, and the speed of change. The speed of change was expressed as T', P' or J' respectively according to the function being considered. Nevertheless since output was a flow, it was also inherently a speed so that T' more accurately denoted acceleration.

---

[9] The term action is used to indicate economic facts or events classified as primary movements, regardless of their underlying causes; an action is a fact or event which is considered an effect (a derived movement). Reactions caused by inertia or friction occur in exactly the same place and time as the action, and express the tendency of certain phenomena to persist in economic states and dampen the modifiying action. This concept derives from physics, rather like the phenomenon of *hysteresis*, and shows how the current value of a variable depends on its past value. Induced reactions are remote in time and space from the action which determined them, and may be followed by similar, amplifying, opposite, dampening or anticipatory effects. Reactions may be guiding or mechanical. Guiding reactions, which are distant in time from the action which determined them, are reactions to expected or desired future events: thus they occur before the action. Mechanical reactions, which are distant in time and space from their respective actions, do not come before and sometimes come after such actions.

Movements in T, P, and J represented actions in the three sectors (primary movements) which in turn gave rise to reactions in other sectors (derived movements)

Exluding the possibility of virtual reactions induced within one sector, because the three sectors were too large, and also excluding potential future reactions arising in the same sector, Amoroso expressed as inertial reactions the correlations between T and T', P and P', and between J and J'. Such reactions corresponded to the more modern phenomenon of hysteresis. The links existing within the system favoured the onset of derived reactions, hence the correlations in the industrial and the trade sectors between T and P' and between P and T'; in the trade and banking sectors between P and J' and between J and P'; and finally in the banking sector and industrial sector, between J and T' and between T and J'. Nine reactions existed in all: three due to inertia and the remainder induced. By adding additional sectors to the model a greater number of reactions could have been observed.

Certain reactions reflected the general state, pre-existing conditions, the strategies adopted and the resulting consequences. This explains the importance of the inertial reactions in the three sectors, which represented past 'roots', thereby conditioning the future. As a result

> *Messe senescit ager*: every change in progress in each of the three sectors gives rise, *in that sector*, to a corresponding opposite reaction. The greater the spirit of initiative, the greater the resistance to natural and technological constraints and the greater the intellectual effort of those who try to protect society from the damage caused by changes in economic relationships. (Rossi, 1963 [authors own italics]: 277).

Reactions induced in the trade sector revealed the dominant market trends: the stabilisation of prices in relation to the future volume of output (a correlation between P and T') and of the price of capital (share prices) in relation to the flow of expected future income *pro tempore* and capitalised at the expected future interest rate (a correlation between P and J'). In the first case, future variations in output were discounted instantaneously, so that prices rose or fell according to the upward or downward output trend. The result was that the action was observed subsequently, and that the opposing reaction which pre-empted the action became the guiding force. The trade reaction to the bank action confirmed the validity of the capitalisation of income principle[10], one of the pivots of the financial system (Amoroso, 1961: 8), so that the value

---

[10] Subsequently Amoroso (1961) focused on share prices, demonstrating that the difference between the market rate and the rate of capitalisation for each share represented the index of mathematical expectation of future income, assumed to be given by the market at a that particular time. This was the origin of the marginal principle in investment, which demonstrated not only how current prices gravitate around the

of capital adjusted to the value of expected income *pro tempore* at the interest rate foreseen in the near future. Since the capital value of income varied inversely with the rate of capitalisation, a reaction inverse to that originating in the banking system was observed. Once again, it was a guiding reaction.

What was crucial about the response of the industrial sector to the trade action, revealed by the correlation between T and P', was the fact that output expanded when prices rose and contracted when prices fell. Consequently the reaction took place in the same direction as the action. In turn the correlation between T and J', namely the industrial reaction to the bank action operated in the opposite direction, since the rising interest rate raised costs and tended to dampen the increase in output. Hence reactions in the industrial system lagged behind out of phase, and were mechanical, being the result of independent adjustment mechanisms.

Demaria (1935) criticised in general terms the acceptability of a positive dependence between the rate of change of the price index (an action) and the output index on the one hand, and a negative dependence between the rate of change of the interest rate (an action) and the output index on the other, both of which lagged out of phase. He also challenged the inaccuracy of the leads and lags expressed in the mathematical model, challenging the efficacy of the three chosen indexes which were the core of the dynamic world. He considered them incapable of representing 'highly intricate constellations' such as the trade situation, the banking system and the output situation. The objections he raised, which centred on the ability of the indexes to represent economic quantities and functional relationships among quantities,

> could be extended to any reasoning on macroscopic variables consisting of economic quantities which vary as part of an intricate play of forces and reactions (Buscarino, 1959: 88).

In the banking system the reaction was the stabilisation of the rate of interest with respect to the immediately preceding price level and the immediately following level of output. The trends converged: banks mitigated the effects of an upward trend, and the market boosted the effects in the opposite case. In the first case (a correlation between J and P'), the bank reaction to the commercial action was emphasised. In fact rising prices pushed interest rates upwards[11],

---

market rate but also that the market rate was the parameter linking the present to the future. See Rossi (1963:. 277-278 and 283).

[11] Price increases led to a decrease in the value of capital plus interest.

while falling prices caused the rate of interest to decline. The reaction occurred in the same direction as the action, and was followed by a process of mechanical adjustment.

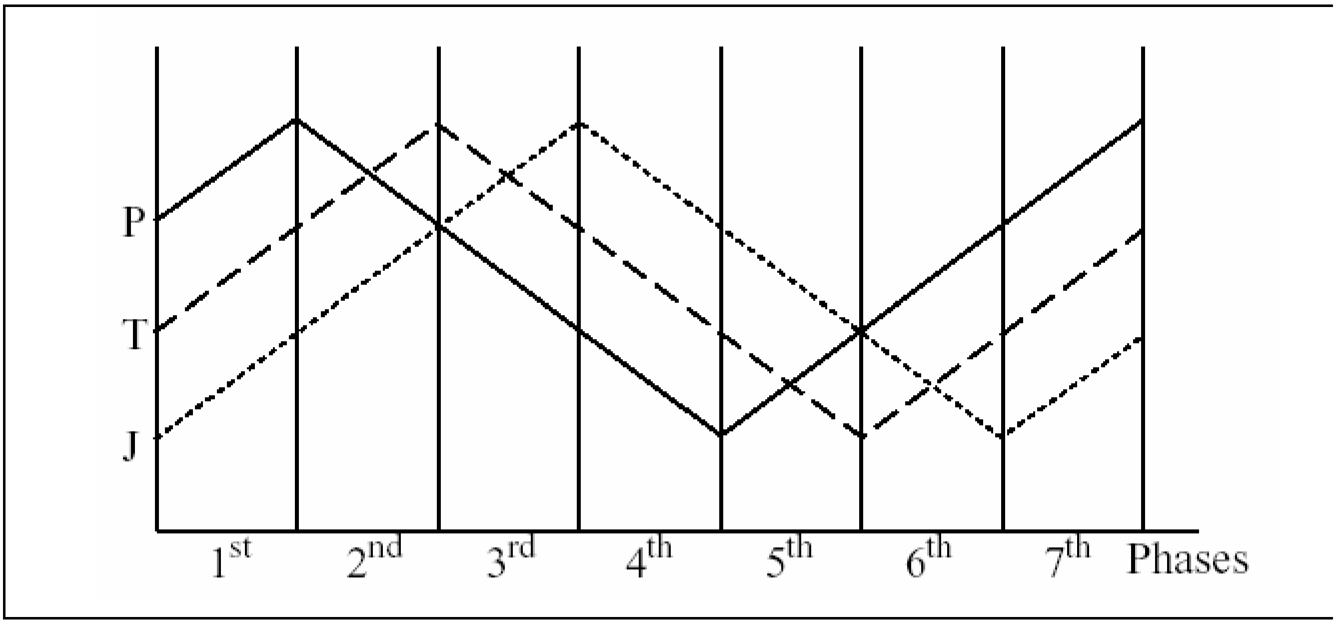
The bank reaction to the industrial action (a correlation between J and T'), by proceeding and guiding the movements which caused it, was ultimately a guiding action [12]. From the behaviour of the banking sector the central role played by interest rate manoeuvre emerged, representing a fundamental strategy for regulating output and the economy in general.

The interrelations which resulted as actions were triggered led to compensatory price movements. The consequent price trends, interest rates and outputs generated waves of expansion or contraction which characterised the entire economic cycle.

4. The phases of economic fluctuations

Our analysis departs from an initial state of activism (phase one) in the system (see fig. 1). Prices, output and the interest rate are inevitably rising and thus their potential curves have an upward path. However the price trend goes against the trend of the other curves. When output rises, prices fall, and similarly when the rate of interest rises, prices react by falling (the capitalisation of income law). In this phase, prices rise, up to a certain point at least.

Fig. 1: *The cyclical trend*



---

[12] The expansion of production called for new plant; this was likely to cause monetary tension, expected by the bank, which reacted by increasing the cost of money. Naturally the inverse arose in the opposite case.

Legend:

t: time
P: trade reaction (prices);
T: industrial reaction (output);
J: bank reaction (interest rate).

This conflicting situation cannot last: the price curve turns down and the system enters a phase of tension (phase two). At this phase analysis focuses on the industrial sector, since growth in the level of output, unlike the other two movements, cannot last indefinitely. Output tends to decrease, in view of falling prices, the effects of which are exacerbated by higher interest rates, in turn associated with greater costs. The end result is a downturn in the industrial output curve, leading us to the liquidation phase (phase three).

The process of liquidation boosts the demand for money, which in turn pushes up the interest rate: this contradicts with events in the industrial and trade systems, since the interest rate should fall. The turnaround is caused by the inevitable drop in the interest rate, so the system enters a phase of depression (the fourth phase). Thus

> Not even the recession is eternal. If plant is idle, it means that we are consuming hoarded goods (Amoroso, 1938: 8).

Nevertheless the fall in prices contradicts the reaction triggered by the drop in output and the declining interest rate. Thus prices subsequently increase, signalling the start of the recovery (the fifth phase). In the industrial sector the rising price trend conflicts with the low level of output and falling interest rates, and the result is an upturn (phase six). The abundance of money which results lasts until growth absorbs the new monetary savings

> Despite the contrast between the falling interest rate and the reactions that the growth in prices and output trigger in the banking system, when this conflict reaches the critical point the money curve turns down as well (Rossi 1963: 284).

Thus we witness a return to activism (seventh and first phases) and the cycle starts over again. Six successive phases in the cycle can be identified, as indicated in table 1.

Tab. 1: *Phases of the economic cycle*

	Speculation curve	Business Curve	Money Curve
I. Activism	rises	rises	rises
II. Tension	falls	rises	rises
III. Liquidation	falls	falls	rises
IV. Recession	falls	falls	falls
V. Recovery	rises	falls	falls
VI. Expansion	rises	rises	falls

Despite its simplicity and elegance, Amoroso's scheme was so restricted to mathematical conceptions that it dodged certain inevitable economic considerations. It is singular that with rising output prices tended to fall, just as they decreased when interest rates rose. Apart from the possibility of demand-pull inflation[13], prices could be affected by higher costs resulting from trade union disputes and raw material costs, and hence cost inflation: in the case in question wage inflation and imported inflation. Furthermore the existence of forms of monopoly could exert pressure on prices and fuel inflation. But even an increase in the interest rate on borrowed capital could have a negative impact on company finances, and companies may in turn pass on the higher costs in the form of higher prices. This approach may appear paradoxical if we consider that raising the interest rate is a key instrument of anti-inflationary monetary policy. Pressure on inflation could also result if the monetary authority were to neutralise the contraction in the money supply, maintaining it constant or even expanding it. Fiscal policy may also kindle inflation by influencing the level of development and income growth. The periods of stagflation and slumpflation witnessed in the 1970s (long after Amoroso's time) disproved this belief: sudden price increases were coupled with stagnation or falling output[14]. In Amoroso's view, price increases boosted output, while falling prices, which were generally considered positive, generated higher savings.

---

[13] In Amoroso's work, inflationary dynamics were related to the quantity of money in circulation, an independent variable. Deflation was related to the volume of commercial transactions, the result of a vast number of asymmetric movements, and certainly could not be considered an independent variable (Amoroso, 1950: 96).

[14] For an analysis about these particular situations, see among others, Nardi Spiller (2003).

5. From a cycle theory to a dynamic theory

Having traced an outline of his economic cycle theory, Amoroso never returned to the model which he considered more or less complete. In the late 1930s and early 1940s he resumed his initial research programme dedicated to the formal construction of a dynamic system comparable to Pareto’s static version. In his 1938 essay *La teoria matematica del programma economico [A mathematical theory of economic plans]* he offered an initial, partial treatment, which he later perfected with the publication in 1942 of his weighty *Lezioni di Meccanica Economica [Lectures on Economic Mechanics]*. Amoroso’s ideas received international acclaim in 1940 with the publication of an article in *Econometria*, *The Transformation of Value in the Productive Process.*

Amoroso’s starting point was his dissatisfaction with static general equilibrium theory. Although Pareto’s model was unrivalled, the theory was static and therefore detached from real phenomena. The greatest need for a dynamic vision was in the savings sector or in the accumulation of capital as we would define it today. In his words

> If there is one thing that characterises economic life and modern industrial society in particular, it is the continuous growth of plant, and of capital in general, the tool which enables man to prevail over nature. But new capital is generated by saving: hence the importance attributed to the process of saving in economic theory. […] This mutilation is grave, and authorises us to conclude that in this sector equilibrium theory is, by its very nature, *incapable of representing the core of this concrete phenomenon* (Amoroso, 1942: 119).

Such incongruencies emerged not only in the analysis of saving, but also in trade and distribution: here the assumption of constant prices, a requisite for static equilibrium, clearly conflicted with reality. Thus Amoroso considered that if economics were to be an empirical science, static treatments of the economy had to be abandoned in favour of dynamic representations of concrete economic phenomena.

A rule in economics in general, and in the study of dynamic phenomena in particular, is that every theoretical development brings with it new analytical tools. In developing his new theory Amoroso introduced significant analytical innovations, namely the calculus of variations, once again derived from rational mechanics. Within a genuinely intertemporal context, traditional methods of statistical optimisation were inadequate, and there was a need for a mathematical tool which took account of the fact that variables alter an established path in the long term. The optimal solution to dynamic problems was not a vector representing the chosen basket of goods but rather a function describing the optimum path of the basket of goods in the relevant time period. The calculus of variations was a mathematical tool suited to the analysis of this type of problem. Amoroso adhered strictly to Pareto’s methodological principles in proposing a microeconomic theory of dynamic choice, in terms of the individual consumer and the individual firm. In the following paragraph we will examine consumption choices, and subsequently we will turn to production choices which allowed Amoroso to take the analogy between physical and economic phenomena to the extreme.

In the dynamic model the approach to consumer choice differed from the traditional approach, in so far as the choice variable was not represented by a single basket but rather by a consumption path indicating the optimum basket at each moment in time. The solution proposed by Amoroso as early as his 1938 paper was to increase the number of elements in the traditional utility function, which he related not only to the vector of quantities consumed x, but also to its variations in a single moment of time, $\dot{x}$ (where the dot over the variable denotes its derivative with respect to time). The latter term thus indicated acceleration in the flow of consumption, for which Amoroso offered a psychological explanation: individuals were constrained their habits which changed slowly over time. The dynamic utility function thus became

$$U = U(x, \dot{x}) \qquad [1]$$

Amoroso called [1] Lagrangian ophelimity, to distinguish it from Pareto ophelimity. It could well have been considered an extension of the latter: if $\dot{x} = 0$ the two utility functions coincide and there was a return to the static case.

Rational consumers attempted to determine the maximum value of [1] subject to constraints such as the vector of prices and income allocated within the time period under consideration. In mathematical terms, this involves establishing the maximum value of the following functional:

$$\int_{t_0}^{t_1} U(x, \dot{x})dt \qquad [2]$$

The determination of the optimum trajectory implied that in a given time period the integral denoted by [2] was a typical problem of the calculus of variations. It could be solved by taking into account the maximum conditions that confirmed Euler's theorem:

$$\frac{\partial U}{\partial x} - \frac{d}{dt}\left(\frac{\partial U}{\partial x'}\right) = 0 \qquad [3]$$

Amoroso labelled this expression marginal Lagrangian ophelimity. The first term in [3] represented traditional marginal utility[15], whereas the second, which captured the dynamic element, represented a loss of utility due to the inertial forces present in the system. If consumers had to allocate their income between several goods, the equilibrium conditions implied that maximum utility would be achieved when Lagrangian marginal utilities were equal.

Two aspects of Amoroso's dynamic theory of consumption are worthy of particular attention. On the one hand, the mathematical nature of the problem was well identified; on the other the model offered a common-sense explanation. The final result was an extension of the traditional rule of levelling marginal utility, totally disrupted by what we might call inertial resistance.

Amoroso's procedure lent itself to further interpretation from a macroeconomic standpoint: this became popular in the 1960s within the context of optimum growth theory models. Amoroso himself wrote an essay in 1938: [3] became the fundamental equation in a theory of economic planning in which the state endeavoured to identify the means required to progress from an initial configuration to a previously determined result. Amoroso intended to overturn the economic logic underlying the authoritarian policy makers, while in the Unites States attention focused on more effective economic policies to sustain growth, albeit within the same analytical framework. Amoroso abandoned this vision of a dynamic process, so alien to his intellectual style, as early as 1942, and returned firmly to the market and the forces of supply and demand.

6. An energetic approach to the production process

---

[15] Basically changes in total utility.

Amoroso's 1942 lectures did not only produce a formal solution to the problem of rendering Pareto's system dynamic. Amoroso also achieved one of the most important results of his entire research programme by demonstrating the complete analogy between economic and mechanical phenomena. Whilst in his 1921 lectures he presented a basic outline of the analogies between pure economics and rational mechanics based on Pareto and Fisher before him (1886), in his 1942 lectures Amoroso demonstrated a deeper link by means of an energetic interpretation of economic action in the field of production. Within the sphere of production it was possible, with slight adjustments and by taking certain differences into account, to detect the workings of the principle of energy conservation, one of the fundamental principles of classical dynamics.

The question at the base of Amoroso's research was the very same that Fisher posed in his doctoral thesis in 1882: in economic sectors to what extent did equivalent concepts to the concept of energy exist? Fisher limited his treatment to static chaos in consumption (Mirowski, 1989), and concluded that utility could be interpreted as kinetic energy; budget constraints, assuming fixed prices, could be interpreted as a form of kinetic energy. In the process of rational choice, the principle of energy conservation assured that their sum was constant. In other words economic action was subjected to the action of conservative forces (marginal utilities) the final result of which was independent of time. Amoroso extended this approach to production:

> Is it possible to combine the economic concept of value and the mechanical concept of energy in such way as to link the transformation (of value) which is effected in the productive process to the transformation (of energy) which is displayed in the mechanical process? (Amoroso, 1940: 6).

In order to identify the principal of energy conservation in the field of production it is necessary to define the two variables in question: potential energy and kinetic energy.

Here we follow Amoroso's ingenious procedure, which introduced the new variable of potential, V. He observed how each industrial plant has a configuration which enables it to maximise production, M_S. If the quantities actually produced are denoted x_S, the initial value of production (with q_S denoting the given vector of prices) becomes:

$$V = e^{-rt}\int q_S(M_S - x_S)dt \qquad [4]$$

If we denote total costs as Θ and the maximum potential as V_0 (when $q_s = 0$) [4] can be expressed alternatively to make it more congenial to an energetic interpretation:

$$V + \Theta = e^{-rt} V_0 \tag{5}$$

The difference between the maximum level of production and actual production can be interpreted in physical terms as a drop in potential, the competitive market value of which is exactly measured from total cost. In the simpler case of a steady state, the right-hand term in [5] is constant and the differential expression becomes:

$$d\Theta = dV \tag{6}$$

On the basis of [6], we can interpret marginal cost $d\Theta$ (with the exception of the sign) in energetic terms as a fall in potential with respect to V, which in economic terms represents the highest possible level of production, and in physical terms the potential energy in the system.

To complete the energy metaphor in the economic interpretation we need to add a second variable with the characteristics of kinetic energy. Amoroso singled out the monetary value of production, total income T. The problem faced by the firm was thus to maximise the integral:

$$I = \int (T - \Theta) dt \tag{7}$$

Considering that income depends not only on the quantity sold but also on changes in the latter, just as in the case of consumption we obtain the following expression:

$$\frac{\partial \mathrm{T}}{\partial \mathrm{x}} - \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{x}'}\right) = \frac{\partial \Theta}{\partial \mathrm{x}} \tag{8}$$

According to [8] marginal income, appropriately redefined, is the same as marginal cost. In the static case which interested Amoroso, the second part of the first term cancels out and becomes:

$$\frac{\partial T}{\partial x} = \frac{\partial \Theta}{\partial x} \qquad [9]$$

After these analytical distortions, Amoroso was able to bring the economic explanation closer to the physical one. Thanks to [9], equation [6] can be redefined as follows:

$$dT + dV = 0 \qquad [10]$$

Or in more general terms:

$$U + V = Cost. \qquad [11]$$

The latter equation implies that in the event of competition, such that price equals marginal utility, the theorem of energy conservation holds true. The value of production represents kinetic energy, while costs represent potential energy. The mechanical principle of energy conservation, which was so important in nineteenth century mechanics, can also be found in the field of economic processes. In Amoroso's words

> If the prices are supposed to be invariable and if the effect of interest is neglected, the transformation in value which is affected in the productive cycle may be interpreted in the sense of the sum of the values expressed by the potential energy of the plant and of the kinetic energy is constant. This means that every variations of cost up or down- must find its exact equivalent in the increase or in decrease of the latent product [marginal cost] (Amoroso, 1940: 9).

In Amoroso's view the rule of equality between marginal cost and marginal income merely denoted the result of rational behaviour: it indicated that in the process of transformation from one form of energy to the other, nature itself acted in the same way as a producer, and that in a steady state the main aim was to maximise the efficiency of production (in terms of energy).

In his 1942 lectures, just as in the article published in *Econometria* in 1940, Amoroso drew the final conclusions of his work on dynamic analysis. At an international level, no further mention was made of energetic interpretations of economic production theory until Samuelson and Solow's essay in 1956. The latter authors attempted to demonstrate the correspondence between the principle of energy conservation and optimum capital theory. Sporadic references were also found in literature on optimum growth (Shell, 1969) in the late 1960s, but in later years the theme of the analogy between economic and mechanical phenomena failed to arouse scholarly interest.

7. The limits of Amoroso's research on economic dynamics

Having examined the salient features of Amoroso's research, we are now ready to appraise its power and validity. Despite the originality and interest of the work, Amoroso undoubtedly encountered a number of difficulties of both a theoretical and an interpretative nature. Combined with other, external factors, these difficulties undermined the importance of the work both in Italy and abroad.

Dominedò highlighted one of the model's first weaknesses as early as 1942. However due to wartime publishing difficulties he succeeded in publishing his paper only much later in 1966. The author observed that the analogy between economic phenomena and mechanical phenomena as a result of energy conservation was not in itself an appropriate foundation for economic dynamics, but was simply an expedient used for the purpose of argumentation, almost a tribute to the nineteenth century tradition which considered economics a science only in so far as it resembled rational mechanics. Leaving aside certain intrinsic difficulties involved in the comparison of the traditional categories of classical mechanics with those of economic reasoning, acknowledged by Amoroso himself, the real problem lay in the fact that the analogy between the two disciplines was only valid in a steady state, in which all variables were by definition constant. In the light of this observation Dominedò underlined that the principle of energy conservation did not in any way further the understanding of economic phenomena. What it did provide was an interesting, albeit limited, physical metaphor: production was seen as a form of energy transformation. The plan to arrive at a dynamic theory by this route was wholly unsatisfactory: other paths, such as the role of expectations, needed to be pursued.

Dominedò's criticism concerning the irrelevance of the physical interpretation of economic phenomena leads us directly to a second critical aspect of Amoroso's work. Even its sophisticated analytical formulation based on the calculus of variations failed to offer a solution to Amoroso's original problem: the formulation of a theory of saving and capital accumulation. In the 1942 model the consumer chose between numerous goods, evaluating the present benefit and future cost of his habits: no particular role was assigned to saving. However if there is no trade-off between consumption and saving, which the neoclassical model automatically converted into investment spending, it becomes impossible to arrive at a theory of economic growth capable of accounting for the increasing accumulation of capital. It is no coincidence that this aspect was central to growth theory in the 1960s, which started not from Amoroso but from Ramsey-Keynes' genuinely macroeconomic perspective of the previous decade.

Why did Amoroso fail to arrive at a formal theory of growth, and hence capital accumulation? Capital accumulation was the linchpin of the theory of long-term economic dynamics, and Amoroso possessed all the necessary analytical tools. One possible answer is that he was firmly bound to a Paretian approach to general equilibrium theory in terms of a set of interrelated markets. In his 1942 lectures Amoroso was satisfied that from the principle of equality between Lagrangian ophelimity of consumption and of production it was possible to arrive at a set of supply and demand curves which intersected in the market thereby determining the relevant variables, the price system and equilibrium quantities. Since in a Paretian approach, the number of equations equals the number of unknowns, the system was defined and Amoroso considered his task to be done. However without a genuinely macroeconomic framework, Amoroso's dynamic framework remained an ingenious abstraction void of any practical or operational relevance. This is why it was no longer cited, except by his scholar Palomba (1956).

8. Concluding Remarks

The formulation of a complete theory of economic dynamics was one of the tasks left to future generations by the first generation of marginalist economists. Among those who took up the challenge in Italy, Amoroso deserves pride of place: he was considered a pioneer, both in terms of his analytical techniques and for certain interpretations he offered during the golden days of dynamic theory in the 1950s and 1960s.

The orthodox doctrine of economic equilibrium hinges on the assumption that equilibrium is reached instantly, and that once it has been reached it endures indefinitely, on condition that the underlying conditions remain constant over time. Amoroso endeavoured to arrive at a dynamic theory of equilibrium, based on the reciprocal actions and reactions in the industrial, trade and banking sectors. Forces tend to evolve, generating opposing trends, and the dynamics in an economy are thus the result of reciprocal actions and reactions in the three sectors.

Amoroso's conception was grounded in corporative economics, and this induced him to emphasise state intervention. However he failed to include the latter as a corrective measure with which to counteract cyclical dysfunctions in the economy. It was not until the advent of Keynesian deficit spending that certain anomalies would be overcome.

Despite his considerable intuition and outstanding analytical skills, Amoroso never succeeded in going beyond Paretian schemes and arriving at a macroeconomic view of general economic equilibrium from a dynamic relationship between savings and investment, whether collective as in neoclassical tradition or capitalist as in the classical tradition. It was from this relationship that post-war formalizations of growth theory departed. In seeking to arrive at an alternative dynamic theory to the Ramsey-Keynes approach, Amoroso was beset by numerous intrinsic difficulties which hindered his ambitious analytical and epistemological research programme. He was thus identified as a successor to the Paretian tradition rather than as one of the founders of neoclassical economic dynamics, and alas not only in Italy.

Selected Bibliography

Amoroso L. (1932a) *Contributo alla teoria matematica della dinamica economica,* Nuova Collana degli Economisti, vol. v. Turin: UTET, 1932.

Amoroso L. (1932b) *Economia e politica* in Università degli Studi di Roma, *Annuario per l'Anno Accademico 1932-33.* Rome: Tipografia Pallotta, 1932.

Amoroso L. (1933a), *Il problema demografico in Italia e nel mondo*, in AA.VV., *Popolazione e fascismo*, Parte Prima, «L'economia italiana», Anno XII, December, 19-21.

Amoroso L. (1933b), *La dinamica dei prezzi* in L. Amoroso, *Ciclo, circolazione, politica monetaria*, ed. L. Venturi, introduction by S. Vinci. Turin: UTET: 1-76.

Amoroso L. (1938), *Principii di economia corporativa*, Bologna, Nicola Zanichelli Editore.

Amoroso L. (1940) 'Riflessioni sulla dinamica dei prezzi', *Rivista Italiana di Scienze Economiche,* May: 583-589.

Amoroso L. (1942), *Meccanica economica.* Città di Castello: Macrì.

Amoroso L. (1950), *Riflessioni sulla politica monetaria* in L. Amoroso, *Ciclo, circolazione, politica monetaria*, ed. L.Venturi, introduction by S. Vinci. Turin: UTET, 91-104.

Amoroso L. (1961),'Si livellano i valori azionari al saggio d'interesse di mercato?', *Economia Internazionale*, 14/2: 3-13.

Amoroso L. (1999), *Ciclo, circolazione, politica monetaria*, ed. L.Venturi, introduction by S. Vinci. Turin: UTET.

Buscarino G., 1959, *Le matematiche e l'economia politica nel pensiero scientifico moderno,* Milano, Giuffrè.

Demaria G. (1935), *Osservazioni sulla teoria statistica delle serie dinamiche. Giornale degli Economisti*, 8: 1-45.

Fanno M. (1956), *La teoria delle fluttuazioni economiche.* Turin*:* UTET.

Faucci R. (1990), *Materiali e ipotesi sulla cultura economica italiana fra le due guerre mondiali,* in G. Becattini, *Temi, problemi e scuole.* Turin: UTET: 183-231.

Keynes JM. (1936), *Teoria generale dell'occupazione, dell'interesse e della moneta e altri scritti,* ed. A. Campolongo. Turin: UTET,1978.

Nardi Spiller C. (1990), *Struttura produttiva e dinamica dei prezzi,* introduction by M. Baranzini. Padua: Cedam.

Nardi Spiller C. (2003), *The Dynamics of the Price Strucuture and the Business Cycle. The Italian Evidence from 1945 to 2000*, Heidelberg-New York, Physica-Verlag.

Palomba G. (1966), 'Il pensiero scientifico di Luigi Amoroso', *Rivista di Politica Economica*, 56/2: 387-421.

Pareto V. (1896-97), *Corso di economia politica*. Turin: Einaudi, 1942.

Pareto V. (1906), *Manuale di economia politica.* Milan: Soc. Ed. Libreria. Reprinted from the French edition with an introduction by L. Amoroso. Rome: Bizzarri, 1965.

Pareto V. (1973), *Epistolario 1890-1923*, 2 vols., ed. G. Busino. Accademia Nazionale dei Lincei: Roma.

Rossi L. (1963), *Elementi di economia*, vol. 1: *L'economia politica.* Padua: Cedam.

Vinci F. (1934), 'Significant Developments in Business Cycle Theory', *Econometrica*, April: 136-139.

Vinci F. (1940) 'Prezzi e scorte', *Econometrica*, August: 1031-1037.

Vinci S. (1999), *Introduzione*, in L.Amoroso, 1999, *Ciclo, circolazione, politica monetaria*, ed. L. Venturi, introduction by S. Vinci. Turin: UTET: XIII-XXVIII.