Do redistributive schemes reduce inequality between individuals?

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Abstract

Redistribution schemes (taxes or benefits) are generally performed at the household level. The issue is to know whether intra-household inequality magnifies or hampers the redistributive effect of the transfers, when the policy-maker focuses on the inequality at the individual level. Depending on the type of the transfer, three properties capturing the idea that the more wealthy the household is, the more unequally it behaves, have been shown to matter. In the *moving away* approach, the deviation with the equal split make a difference, in the *star-shaped* approach, the average share counts while the marginal share is relevant for *concavity.* We complete the analysis by showing how these properties of the intra-household allocation may be recovered through a bargaining model of the household. Then, the DARA and DRRA properties of the utility function emerge as the key conditions for the recovery.

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1 Introduction

Whenever the (relative) Lorenz criterion is adopted for inequality comparisons, a progressive tax schedule guarantees that the post-tax income distribution is at least as egalitarian as any initial income distribution (Jacobsson [7]). This inequality-reducing property of income tax systems has been deeply analyzed in the literature (see for instance, Eichhorn et al. [5], Le Breton et al. [9]). Nevertheless, its ultimate effect is still unknown, when we deal with inequality among *individual*s. In fact, taxation is usually performed at the level of households, despite the fact hat discrimination in the resources allocation (e.g. money or leisure time) within household members is present in many societies (Sen [16], Haddad and Kanbur [6], Anand and Sen [1]). The important question which then arises is whether intra-household inequality magnifies or hampers the redistributive effect of taxation, when the policy-maker focuses on inequality at the individual level.

In this paper we retain the setup of Peluso and Trannoy ([12]), considering a population of couples where each household contains a *dominant* individual and a *dominated* one. The dominant receives more than one half of the family budget while the dominated receives less than one half. In the first part of the paper, the intra-household allocation is described through a *sharing function* which can be considered as a reduced form of a structural model of the allocation among family members. The sharing function gives the income received by the *dominated* type as a function of the household income: To study the properties of the sharing function is tantamount to analyze the way intra-household inequality is related to household income. We analyze the effectiveness of redistributive policies designed at the household level on the inequality among individuals. Our results illustrate that when the domain of redistributive policy schemes shrinks, the class of sharing functions which keep the inequality-reducing effect of progressive taxation becomes larger. We first show that concavity of the sharing functions is both necessary and sufficient to extend the effects of any inequality-reducing redistributive scheme among couples to the individual level. Since empirical evidence are far from confirming that a strong property as the global concavity of sharing functions is really satisfied (see Kanbur and Haddad [8], Couprie et al. [3]), we proceed by looking for redistributive policies which are inequality-reducing for individuals, under less demanding conditions on the curvature of the sharing function . By focusing on some important cases such as a *poll-tax* and a *flat tax*, which are "neutral" − in the sense that they do not affect inequality among households when Absolute (respectively Relative) Lorenz criterion are used for inequality comparisons) − some clear-cut results may be established. Indeed, these basic taxation schemes reduce the inequality at the level of individuals for classes of sharing functions larger than the concave one: respectively the *moving away* class, (for which the deviation of individual expenditures with the equal split increases) and the *star-shaped* class (where the average share received by the dominated type is non-decreasing). These results are consistent with the general intuition that the pattern of individual inequality is the same as that of household inequality, when the poorer the household is, the more egalitarian it behaves. The purpose of the second part of the paper is to recover such classes of sharing functions starting from the primitives of one of the canonical model of family decision making. By considering a symmetric Nash bargaining model between the two spouses, it is shown that the shapes of the sharing function mentioned above are induced by some well-known properties of the individual utility functions under risk, as DARA (decreasing absolute risk aversion) and DRRA (decreasing relative risk aversion).

2 Basic concepts

We consider a population composed of n households indexed by $i = 1, ..., n$, with $n \geq 2$. Let $\mathbf{y}$ be a generic vector of strictly positive household incomes, ordered in an increasing way. The feasible set of $\mathbf{y}$ is $\mathbb{Y}_n = \left\{ \mathbf{y} \in \mathbb{R}^n_+ \mid a \leq y_1 \leq y_2 ... \leq y_n, \text{ for some } a > 0 \right\}$. The unit vector in $\mathbb{R}^n_+$ is denoted $\mathbf{e}_n$. The quasi-orders we consider for inequality comparisons are the *Lorenz* (L) criterion, the *Relative Lorenz* (RL) and the *Absolute Lorenz* test (AL). For the sake of completeness, we recall the definitions.

Definition 1 *Given* $\mathbf{y},\mathbf{y}'\in\mathbb{Y}_n$,

i) $\mathbf{y}$ *dominates* $\mathbf{y}'$ *according to the Lorenz criterion (L), denoted by* $\mathbf{y}\succcurlyeq_L\mathbf{y}'$, *if*

$$\sum_{i=1}^{k} y_i \geq \sum_{i=1}^{k} y_i', \text{ for } k=1,.,n \text{ and } \mu_{\mathbf{y}}=\mu_{\mathbf{y}'}.$$

ii) $\mathbf{y}$ *dominates* $\mathbf{y}'$ *according to the Relative Lorenz criterion (RL), denoted by* $\mathbf{y}\succcurlyeq_{RL}\mathbf{y}'$, *if*

$$\frac{1}{n}\sum_{i=1}^{k}\frac{y_i}{\mu_{\mathbf{y}}} \geq \frac{1}{n}\sum_{i=1}^{k}\frac{y_i'}{\mu_{\mathbf{y}'}}, \text{ for } k=1,.,n.$$

iii) $\mathbf{y}$ *dominates* $\mathbf{y}'$ *according to the Absolute Lorenz criterion (AL), denoted by* $\mathbf{y}\succcurlyeq_{AL}\mathbf{y}'$, *if*

$$\frac{1}{n}\sum_{i=1}^{k}(y_i-\mu_{\mathbf{y}}) \geq \frac{1}{n}\sum_{i=1}^{k}(y_i'-\mu_{\mathbf{y}'}), \text{ for } k=1,..,n.$$

We adopt a very comprehensive definition of redistributive taxation schemes, borrowing some concepts from Le Breton et al. (1996).

Definition 2 *A redistributive scheme is a mapping* $G:\mathbb{R}_+\to\mathbb{R}_+$,*that associate a post-redistribution income* $G(y)$ *to the pre-redistribution income* y.

Let $\mathcal{G}$ be the general set of redistribution schemes defined on $\mathbb{R}_+$.

A reasonable property of a redistributive scheme is that it does not affect the ranking of individuals in the pre-tax income distribution (see Le Breton et al. [9] for a discussion). Usually, we also impose a notion of *consistency,* excluding the case of redistribution of some external wealth. We give more formal definitions of these concepts.

Definition 3 *A redistributive scheme* $G\in\mathcal{G}$ *is*

- *rank-preserving if* G *is non-decreasing over* $\mathbb{R}_+$.

- *consistent if* $\sum G(y_i)\leq\sum y_i$, *for any* $\mathbf{y}\in\mathbb{Y}_n$

- *progressive if* $G(y)/y$ *is non-decreasing over* $\mathbb{R}_{++}$

Let us call $\mathcal{G}^*$ the set of rank-preserving, consistent and progressive redistributive schemes on $\mathbb{R}_{++}$. In general, a redistribution scheme can be the result of many different policies. Among the redistributive schemes, some present salient properties.

Definition 4 *A redistributive scheme $G \in \mathcal{G}$ is*

- *a taxation scheme if $G(y) \leq y$, for any $y \geq 0$*
- *a benefit scheme if $G(y) \geq y$, for any $y \geq 0$.*
- *linear if $G(y) = (1-t)y + q$, for $t \in [0,1]$.*

We will consider some important cases of linear taxation: if $q = 0$, then G is a *flat tax.* If $t = 0$, when $q < 0$ we have a *poll tax*, when $q > 0$ a *basic income.*

2.1 Sharing Functions

We follow here the setup of Peluso and Trannoy [12]. Without loss of generality, we consider a population of couples and we suppose that each household is composed of two equally needy individuals. In spite of their equal need, the intra-household allocation is unequal. A *dominated* individual, receives at most an income share equal to the share received by the *dominant* one. Thus dominant individuals are the 'rich' within the household and the dominated are the 'poor'. Let $p_i = f_p^i(y_i)$, be the amount received by the dominated individual in household i. The amount r_i received by the dominant is defined by $r_i = y_i - f_p^i(y_i)$.

Given a vector $\mathbf{y}$ of household incomes, $\mathbf{p}(\mathbf{y}) = (p_1, .., p_j, .., p_n)$ designates the income vector for dominated individuals, $\mathbf{r}(\mathbf{y}) = (r_1, ..., r_j, ..., r_n)$ the income vector for dominant individuals, and $\mathbf{x}(\mathbf{y}) = (\mathbf{p}(\mathbf{y}), \mathbf{r}(\mathbf{y}))$.

The sharing functions f_p^i are the same among households, that is, a common bias due to a social norm induces a homogeneous intra-household discrimination in the population considered.

Assumption 1 *The functions $f_p^i : D \rightarrow \mathbb{R}_+$ are identical across households and such that $f_p(y) \leq \frac{1}{2}y$.*

Let us designate by $\mathcal{F}$ the set composed of sharing functions satisfying Assumption 1 above. A first remark confirms the rather obvious intuition that a lower intra-household discrimination implies a more equally income distribution among individuals.

Remark 1 *Let f_p and $f_p' \in \mathcal{F}$.*

$$f_p(y) \geq f_p'(y) \ \forall y \in \mathbb{R}_+ \iff \mathbf{x}(\mathbf{y}) \succcurlyeq_L \mathbf{x}'(\mathbf{y}) \text{ for all } \mathbf{y} \in \mathbb{Y}_n.$$

Proof. $\Longrightarrow$ If we compare the individual incomes generated by the different sharing functions f_p and f_p' for the household i, then the second distribution can be deduced from the first one through a sequence of regressive transfers. Therefore

$$(f_p(y_i), f_r(y_i)) \succcurlyeq_L (f_p'(y_i), f_r'(y_i)). \tag{1}$$

Moreover, given that (1) holds for every household, applying Proposition A.7 (i), p. 121 of Marshall-Olkin [10], we obtain $\mathbf{x}(\mathbf{y}) \succcurlyeq_L \mathbf{x}'(\mathbf{y})$.

$\Longleftarrow$ Suppose by contradiction $f_p'(a) > f_p(a)$ for some $a > 0$. By considering the distribution $\mathbf{y} = (a, ..., a)$, it is easy to see that $\mathbf{x}(\mathbf{y})$ may be obtained from $\mathbf{x}'(\mathbf{y})$ through some regressive transfers and then $\mathbf{x}(\mathbf{y}) \succcurlyeq_L \mathbf{x}'(\mathbf{y})$ is false. ■

There are two ways to capture the idea that the situation of the dominated weakens when the household income increases: we may think either in *relative* or in *absolute* terms.

To catch the *relative* point of view, let us define $\mathcal{S}_\infty$ as the subset of $\mathcal{F}$ of *star-shaped at* $+\infty$ sharing functions, composed of functions satisfying: $f_p(\alpha y) \geq \alpha f_p(y)$, $\forall \alpha \in [0, 1]$ and $\forall y \in \mathbb{R}_+$. Within this class, the income share of the dominated is decreasing with household wealth, i.e. $\dfrac{f_p(y)}{y}$ decreases with y for $y > 0$.

The opposite case, $f_p(\alpha y) \leq \alpha f_p(y)$, $\forall \alpha \in [0, 1]$ and $\forall y \in \mathbb{R}_+$ describes *star-shaped at* 0 sharing functions.

In order to apprehend the *absolute* point of view, we introduce the class $\mathcal{MA} \subset \mathcal{F}$, composed of *moving away* functions. Namely, $\forall a, b \in \mathbb{R}_+$ with $a < b$, $f_p(b) - f_p(a) \leq \dfrac{b-a}{2}$. In order to give an interpretation of this class, let us consider the deviation between the equal split income and the amount devoted to the 'poor guy', $\dfrac{y}{2} - f_p(y) = \psi(y)$. Requiring ψ to be non-decreasing is tantamount to restrict its attention to the $\mathcal{MA}$ class.

A *moving closer* sharing function goes on the opposite way. The division of the family cake is moving closer to the equal split as the household income increases along.

A result of this paper is related to a third set $\mathcal{C} \subset \mathcal{F}$ composed of concave functions. The functions of $\mathcal{C}$ are continuous and non-decreasing (see Moyes [11], Lemma 3.2). On the opposite, classes $\mathcal{S}_\infty$ and $\mathcal{MA}$ allow for non-monotonic sharing functions. An illustration is provided below.

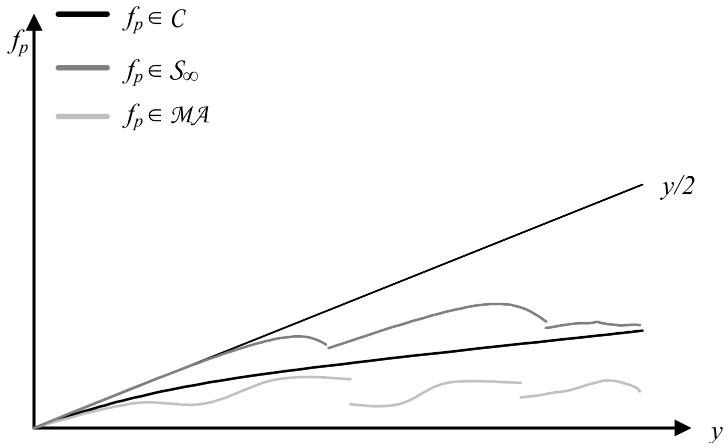

Figure 1: The classes of sharing functions

It is well established that $\mathcal{C}$ is a subset of $\mathcal{S}_\infty$ (see Marshall and Olkin [10] p. 453). We prove the following remark:

Remark 2 $\mathcal{S}_\infty \subset \mathcal{MA}$.

Proof. Suppose by contradiction that f_p does not belong to $\mathcal{MA}$. Then, for some $a, b \in \mathbb{R}_{++}$, with $a < b$, we get $f_p(b) - f_p(a) > \dfrac{b-a}{2}$. This is equivalent to:

$$b(\frac{1}{2} - \frac{f_p(b)}{b}) < a(\frac{1}{2} - \frac{f_p(a)}{a}). \tag{2}$$

Both $(\frac{1}{2} - \frac{f_p(b)}{b})$ and $(\frac{1}{2} - \frac{f_p(a)}{a})$ are non-negative by Assumption 1. If they are both positive, since $b > a$, we get $\frac{1}{2} - \frac{f_p(b)}{b} < \frac{1}{2} - \frac{f_p(a)}{a}$. Then $\frac{f_p(b)}{b} > \frac{f_p(a)}{a}$, that contradicts $f_p \in \mathcal{S}_\infty$. If $\frac{1}{2} - \frac{f_p(a)}{a} = 0$, from (2) we get $\frac{b}{2} < f_p(b)$, which is impossible. Finally, if $\frac{1}{2} - \frac{f_p(b)}{b} = 0$ and $f_p \in \mathcal{S}_\infty$, then $\frac{1}{2} - \frac{f_p(a)}{a} = 0$ and (2) gives $0 < 0$. ■

3 Redistributive schemes and inequality among individuals

In this section, we investigate the main effects of redistributive policies performed at the household level on the individual inequality. The properties of the sharing function play a decisive role. The following proposition establishes that a concave sharing rule extends the inequality-reducing properties of a progressive scheme to the individual level .

Proposition 1 *Let* $f_p \in \mathcal{F}$*.*

$f_p \in \mathcal{C} \iff \forall\ \mathbf{y} \in \mathbb{Y}_n,\ \forall G \in \mathcal{G}^*,\ \mathbf{x}(G(\mathbf{y})) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y})$.

Proof. $\Longrightarrow$ A direct consequence of Corollary 1 in Peluso and Trannoy [12] and Proposition 3.1 in Le Breton et al. [9].

$\Longleftarrow$ Assume by contradiction that f_p is not concave. From a Yaari's lemma (see [17], Lemma p.1184) it follows that for some $y^* \in \mathbb{R}_{++}$, there exists $\zeta > 0$ such that, for any ε with $0 < \varepsilon < \zeta$

$$2f_p\,(y^*) < f_p(y^* - \varepsilon) + f_p(y^* + \varepsilon). \tag{3}$$

Furthermore, (3) combined with $f_p(y^*) \leq \frac{1}{2}y^*$ implies $f_p(y^*) - f_r(y^*) < 0$.[1] Then, by continuity, there exists $\bar{\zeta} > 0$ such that, for every ε satisfying $0 < \varepsilon < \bar{\zeta}$, (3) holds and

$$f_p(y^* + \varepsilon) < f_r(y^* - \varepsilon). \tag{4}$$

We now choose $\mathbf{y} = (y_1, .., y_{n-2},\ y^*, y^*)$ and $\mathbf{y}' = (y_1, .., y_{n-2},\ y^* - \varepsilon, y^* + \varepsilon)$, such that $f_r(y_{n-2}) \leq f_p(y^* - \varepsilon)$. By construction, $\mathbf{y} \succcurlyeq_{RL} \mathbf{y}'$. From (3), we deduce $f_p\,(y_{n-1}) + f_p\,(y_n) < f_p(\ y'_{n-1}) + f_p(y'_n)$ which gives, combined with (4), $\frac{1}{n\mu}\sum_{j=1}^{2n-2} x_j < \frac{1}{n\mu}\sum_{j=1}^{2n-2} x'_j$. Hence, $\mathbf{x}(\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}')$ is contradicted. ■

A policy maker can ignore intra-household inequality when designing a redistribution scheme at the household stage if and only if the sharing function is concave. The rationale behind this result may be captured from the proof above: without a concave sharing function, the benefit received by a poor household further to a transfer from a richer household may increase the

[1] Indeed, by assumption $f_p(y^*) \leq f_r(y^*)$. Suppose now that $f_p(y^*) - f_r(y^*) = 0$. This is equivalent to $f_p(y^*) = \frac{1}{2}y^*$. From (3) and $f_p(y) \leq \frac{1}{2}y\ \forall y$, we get $y^* < f_p(y^* - \varepsilon) + f_p(y^* + \varepsilon) \leq y^*$, which is impossible.

inequality among individuals. Less restrictive results may be obtained whenever we focus on taxation schemes, excluding the possibility of benefits. In the following proposition, we show that, under sharing functions star-shaped at ∞, a flat-tax at the household level reduces the inequality among individuals.

Proposition 2 *Let* $f_p \in \mathcal{F}$

$$f_p \in \mathcal{S}_\infty \iff [\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}), \forall\alpha \in [0,1] \text{ and } \forall \mathbf{y} \in \mathbb{Y}_n] .$$

Proof. $\Longrightarrow$ By assumption $f_p(\alpha y) \geq \alpha f_p(y)$, $\forall\alpha \in [0,1]$ and $\forall y \geq 0$. This implies that for every household i

$$(f_p(\alpha y_i),\ f_r(\alpha y_i)) \succcurlyeq_{GL} (\alpha f_p(y_i),\ \alpha f_r(y_i)) . \tag{5}$$

Since (5) holds for any household i, by applying Proposition A.7 (iii), p. 121 of Marshall-Olkin [10], we obtain $\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{GL} \alpha\mathbf{x}(\mathbf{y})$. Dividing both vectors by $\alpha\frac{\mu_y}{s}$, we get: $\dfrac{\mathbf{x}(\alpha\mathbf{y})}{\alpha\frac{\mu_y}{s}} \succcurlyeq_{GL} \dfrac{\mathbf{x}(\mathbf{y})}{\frac{\mu_y}{s}}$, equivalent to $\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y})$.

$\Longleftarrow$ Suppose that $f_p \notin \mathcal{S}_\infty$. Then $\exists\alpha \in (0,1)$ and $\exists y^* > 0$, such that: $f_p(\alpha y^*) < \alpha f_p(y^*)$. By picking $\mathbf{y} = (y^*, .., y^*)$ and $\mathbf{y}' = \alpha\mathbf{y}$, we have $\mathbf{y}' \succcurlyeq_{RL} \mathbf{y}$ by construction and $\mu_{\mathbf{y}} = \frac{\mu_{\mathbf{y}'}}{\alpha}$. Since $\frac{f_p(\alpha y^*)}{\frac{\alpha y^*}{n}} < \frac{f_p(y^*)}{\frac{y^*}{n}}$, then $\sum_{j=1}^{n/2} x_j < \sum_{j=1}^{n/2} x'_j$. Hence $\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y})$ does not hold. ∎

In particular, it may be observed that the inequality among individuals is strictly reduced for all non-linear $f_p \in \mathcal{S}_\infty$. This result has a double interest. On the one hand, unexpectedly, it affirms that a policy-maker may strictly mitigate the inequality at the level of individuals even avoiding a progressive taxation of households (a flat tax does not modify the inequality among households). On the other hand, the key-condition about intra-household allocation, that is, a sharing function star-shaped at ∞, appears more plausible than concavity, at least according to the first empirical analysis in this direction (Couprie et al. [3]).

In a similar way, it is possible to show that a benefit scheme, which increases household incomes in the same proportion, reduces the inequality among individuals in relative terms whenever the sharing function is star-shaped at 0. Corresponding results emerge whenever absolute differences count for the appraisal of inequality. The next proposition emphasizes the interest of a *poll-tax*, which leaves unchanged the inequality among households (according to the AL criterion) but reduces the inequality among individuals under the large class of the *moving away* sharing functions. We assume that the *poll-tax* may be paid by all households, that is $t < a$.

Proposition 3 *Let* $f_p \in \mathcal{F}$.

$$f_p \in \mathcal{MA} \iff [\mathbf{x}(\mathbf{y}-t\mathbf{e}_n) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y}), \forall \mathbf{y} \in \mathbb{Y}_n \text{ and } \forall t < a]$$

Proof. $\implies$ Since $f_p \in \mathcal{MA}$, then $f_p(y_i) - f_p(y_i - t) \leq \frac{t}{2}$, $\forall t > 0$, which may be rewritten as $f_p(y_i) - \frac{\mu_{\mathbf{y}}}{2} \leq f_p(y_i - t) - \frac{\mu_{\mathbf{y}-t\mathbf{e}_n}}{2}$. We get, for any household i,

$$\left(f_p(y_i - t) - \tfrac{\mu_{\mathbf{y}-t\mathbf{e}_n}}{2}, f_r(y_i - t) - \tfrac{\mu_{\mathbf{y}-t\mathbf{e}_n}}{2}\right) \succcurlyeq_{GL} \left(f_p(y_i) - \tfrac{\mu_{\mathbf{y}}}{2},\ f_r(y_i) - \tfrac{\mu_{\mathbf{y}}}{2}\right). \tag{6}$$

Let $\check{\mathbf{x}}(\mathbf{y})$ designate the centered vector of individual incomes. Then, we can deduce from (6) that $\check{\mathbf{x}}(y_i - t) \succcurlyeq_L \check{\mathbf{x}}(y_i)$ for any i. Proposition A.7 (i), p. 121 of Marshall and Olkin [10], gives $\check{\mathbf{x}}(\mathbf{y}-t\mathbf{e_n}) \succcurlyeq_L \check{\mathbf{x}}(\mathbf{y})$, that is $\mathbf{x}(\mathbf{y}-t\mathbf{e_n}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y})$.

$\impliedby$ Suppose that $f_p \notin \mathcal{MA}$. Then, $\exists t > 0$ and $\exists y^* > 0$, such that: $f_p(y^*) - \frac{y^*}{2} > f_p(y^* - t) - \frac{y^*-t}{2}$. Let us consider $\mathbf{y} = (y^*, .., y^*, y^*)$ and $\mathbf{y'} = \mathbf{y}-t\mathbf{e}_n$. By construction $\mu_{\mathbf{y'}} = \mu_{\mathbf{y}} - \alpha$ and $\mathbf{y'} \succcurlyeq_{AL} \mathbf{y}$. Since $f_p(y^*) - \frac{ny^*}{n2} > f_p(y^* - t) - \frac{n(y^*-t)}{n2}$, then $\sum_{j=1}^{n}(x_j - \mu_{\mathbf{x}}) > \sum_{j=1}^{n}(x'_j - \mu_{\mathbf{x'}})$ and consequently $\mathbf{x}(\mathbf{y'}) \succcurlyeq_{AL} \mathbf{x}(\mathbf{y})$ does not hold. ■

By the same token, it may be shown that a basic income reduces the inequality in absolute terms whenever the sharing function is of the *moving closer* type.

4 Recovering sharing functions through a bargaining model

The sharing functions are an intermediate product of the analysis, an output of theories of family and an input of the inequality analysis. The purpose of the present section is to recover the relevant classes of sharing functions when the allocation of income within the household is described through a bargaining model. The price of the good is normalized to 1, public goods and externalities are neglected. Following Kanbur and Haddad [8], we introduce a symmetric Nash bargaining model.

Assumption 2 *Household behavior is described by the following program*

$$\max_{x_1,x_2}(u(x_1)-\bar{u}_1)(u(x_2)-\bar{u}_2) \tag{7}$$

$$s.t.\ x_1+x_2=y.$$

If the individuals share the same utility function, they are differentiated by their threats points $\bar{u}_1$ and $\bar{u}_2$ (we denote the utility pair $(\bar{u}_1,\bar{u}_2)=\bar{\mathbf{u}}$). Then, the solution of (7), $x_1^*(y,\bar{\mathbf{u}})$, represents the sharing function f_p as a function of the household income and of the threats points.

Our aim is to clarify how the intra-household different bargaining power interacts with individual attitude towards risk in order to generate the properties of the sharing function. The tools of the risk theory we use are the well-known Pratt [13] coefficients. We designate by $R(x)=-u''(x)/u'(x)$ the *absolute risk-aversion* coefficient and by $r(x)=-xu''(x)/u'(x)$ the *relative risk-aversion* coefficient. We assume that u belongs to be the class of (strictly) increasing and strictly concave Bernoulli utility functions two times differentiable.

4.1 *DARA* utility functions and moving away sharing rule

A marginal rise in the household income increases more the expenditure of the dominant individual if and only if individuals have a non-decreasing absolute risk aversion (utility belongs to the *DARA* class). In other terms, in the set-up of our bargaining model, *DARA* utility functions generate *moving away* sharing functions.

Proposition 4 *Suppose that a household follows Assumption 2 and that $x_1^*(y,\bar{\mathbf{u}})$ is twice continuously differentiable.*

Then $x_1^*(y,\bar{\mathbf{u}})\in\mathcal{MA}$ for all $\bar{u}_1\leq\bar{u}_2\Longleftrightarrow u\in DARA$.

Proof. From the f.o.c. of (7) we get:

$$\frac{u'(x_1^*)}{u'(x_2^*)} = \frac{u(x_1^*) - \bar{u}_1}{u(x_2^*) - \bar{u}_2}. \tag{8}$$

This guarantees that, if $\bar{u}_1 < \bar{u}_2$, then $x_1^* < x_2^*$. By differentiating the f.o.c. of (7) with respect to y and solving a standard comparative static problem, we get:

$$\begin{aligned} \frac{\partial x_1^*}{\partial y}(y, \bar{\mathbf{u}}) &= \frac{A}{A+B} \\ \frac{\partial x_2^*}{\partial y}(y, \bar{\mathbf{u}}) &= \frac{B}{A+B} \end{aligned} \tag{9}$$

where:

$A = (u(x_1^*) - \bar{u}_1)u''(x_2^*) - u'(x_1^*)u'(x_2^*)$;

$B = (u(x_2^*) - \bar{u}_2)u''(x_1^*) - u'(x_2^*)u'(x_1^*)$.

Under a differentiable sharing rule, the moving away property results in $\frac{\partial x_1^*(y,\bar{\mathbf{u}})}{\partial y} \leq \frac{1}{2}$. It follows:

$\frac{A}{A+B} \leq \frac{1}{2} \Longleftrightarrow \frac{A}{A+B} \leq \frac{B}{A+B} \Longleftrightarrow (u(x_1^*) - \bar{u}_1)u''(x_2^*) \geq (u(x_2^*) - \bar{u}_2)u''(x_1^*)$.

Using (8), we conclude:

$$\frac{\partial x_1^*}{\partial y}(y, \bar{\mathbf{u}}) \leq \frac{1}{2} \Longleftrightarrow -\frac{u''(x_1^*)}{u'(x_1^*)} \geq -\frac{u''(x_2^*)}{u'(x_2^*)}.$$

This equivalence is sufficient to show the sufficiency part. By considering all $\bar{\mathbf{u}}$ such that $\bar{u}_1 \leq \bar{u}_2$, the necessity part is also implied. ■

4.2 DRRA utility functions and progressive sharing rule

We now establish a relation between *star shaped at* ∞ sharing functions and individual utility with decreasing relative risk aversion (DRRA class*)*.

Proposition 5 *Suppose that a household follows Assumption 2 and that $x_1^*(y, \bar{\mathbf{u}})$ is twice continuously differentiable.*

$x_1^*(y, \bar{\mathbf{u}}) \in \mathcal{S}_\infty$ *for all* $\bar{u}_1 \leq \bar{u}_2 \Leftrightarrow u \in DRRA$.

Proof. A differentiable star-shaped at ∞ is characterized by $\frac{\partial x_1^*}{\partial y}(y, \bar{\mathbf{u}}) \leq \frac{x_1^*(y,\bar{\mathbf{u}})}{y}$. From (9), it is equivalent to $\frac{A}{A+B} \leq \frac{x_1^*(y,\bar{\mathbf{u}})}{y} \iff Ax_2^* \geq Bx_1^*$. This implies $(u(x_1^*) - \bar{u}_1)u''(x_2^*)x_2^* \geq (u(x_2^*) - \bar{u}_2)u''(x_1^*)x_1^*$, since $x_2^* \geq x_1^*$.

Using (8), we get

$-\frac{u''(x_2^*)}{u'(x_2^*)}x_2^* \leq -\frac{u''(x_1^*)}{u'(x_1^*)}x_1^*$,which is precisely the requirement of DRRA. By considering all $\bar{\mathbf{u}}$ such that $\bar{u}_1 \leq \bar{u}_2$, the necessity part is proved.

In order to show the sufficiency of DRRA, we start from $Ax_2^* \geq Bx_1^*$, which is equivalent to $\frac{x_2^*\left[(u(x_1^*)-\bar{u}_1)u''(x_2^*)-u'(x_1^*)u'(x_2^*)\right]}{u'(x_1^*)u'(x_2^*)} \geq \frac{x_1^*\left[(u(x_2^*)-\bar{u}_2)u''(x_2^*)-u'(x_1^*)u'(x_2^*)\right]}{u'(x_1^*)u'(x_2^*)}$

$\Leftrightarrow\ x_2^*\left[-\frac{(u(x_1^*)-\bar{u}_1)}{u'(x_1^*)}R(x_2^*) - 1\right] \geq x_1^*\left[-\frac{(u(x_2^*)-\bar{u}_2)}{u'(x_2^*)}R(x_1^*) - 1\right]$, which can be written, using again (8) and setting $k(x_1^*, x_2^*) = \frac{(u(x_1^*)-\bar{u}_1)}{u'(x_1^*)} = \frac{(u(x_2^*)-\bar{u}_2)}{u'(x_2^*)}$, as:

$$\frac{x_1^*}{x_2^*} \geq \frac{-R(x_2^*) - \frac{1}{k(x_1^*,x_2^*)}}{-R(x_1^*) - \frac{1}{k(x_1^*,x_2^*)}}. \tag{10}$$

DDRA means that $1 \geq \frac{x_1^*}{x_2^*} \geq \frac{R(x_2^*)}{R(x_1^*)}$ and therefore implies (10). ■

The interpretation of these results is quite intuitive: in a bargaining problem, players know that negotiations may randomly break down. Consequently, a more risk-averse agent may in general accept a solution which advantages the other player (see Roth and Rothblum [15] for a rigorous treatment). Since here risk aversion decreases with income, the dominated individual is also the more (locally) risk-averse. Then he asks for a larger share of the cake in case of low levels of income, by accepting a lower part of income (in absolute or relative terms) when the state of the world is more favorable.

5 Conclusion

In this paper, we study the effectiveness of redistributive schemes to reduce inequality at the individual level, when they are performed at the household level. Combining the results presented in Section 3, a little more complex picture emerges than one would expect, even if the bottom line is that a positive correlation between within-inequality and household income enforces the redistributive effects of taxation. Three properties capturing the idea that the more wealthy the household is, the more unequally it behaves, have been shown to matter. In the *moving away* approach, the deviation with the equal split make a difference, in the *star-shaped* approach, the average share counts while the marginal share is relevant for *concavity.* When general redistribution schemes mixing taxes and benefits are considered, the concavity property is needed. When we focus the attention on simple taxation schemes such as a poll tax or a flat tax, the moving away and star-shaped functions prove to be the relevant ones. If we move to the pure-benefit side of the redistribution, the view-point is completely reversed: the more wealthy the household is, the more equally it must behave. If the empirical evidence brings some support to this last statement, the policy-maker will have no choice but to target benefits at the individual level to make his redistribution policy effective. More generally, the policy-maker should analyze the link between intra-household inequality and household wealth in order to know for which kind of taxation scheme (taxation or benefit) the individual level proves to be the unescapable level of design of the policy. A different application of our result is suggested by Ravallion (2001) and Dollar and Kraay (2002). Among others, they find that growth tends to be distribution neutral in relative terms. Then, under star-shaped at 0 sharing functions, the idea of a pro-poor growth could be strengthened. This suggests a further exploration of poverty *among individuals.*

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