

Product Development and Market Expansion: a Valuation Approach Based on Real Options

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Abstract

In this paper we investigate the valuation and optimal timing of the launch of two complementary/substitute products (or projects), one of which is a pilot product. As a first step, we study the problem from a strategic point of view and analyze the ability of the pilot product per se to create shareholder value. Next we provide a model to evaluate the option to launch the pilot product in order to create the option to launch the key product.

The model is designed to analyze the value of the pilot product and the optimal timing to invest as functions of the degree of complementarity/substitutability among the two products. As a specification, we analyze under what conditions it is worth investing in a pilot product with a negative NPV, whose option to invest is worthless due to perfect competition.

The pilot product and the key product are driven by two different but correlated sources of uncertainty. The case is enriched by the presence of a patent on the key product and by the related consideration of a change in the competition due to patent expiry. By resorting to numerical analysis, we evaluate the embedded options and determine the optimal investment policy for both products.

Our analysis allows us to capture the value of the options to invest in both the key and pilot products (as a compound option) and shows how the investment decision is affected by a higher correlation among products and a higher degree of substitutability. The major outcome of the model is that a higher correlation and a higher degree of substitutability among products increase the option value and expand the investment thresholds of the whole investment project, thus making the investment relatively less likely in the case of either a lower correlation or a lower degree of substitutability.

Keywords: Marketing strategy, Real options, Option interactions, Binomial lattices.

JEL classification: C61, G31, O32.

1 Introduction

This study investigates the value and optimal timing to invest in a pilot product whose major purpose is to open up the market opportunity to launch a complementary/substitute key product that will play a critical role in the investment strategy of a corporation. The analysis is entirely based upon the assumption that *the pilot product is the required step to acquire the opportunity to invest in the key product*. Such assumption may be supported by one of the two following fundamental (and extreme) economic cases:

- The company is developing a family of complementary products to be jointly launched in order to increase their overall utility to customers. In this sense, both the pilot and key products may be seen as the object of a broad marketing strategy whose final aim is to optimize the market position of a new family of complementary products;
- The company is faced with a rapidly decreasing life-cycle of products requiring a constant development of updated versions of existing products in order to sustain competitiveness. Their utility to customers decreases rapidly over time and new substitute products are needed to revitalize their utility.

Under this scheme, the main purpose of the paper is to determine the value and the optimal rule to invest in the pilot product as well as the optimal investment policy for the key product.

Real life evidence offers many examples of this business case. Generally speaking, it applies to each situation where a company implements a strategy of product-line diversification based on sound capital investments in a new technology/area characterized by considerable market uncertainty. In such cases, a pilot product based on a reduced scale investment allows the company to test the market in order to better understand some critical aspects such as the demand potential, the competitive reaction and a feasible target price.

It is usually right before the completion of the late and most R&D intensive stage that particularly valuable marketing options are considered. These typically include: Options to enter new geographical markets, contingent on the success of the initial target market; options to create product line extensions and get new applications on the same product, contingent on the initial success of the base version; options to develop and launch new products after the pilot one has proven to be successful.

Recently, other Authors have addressed the same issue. An early contribution was provided by Trigeorgis (1993) who first developed the concept of (real) option interaction, so that the value of a portfolio of managerial opportunities can deviate from the sum of the values of the individual opportunities. Pope and Stark (1997) model the firm's value as a portfolio of two major options: the option to use and the option to invest in new production capacity. The first type of option is mainly meant to *realize* the option value whereas the second one is mainly utilized to *develop* new option value. Also Childs, Mauer and Ott (2000), with the purpose of analyzing the interaction between investment and financing decisions, propose a model for evaluating a firm's growth option assuming that this option depends on an asset whose value is correlated with the asset in place. Other contributions have been given by Abel, Dixit, Eberly and Pindyck (1996) and by Kandell and Pearson (2002), who study the effect of (partial) reversibility on the value of the investment project and on the investment policy under different market situations, ranging from perfect competition to monopoly, through oligopoly. As far as technology investment is concerned, Grenadier and Weiss (1997) and Huisman and Kort (2003) present a model to incorporate into the option to invest in the current technology, the value of the option to invest in future, better technologies, which may replace current technologies, also bearing in mind a competitive environment. The dynamics of R&D investment have been studied in a contingent claim framework by Childs and Triantis (1999), who provide a model incorporating the possibility to invest in many competing projects at an early stage, allowing only one of them to prevail. Also Schwartz and Moon (2000) address the problem of R&D investment, by modelling a multistage investment, taking into account the risk of failure in the research process.

In contrast with the above-mentioned approaches, we concentrate on the marketing stage, explicitly modelling the effect of complementarity/substitution between products and the effect of a transition from a monopolistic to a competitive market on both the value of the project and on the investment policy. Moreover we provide a different numerical methodology and a different perspective for analyzing the results, by comparing the investment thresholds for the different real options.

The next section of this paper describes some business cases involving a pilot project/product and a main project/product. The (gross) values of the two projects are different but correlated, and the two projects are, from a strategic viewpoint, either complementary or substitute. Section 3 introduces notations and discusses the model, deriving both the valuation formula for the option to launch the pilot project and the one for the main project. In Section 4 we present a numerical method to evaluate these options

and the optimal investment policy and then we discuss numerical results. Section 5 presents two possible real-life applications of the model introduced in this paper. We relegate proof and derivation of the valuation formulas to Appendix A. Details of the numerical method used in this paper are presented in Appendix B.

2 Flexibility options, complementarity and substitutability among different products

The problem we address in this paper can be outlined as follows: a company is evaluating the opportunity to launch a pilot product called “Minprox” which is a necessary step for opening up the opportunity to launch a key product, called “Newprox”. The completion of the investment in Minprox not only allows the company to benefit from the product potential itself, but also gives the option to conclude the investment program for Newprox. Therefore, Minprox is the necessary bridge leading to the key product. We assume that the most relevant uncertainty driving the above options is due to the amount of units sold.¹ Two different (but correlated) stochastic processes model the expected units sold of the two products.

The present work also considers the problem of Newprox’s patent expiry, and, more generally, any other finite horizon maturity due to exogenous reasons, and the related change in the market structure as far as competition is concerned. This assumption limits the time value of the embedded flexibility options and introduces an interesting concept of opportunity cost. Therefore, the relevant question is to see how the company’s profit opportunity behaves in the light of the patent expiry. In order to account for this, we assume that the time horizon is divided into two periods. The first period goes from zero to the patent expiry, when the company is assumed to be in a monopolistic position. If Newprox were launched, an excess market return would be gained. In the same period, the price of both products is assumed to be constant over time. The second period goes from the patent expiry onwards. The company can still enter the market by optimally launching both products, but it operates in a (perfectly) competitive environment, where the profit of the marginal firm is null. The market price of both products is (rapidly) reduced by competitive pressure as soon as the patent expires and remains constant thereafter. To present a more interesting situation, we assume that the current market for Minprox is perfectly competitive, so that

¹Yet, it is straightforward to extend this model to incorporate price uncertainty, as an alternative to uncertainty about the units sold. On this, see also footnote 7 below.

the marginal firm invests when the NPV of Minprox is null and the option to launch Minprox is worthless (as considered in isolation with respect to the option to launch Newprox), as shown in Dixit and Pindyck (1994), Ch. 8.

[Figure 1 about here]

An interesting aspect worth analyzing is the complementarity and substitutability characteristics of these products in relation to the correlation between the quantities sold for both products. At the outset, we can foresee at least four main combinations, as described in Figure 1.

In the first quadrant, case 1, both products are totally complementary, in the sense that their combination increases the total utility to customers, and their revenues are positively correlated. In such case the success of the pilot product would certainly be a trigger to launch the key product, the positive interaction among them being the key aspect of the decision making process.

Case 2 offers an example of “cannibalization” among products. In fact, the substitutability among products, i.e. the ability of a product to replace another one, and the positive correlation allows the launch of the key product only at the expense of the pilot product. Moreover, this would happen under the same scenario (i.e, best-best or worst-worst case), thus increasing the opportunity cost of the launch of the key product.

In the third case the company would experience a diversification effect. Since the two products are complementary, the negative correlation allows the company to reduce its business risk by using one product as a hedge that limits the downward potential in case the other one falls under the worst scenario.

Lastly, the fourth case could describe the rapidly-evolving landscape of highly competitive industries where new versions of (possibly) the same products are needed in order to stay at the cutting edge of technological breakthrough.

This intuition can be enriched by considering how the optimal investment policy of the firm regarding the two products is related to the to different combinations of the two attributes, as it is shown in Figure 2. Two major rules appear relevant:

- A weaker (negative) correlation and a higher degree of complementarity among products lead the firm to implement the investment in Minprox with relatively lower expected cash flows. This is mainly due to the relatively low opportunity cost in launching the key product that does not cannibalize the pilot product and works better under different scenarios.

- A stronger (positive) correlation among products and a stronger substitution effect require a relatively higher amount of expected sales of Minprox before launching it. In such a case, the decision to invest must be based on a higher intrinsic value of the whole project that compensates for the higher opportunity cost due to the more likely loss of sales of Minprox.

The above is quite obvious as far as the decision about Newprox is concerned, following the launch of Minprox has taken place. It is far less obvious where the investment policy for both products is concerned. These intuitive concepts will be confirmed by numerical analysis and will serve as a basis of the application of the valuation model to real-life business cases.

Intuition is of little help when considering the cases where: (a) complementary products are positively correlated; and (b) perfectly substitutable products are negatively correlated. In these cases, numerical results are needed to address the problem.

Next, we provide two real life examples/applications for the basic intuition underlying the model: one from the pharmaceutical (complementary products) and one from the information technology (IT) industry (substitutive products).

[Figure 2 about here]

2.1 The Pharmaceutical industry and the complementarity among products

An example of complementary products can be taken from the pharmaceutical industry. As will be clearer later, one important driver of a pharmaceutical company's likely future success is the number of products currently under active research and development. In particular, the competitive dimension in the pharmaceutical industry is based upon the concept of "families of products", meaning that a successful marketing strategy in a given therapeutic area requires a wide variety of products able to be jointly prescribed to cure various forms of the base pathology. Table 1 represents the R&D pipeline, ranking them according to the number of products, of the top ten companies.

[Table 1 about here]

With regards to this, several diseases (e.g., infections, allergies, disorders of the central neural system etc.) are the consequence of others and may require the joint prescription of different products/treatments. In fact, in

prescribing medicines, doctors are usually influenced by the brand equity of a certain firm and the size of its portfolio of complementary products which can be jointly prescribed in order to increase the effectiveness of a certain treatment.

The value of a pilot product in the pharmaceutical industry becomes clearer after focusing on the entire R&D process. The R&D process of a new pharmaceutical product is quite complex, and can be analyzed through 4 major phases lasting approximately 8-10 years: primary research, exploratory development, full development and marketing strategy. The two central phases are fundamentally based on clinical trials, which represent the necessary technical tests aimed at obtaining the final approval from the FDA commission.

[Table 2 about here]

There are three standard stages of clinical trials, whose characteristics are described in Micalizzi (1999) and summarized in Table 2.² Although the failure rate decreases sharply after the first and second phases, the large number of patients and scope of phase three causes a large increase in costs approaching \$100 million.

This makes the third phase of clinical trials the most suitable time for optimizing the drug's development process and designing alternative marketing strategies. In fact, at this stage some flexibility options become particularly valuable. Among them, it is worth mentioning the opportunity to abandon the R&D process for its scrap value (usually paid by biotech firms), to optimize the timing of the launch and to widen the spectrum of applications by conducting more extensive trials, thus increasing the size of its market potential.

Consequently, when entering new therapeutic areas, pharmaceutical/biotech firms usually launch pilot products as drivers that open up market opportunities for other products. The R&D expenditures to complete and launch the pilot product can be interpreted as a "premium" to acquire a series of nested options that include the opportunity to launch new versions of the same product or new products belonging to the same family.

2.2 The Data Storage industry and the substitutability among products

An example of substitutive products can be derived from the data storage industry. The increased use of open-system computing environments, which

²On this, see also Schwartz and Moon (2000).

link multiple applications, files and databases to networked computers, makes the task of data management increasingly difficult. As a result, data storage products and services have accounted for an increasing percentage of most organizations' (IT) budgets and management resources.

Enterprises have historically attempted to support the management of data requirements by directly attaching storage devices to the individual servers on a local area network. Servers communicate in this directly-attached environment using the Small Computer System Interface (SCSI). The major drawbacks of such a protocol include short transportation distance, limited configuration flexibility and limited connections. These limitations restrict the capabilities of traditional storage architectures and result in a significant bandwidth bottleneck between storage systems and servers.

To address these limitations, fibre channel technology and interconnection standards evolved in the early 1990s to enable new high-performance connectivity. Fibre channels overcame the SCSI's limitations and offer high performance and increased capacity needed for I/O applications. Unfortunately, fibre channels are an expensive storage solution, both in terms of installation and maintenance, and are typically only implemented for use in critical projects or enterprise applications.

For this reason many vendors are exploring iSCSI, a new protocol that allows storage devices to be connected to a network without the need of a fibre channel, moving storage data over the Internet. In fact, iSCSI runs over an existing Ethernet network but at speeds significantly faster than SCSI. IDC estimates (see Rancourt, *et al.* (2001)) that this market will grow at a CAGR of over 400% between 2001 and 2005. The empirical evidence says that first movers who establish themselves with customers stand to benefit when the market rumps up and new technologies can be offered to the already established client base.³

Hence, the rapid evolution of such technological breakthrough leads players to move from one product to another and face a significant substitution effect that cannibalizes the existing products in favor of the new and better performing technologies. Pilot products become key in these cases to let the firm acquire an initial client-base to be shifted across to the next technologies.

3 The valuation model

As a general valuation procedure, we will start by calculating the present value of Newprox (V) and Minprox (W) as a function of units sold (x and

³IDC estimates the market will be almost \$2.5 million by 2005 (see Rancourt, *et al.* (2001)).

y , respectively). Then, we calculate the value of the opportunity to invest in Newprox ($F(V)$) as a single product (assuming that Minprox has already been launched), considering that the company can benefit from the optimal timing to invest due to the uncertainty over the units sold. Therefore, we move from the product to the portfolio perspective and consider both investment opportunities as a compound option. We will, initially, calculate the value of the opportunity to complete the investment in Minprox as a function of Minprox units sold and of Newprox ($G(V, W)$). This calculation will provide the answer to the fundamental issue of how much a pilot product is worth, also considering its role as driver product for subsequent complementary/substitutable investment opportunities. The determination of G will also provide the optimal rule of investment in Minprox ($W^*(V)$) as the value of Newprox changes, as well as the optimal investment rule in Newprox (V^*).

In what follows, we will assume that both investment projects are all equity financed, i.e., we will not explore how the cost of capital influences the value of the options and the investment policy.

3.1 The value of the assets

In this section, we will derive the value of Newprox denoted V , at time t , as a function of $X(t)$, the sales of Newprox.⁴ The analysis is enriched by the presence of a patent on Newprox: before the expiry of the patent, the firm can obtain extra profits from the product because of a monopolistic position; after the patent's expiry, the extra profits are worn away by perfect competition. The patent expires in T years.

We will also derive the value of Minprox, denoted W , as the present value at the time of its launch of the cash flows from the sales of this product, $Y(t)$. To keep the analysis reasonably simple, we will assume that there is no patent for Minprox.⁵ Since we assume the financial markets to be sufficiently rich, so that the uncertainties about Minprox and Newprox are spanned by other traded securities, we will describe the relevant state variables of the model under the equilibrium martingale measure. Let $Y(t)$, the quantity of Minprox

⁴In the case of a pharmaceutical product, V is the value of the product when the last phase of clinical trials is successfully performed or the FDA approve the product. It is reasonable to assume that the successful event is independent of the sales of both products and hence of market risk. If we assume that the probability of success, denoted π , is known and time-independent, then the value of the product (before the outcome of the clinical trial is known or FDA approval is obtained) is replaced by πV . This line of reasoning can also be applied to other R&D projects.

⁵Otherwise, we could simply carry out for Minprox the same analysis proposed for Newprox.

sold at t , be a variable following a geometric Brownian motion:

$$\frac{dY(t)}{Y(t)} = \mu dt + \zeta dZ_y(t), \quad Y(0) = y \quad (3.1)$$

where μ is the (risk-adjusted; i.e., net of the risk premium) drift of the sales of Minprox and ζ is the volatility. Let $X(t)$, the quantity of Newprox sold at time t , be a stochastic process given by the solution of the following equation:

$$\frac{dX(t)}{X(t)} = \nu(t)dt + \sigma(t)dZ_x(t), \quad X(0) = x, \quad (3.2)$$

where

$$\nu(t) = \begin{cases} \nu_1 & \text{if } t < T \\ \nu_2 & \text{if } t \geq T \end{cases} \quad \sigma(t) = \begin{cases} \sigma_1 & \text{if } t < T \\ \sigma_2 & \text{if } t \geq T. \end{cases}$$

(Also the drift of Newprox is net of the relevant risk premium.) This is to model a possibly different (lower) growth rate and a different (lower) uncertainty on sales of Newprox when the patent expires.

Let $\mathbb{E}[dZ_x(t) dZ_y(t)] = \gamma dt$ and $\mu, \nu_2 < r$ in order to ensure a finite fundamental value for both investment projects. In such an analysis, γ will then represent the correlation among the increment in sales of the two products.

The value of Newprox is the sum of two components: the present value of the cash flows prior to patent expiry, T , plus the present value of the cash flows after T . During the first phase, in $[0, T]$, the firm is in a monopolistic position. In this time period there could be value in waiting for more favorable market conditions before launching the key product. In the second phase, after the patent expiry, Newprox⁶ can also be sold by competitors and the monopolistic market turns into a competitive one. For simplicity, we assume that after T any firm can enter the market and the competition is perfect. Hence, after T , the analysis will follow Dixit and Pindycks' model (see Dixit and Pindyck (1994), Ch. 8). Since all competitors in this market/niche face the same risk, according to Dixit and Pindycks' model, perfect competition and equilibrium put an upper boundary on the price of the product, so that, as long as the price fluctuates below the boundary, no new firm can enter the market and, as soon as the price touches (from below) the boundary, a new firm can enter changing the supply curve for this product and reducing its price. Hence, the upper boundary is a reflecting barrier for the stochastic process of pricing. Let p denote the unit price for Newprox. Given an inverse demand function $p = p(x)$, the upper reflecting barrier for price corresponds to an upper boundary for product sales, denoted κ . This

⁶Or, in the pharmaceutical case, a product based on the same formula.

means that, after T , the competitive pressure reduces the market share of our firm. κ is a reflecting barrier for $X(t)$; that is, $X(t)$ reaching κ represents a signal that leads a new firm to enter. As soon as this happens, Newprox's sales drop below κ . Since in perfect competition a firm cannot significantly affect the total supply and act as a price taker, with no loss of generality we may assume that, after T , from the perspective of our firm, price is given and settled in competitive equilibrium. Moreover, since before T our firm is in a monopolistic position with respect to Newprox, we assume that price p is a constant and that the product is sold at a premium, so that when the patent expires, the price of Newprox is reduced by a fixed percentage $(1 - \lambda)$, $0 < \lambda < 1$.

To summarize, in this model, before T price is p , after T price is λp . For different reasons, the price is given in both time periods. The above assumptions permit us to model our project assuming that sales are the source of uncertainty, both before and after T .⁷

Let Φ denote the present value of the cash flows⁸ in the first phase. Putting $\delta_1 = r - \nu_1$ and $\tau = T - t$, it can be shown that⁹

$$\Phi(x, t) = \begin{cases} px \frac{1 - e^{-\delta_1(T-t)}}{\delta_1} & \text{if } t < T \\ 0 & \text{if } t \geq T. \end{cases} \quad (3.3)$$

$\Phi(x, t)$ is a decreasing and continuous function of t .

Let Ψ be the value of the cash flows obtained from Newprox after the expiry, T :

$$\Psi(x, t) = \begin{cases} \psi_1(x, t) & \text{if } t < T \\ \psi_2(x) & \text{if } t \geq T. \end{cases} \quad (3.4)$$

In Equation (3.4), letting $\delta_2 = r - \nu_2$,

$$\psi_1(x, t) = \frac{\lambda p}{\delta_2} \left[x e^{-\delta_1(T-t)} \mathcal{N} \left(z(x, t) - \sigma_1 \sqrt{T-t} \right) - \right. \quad (3.5a)$$

$$\left. - \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho e^{(\nu_1 \varrho + \frac{1}{2} \sigma_1^2 \varrho(\varrho-1) - r)(T-t)} \mathcal{N} \left(z(x, t) - \sigma_1 \varrho \sqrt{T-t} \right) + \right. \quad (3.5b)$$

$$\left. + \kappa \frac{\varrho - 1}{\varrho} e^{-r(T-t)} \mathcal{N} \left(-z(x, t) \right) \right] \quad (3.5c)$$

⁷ Implicit in the above discussion is the possibility to extend the model in order to accommodate price uncertainty instead of quantity uncertainty, as in Dixit and Pindyck (1994), Ch. 8.

⁸ If variable costs are considered, p is the marginal rate of contribution (or unit cash flow) of Newprox. The same can be said for q , the price of Minprox.

⁹ In what follows, we assume that $\nu_1 \neq r$. If $r = \nu_1$, we just need to replace $\Phi(x, t) = px(T-t)$ if $t < T$ in Equation (3.3).

where $\mathcal{N}(\cdot)$ is the cumulative Normal distribution and

$$z(x, t) = \frac{\log \frac{\kappa}{x} - \left(\nu_1 - \frac{1}{2}\sigma_1^2\right)(T - t)}{\sigma_1 \sqrt{T - t}}$$

where

$$\varrho = \frac{1}{2} - \frac{\nu_2}{\sigma_2^2} + \sqrt{\left(\frac{\nu_2}{\sigma_2^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma_2^2}} > 1. \quad (3.6)$$

$\psi_1(x, t)$ is the present value at $t < T$ of $\psi_2(x, t)$, the value of the cash flows after T as computed below. In Equation (3.4),

$$\psi_2(x) = \begin{cases} \frac{\lambda p}{\delta_2} \left(x - \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho\right) & \text{if } x < \kappa \\ \frac{\lambda p}{\delta_2} \frac{\varrho-1}{\varrho} \kappa & \text{if } x \geq \kappa. \end{cases} \quad (3.7)$$

The value of Newprox, at (x, t) is:

$$V(x, t) = \Phi(x, t) + \Psi(x, t). \quad (3.8)$$

Since there is no patent on Minprox, this market is competitive. Following the above discussion on perfect competition, given an inverse demand function for Minprox, our firm takes the price of Minprox as a constant, q . Hence, W , the present value of cash flows from Minprox, can be found by applying the same model used for Newprox after the expiry of the patent. Given θ , the reflecting barrier for the stochastic process of Minprox sales, the value of Minprox at (y, t) is the expectation of the present value of the cash flows from the sales of Minprox:

$$W(y) = \begin{cases} \frac{q}{r-\mu} \left(y - \frac{\theta^{1-\varphi}}{\varphi} y^\varphi\right) & \text{if } y < \theta \\ \frac{q}{r-\mu} \frac{\varphi-1}{\varphi} \theta & \text{if } y \geq \theta. \end{cases} \quad (3.9)$$

where r is the instantaneous risk-free interest rate, assumed to be constant and

$$\varphi = \frac{1}{2} - \frac{\mu}{\zeta^2} + \sqrt{\left(\frac{\mu}{\zeta^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\zeta^2}} > 1. \quad (3.10)$$

Note that, if θ is very large ($\theta \rightarrow \infty$), this model can be used also to describe absence of competition for Minprox.

3.2 The value of the option to launch the key product

In this section we will derive the value of the option to launch Newprox.

Let F be the value of the investment opportunity in Newprox. Since the option to launch the key product is exercised by giving up a share η of the business in Minprox, F is a function of $V(X(t), t)$, $W(Y(t))$ and of time t , because as the patent approaches its maturity, the value of delaying the investment declines. $\eta = 1$ implies that Newprox is a substitute for Minprox, whereas $\eta = 0$ means that Newprox is a complementary product for Minprox.

At (x, y, t) the firm faces the optimal stopping problem

$$F(x, y, t) = \max_{\tau} \left\{ \mathbb{E}_{x,t} \left[e^{-r(\tau-t)} (V(X_{\tau}, \tau) - I_V - \eta W(Y_{\tau})) \right] \right\} \quad (3.11)$$

where I_V is the investment cost,¹⁰ and $\tau \geq t$ is a stopping time with regards to the information on the state variables. According to (3.11), the firm faces the choice of the optimal time to invest in Newprox, and F is the value function of this problem.

By standard arguments (see Dixit and Pindyck (1994), p. 128 et ss), there is a waiting region \mathcal{C}_N , defined as

$$\mathcal{C}_N = \{(x, y, t) \mid F(x, y, t) > V(x, t) - I_V - \eta W(y)\}$$

where it is optimal to wait to launch Newprox, and a region $\mathcal{S}_N = \mathbb{R}_{++}^3 \setminus \mathcal{C}_N$ where it is optimal to invest. The continuation (and the stopping) region is determined as a part of the solution of (3.11). We denote $\Omega_N = \partial \mathcal{C}_N$ the frontier of \mathcal{C}_N . Ω_N is a surface in \mathbb{R}_{++}^3 .

Let us define the operator

$$\begin{aligned} \mathcal{L} = \frac{1}{2} \left(\sigma(t)^2 x^2 \frac{\partial^2}{\partial x^2} + 2\gamma\sigma(t)\zeta xy \frac{\partial^2}{\partial x \partial y} + \zeta^2 y^2 \frac{\partial^2}{\partial y^2} \right) + \\ + \nu(t)x \frac{\partial}{\partial x} + \mu y \frac{\partial}{\partial y} + \frac{\partial}{\partial t} - r. \end{aligned}$$

The solution of problem (3.11) satisfies conditions¹¹

$$\mathcal{L}F(x, y, t) = 0 \quad (x, y, t) \in \mathcal{C}_N \quad (3.12a)$$

$$F(x, y, t) = V(x, t) - I_V - \eta W(y) \quad (x, y, t) \in \Omega_N \quad (3.12b)$$

$$F_x(x, y, t) = V_x(x, t) \quad (x, y, t) \in \Omega_N \quad (3.12c)$$

$$F_y(x, y, t) = -\eta W_y(y) \quad (x, y, t) \in \Omega_N \quad (3.12d)$$

$$F(0, y, t) = 0 \quad \text{for all } t \quad (3.12e)$$

¹⁰In the pharmaceutical case, I_V is the cost of the last phase of clinical trials, whereas, in the IT case, it is the cost of the new technology.

¹¹This is the Verification Theorem (see Brekke and Øksendal (1991) and Øksendal (1998)).

where subscripts of F denote partial derivatives.

Problem (3.12) cannot be solved analytically. Some numerical methods are needed to obtain both the value of the investment opportunity $F(x, y, t)$ and the frontier Ω_N . We postpone the analysis until Section 4.

Nevertheless, an analytical solution can be found if $t \geq T$, and $\eta = 0$. Actually, in this case $V(x, t) = \psi(x)$ according to Equation (3.7) and the solution is independent on time and on y , $F(x)$. Hence, the unknown threshold is $x_t^* = x^*$ and conditions (3.12) reduces to¹²

$$\begin{aligned} \frac{1}{2}\sigma_2^2 x^2 F_{xx} + \nu_2 x F_x - rF &= 0 && \text{for } x < x^* \\ F(x^*) &= \psi_2'(x^*) - I_V && (3.13a) \\ F_x(x^*) &= \psi_2'(x^*) && (3.13b) \\ F(0) &= 0. \end{aligned}$$

By standard algebra we obtain the investment threshold

$$x^* = \frac{\varrho}{\varrho - 1} \frac{\delta_2}{\lambda p} I_V \quad (3.14)$$

with ϱ from Equation (3.6), and the value of the option to invest in Newprox is

$$F(x) = \begin{cases} Ax^\varrho & \text{for } x < x^* \\ \psi_2(x) - I_V & \text{for } x \geq x^* \end{cases}$$

with

$$A = \frac{\lambda p}{\varrho \delta_2} [(x^*)^{1-\varrho} - \kappa^{1-\varrho}]$$

and $\kappa = x^*$. In fact, since κ is a reflecting barrier, x^* cannot be higher than κ to make the model meaningful. Hence, $F(x) \leq 0$. Moreover, since $F(x)$ is the value of an opportunity, it cannot be negative: $F(x) \geq 0$. This implies that $F(x) = 0$, that is $A = 0$ and $\kappa = x^*$. Following Dixit and Pindyck (1994), p. 252 et ss. (under slightly different hypotheses), this result has two implications: the option to invest after the expiry of the patent is worthless and the firm enters the market only if $X(t)$ reaches the ceiling κ ; the value of κ depends on the investment cost of the project, as can be easily observed in Equation (3.14). This means that, after patent expiry, also our firm obeys the (optimal) rule of investing only when the sales hit the level κ and the option to defer the launch of the key product is worthless ($A \equiv 0$) and so

$$F \equiv 0 \equiv \psi_2(x^*) - I_V :$$

¹²In real world cases, the investment cost after T is generally lower than I_V ; hence, also the investment threshold to enter the market becomes lower after patent expiry. In what follows, for the sake of simplicity, we keep I_V constant.

after patent expiry there is no value in waiting nor in investing in Newprox.

3.3 The value of the option to launch a pilot product

In this section we propose a model to evaluate the option to invest in Minprox. The value of this opportunity depends both on W , the present value of the cash flows given by Minprox, and F , the value of the option to launch the key product.

The cost of the investment in Minprox, denoted I_W , is assumed to be constant,¹³ and is the exercise price of the option to launch the pilot product in order to obtain the “right” to launch Newprox. The firm’s investment opportunity, considering both products, is an American-type compound option with two underlying assets, V and W and a payoff given by two components: the net present value of Minprox, and the value of the option to invest in Newprox.

Let G denote the option to invest in Minprox. G is given by selecting the best time for launching the pilot product. At (x, y, t) ,

$$G(x, y, t) = \max_{\tau} \left\{ \mathbb{E}_{x,y,t} \left[e^{-r(\tau-t)} (W(Y_{\tau}) - I_W + F(X_{\tau}, Y_{\tau}, \tau)) \right] \right\}. \quad (3.15)$$

The continuation region for this problem is

$$\mathcal{C}_M = \{(x, y, t) \mid G(x, y, t) > W(y) - I_W + F(x, y, t)\}$$

where it is optimal to postpone the investment in Minprox and a stopping region $\mathcal{S}_M = \mathbb{R}_{++}^3 \setminus \mathcal{C}_M$. Hence, there is an unknown investment threshold $\Omega_M = \partial \mathcal{C}_M$ given by the frontier of \mathcal{C}_M .

The solution to problem (3.15) is characterized by conditions

$$\mathcal{L}G(x, y, t) = 0 \quad (x, y, t) \in \mathcal{C}_M \quad (3.16a)$$

$$G(x, y, t) = W(y) - I_W + F(x, y, t) \quad (x, y, t) \in \Omega_M \quad (3.16b)$$

$$G_x(x, y, t) = F_x(x, y, t) \quad (x, y, t) \in \Omega_M \quad (3.16c)$$

$$G_y(x, y, t) = W_y(y) \quad (x, y, t) \in \Omega_M. \quad (3.16d)$$

The optimal threshold, Ω_M , is given as part of the solution. Problem (3.16) can not be solved analytically. In the next section, we will provide a different approach to solving Problem (3.16) numerically.

¹³In the pharmaceutical case, I_W is given by the last stage of clinical trials needed to obtain the approval for the launch of the product whereas in the data storage industry I_W can be thought of as the capital investment needed to complete the deployment of the communication network.

In what remains of this section, we will analyze two interesting cases: $x = 0$ and $\eta = 0$; and $y = 0$.

First, let $x = 0$ and $\eta = 0$. This is the case when there is only the pilot product. Since the value of the option to launch the pilot product is a function only of y and is independent of time, then we denote it $G(y)$. Hence, G is characterized by

$$\frac{1}{2}\zeta^2 y^2 G_{yy} + \mu y G_y - rG = 0 \quad (3.17a)$$

$$G(y^*) = W(y^*) - I_W \quad (3.17b)$$

$$G_y(y^*) = W_y(y^*) \quad (3.17c)$$

$$G(0) = 0 \quad (3.17d)$$

where y^* is the (unknown) free boundary when $x = 0$. Applying the usual methods we have

$$G(0, y) = G(y) = \begin{cases} My^\varphi & \text{for } y < y^* \\ W(y) - I_W & \text{for } y \geq y^* \end{cases}$$

where φ is defined in Equation (3.10), and, as in the analysis of the option to launch Newprox after T ,

$$y^* = \frac{\varphi}{\varphi - 1} \frac{r - \mu}{q} I_W = \theta, \quad M = 0, \quad W(y^*) - I_W = 0 :$$

the option to invest in Minprox is always worthless, so there is no point in waiting. Moreover, the net present value of Minprox cash flows is zero at the investment threshold, so there is no point in launching Minprox *per se*. Hence, the only incentive for a firm to enter this market is the subsequent opportunity to launch a main product (Newprox) starting from Minprox.

On the other hand, let $y = 0$. In this case, since the value of the option is a function of x and t , we denote it $G(x, t)$. Since it has been assumed that Minprox is a necessary step to obtain the opportunity to invest in Newprox, for some values $x \neq 0$ it could be worth investing in Minprox, even if this is valueless ($W(0) = 0$). Following the usual line of reasoning, G satisfies equations

$$\frac{1}{2}\sigma^2(t)x^2 G_{xx} + \nu(t)xG_x + G_t - rG = 0 \quad (3.18a)$$

$$G(0, t) = 0 \quad (3.18b)$$

$$G(x_t^{**}) = -I_W + F(x_t^{**}). \quad (3.18c)$$

$$G_x(x_t^{**}) = F_x(x_t^{**}) \quad (3.18d)$$

where x_t^{**} is the (unknown) boundary. This problem must be solved numerically. Nevertheless, if $t \geq T$, G does not depend on t any more ($G_t = 0$), and $F \equiv 0$ from problem (3.13). Hence, the solution to (3.18) is $G(x, 0, t) = 0$ at any x , because it is an option on a worthless option which is costly (I_W) to obtain.

4 Numerical analysis

In this section we provide a numerical solution of problems (3.11) and (3.15) employing a log-transformed binomial lattice approximation of the diffusions of the state variables, according to Gamba and Trigeorgis (2002). This approach proves to be more efficient than other lattice methods, such as Boyle, Evnine and Gibbs (1989) and Ekvall (1996), as far as multidimensional geometric Brownian motion processes are concerned. This feature of the log-transformed binomial lattice approximation is extremely important in this contest, since the analysis below is computationally intensive. An outline of this lattice approach and of the valuation procedure is described in Appendix B.

We also provide a description of G and F as a function of x and y , for different values of the model parameters, in order to describe how the value of the investment project, the value of the option to invest, and the investment decision rule change as a function of parameters. This analysis is performed by computing $G(x, y, t)$ (and $F(x, y, t)$), at a specific time t , in a regular grid within the rectangle $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$.¹⁴

As far as the calculation of the free-boundaries is concerned, the curves are obtained by searching an approximate 0-level sections of $G(x, y, t) - W(y) - I_W + F(x, y, t)$.¹⁵ In fact, the search for exact 0-level curve is very difficult because the exact 0-level curve degenerates in a region in $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$.

For definiteness, the base case parameters are given in Table 3.

[Table 3 about here]

In Figure 3 we plot the expanded-NPV (E-NPV) of the whole project, i.e. the value of the project including the value of the option to defer the investment, as a function of x and y , which represent the current values of the stochastic processes $X(t)$ and $Y(t)$. E-NPV is increasing both in x and

¹⁴We compute the value of F and G in a grid with 51×51 points, and, for each computation, we use the log-transformed binomial lattice scheme with 40 time steps.

¹⁵In detail, the graph show an ϵ -level section for $\epsilon = 0.1$.

y and at $x = 0$ is worthless for any y . In fact, since the market for Minprox is in equilibrium under the hypothesis of perfect competition, the NPV is negative when $y < \theta (\approx 7.62)$ and equal to zero when $y \geq \theta$.

[Figure 3 about here]

Moreover, Figure 3 shows the (time-) value of the option to delay the investment in Minprox, computed as the difference between the E-NPV of Minprox and the net present value of a committed investment in Minprox, that equals $W(y) + F(x, y, t) - I_W$, with $F(x, y, t)$ obtained by solving problem (3.11) numerically at the points of the grid within $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$. As expected, the option value decreases as either x or y increase (the other one being constant).¹⁶

Lastly, in Figure 3 we plot the investment threshold for Minprox at $t = 0$, i.e., the set of triples $(x, y, 0)$ in Ω_M . The set below the investment threshold (which falls in the positive orthant) is the continuation region of problem (3.15) at $t = 0$, i.e., the set of (x, y, t) in \mathcal{C}_M for which it is optimal to postpone the launch of the pilot product. The continuation region is below the threshold because the payoff of the option to launch Minprox at any given t is an increasing function of both x and y . As we will see below, the threshold curve for G will always have the same shape as the one illustrated in Figure 3, so that, for high values of x , the investment is optimal even if y is low, and viceversa. Moreover, we observe that the region is well set within the rectangle $[0, \kappa] \times [0, \theta]$, where κ and θ are the reflecting barriers of the units sold respectively for Newprox and Minprox in the steady state situation.

Figure 4 provides a comparison of the investment thresholds for Minprox and Newprox at different values of the correlation coefficient among products, γ , and at different values of the complementarity parameter, η . We determine the investment thresholds at $\gamma = 0.99 \approx 1$, $\eta = 0$, $\eta = 1$ and at $\gamma = -0.99 \approx -1$, $\eta = 0$, $\eta = 1$ and compare them to the base case. In the base case (i.e., $\eta = 0.5$ and $\gamma = 0$) the opportunity to invest in Newprox clearly limits the waiting region rightwards for Minprox.

[Figure 4 about here]

On the other hand, the thresholds change for both a different correlation and for a different degree of complementarity/substitutability. Let us look first at the impact on the thresholds due to a higher correlation. The investment threshold of Newprox is shifted leftwards and the continuation region

¹⁶The surface is irregular for smaller values of y because of a poor numerical approximation.

for Newprox is shrunk; i.e., a lower level of expected sales of Newprox is required to make the option exercisable. Alternatively, investment in Newprox becomes more likely and the option value is reduced. Notably, this is true both at $\eta = 1$ and $\eta = 0$. The size of the waiting region for Minprox is increased and as such the option value. Since Newprox's sales are more likely to follow the path of Minprox's sales, the latter need to be higher to provide a positive "context" for the decision to invest in the entire project. In other words, a positive correlation makes the option to delay more valuable and investment in Minprox less likely. Again, this is true for both $\eta = 1$ and $\eta = 0$.

In terms of the effect of complementarity/substitutability, it is worth noticing that in all of the cases shown in Figure 4, a higher degree of substitutability ($\eta = 1$) among products cause each investment threshold to shift rightwards as, *coeteris paribus*, a higher amount of expected sales are always needed to offset the more likely effect of substitution among products.

To summarize, Figure 4 provides an insight on the impact of γ and η on the time value of Minprox (and so, on the value of the whole project). It shows that the time value of Minprox in the case of higher correlation and substitutability is always higher than the case of lower correlation and complementarity among products. Hence, the opportunities to defer the investment in both products are increasingly valuable in a relatively more uncertain context where the substitutability among highly correlated products increases the opportunity cost of launching Newprox. With reference to Figure 2, the above analysis allows us to provide guidance also in cases where intuition is of little help, as noted at the outset of our analysis. Lastly, correlation also has an effect on the probability of a joint launch of both products (i.e., the launch is decided on the same date). A negative correlation expands the continuation region for Newprox and shrinks the continuation region for Minprox; i.e., it is more likely that some time will elapse after the launch of Minprox is decided. The opposite is true for a positive correlation: it is more likely that the launch of Minprox and the launch of Newprox will be decided together.

The impact of a different time to maturity of the patent on the value of the project is remarkable as well. Figure 5 shows the results of a sensitivity analysis of the time-to-maturity of the patent (for 20, 10, 5, 2 years to maturity) on the value of the option to delay the launch of Minprox and on the investment threshold. As we can see, both the value of the option and the continuation region decrease in relation to the time to maturity; i.e., as t increases (respectively, $t = 0, 10, 15, 18$), the launch of Minprox becomes less likely. It should be stressed that this result is strictly related to the choice of parameters.

5 Business applications

5.1 The Schering Plough case

One concrete application of the model presented in this work refers to Schering Plough and its investment strategy in the Asthma-Allergy therapeutic area. Schering Plough has produced for approximately ten years a well-known anti-histamine product (Claritine). Due to the success of this product, the company enjoys a strong image in the anti-histamine area. Since Claritine's patent was about to expire, the company was selecting a new product that would be able to supplant Claritine's role as a major source of the group's revenues.

This is the context in which the company, in the early 1990's, decided to enter the asthma field and started the development of a new product, Asmanex, with the following key characteristics:

- i)* it uses the same molecule as Claritine, so that Schering Plough can take advantage of the positive brand image built over the years, with notable advantages especially in the uptake phase of the new product;
- ii)* it focuses on asthma, a therapeutic area with one of the strongest expected growth rates. In fact, the company expected gross revenues to reach one billion dollars over a five year period;
- iii)* the patent will expire in 15 years.

Asmanex is, therefore, a key product for Schering Plough, and will underpin the company's development strategy for the next decade. Asmanex, however, poses two basic problems.

The first problem is of a marketing nature. Few of the allergy symptoms can be jointly treated with asthma pathologies, and this weakens the relationship between the two products. Moreover, although the product contains the same molecule as Claritine, the asthma pathology is different from allergy, and Schering Plough does not have a significant experience in asthma (currently, Glaxo Wellcome and Astra are the major competitors with significant experience in the area of asthma pathologies). Consequently, Schering Plough is faced with the challenge of bridging the gap between Claritine and Asmanex so as to associate the use of Asmanex with other products of the same therapeutic area.

The second problem is related to the costly and irreversible investment due to the experimentation program required by the FDA. The company is required to conduct tests on approximately 10,000 patients, at a cost of

nearly \$5,000 each. The present value of the total investment, including fixed costs due to facilities, amounts to approximately \$275 million.

One feasible answer to the above problems lies in a product designed to treat nasal congestion due to allergies, Nasonex. Nasonex requires fairly contained fixed costs since the experimentation phase will require tests on 2,000 patients for a period of a few months. Thanks Nasonex the company will be able to manage both problems. In particular, Schering Plough bases its marketing strategy upon the following key success factors:

- a) pointing out the link between nasal congestion due to allergy and asthma;
- b) adopting a portfolio strategy based on complementary products;
- c) actively managing the products' life cycle.

In particular, Nasonex would underline the continuity of investments in the company image and would represent a bridge between allergy and asthma. In fact, the company would emphasize that asthma is nothing more than the result of a poorly cured or an incurable allergy. In this way, Nasonex would have the characteristics to be jointly prescribed with Asmanex, and therefore would become the mechanism through which Schering Plough can actively manage the transition phase between the two pathologies.

Having said that, the valuation model presented in this work fits with the business case of Schering Plough and answers two fundamental questions:

1. what is the value and the optimal rule to invest in the clinical trials of Asmanex (key product)?
2. how valuable is Nasonex as a pilot product and as a required stage to implement a marketing strategy of presenting Nasonex and Asmanex as complementary products?

The correlation between Nasonex and Asmanex is expected to be fairly positive, due to the continuity of the asthma and allergy pathologies. Hence, the current case may belong to Case 1 of Table 1.

5.2 The Adaptec case

Adaptec is a leading supplier of storage access solutions deriving 80% of its revenues from Small Computer Systems Interface (SCSI) products, a market in which Adaptec is one of the leaders. This market relationship has been key to the company's success and is expected to drive revenue in the short-term.

As explained before, the SCSI technology is being replaced by fiber channel in some applications. Adaptec was late in introducing its fibre channel products and has not enjoyed the same market leadership with them as it did with SCSI. However, the company has made several pilot investments in the fiber protocol, analyzed several ways to overcome the constraints of the SCSI technology and explored the real upside potential of the high performance storage connectivity market.

These investments are now becoming even more strategic. In fact, as fibre channel technology continues to take market share from SCSI, Adaptec runs the risk of declining revenues.

In order to overcome this risk, the company is now turning its attention to the next cutting edge technology, the iSCSI (i.e., the standard SCSI where storage data is moved over the Internet), which is considered a key investment program. iSCSI-based Storage Area Network (SAN) is expected to be a disruptive technology, which has the potential to replace fiber channel-based SANs. Due to the considerable expected growth rate of iSCSI over the next five years (400% CAGR), Adaptec's business unit dedicated to SAN products could experience huge growth over the next few years. If IDC estimates are correct, revenues in this segment will offset the diminished income due to the shift away from SCSI technology.

An initial sign of a concrete interest in the iSCSI segment is the recent acquisition of Platys (approximately \$150 million in value), a small player fully dedicated to the development of the iSCSI protocol. By doing so Adaptec has strategically entered the iSCSI market and is now positioned with Cisco and NetApp in developing the new protocol.

Design lock-in is what drives Adaptec's new market. Once an Original Equipment Manufacturers (OEM) decides on a Host Bus Adaptor (HBA) or other card, many engineers work for up to a year writing the drivers and certifying the HBA. This commitment of time and effort locks an OEM into certain devices. When a different vendor comes along with a new device, the new capabilities have to be weighted against the time and effort required to certify the new device. When awarded, design wins usually lock-in an OEM for five years. By winning contracts with major hardware vendors, Adaptec will firmly position itself as a player in this market while validating its Platys acquisition and technology.

The key risk is the timing and degree of iSCSI adoption and Adaptec's execution. In fact, iSCSI success is not assured as its advantages over the incumbent Fibre Channel protocol are not as compelling as previous technology transitions. In particular, in the iSCSI adapter market, the competitive landscape is shaping up to be quite tough with almost all the major players working actively in this space (e.g., Intel, Emulex and QLogic).

Our model may help interpret the investment in the Fiber Channel segment as being a pilot project designed to ramp up the company's ability to serve the demand for high performance storage connectivity. While doing so, Adaptec may acquire critical information on such demand and on how manageable the transition from fiber to iSCSI is. Consequently, the large investment in the iSCSI protocol, as a key investment area, may be optimized over time and gradually implemented as the market demand becomes ready to be shifted away from fiber.

Such an analysis leads us to the conclusion that the current Adaptec case is a good example of Case 4 of Table 1. In fact, the high degree of substitutability is associated with an expectation of a negative correlation among products. This is mainly supported by the empirical evidence of the last few years where breakthrough technologies have shown a negative correlation with the incumbent products.

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A Proofs

We will verify that

$$V(x, t) = \Phi(x, t) + \Psi(x, t)$$

with $\Phi(x, t)$ from equation (3.3) and $\Psi(x, t)$ from equation (3.4). To derive Ψ analytically, we will discuss separately the cases $t < T$ and $t \geq T$.

Let first consider the value of the revenues after the expiration of the patent: $t \geq T$. In this case the horizon is infinite. The present value of the revenues after patent expiry is

$$\psi_2(x) = \begin{cases} \frac{\lambda p}{\delta_2} \left(x - \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho \right) & \text{if } x < \kappa \\ \frac{\lambda p}{\delta_2} \frac{\varrho-1}{\varrho} \kappa & \text{if } x \geq \kappa \end{cases} \quad (\text{A.1})$$

with $\delta_2 = r - \nu_2$ and where

$$\varrho = \frac{1}{2} - \frac{\nu_2}{\sigma_2^2} + \sqrt{\left(\frac{\nu_2}{\sigma_2^2} - \frac{1}{2} \right)^2 + 2 \frac{r}{\sigma_2^2}} > 1 \quad (\text{A.2})$$

is one of the roots of

$$\frac{1}{2} \sigma_2^2 \varrho (\varrho - 1) + \nu_2 \varrho - r = 0$$

as in Dixit and Pindyck (1994), pp. 255-256.

Equation (A.1) can be proved as follows. When $x < \kappa$, the value of Newprox is

$$\psi_2(x) = \frac{\lambda p}{\delta_2} \left(x - \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho \right) \quad (\text{A.3})$$

because κ is a reflecting barrier for $X(t)$. (A.3) is the solution of equation

$$\frac{1}{2} \sigma_2^2 x^2 f_{xx}(x) + \nu_2 x f_x(x) - r f(x) + \lambda p x = 0 \quad (\text{A.4})$$

with condition $\psi(0) = 0$; i.e.,

$$\psi_2(x) = Bx^\varrho + \frac{\lambda p x}{\delta_2} \quad (\text{A.5})$$

where ϱ is defined in (A.2). Since κ is a reflecting barrier, $\psi_2(x)$ in (A.5) satisfies the following condition (see Malliaris, Brock (1983), p. 200):

$$\psi_x(\kappa) = \varrho B \kappa^{\varrho-1} + \frac{\lambda p}{\delta_2} = 0$$

which implies

$$B = -\frac{\lambda p \kappa^{1-\varrho}}{\varrho \delta_2}. \quad (\text{A.6})$$

On the other hand, when $x \geq \kappa$ we have

$$\psi_2(x) = \frac{\lambda p}{\delta_2} \frac{\varrho - 1}{\varrho} \kappa; \quad (\text{A.7})$$

i.e., the value of ψ_2 at κ . Equations (A.5), (A.6) and (A.7) considered altogether provide Equation (A.1).

Next, consider $t < T$. In this case, the value at (x, t) of the revenues obtained after the expiry is the expectation at (x, t) of the value at T , since at that date the (infinite horizon) steady state is immediately reached. Hence, putting $\delta_1 = r - \nu_1$,

$$\psi_1(x, t) = e^{-r\tau} \mathbb{E}_{x,t}^* [\psi_2(X_T)] \quad (\text{A.8a})$$

$$= \frac{\lambda p}{\delta_2} x e^{-\delta_1 \tau} \mathcal{N}(z(x, t) - \sigma \sqrt{\tau}) - \quad (\text{A.8b})$$

$$- \frac{\lambda p}{\delta_2} \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho e^{(\nu_1 \varrho + \frac{1}{2} \sigma_1^2 \varrho (\varrho - 1) - r)\tau} \mathcal{N}(z(x, t) - \sigma_1 \varrho \sqrt{\tau}) + \quad (\text{A.8c})$$

$$+ \frac{\lambda p}{\delta_2} \frac{\varrho - 1}{\varrho} \kappa e^{-r\tau} \mathcal{N}(-z(x, t)) \quad (\text{A.8d})$$

where $\mathcal{N}(\cdot)$ is the cumulative standard normal distribution, $T - t = \tau$, and

$$z(x, t) = \frac{\log \frac{\kappa}{x} - (\nu_1 - \frac{1}{2} \sigma_1^2) (T - t)}{\sigma_1 \sqrt{T - t}}.$$

To prove (A.8), let $f(z)$ be the probability density function of the standard normal distribution. The expectation in Equation (A.8a) is

$$\mathbb{E}_{x,t} [\psi_2(X_T)] = \int_{-\infty}^{+\infty} \psi \left(x e^{(\nu_1 - \frac{1}{2} \sigma_1^2)\tau + \sigma_1 \sqrt{\tau} s} \right) f(s) ds \quad (\text{A.9a})$$

$$= \frac{\lambda p}{\delta_2} x \int_{-\infty}^{z(x,t)} e^{(\nu_1 - \frac{1}{2} \sigma_1^2)\tau + \sigma_1 \sqrt{\tau} s} f(s) ds - \quad (\text{A.9b})$$

$$- \frac{\lambda p}{\delta_2} \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho \int_{-\infty}^{z(x,t)} e^{(\nu_1 - \frac{1}{2} \sigma_1^2)\varrho \tau + \sigma_1 \varrho \sqrt{\tau} s} f(s) ds + \quad (\text{A.9c})$$

$$+ \frac{\lambda p}{\delta_2} \frac{\varrho - 1}{\varrho} \kappa \int_{z(x,t)}^{+\infty} f(s) ds. \quad (\text{A.9d})$$

From expression (A.9b) we have

$$\begin{aligned}
\frac{\lambda p}{\delta_2} x \int_{-\infty}^{z(x,t)} e^{(\nu_1 - \frac{1}{2}\sigma_1^2)\tau + \sigma_1\sqrt{\tau}s} f(s) ds &= \\
&= \frac{\lambda p}{\delta_2} x e^{(\nu_1 - \frac{1}{2}\sigma_1^2)\tau} e^{\frac{1}{2}\sigma_1^2\tau} \int_{-\infty}^{z(x,t)} \frac{e^{-\frac{1}{2}(s - \sigma_1\sqrt{\tau})^2}}{\sqrt{2\pi}} ds = \\
&= \frac{\lambda p}{\delta_2} x e^{\nu_1\tau} \mathcal{N}(z(x,t) - \sigma_1\sqrt{\tau}).
\end{aligned}$$

As far as (A.9c) is concerned, we have

$$\begin{aligned}
\frac{\lambda p}{\delta_2} \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho \int_{-\infty}^{z(x,t)} e^{(\nu_1 - \frac{1}{2}\sigma_1^2)\varrho\tau + \sigma_1\varrho\sqrt{\tau}s} f(s) ds &= \\
&= \frac{\lambda p}{\delta_2} \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho e^{(\nu_1 - \frac{1}{2}\sigma_1^2)\varrho\tau} e^{\frac{1}{2}\sigma_1^2\varrho^2\tau} \int_{-\infty}^{z(x,t)} \frac{e^{-\frac{1}{2}(s - \sigma_1\varrho\sqrt{\tau})^2}}{\sqrt{2\pi}} ds \\
&= \frac{\lambda p}{\delta_2} \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho e^{(\nu_1\varrho + \frac{1}{2}\sigma_1^2\varrho(\varrho-1))\tau} \mathcal{N}(z(x,t) - \sigma_1\varrho\sqrt{\tau})
\end{aligned}$$

Finally, (A.9d) is straightforward. By plugging (A.9a) in (A.8a) we have

$$\begin{aligned}
\psi_1(x,t) &= e^{-r\tau} \frac{\lambda p}{\delta_2} [x e^{\nu_1\tau} \mathcal{N}(z(x,t) - \sigma_1\sqrt{\tau}) - \\
&\quad - \frac{\kappa^{1-\varrho}}{\varrho} x^\varrho e^{(\nu_1\varrho + \frac{1}{2}\sigma_1^2\varrho(\varrho-1))\tau} \mathcal{N}(z(x,t) - \sigma_1\varrho\sqrt{\tau}) + \kappa \frac{\varrho-1}{\varrho} \mathcal{N}(-z(x,t))]
\end{aligned}$$

that is, Equation (A.8).

Summing up, the present value at (x, t) of cash flows from Newprox after the expiry is

$$\Psi(x,t) = \begin{cases} \psi_1(x,t) & \text{if } t < T \\ \psi_2(x) & \text{if } t \geq T. \end{cases}$$

with $\psi_1(x_t, t)$ from Equation (A.8) and $\psi_2(x_t)$ from Equation (A.1).

B Numerical methods

We summarize here the main features of the improved log-transformed binomial lattice approach, suited to price options with payoffs depending on a multidimensional log-Normal diffusion, as proposed in Gamba and Trigeorgis (2002). We specialize it to the two-dimensional setting of our valuation problem. The log-transformed method maintains the stability feature of the one-dimensional approach proposed by Trigeorgis (1991).

Given the dynamics of sales

$$\begin{aligned} dX(t) &= \nu_1 X(t)dt + \sigma_1 X(t)dZ_x(t) \\ dY(t) &= \mu Y(t)dt + \zeta Y(t)dZ_y(t) \end{aligned} \quad (\text{B.1})$$

with correlation coefficient γ , we take $\widehat{X} = \log X$ and $\widehat{Y} = \log Y$, so that

$$\begin{aligned} d\widehat{X}(t) &= a_1 dt + \sigma_1 dZ_x(t), \\ d\widehat{Y}(t) &= a_2 dt + \zeta dZ_y(t), \end{aligned} \quad (\text{B.2})$$

where $a_1 = \nu_1 - \sigma_1^2/2$ and $a_2 = \mu - \zeta^2/2$. Let $a^\top = (a_1, a_2)$,

$$\Sigma = \begin{pmatrix} 1 & \gamma \\ \gamma & 1 \end{pmatrix}, \quad b = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \zeta \end{pmatrix}, \quad \Omega = b\Sigma b^\top = \begin{pmatrix} \sigma_1^2 & \sigma_1\zeta\gamma \\ \sigma_1\zeta\gamma & \zeta^2 \end{pmatrix}.$$

Define

$$\rho_{1,2} = \frac{1}{2} \left(\sigma_1^2 + \zeta^2 \mp \sqrt{\sigma_1^4 + 2(1 - 2\gamma^2)\sigma_1^2\zeta^2 + \zeta^4} \right),$$

the diagonal matrix $\Lambda = (\rho_i)$, and matrix

$$W = \begin{pmatrix} \left(\frac{\rho_1}{\sigma_1\zeta} - \frac{\zeta}{\sigma_1} \right) / (\gamma c_1) & \left(\frac{\rho_2}{\sigma_1\zeta} - \frac{\zeta}{\sigma_1} \right) / (\gamma c_2) \\ 1/c_1 & 1/c_2 \end{pmatrix}$$

where

$$c_i = \sqrt{1 + \frac{(\rho_i - \sigma_2^2)^2}{\gamma^2 \sigma_1^2 \sigma_2^2}}.$$

W is a matrix providing a change of coordinates of the plane \widehat{X} - \widehat{Y} so that the dynamics are uncorrelated. Hence, denoting $y^\top = (\widehat{x}, \widehat{y})$ the vector of variables transformed through W , the diffusion process of y is

$$\begin{aligned} d\widehat{x} &= A_1 dt + B_{11} dZ_1 + B_{12} dZ_2 \\ d\widehat{y} &= A_2 dt + B_{21} dZ_1 + B_{22} dZ_2 \end{aligned}$$

where $B = (B_{ij}) = W^\top b$ and $A = W^\top a$. The covariance matrix of dy is $dydy^\top = \Lambda dt$; that is, $d\hat{x}$ and $d\hat{y}$ are uncorrelated.

We approximate $(d\hat{x}, d\hat{y})$ with a discrete process: given the time interval $[0, T]$ and n , we consider subintervals of width $\Delta t = T/n$. The discrete process is (\tilde{x}, \tilde{y}) with dynamics

$$\begin{aligned}\tilde{x}(t) &= \tilde{x}(t - \Delta t) + \ell_1 U_1(t) \\ \tilde{y}(t) &= \tilde{y}(t - \Delta t) + \ell_2 U_2(t)\end{aligned}\tag{B.3}$$

$t = 1, \dots, n$ where (U_1, U_2) is a bi-variate i.i.d. binomial random variable:

$$(U_1, U_2) = \begin{cases} (1, 1) & \text{with probability } p_1 \\ (1, -1) & \text{w.p. } p_2 \\ (-1, 1) & \text{w.p. } p_3 \\ (-1, -1) & \text{w.p. } p_4 \end{cases}$$

and $\sum_{i=1}^4 p_i = 1$. We assign parameters

$$k_i = A_i \Delta t, \quad \ell_i = \sqrt{\rho_i \Delta t + k_i^2}, \quad L_i = k_i / \ell_i$$

$i = 1, 2$ and probabilities

$$p(s) = \frac{1}{4} (1 + \Gamma_{12}(s)L_1L_2 + \Gamma_1(s)L_1 + \Gamma_2(s)L_2) \quad s = 1, 2, 3, 4, \tag{B.4}$$

where

$$\Gamma_i(s) = \begin{cases} 1 & \text{if state variable } i \text{ jumps up} \\ -1 & \text{if state variable } i \text{ jumps down} \end{cases}$$

for $i = 1, 2$, and $\Gamma_{12}(s) = \Gamma_1(s)\Gamma_2(s)$.

For the discrete-time process, we have the following:

$$\begin{aligned}\mathbb{E}[\Delta\tilde{y}_i] &= k_i = A_i \Delta t & i = 1, 2 \\ \text{Var}[\Delta\tilde{y}_i] &= \ell_i^2 - k_i^2 = \rho_i \Delta t & i = 1, 2 \\ \text{Cov}[\Delta\tilde{y}_1, \Delta\tilde{y}_2] &= 0.\end{aligned}$$

Hence, this discrete process is consistent with the continuous process for any time step.

We want to evaluate an option whose payoff, Π , is a non-linear function of $(X(t), Y(t))$. According to the change of variable in (B.2), the payoff becomes

$$\Pi(X(0)e^{\hat{X}(t)}, Y(0)e^{\hat{Y}(t)}).$$

We can make the derivative security dependent on $y^\top = (\hat{x}, \hat{y})$ by changing the payoff function as follows:

$$\hat{\Pi}(\hat{x}(t), \hat{y}(t)) = \Pi(X(0)e^{(Wy(t))_1}, Y(0)e^{(Wy(t))_2})$$

where $(Wy(t))_i$ is the i -th component of vector $Wy(t)$. The risk-neutral price of $\hat{\Pi}$, denoted \hat{F} , is equal to the risk-neutral price of Π , denoted F (we refer to Gamba and Trigeorgis (2002) for details):

$$\begin{aligned}\hat{F}(\hat{x}(t), \hat{y}(t)) &= e^{r(T-t)} \hat{\mathbb{E}} \left[\hat{\Pi}(\hat{x}(T), \hat{y}(T)) \right] \\ &= e^{r(T-t)} \mathbb{E} [\Pi(X(T), Y(T))] = F(X(t), Y(t))\end{aligned}$$

where $\hat{\mathbb{E}}[\cdot]$ denotes the risk neutral expectation with respect the martingale probability of the process $\{(\hat{x}, \hat{y})\}$, and $\mathbb{E}[\cdot]$ is the expectation w.r.t. the martingale probability of the process $\{(X, Y)\}$.

In order to compute the value of the option to launch the two products when a closed form formula is not available, we exploit the above illustrated extended log-transformed binomial lattice approximation of the diffusion in (B.1). Hence, by approximating (\hat{x}, \hat{y}) with (\tilde{x}, \tilde{y}) , as of (B.3), the value of the option to launch Newprox is obtained by backward induction: at $t = T$

$$\hat{F}(\tilde{x}(T), \tilde{y}(T)) = \max \{ \Pi_1(\tilde{x}(T), \tilde{y}(T)), 0 \}$$

and at $t < T$

$$\hat{F}(\tilde{x}(t), \tilde{y}(t)) = \max \left\{ \Pi_1(\tilde{x}(t), \tilde{y}(t)), e^{-r\Delta t} \tilde{\mathbb{E}}_t \left[\hat{F}(\tilde{x}(t + \Delta t), \tilde{y}(t + \Delta t)) \right] \right\}$$

where

$$\Pi_1(\tilde{x}(t), \tilde{y}(t)) = V(\tilde{x}(t), t) - I_V - \eta W(\tilde{y}(t))$$

and $\tilde{\mathbb{E}}_t[\cdot]$ denotes conditional expectation, at t , according to discrete probability in (B.4).

The same can be done to compute the value of Minprox: at $t = T$

$$\hat{G}(\tilde{x}(T), \tilde{y}(T)) = \max \{ \Pi_2(\tilde{x}(T), \tilde{y}(T)), 0 \}$$

and at $t < T$

$$\hat{G}(\tilde{x}(t), \tilde{y}(t)) = \max \left\{ \Pi_2(\tilde{x}(t), \tilde{y}(t)), e^{-r\Delta t} \tilde{\mathbb{E}}_t \left[\hat{G}(\tilde{x}(t + \Delta t), \tilde{y}(t + \Delta t)) \right] \right\}$$

where

$$\Pi_2(\tilde{x}(t), \tilde{y}(t)) = W(\tilde{y}(t)) - I_V + F(\tilde{x}(t), \tilde{y}(t), t).$$

The extension of this methodology to a multidimensional problem (with more than two underlying assets) and more compounded options (i.e., more interacting products/projects) is straightforward.

Exhibits

<p>Case 1 Positive correlation Complementarity</p> <p><i>Interaction</i></p>	<p>Case 2 Positive correlation Substitutability</p> <p><i>Cannibalization</i></p>
<p>Case 3 Negative correlation Complementarity</p> <p><i>Diversification</i></p>	<p>Case 4 Negative correlation Substitutability</p> <p><i>Product rotation</i></p>

Figure 1: Complementarity/Substitutability vs correlation

<p>Case 1 Positive correlation Complementarity</p> <p>?</p>	<p>Case 2 Positive correlation Substitutability</p> <p><i>Need more units sold</i></p>
<p>Case 3 Negative correlation Complementarity</p> <p><i>need less units sold</i></p>	<p>Case 4 Negative correlation Substitutability</p> <p>?</p>

Figure 2: Complementarity/Substitutability and effect on investment decisions

Table 1: R&D product status of the top 10 pharmaceutical companies ranked by number of products in R&D, 1997

Rank	Company	Preclinical studies	Clinical studies			FDA filing	Total
			P1	P2	P3		
1	Merck&Co	137	17	22	10	4	190
2	Hoechst	106	15	24	9	7	161
3	Novartis	82	27	33	12	5	159
4	Roche	82	21	36	10	8	157
5	Pharmacia & Upjohn	83	16	14	16	10	139
6	American Home Products	53	17	32	21	12	135
7	Lilly	84	11	12	7	0	114
8	Bristol Myers-Squibb	65	9	16	13	4	107
9	Glaxo Wellcome	56	11	25	6	4	102
10	SmithKline Beecham	45	14	19	18	6	102

(Source: Datamonitor, ADIS International, 1997)

Table 2: Efficiency of the drug development process

Stage	Preclinical studies	Clinical studies			FDA review	Total
		P1	P2	P3		
Avg. Duration years	5-7	1	2	3	1.5-2.5	12-15
% of success at each stage	< .01%	70 %	47 %	75 %	80 %	< .001%
Avg. cost per stage (\$ m)	6	12	12	100	40	170

(Source: Datamonitor, Lehman Brothers "New Drug Discovery Technologies", 3/1997)

Table 3: Base case parameters

r	risk-free rate	0.05	
μ	drift of sales of Minprox	0.01	
ζ	volatility of sales of Minprox	0.1	
ν_1	drift of sales of Newprox before T	0.1	
ν_2	drift of sales of Newprox after T	0.02	
σ_1	volatility of sales of Newprox before T	0.25	
σ_2	volatility of sales of Newprox after T	0.15	
γ	correlation coefficient	0	
T	patent expiry	20	(years)
I_W	capital expenditure for Minprox	30	(millions of \$)
I_V	capital expenditure for Newprox	400	(millions of \$)
p	price per unit of Newprox	1	(\$)
q	price per unit of Minprox	0.25	(\$)
λ	price reduction after patent expiry	0.8	
η	complementarity coefficient	0.5	

For these parameters, $\varrho = 1.75486$, $\phi = 2.70156$, $\kappa = 34.8711$, $\theta = 7.62094$.

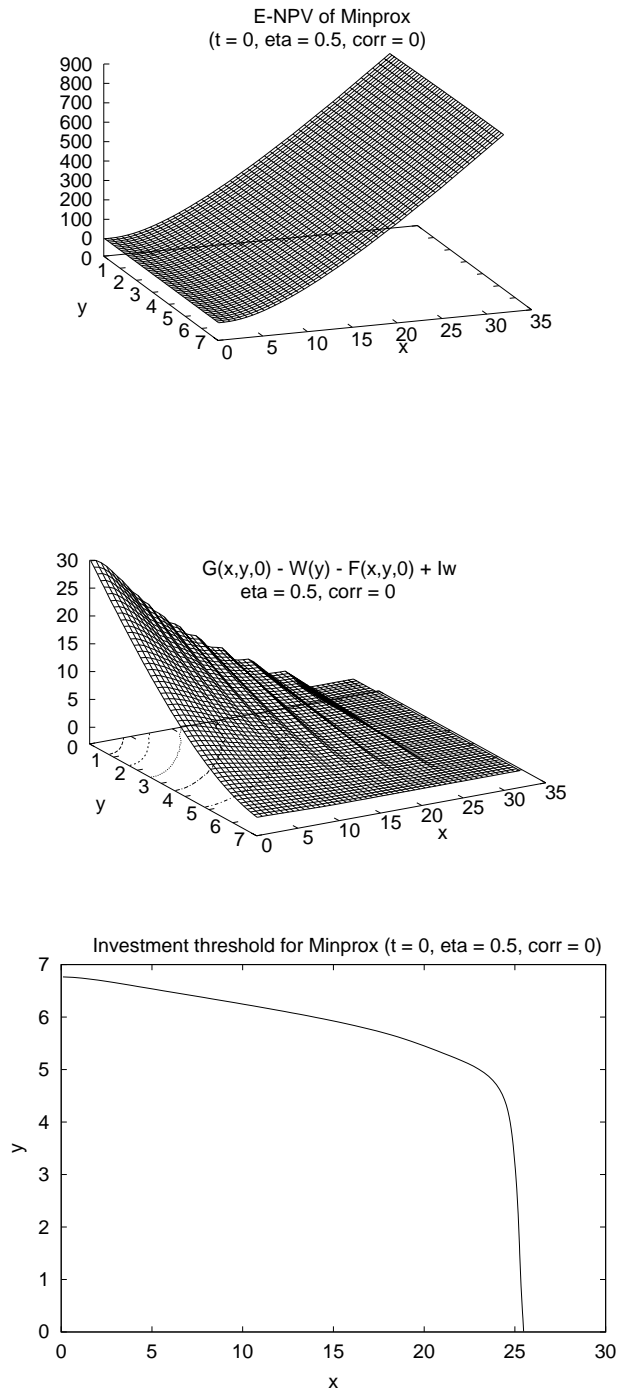


Figure 3: Base case ($\eta = 0.5, \gamma = 0$), at $t = 0$ (from top): expanded NPV of Minprox, $G(x, y, t)$; value of the option to invest in Minprox; investment threshold for Minprox, Ω_N .

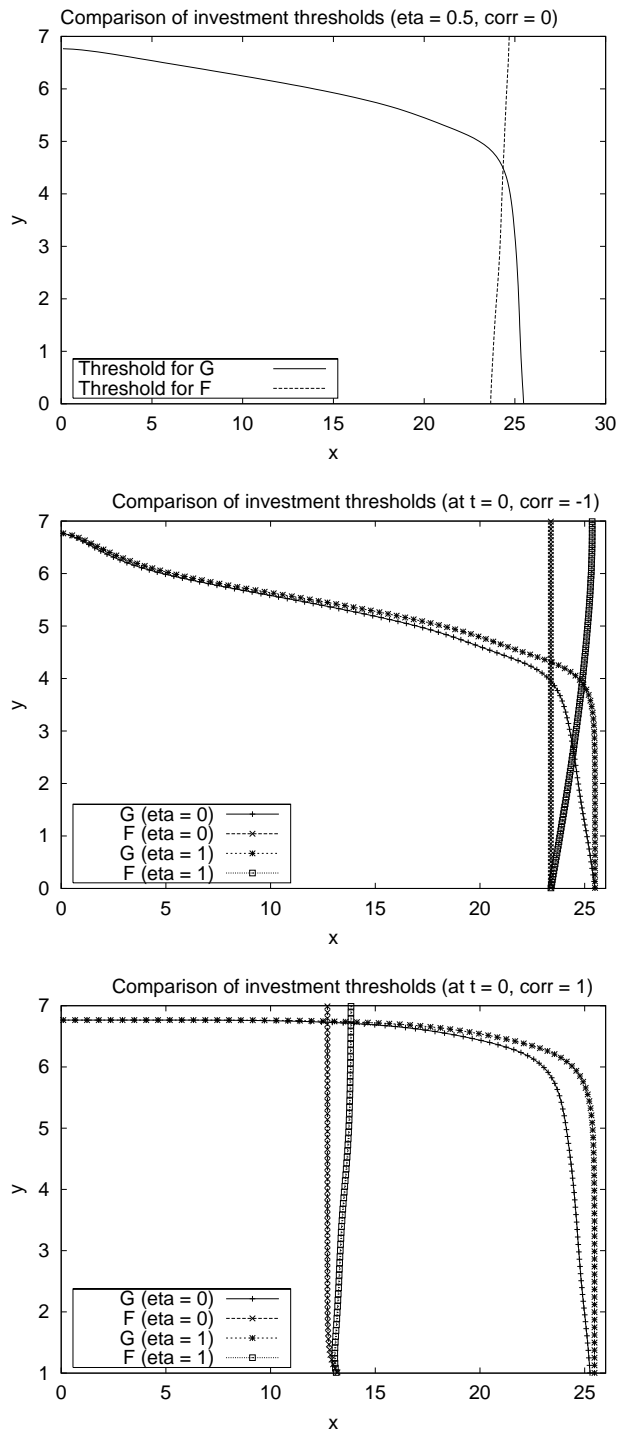


Figure 4: Comparison of trigger functions for Minprox and Newprox at different values of η and γ at $t = 0$.

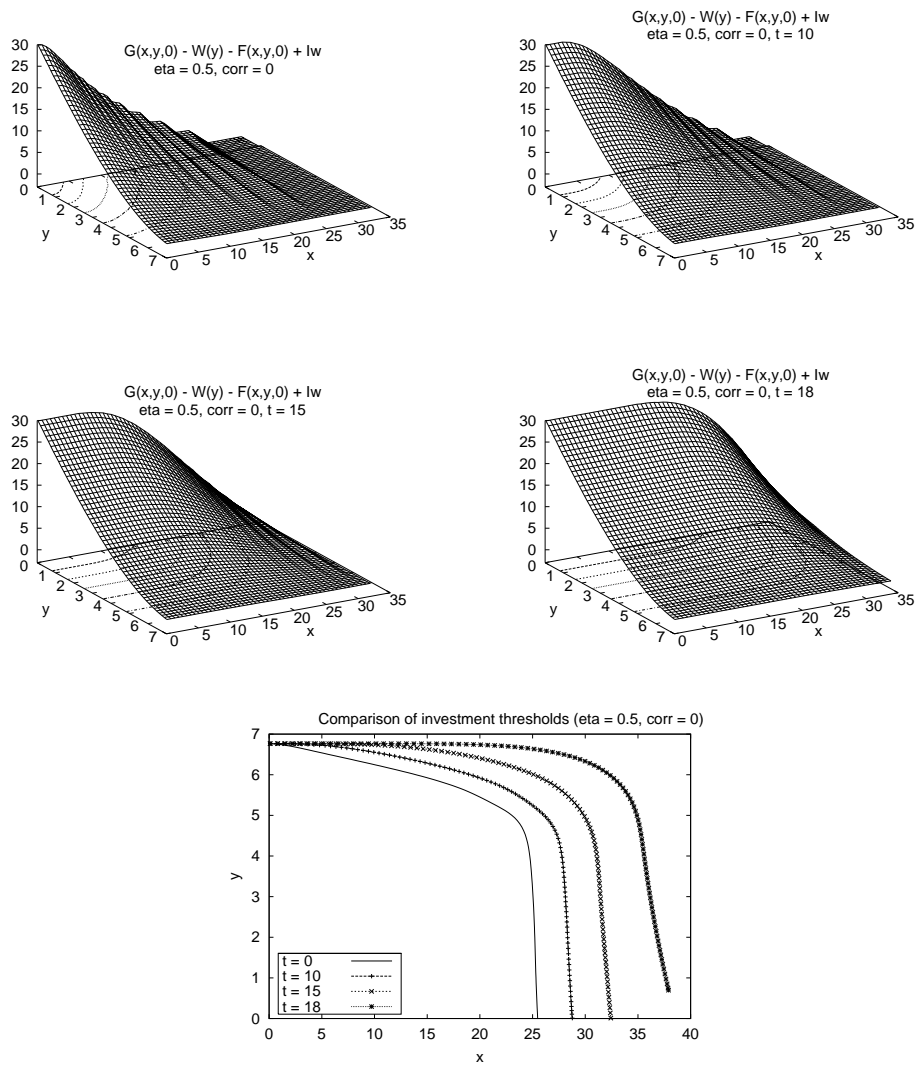


Figure 5: Sensitivity analysis on t ($\eta = 0.5$ and $\gamma = 0$).