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# Infation and Growth in the Long Run: A New Keynesian Theory and Further Semiparametric Evidence

## Abstract

This paper explores the influence of inflation on economic growth both theoretically and empirically. We propose to merge an endogenous growth model of learning by doing with a New Keynesian one with sticky wages. We show that the intertemporal elasticity of substitution of working time is a key parameter for the shape of the inflation-growth nexus. When it is set equal to zero, the inflation-growth nexus is weak and hump-shaped. When it is greater than zero, inflation has a sizeable and negative effect on growth. Endogenizing the length of wage contracts does not lead to inflation superneutrality in presence of a fixed cost to wage resetting. Once adopting various semiparametric and instrumental variable estimation approaches on a cross-country/time-series dataset, we show that increasing inflation reduces real economic growth, consistently with our theoretical model with a positive intertemporal elasticity of substitution of working time.

**Keywords:** inflation, growth, wage-staggering, learning-by-doing, semi-parametric estimator.

**JEL Codes:** E31, E51, E52, O42, C14.

# 1 Introduction

This paper offers a new theoretical model and new empirical evidence on the connection between inflation and growth.

In the theoretical part, we show that changes in the inflation rate can produce permanent changes in the growth rate of output - even though the model here proposed contains no money illusion, no permanent nominal rigidities, and no departure from rational expectations. We do so by extending the model by Graham and Snower (2004) from the inflation-output level domain to the inflation-output growth one. As a consequence, we merge an endogenous growth model of learning-by-doing with a New Keynesian one with sticky wages allowing to explore how the effect of inflation on output growth is connected not only to the direct effects of inflation on capital accumulation, but also to indirect ones passing through the labour market.

In this context, when money supply grows in the presence of temporary nominal frictions (in the form of staggered nominal contracts), nominal adjustments never have a chance to work themselves out fully. Thus, in an endogenous growth context, changes in money growth affect the marginal product of capital and thereby the rate of economic growth.

The basic intuition for the model here proposed can be described as follows.

- When nominal wage contracts are staggered, current contract wages depend on the current and future prices that prevail over the contract

period. The current contract wage is affected more by current prices than by expected future prices, due to time discounting.

- The greater is the rate of money growth, the faster prices rise, and the more the contract wage lags behind the current price level. Thus the lower is the average real wage over the contract period. Consequently, the more labor is demanded by firms.

In our endogenous growth model, an increase in the labour input raises the marginal product of capital, leading to faster economic growth. There is however an important countervailing effect:

- As money growth - and consequently inflation - increases, relative prices become more volatile, i.e. the real wage varies more over the contract period, since the nominal wage of each cohort is constant over the contract period whereas the aggregate price level rises gradually through time.
- This real wage volatility induces employment fluctuations, i.e. "employment cycling". Since there are diminishing returns to labor, this is inefficient leading to a reduction in the marginal product of capital and therefore in output growth.<sup>2</sup>

These influences imply a negative long-run relation between inflation and output growth.

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<sup>2</sup>Note that employment cycling takes place over a time period short enough (the contract period) for diminishing returns to be relevant.

We show, in a numerical example, that, when the intertemporal elasticity of substitution of working time is set to zero, the discounting effect is dominant at low money growth rates whereas the employment cycling effect is dominant at high money growth rates. By implication, the long-run relation between inflation and output growth is backward bending: growth rises with inflation at low inflation rates, but falls with inflation at high inflation rates.

However, once allowing for a positive intertemporal elasticity of substitution of working time, inflation has a negative impact on growth, because households, having a greater preference for labour supply smoothing, respond to increases in money growth - which entail more employment cycling - by raising their wage, reducing employment and output growth.

Finally, endogenizing the length of the labour contract reduces the strength of the effect of inflation on growth, but, in presence of a fixed cost to wage resetting, the negative effect of inflation on growth does not disappear. This is because households can reduce labour cycling by paying the wage resetting cost more often. So they choose wage flexibility only for a low level of this cost. The welfare implications of these different model parametrizations are also discussed in the paper.

The empirical part of the paper is mainly based on semiparametric estimation methods applied to a cross-country/time-series dataset, similarly to Vaona and Schiavo (2007). However, we follow a different research strategy. We first use various model selection criteria for semiparametric power series estimation approaches. Then, we check whether our results have an endo-

generosity bias and we conduct various sub-sample stability tests. Finally, instead of adopting a Nadaraya-Watson Kernel estimator we use a local linear one with a variable window width. Fan and Gijbels (1992) list a number of advantages of the latter estimator over the former one. In particular, it does not lose precision near the boundary of the observation interval (boundary effects). We find that higher inflation just harms real economic growth. The empirical results we obtain help us to choose our preferred calibration for the theoretical model here proposed.

The paper is organized as follows. Section 2 relates our contribution to the existing literature. Section 3 presents the underlying model. Section 4 sketches the model solution, which is illustrated in detail in Appendices A and B. Section 5 contains our results regarding the relations between inflation and growth and between inflation and welfare. Section 6 generalizes households' preferences and endogenizes the frequency of nominal adjustments. Section 7 shows our empirical results and Section 8 concludes.

## **2 Relation to the Literature**

### **2.1 Theoretical Literature**

With the exception of a few recent studies, the theoretical literature mainly produced models where inflation has a negative impact on growth.

One of the first theoretical studies concerning inflation and output is Tobin (1965), according to which inflation is beneficial to the output level

because it lowers the interest rate and therefore the opportunity cost to invest. This increases the capital-labour ratio and therefore output<sup>3</sup>. Stockman (1981) pointed to the possible existence of an inverse Tobin effect, whereby an increase in the inflation rate causes the capital stock to decrease, once supposing a cash in advance constraint for capital accumulation and given that inflation raises the cost of money holding.

More recently, the literature has shifted from the level of output to the output growth rate and from Solow models to endogenous growth ones. Gillman and Kejak (2005a) proposed to distinguish between physical capital models, labelled as  $Ak$ , human capital ones, labelled as  $Ah$ , and combined models, with both human and physical capital. The results have been mixed. Some contributions have produced insignificant long-run inflation-growth effects, such as the  $Ak$  models of Ireland (1994) and Dotsey and Sartre (2000) and the combined model of Chari *et al.* (1996). Other contributions have produced a negative and significant inflation-growth effect, such as the  $Ak$  models of Haslag (1998) and Gillman and Kejak (2004), the  $Ah$  model of Gillman *et al.* (1999), the combined models of Gomme (1993) and that of Gillman and Kejak (2002, 2005b). Gillman and Kejak (2005a) propose a model that nests most of the models advanced before. Their result is that in the  $Ak$  model inflation works as a tax on physical capital, implying a negative Tobin (1965) effect, whereas in the  $Ah$  model inflation works as a tax

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<sup>3</sup>For this definition of Tobin (1965) effect I rely on Gillman and Kejak (2005a). It is worth noting that a positive effect on inflation and the level of output might not in principle exclude a negative one on the growth rate of output and viceversa.

on human capital, implying a positive Tobin (1965) effect. Finally, in the combined model, inflation works more like a tax on human capital than on physical capital implying a positive Tobin (1965) effect.

Within a pecuniary transaction costs model, Zhang (2000) showed that an increase in money growth leads to lower steady state production inputs, consumption and real money balances.

Finally, Gylfason and Herbertsson (2001) proposed to insert real money balances into the production function, as a proxy of the effect of financial depth on production (about which see King and Levine, 1993; Levine, 1997; Gylfason and Zoega, 2006), and they found a negative effect of inflation on growth through three channels: it lowers the real interest rate, and therefore savings, it reduces efficiency by driving a wedge between the returns to real and financial capital and, finally, it reduces financial depth harming output.

In Wang and Yip (1992) inflation is negatively related to growth, because a reduction in real balances arising from an increase in the rate of monetary growth raises transaction time and therefore transaction costs. On the contrary Mino and Shibata (1995), in an overlapping generation framework, show that inflation may have a redistributive impact from one generation to the other and foster capital accumulation. Bonatti (2002a, b) argue that, when multiple balanced growth paths exist in a non-monetary economy, inflation targeting cannot resolve the resulting indeterminacy, whereas a fixed monetary growth rule can do it and it also determines the growth path of the economy. Furthermore, a restrictive monetary policy may select a lower

growth path than a more expansive one.

In Paal and Smith (2000), the relationship between money growth and real growth is showed to be characterized by a threshold. At low money growth rates, banks perceive a small opportunity cost in detaining reserves instead of lending funds for investments. As money growth rises, the nominal interest rate rises too increasing the opportunity cost of holding reserves and spurring lending and therefore investment and growth. When the nominal interest rate grows beyond a certain threshold level credit rationing badly affects lending, reducing capital accumulation and growth. Also Bose (2002) is concerned with the impact of inflation on growth passing through the credit market. This paper argues that, under asymmetric information between lenders and borrowers, there might be two lending regimes. In the former one, high and low risk borrowers are separated by credit rationing (rationing regime). In the latter one, separation takes place through costly screening (screening regime). A rise in the inflation rate increases the cost of screening or the incidence of credit rationing or it may even produce a change from the screening to the rationing regime. All these three effects are harmful to growth.

On the other hand, Funk and Kromen (2005, 2006) investigated the connection between inflation and growth in a Schumpeterian framework with short-run price rigidity. They also found an hump-shaped inflation-growth locus due to the distortionary effects produced by inflation on the incentive to innovate.

It is possible to conclude that the theoretical literature focused on the effect of inflation on growth passing through the accumulation of either human or physical capital, through the credit market or through the product market. This study, instead, deals with the effect of inflation on real growth passing through the labour market in presence of wage staggering. Furthermore, this study gives different insights into the inflation-growth nexus than Funk and Kromen (2005, 2006) as we use a learning by doing model and not a Schumpeterian framework, wage-staggering and not price-staggering and, more importantly, labour supply is not exogenously given, but determined by the optimizing behavior of economic agents.

Temple (2000) offers a broader review of the theoretical literature.

## 2.2 Empirical Literature

The literature review that follows gives special weight to the studies on the inflation-growth nexus after 2000, as Bullard (1999) and Temple (2000) offered two surveys of previous work. For the studies before 2000, only the key ones are mentioned hereafter.

Regarding the time-series literature, Bullard (1999) and Temple (2000) agree that, being mainly based on unit root testing, its results might be questionable due to the low power of these tests in finite samples. One of its main results is that inflation and growth are not connected in the long-run, given that the former would be non-stationary and the latter stationary<sup>4</sup>.

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<sup>4</sup>This conclusion is in contrast with Christopoulos and Tsionas (2005), who, using panel

The presence of a unit root in inflation might be re-assessed once considering that many studies of the inflation persistence network of the ECB found that, allowing for structural breaks in the mean of the inflation process, inflation appears to be stationary as well (Altissimo et al., 2006).

Many cross-sectional and panel data studies found a non-linear relationship between inflation and real growth. Gylfason and Herbertsson (2001) refer to various studies pointing out that an increase in inflation from 5% to 50% decreases the real growth rate. However they found that this effect is non-linear and that inflation rates below 10% are positively correlated with growth, but the opposite holds for inflation rates above 10%.

Similar results were found by Chari *et al.* (1996) and Barro (2001). From the former study would result that an increase in inflation from 10 to 20% would decrease growth by an amount ranging from 0.2 to 0.7%. Kahn and Senhadji (2001) found the threshold to be around 1% for industrialized countries and 11% for developing ones. The results of Ghosh and Phillips (1998) would point it to be at 2.5%, whereas Judson and Orphanides (1999) at 10% and Pollin and Zhu (2006) at about 13-15%.

A threshold effect was also found by Thirlwall and Barton (1971) at an annual inflation rate ranging from 8% to 10%. A similar value was suggested by Sarel (1996) as well. Gylfason (1991) found that economies with inflation

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unit root and cointegration techniques, found both inflation and (productivity) growth to be I(1), though they could not find a negative link between them across all the countries included in their sample. They measured productivity as the ratio between real gross output and employment and not as GDP per capita.

above 20% grew less rapidly than economies with inflation below 5% a year. Bruno and Easterly (1998) report that inflation rates above 40 percent a year for at least two years in a row are generally harmful to growth. Also Fischer (1993) noted the existence of a positive relationship at low inflation rates and a negative one as inflation rises. Burdekin (2004) found that, for industrialized countries, inflation rates below 8% have an insignificant effect on growth and a negative one above that rate. In developing countries, inflation harms growth above 3%. However, above 50% the marginal cost of inflation decreases substantially. Finally, Vaona and Schiavo (2007) used both a nonparametric and a semiparametric instrumental variable estimators, which have the advantage of letting as much as possible the data speak. They showed that, for developed countries, low inflation rates have hardly any real effect and high inflation rates have negative real effects. For developing countries they found that too high variability does not allow to reach clear-cut results.

Guerrero (2006) used previous hyperinflationary experience as instrument for inflation, finding that inflation has a negative impact on growth. Arai et al. (2004), using dynamic panel data methods on a dataset of 115 countries over the period 1960-1995, did not find any evidence that inflation is harmful to growth. On the contrary, the negative correlation between inflation and growth can be explained by oil price shocks. However, Kim et al. (2000), using cross-country/time-series data, found that the negative effect of inflation on growth is greater in developed countries than in developing ones and that

the inclusion in the model of oil supply shocks weakens the inflation-growth nexus, which, however, does not disappear.

Boyd et al. (2001), Haslag and Koo (1999) and Andrés et al. (2004) focused on the interconnections between inflation, financial development and growth. The first two studies support the validity of the growth-financial development link but not of the inflation-growth one. On the other hand, Andrés et al. (2004) challenge the importance of financial markets in understanding the impact of inflation on growth.

All in all, it is possible to state that high inflation is detrimental to growth, but there are some chances that a low inflation rate might have a positive impact on growth, though how low is a question the literature has had many difficulties in replying to.

### **3 A Model of Nominal Rigidities and Endogenous Growth**

In our analysis, the nominal rigidity takes the form of staggered Taylor wage contracts<sup>5</sup>. Our model economy contains a continuum of households, sup-

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<sup>5</sup>As noted by Graham and Snower (2008), for "sufficiently high levels of money growth, Calvo contracts are not appropriate. The reason is straightforward. With Calvo contracts, some households keep their nominal wage unchanged for a very long period of time, which means that, in the presence of inflation, the real value of this wage approaches zero. This implies that the firm will wish to hire as much of the labor of these households as possible, and as little of the other households. This is very inefficient so output approaches zero". This shortcoming of Calvo contracts could be overcome by introducing inflation indexation (on this issue see, for instance, the papers quoted in Minford and Peel, 2004). However, such a contracting device is not as widespread as in the past in developed (*continues*)

plying differentiated labor, and a large number of identical firms, producing output by means of all labor types and capital. The labor types are imperfect substitutes in the production function, exhibiting diminishing returns to labor but increasing returns to scale. Thus each household faces a downward-sloping labour demand curve in the short run. The government prints money and it returns its seigniorage proceeds to households in the form of lump sum tax rebates.

### 3.1 The supply side of the economy

As far as the supply side of the economy is concerned, we assume the existence of two good sectors. In the final good sector a set of perfectly competitive firms transforms a continuum of horizontally differentiated inputs into an homogeneous good. In the intermediate good sector a continuum of monopolistically competitive firms produces different varieties of a horizontally differentiated good using both labour and capital inputs. The continuum of monopolistically competitive firm, indexed by  $f$ , is normalized on the  $[0, 1]$  interval.

We further suppose that there exist a capital and a labour market. Capital is a homogeneous production factor. In the labour market, households belonging to different cohorts set their wages in a monopolistically competi-

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countries. Furthermore, in Calvo contracts, economic agents have in each period the same probability to reset wages/prices. This is rather unrealistic when applied to the labour market, where contracts with fixed durations are more widespread and the timing of contract renewals is usually characterized by a high degree of coordination. For these reasons we prefer the Taylor specification.

tive environment and sell their labour to the continuum of monopolistically competitive firms of the intermediate good sector. The continuum of monopolistically competitive households, indexed by  $h$ , is normalized on the  $[0, 1]$  interval.

### 3.1.1 The final good sector

In the final good sector, due to the existence of a set of differentiated inputs with a constant elasticity of substitution ( $\theta_p$ ), the production function assumes the form of a CES aggregator  $y_t = \left( \int_0^1 y_{ft}^{\frac{\theta_p-1}{\theta_p}} df \right)^{\frac{\theta_p}{\theta_p-1}}$ , namely in the final good sector firms just use intermediate inputs,  $y_{ft}$ , to produce their output,  $y_t$ . Firms maximize profits subject to their production function:

$$\begin{aligned} \max_{\{y_{ft}\}} \quad & p_t y_t - \int_0^1 p_{ft} y_{ft} df \\ \text{s.t. } y_t = \quad & \left( \int_0^1 y_{ft}^{\frac{\theta_p-1}{\theta_p}} df \right)^{\frac{\theta_p}{\theta_p-1}} \end{aligned}$$

where  $p_t$  is the aggregate price level and  $p_{ft}$  is the price of the  $f$ -th intermediate input. By solving this maximization problem it is possible to get the demand function for each good variety:

$$y_{ft} = \left( \frac{p_{ft}}{p_t} \right)^{-\theta_p} y_t \tag{1}$$

Furthermore, by imposing the zero profit condition and substituting (1) into the profit equation it is possible to obtain the price index:

$$p_t = \left( \int_0^1 p_{ft}^{1-\theta_p} df \right)^{\frac{1}{1-\theta_p}} \quad (2)$$

### 3.1.2 The intermediate good sector

In the intermediate good sector each firm  $f$  buys capital and labour to produce one variety of good. In so doing, it minimizes costs subject to the constraint of its production function and it maximizes the spread between the price it charges and its marginal cost.

Given that labour is horizontally differentiated, we apply a two-stage budgeting approach after Chambers (1988, pp. 112-113) and Heijdra and Van der Ploeg (2002, pp. 360-363). First, firms choose their desired quantity of capital and of a composite labour input,  $\hat{n}_{ft}$ , which is defined as follows

$$\hat{n}_{ft} = N^{-\frac{1}{\theta_w-1}} \left\{ \int_0^1 [n_t(h)]^{\frac{\theta_w-1}{\theta_w}} dh \right\}^{\frac{\theta_w}{\theta_w-1}}$$

where  $h$  is the household index and  $\theta_w$  is the elasticity of substitution between different labour kinds. We suppose that households are grouped into  $N$  cohorts and that there are  $\frac{1}{N}$  households per cohort. It is plausible to assume that the presence of a larger or a smaller number of cohorts does not have an impact on the efficiency of the production process. Therefore, the term

$N^{-\frac{1}{\theta_w-1}}$  avoids economies of specialization in different labour kinds<sup>6</sup>, which in the present setting coincide with cohorts. Households within cohorts are symmetric and so the composite labour input can be re-written as follows

$$\hat{n}_{ft} = N^{-\frac{1}{\theta_w-1}} \left[ \sum_{j=0}^{N-1} \left( \frac{n_{j,t}}{N} \right)^{\frac{\theta_w-1}{\theta_w}} \right]^{\frac{\theta_w}{\theta_w-1}} \quad (3)$$

where  $j$  is the cohort index.

In the first stage of budgeting, firms solves the following problem:

$$\begin{aligned} \min_{\{\hat{n}_{ft}, k_{ft}\}} \quad & \frac{w_t}{p_t} \hat{n}_{ft} + r_t k_{ft} \\ \text{s.t.} \quad & y_{ft} = (\hat{n}_{ft} k_t)^\alpha (k_{ft})^{1-\alpha} \end{aligned} \quad (4)$$

where  $w_t$  is the nominal aggregate wage rate,  $r_t$  is the remuneration for capital services,  $k_{ft}$  is the amount of capital used by firm  $f$ ,  $k_t$  is the aggregate capital stock and  $\alpha$  is a parameter. (4) is a typical production function with knowledge externalities, whereby an increase in a firm's capital stock leads to an increase in its stock of knowledge. Assuming that each firm's knowledge is a public good, the aggregate increase in knowledge is proportional to the aggregate capital stock (Barro and Xala-i-Martin, 1995)<sup>7</sup>.

<sup>6</sup>See also Heijdra and Van der Ploeg (2002), p. 361.

<sup>7</sup>Note that (4) is not defined over the number of employees, rather over a composite index of working hours, whose stationarity is consistent with the Kaldor facts (Cooley and Prescott, 1995, p. 16). Therefore, our model of endogenous growth is not undermined by the Jones (1995) critique, according to which the real growth rate, being a stationary variable, cannot be explained by non-stationary variables, like, in his case, the investment

The first order conditions for labour and capital are:

$$\frac{w_t}{p_t} = mc_t \alpha (\hat{n}_{ft} k_t)^{\alpha-1} (k_{ft})^{1-\alpha} k_t \quad (5)$$

$$r_t = mc_t (1 - \alpha) (\hat{n}_{ft} k_t)^\alpha (k_{ft})^{-\alpha} \quad (6)$$

where  $mc_t$  is the real marginal cost.

Having chosen the amount of labour and capital that minimize costs, firms of the intermediate output sector maximize profits by maximizing the spread between the price they charge and their marginal cost under the constraint of the demand for the specific good variety they produce (1):

$$\max_{\{p_{ft}\}} \frac{p_{ft} - mc_t^n}{p_t} y_{ft} \quad (7)$$

$$s.t. y_{ft} = \left( \frac{p_{ft}}{p_t} \right)^{-\theta_p} y_t \quad (8)$$

where  $mc_t^n$  is the nominal marginal cost.

By solving the problem (7)-(8), it is possible to show that the price charged by each firm is just a mark-up over the real marginal cost:

$$\frac{p_{ft}}{p_t} = \frac{\theta_p}{\theta_p - 1} mc_t \quad (9)$$

Recalling that firms are symmetric and their number is normalized to  


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rate and R&D employment.

one, the  $f$  index in  $\hat{n}_{ft}$  can be dropped as

$$\int_0^1 \hat{n}_{ft} df = \hat{n}_t$$

In the second stage of budgeting, firms "construct" optimally the composite labour input by choosing the amount of working time of cohort  $j$ ,  $n_{j,t}$ , solving the following problem

$$\max_{\{n_{j,t}\}} N^{-\frac{1}{\theta_w-1}} \left[ \sum_{j=0}^{N-1} \left( \frac{n_{j,t}}{N} \right)^{\frac{\theta_w-1}{\theta_w}} \right]^{\frac{\theta_w}{\theta_w-1}} \quad (10)$$

$$s.t. \quad w_t \hat{n}_{ft} - \sum_{j=0}^{N-1} \frac{w_{j,t}}{\mu^j} \left( \frac{n_{j,t}}{N} \right) = 0 \quad (11)$$

The term  $\frac{w_{j,t}}{\mu^j}$  is the wage of cohort  $j$  that has been detrended because money is growing in steady state but the wage of each cohort is staggered.

Solving the problem above after Heijdra and Van der Ploeg (2002, p. 363), it is possible to find the demand function for each cohort's labour services (19) and the aggregate wage index:

$$w_t = \left[ \frac{1}{N} \sum_{j=0}^{N-1} \left( \frac{w_{j,t}}{\mu^j} \right)^{1-\theta_w} \right]^{\frac{1}{\theta_w-1}} \quad (12)$$

### 3.2 The household's problem

We consider a deterministic setting. Suppose there are  $N$  cohorts of households. Each household maximizes the present value of its utility ( $U$ ) with

respect to its consumption ( $C$ ), wage ( $W$ ), real money balances ( $M/P$ ), capital ( $K$ ) and private bond holdings ( $B$ ), subject to its budget constraint, its labor demand function and the capital law of motion. Nominal wages are set for  $N$  periods being households grouped into  $N$  wage setting cohorts.

Specifically, the problem of household  $h$  is

$$\max_{\{C_{t+j}(h), M_{t+j}(h), W_{t+jN}(h), K_{t+j+1}(h), B_{t+j}(h)\}} \sum_{j=0}^{\infty} \sum_{j=iN}^{(i+1)N-1} \beta^j U \left( C_{t+j}(h), \frac{M_{t+j}(h)}{P_{t+j}}, n_{t+j}(h) \right) \quad (13)$$

$$s.t. \sum_{j=0}^{N-1} \tilde{\rho}_{t,t+j} \left[ C_{t+j}(h) + \frac{M_{t+j}(h)}{P_{t+j}} - \frac{M_{t+j-1}(h)}{P_{t+j}} + \frac{1}{1+r_{t+j+1}^b} \frac{B_{t+j}(h)}{P_{t+j}} - \frac{B_{t+j-1}(h)}{P_{t+j}} + I_{t+j}(h) - \frac{T_{t+j}(h)}{P_{t+j}} \right] = \sum_{j=0}^{N-1} \tilde{\rho}_{t,t+j} \left[ \frac{W_t(h)n_{t+j}(h)}{P_{t+j}} + \Pi_{t+j}(h) + r_{t+j}K_{t+j}(h) \right] \quad (14)$$

$$n_{t+j}(h) = \left( \frac{W_t(h)}{W_{t+j}} \right)^{-\theta_w} \hat{n}_{t+j} \quad (15)$$

$$K_{t+j+1}(h) = I_{t+j}(h) + (1 - \delta) K_{t+j}(h) \quad (16)$$

where  $\beta$  is the discount factor,  $I$  is investment in physical capital,  $T$  are lump sum transfers,  $\Pi$  is profit income,  $r^b$  is the interest rate paid to bond holders,  $r$  is the user's cost of capital,  $\tilde{\rho}_{t,t+j} = \prod_{l=0}^j \frac{1}{1+\tilde{r}_{t+l}}$ , where  $\tilde{r}$  is the market interest rate, and  $\delta$  is the depreciation rate. The first constraint is the household's budget constraint: the sum of the present values of the household's expenditures on consumption, money balance accumulation, bond accumulation and

investment are equal to disposable income, namely the sum of the household's wage, profit and capital incomes. The second constraint is the labor demand for the cohort's services (derived above), and the final constraint is the law of motion of capital. The transversality condition is  $\lim_{t \rightarrow \infty} \Lambda_t K_{t+1} = 0$ , where  $\Lambda_t$  is the non-detrended Lagrangian multiplier.

After detrending nominal variables for nominal growth ( $\mu$ ) and real variables for real growth ( $\gamma$ ), it is possible to restate problem (13) - (16) in the following form:

$$\max_{\{c_{t+j}(h), m_{t+j}(h), w_{t+jN}(h), k_{t+j+1}(h), b_{t+j}(h)\}} \sum_{j=0}^{\infty} \sum_{i=N}^{(i+1)N-1} \beta^j U \left[ c_{t+j}(h), \frac{m_{t+j}(h)}{p_{t+j}}, n_{t+j}(h) \right] \quad (17)$$

$$\begin{aligned} s.t. \quad & \sum_{j=0}^{N-1} \tilde{\rho}_{t,t+j} \left[ c_{t+j}(h) + \frac{m_{t+j}(h)}{p_{t+j}} - \frac{m_{t+j-1}(h)}{p_{t+j}} \frac{1}{\mu\gamma} + \frac{1}{1+r_{t+j+1}^b} \frac{b_{t+j}(h)}{p_{t+j}} - \right. \\ & \left. - \frac{b_{t+j-1}(h)}{p_{t+j}} \frac{1}{\mu\gamma} + i_{t+j}(h) - \frac{t_{t+j}(h)}{p_{t+j}} \right] = \sum_{j=0}^{N-1} \tilde{\rho}_{t,t+j} \left[ \frac{w_t(h)}{\mu^j p_{t+j}} n_{t+j}(h) + \pi_{t+j}(h) + r_{t+j} k_{t+j}(h) \right] \end{aligned} \quad (18)$$

$$n_{t+j}(h) = \left[ \frac{w_t(h)}{w_{t+j}} \frac{1}{\mu^j} \right]^{-\theta_w} \hat{n}_{t+j} \quad (19)$$

$$\gamma k_{t+j+1}(h) = i_{t+j}(h) + (1 - \delta) k_{t+j}(h) \quad (20)$$

Lower-case letters denote de-trended variables.

To start with, preferences are assumed to be as follows:

$$U = \sum_{j=0}^{\infty} \sum_{j=iN}^{(i+1)N-1} \beta^j \left\{ \log c_{t+j}(h) - \varsigma n_{t+j}(h) + V \left[ \frac{m_{t+j}(h)}{p_{t+j}} \right] \right\} \quad (21)$$

where  $\varsigma$  is a parameter and  $V[\cdot]$  the partial utility function of real money balances. The first order conditions for consumption, capital and each cohort's wage are of particular importance for the solution of the model. Given that there exist complete asset markets, that the only heterogeneity among households is due to wage staggering and that households face identical  $N$  sub-problems, they can be written as follows:

$$\frac{\beta^j}{c_{t+j}} = \lambda_{t+j} \tilde{\rho}_{t,t+j} \quad (22)$$

$$\gamma \lambda_{t+j} = \tilde{\rho}_{t,t+1} \lambda_{t+j+1} (1 - \delta + r_{t+j}) \quad (23)$$

$$\sum_{j=0}^{N-1} n_{t+j} \beta^j = \frac{\theta_w - 1}{\theta_w \varsigma} \sum_{j=0}^{N-1} \frac{\tilde{\rho}_{t,t+j} w_{j,t} n_{t+j}}{p_{t+j} \mu^j c_{t+j}} \quad (24)$$

where  $\lambda_{t+j}$  is the de-trended Lagrange multiplier.

### 3.3 The Government

For simplicity, the government is assumed to distribute its seigniorage in the form of lump-sum transfers to households:

$$m_{t+j} - m_{t+j-1} \frac{1}{\mu \gamma} = t_{t+j} \quad (25)$$

## 4 The Model Solution

The macroeconomic model is constituted by the equations: (1), (2), (4), (5), (6), (9), (12), (19), (22), (23), (24) and (25). The procedure to solve the model is outlined in more detail in Appendix A.

Note that Graham and Snower (2004) showed that in steady state  $\tilde{\rho}_{t,t+j} = \beta^j$ . Furthermore in equilibrium there is no arbitrage between alternative assets and so  $\tilde{r} = r^b = r - \delta$ . It is possible to obtain the solution for the steady state growth rate of the economy, by using (4), (9), (6), (22) and (23):

$$\gamma = \beta \left[ 1 - \delta + \frac{\theta_p - 1}{\theta_p} (1 - \alpha) (\hat{n})^\alpha \right] \quad (26)$$

The non-detrended version of (23) implies that the law of motion of the non-detrended Lagrangian multiplier is

$$\Lambda_{t+j} = \left[ \frac{1}{\tilde{\rho}_{t,t+1}(1 - \delta + r_{t+j})} \right]^j \Lambda_t$$

which, together with the transversality condition and the equality  $\tilde{\rho}_{t,t+j} = \beta^j$ , imposes the restriction on the percentage growth rate to be non-negative.

At this stage it is necessary to find the steady state level of the composite labour input in order to solve the model. In the appendix we show that, assuming  $\alpha = \frac{2}{3}$ , the composite labour input is given by the roots of the

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<sup>8</sup>Their demonstration applies also here as  $c_{t+j}$  is detrended consumption.

following polynomial

$$A^3 \hat{n}^3 - 3A^2 B \hat{n}^2 + (3AB^2 - C^3) \hat{n} - B^3 = 0 \quad (27)$$

where  $A$ ,  $B$  and  $C$  are defined as follows:

$$A : = \left[ 1 - \beta \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right] \quad (28)$$

$$B : = \left( N \frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right) \frac{1 - \beta \mu^{\theta_w}}{1 - (\beta \mu^{\theta_w})^N} \frac{\theta_w - 1}{\theta_w \varsigma} \left( \frac{1}{N} \frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{\theta_w}{\theta_w - 1}} \quad (29)$$

$$C : = (1 - \delta) (\beta - 1) \quad (30)$$

The possibility to have multiple steady states within a learning-by-doing model is well known in the literature (Benhabib and Farmer, 1994). We calibrate the model using standard parameter values ( $\beta = 0.96^{\frac{1}{N}}$ ,  $N = 52$ ,  $\delta = 1 - 0.92^{\frac{1}{N}}$ ,  $\theta_p = 10$ ,  $\alpha = 0.67$ ,  $\theta_w = 2$ ) similarly to Ascari (2003), Graham and Snower (2004) and Huang and Liu (2002).  $\varsigma$  has been adjusted for the model to produce realistic growth rates. Under this calibration two of the three roots of (27) turn out to be complex numbers which can be ruled out being without economic meaning. This leaves us with a unique solution for the model.

Figures 1 and 2 show how the long-run growth rate of the economy changes as a function of  $\theta_p$  and  $\theta_w$ , once setting  $\mu = 1.02$ . An increase in  $\theta_p$  reduces monopolistic rents in the intermediate product market, imply-

ing a positive income effect and enhancing economic growth. This effect is captured by terms depending on  $\theta_p$  in (26), (28) and (29). The same happens for  $\theta_w$ , even though its impact on long run growth is the result of a number of different mechanisms. First of all, an increase in  $\theta_w$  reduces monopolistic rents on the labour market, entailing a greater labour supply. However, due to wage staggering, this is not the complete story. Consider the ratio between the labour demanded to cohort 0 and cohort  $j$ :

$$\frac{n_0}{n_j} = \mu^{-\theta_w j} \quad (31)$$

An increase in  $\theta_w$  entails more labour cycling, which produces more inefficiencies. This negative effect, however, does not offset the beneficial reduction of rents leading to a net increase in the composite labour input and therefore in economic growth.

## 5 The Effects of Monetary Policies

### 5.1 The Real Effects of Monetary policy

Figure 3 shows that the relationship between money and real growth is clearly non-linear. There is a threshold: increasing the money growth rate has a tiny positive impact on real growth until about 2% and a negative one above this value. As showed by Graham and Snower (2004) analysing structural equatons, the discounting and the employment cycling effects underlie these

non-linearities. At low inflation rates, the time discounting effect prevails leading to a greater labour supply and therefore to faster capital accumulation and growth. On the contrary, at high inflation rates the employment cycling effect is stronger leading to less labour demand and therefore to slower growth. The labour cycling effect is due to the fact that firms substitute between different kinds of labour because agents belonging to different cohorts have different wages, being some of them locked in past contracts.

The analysis above, regarding how the elasticity of substitution between different goods and different labour kinds affect the real growth rate, already gives some insights on how they affect the long-run relationship between money growth and real growth. Figures 4 and 5 show that the money growth-real growth locus moves upward as either  $\theta_p$  or  $\theta_w$  increases. Under the present calibration, economic growth appears to be much more sensitive to the structural parameters of the model than to money growth, as the discounting and the employment cycling effects tend to offset one another.

## 5.2 Optimal Monetary Policy

To individuate the optimal rate of money growth we use a specification of the welfare function similar to Woodford (1998), Benigno (2004) and Aoki (2001) where the weight of money holdings is assumed to be so small to be negligible:

$$W = \sum_{j=0}^{\infty} \beta^j [\log C_{t+j} - \varsigma n_{jt+j}]$$

$C_{t+j}$  is growing at the rate  $\gamma$ , therefore  $C_{t+j} = c_t \gamma^j$ .

As showed in Appendix B, maximizing  $W$  is equivalent to maximizing the criterion

$$\begin{aligned} W' = & \frac{1}{1 - \beta^N} \left[ \beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta} \right] \log \gamma - \\ & - \varsigma n_0 \frac{1}{1 - \beta^N} \frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}} + \frac{1}{1 - \beta^N} \log (\hat{n}^\alpha - \gamma + 1 - \delta) \end{aligned} \quad (32)$$

Figure 6 shows that money growth does not affect welfare to a great extent. However, under the present calibration the optimal inflation rate would be above 20%, a result that does not hold for more realistic calibrations below. This happens because money growth affects  $W'$  through four channels: the real growth rate of the economy, the working time of cohort zero, the term  $\frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}}$  and the initial level of consumption. The second channel can be taken to represent the labour cycling effect<sup>9</sup>, while, following Graham and Snower (2004, p. 18), the third channel can be taken to represent the time discounting effect. To understand which channel dominates we decompose  $W'$  in three terms

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<sup>9</sup>The more the working time of cohort zero shrinks, the more labour demand will shift to other cohorts.

$$\begin{aligned}
W'_1 &= \frac{1}{1 - \beta^N} \left[ \beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta} \right] \log \gamma \\
W'_2 &= \varsigma n_0 \frac{1}{1 - \beta^N} \frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}} \\
W'_3 &= \frac{1}{1 - \beta^N} \log (\hat{n}^\alpha - \gamma + 1 - \delta)
\end{aligned}$$

and we compute six percentage variations  $\% \Delta W'_{1a} = \frac{W'_{1\max} - W'_{1ff}}{W'_{ff}} * 100$ ,  $\% \Delta W'_{2a} = \frac{W'_{2\max} - W'_{2ff}}{W'_{ff}} * 100$ ,  $\% \Delta W'_{3a} = \frac{W'_{3\max} - W'_{3ff}}{W'_{ff}} * 100$ ,  $\% \Delta W'_{1b} = \frac{W'_{1end} - W'_{1\max}}{W'_{\max}} * 100$ ,  $\% \Delta W'_{2b} = \frac{W'_{2end} - W'_{2\max}}{W'_{\max}} * 100$ ,  $\% \Delta W'_{3b} = \frac{W'_{3end} - W'_{3\max}}{W'_{\max}} * 100$ , where the subscript "max" indicates values taken at the maximum of  $W'$ ,  $ff$  the values taken with flexible wages and  $end$  the last computed values. After for instance Wolff (2003), we further decompose  $\% \Delta W'_{2a}$  and  $\% \Delta W'_{2b}$  to isolate the effect of  $n_0$  from that of  $\frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}}$  using the fact that, for a given variable  $z = b * f$ , one has that  $z_2 - z_1 = b^* (f_2 - f_1) + f^* (b_2 - b_1)$  where  $b^* = \frac{b_2 + b_1}{2}$  and  $f^* = \frac{f_2 + f_1}{2}$ , where 1 and 2 indicate different values of the variables.

Under the present parametrization,  $\% \Delta W'_{1a} = -0.0328\%$ ,  $\% \Delta W'_{2a} = -0.0715\%$  and  $\% \Delta W'_{3a} = -0.0072\%$ , while  $\% \Delta W'_{1b} = -0.0263\%$ ,  $\% \Delta W'_{2b} = -0.0292\%$ ,  $\% \Delta W'_{3b} = -0.0057\%$ . So the second term in (32) appears to be the most responsive to changes in money growth. However,  $\% \Delta W'_{1a}$  and  $\% \Delta W'_{3a}$  cannot compensate for  $\% \Delta W'_{2a}$ , though  $\% \Delta W'_{1b}$  and  $\% \Delta W'_{3b}$  can compensate for  $\% \Delta W'_{2b}$ , eventually leading  $W'$  to fall. The changes in  $\% \Delta W'_{2a}$  and  $\% \Delta W'_{2b}$  attributable to  $n_0$  and  $\frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}}$  are about of the

same magnitudes, though the latter tends to be somewhat greater. Indeed,  $\% \Delta W'_{2a} = -0.0715\%$  can be decomposed into two further percentage variations: one, attributable to  $n_0$ , equals  $-16.9854\%$  and the other, attributable to  $\frac{1-\beta^N \mu^{\theta_w N}}{1-\beta \mu^{\theta_w}}$ , equals  $16.9140\%$ . For  $\% \Delta W'_{2b}$  these two changes are equal to  $-5.3833\%$  and  $5.3542\%$  respectively.

To sum up, under the present parametrization, money growth has a tiny nonlinear effect on real growth and welfare because the time discounting and labour cycling effects tend to cancel each other out. However, welfare tends to increase up to very high inflation rates because the effects of real growth, time discounting and the initial level of consumption cannot completely offset the labour cycling effect, which leads to a welfare enhancing decrease in working time. We will see below that this result does not hold for more general preferences.

## 6 Extensions

### 6.1 Generalized Preferences

In this Section we relax the hypothesis of linear preferences in labour time. And we suppose that the intertemporal elasticity of substitution of working time is equal to  $\phi$ :

$$U = \sum_{j=0}^{\infty} \beta^j \left\{ \log c_{t+j}(h) - \varsigma \frac{n_{j,t+j}^{1+\phi}(h)}{1+\phi} + V \left[ \frac{m_{t+j}(h)}{p_{t+j}} \right] \right\} \quad (33)$$

Consequently the first order condition of the consumer's maximization problem and  $B$  will respectively be as follows

$$\sum_{j=0}^{N-1} n_{j,t+j}^{(1+\phi)} \beta^j = \frac{\theta_w - 1}{\theta_w \varsigma} \sum_{j=0}^{N-1} \frac{\tilde{p}_{t,t+j} w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j c_{t+j}} \quad (34)$$

$$B := \left( N \frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right) \frac{1 - \beta \mu^{(1+\phi)\theta_w}}{1 - [\beta \mu^{(1+\phi)\theta_w}]^N} \frac{\theta_w - 1}{\theta_w \varsigma} \left( \frac{1}{N} \frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{\theta_w}{\theta_w - 1}}$$

The welfare criterion is now

$$W' = \frac{1}{1 - \beta^N} \left[ \beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta} \right] \log \gamma - \quad (35)$$

$$-\varsigma n_0 \frac{1}{1 - \beta^N} \frac{1 - \beta^N \mu^{\theta_w(1+\phi)N}}{1 - \beta \mu^{\theta_w(1+\phi)}} + \frac{1}{1 - \beta^N} \log(\hat{n}^\alpha - \gamma + 1 - \delta) \quad (36)$$

In our benchmark model, we set  $\phi = 5$ , consistently, for instance, with Huang and Liu (2002), but we also check the sensitivity of our results fixing  $\phi$  to 2.5. Figures 7a and 7b show the long-run relationship between inflation and growth and the welfare level associated with different inflation rates for this new specification of the model. Similarly to Graham and Snower (2004) for the relationship between inflation and the level of output, this is due to the labour supply smoothing effect. With  $\phi > 0$  households have a greater preference for smoothing their labour supply over the contract period. As a consequence, being adversely affected by increases in money growth that lead to more employment cycling due to wage staggering, they raise their

wage, reducing labour demand, the aggregate quantity of labour input and, so, output growth. The greater is  $\phi$ , the steeper is the real growth-money growth locus.

The money growth-welfare locus is now downward sloping as the real growth, the labour cycling and the initial consumption effect more than offset the time discounting effect.

## 6.2 Endogenizing the length of the labour contract

This Section is devoted to endogenizing the frequency of nominal adjustments. As in Graham and Snower (2004), we suppose that  $N$  is set in order to maximize households' welfare:

$$\max_N \sum_{j=0}^{\infty} \beta^j \left\{ \log C_{t+j} - \varsigma \frac{n_{j,t+j}^{1+\phi}}{1+\phi} - F \right\} \quad (37)$$

under (14), (15) and (16).  $F$  is a fixed cost to changing wages.

Reasoning along the lines of Appendix B, (37) can be rewritten as

$$\max_N \sum_{j=0}^{\infty} \sum_{l=Nj}^{N(j+1)-1} \beta^l \left\{ \log C_{t+l} - \varsigma \frac{n_{l,t+l}^{1+\phi}}{1+\phi} - F \right\} \quad (38)$$

(38) in steady state is an infinite sum of  $N$  periods problems. Therefore, it becomes

$$\max_N \frac{1}{1 - \beta^N} \left[ \beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta} \right] \log \gamma + \frac{1}{1 - \beta^N} \log (\hat{n}^\alpha - \gamma + 1 - \delta) - \frac{1}{1 - \beta^N} \zeta n_0 \frac{1}{1 + \phi} \frac{1 - [\beta \mu^{(1+\phi)\theta_w}]^N}{1 - \beta \mu^{(1+\phi)\theta_w}} - \frac{1}{1 - \beta^N} F$$

The problem can be solved numerically. Again following Graham and Snower (2004), we assume that, at a 4% inflation rate, wages are set for one year. Results for different values of  $F$  are shown in figures 8a and 8b. First, under the assumption  $F \rightarrow 0$ , endogenizing  $N$  does not lead to money superneutrality. This happens because households have a preference against labour cycling and so for wage flexibility, but they can achieve it only by paying more frequently  $F$ . So, in the end, they face a trade-off and they prefer not to set  $N = 1$ . As a matter of consequence, for  $F \rightarrow 0$ , the length of the wage contract converges to 1 and money becomes super-neutral.

## 7 New semiparametric evidence on the inflation-growth nexus

The present Section offers new empirical evidence on the inflation-growth nexus following the footsteps of Vaona and Schiavo (2007). In order to obtain comparable results to theirs we use the same model specifications and dataset, which, on their own, build on the work of Khan and Senhadji (2001).

Our dependent variable is the growth rate of GDP in constant local currency units. The independent variables of Specification I are the level of inflation, gross fixed capital formation (or gross capital formation when the former is not available) over GDP, the log of initial per capita GDP in PPP adjusted dollars, population growth, the average schooling years in the total population aged 15 or more, and the share of government expenditure over GDP. Specification II controls also for the growth rate of the terms of trade and their 5-year standard deviation<sup>10</sup>. In both the specifications we include a set of country and time dummies to control for possible unobserved heterogeneity.

We resort to different data sources. For per capita GDP and the share of government consumption we revert to Penn World Tables (PWT). Our education measure is obtained from the Barro and Lee dataset on educational attainment. We use the export and import unit value series from the IMF's International Financial Statistics (IFS) to build terms of trade data.

Our dataset spans from 1960 to 1999 over 167 countries. After Temple (2000), we experiment with different time frequencies but our baseline results are based on 5-years means<sup>11</sup>. Missing values reduce the number of available observations (especially for developing countries). We also drop from the sample all the observations with an inflation rate above 40% as in Kahn and

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<sup>10</sup>Vaona and Schiavo (2007) performed robustness checks also including financial development indicators. Given that their results were stable, this line of research is not pursued any further here.

<sup>11</sup>Using medians instead of means would produce similar results to those presented below.

Senhadji (2001)<sup>12</sup>. Our final sample includes 85 countries and 421 observations. 119 observations belong to 19 industrial countries. Considering the terms of trade variables reduces the number of developing countries from 66 to 25, leaving almost unaffected developed countries.

We follow a different research strategy than Vaona and Schiavo (2007). First, after Li and Racine (2007, ch. 15), we use a semiparametric power series estimator and we look for the best fitting model specification by using several criteria. Then we check for endogeneity bias by implementing a two stage least square estimator (2SLS) and a Hausman test. Finally we perform various robustness controls for our results by splitting our sample between developed and developing countries, by using 8 years averages and by resorting to a semiparametric local linear estimator. This research strategy allows us to test whether an instrumental variable estimator is appropriate, rather than assuming it like in Vaona and Schiavo (2007).

Consider the following econometric model

$$Y_i = X_{1i}'\beta + \phi(X_{2i}) + u_i$$

$Y_i$  is the dependent variable,  $X_{ji}$  for  $j = 1, 2$  are two sets of independent variables,  $\beta$  is a vector of coefficients to be estimated,  $\phi(\cdot)$  is a non-linear function of unspecified form,  $i$  is the sub-script for the  $i$ -th observation,  $u_i$  is the disturbance. We here suppose that  $X_{2i}$  is a scalar. It is possible to

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<sup>12</sup>Gillman et al. (2004) show that using different truncation points generate negligible differences in the results.

approximate  $\phi(\cdot)$  by using a power series function

$$\{p_s(X_{2i}) = X_{2i}^s\}_{s=1}^K$$

However, it is necessary to choose  $K$ . The literature has proposed several criteria to help this choice. Two prominent examples are the Akaike and Schwarz parametric criteria. We also consider three nonparametric procedures: Mallows'  $C_L$ , generalized cross-validation and leave-one-out cross-validation.

Mallows'  $C_L$  consists in selecting  $K$  to minimize

$$N^{-1} \sum_i^N \left[ Y_i - X'_{1i} \hat{\beta} - \hat{\phi}(X_{2i}) \right]^2 + 2\hat{\sigma}^2 \left( \frac{K}{N} \right)$$

where  $N$  is the number of observations,  $\hat{\beta}$  and  $\hat{\phi}$  are the estimated counterparts of  $\beta$  and  $\phi$ ,  $\hat{\sigma}^2 = \frac{\sum_i^N \hat{u}_i^2}{N}$  and  $\hat{u}_i = Y_i - X'_{1i} \hat{\beta} - \hat{\phi}(X_{2i})$ .

According to generalized cross-validation, one selects  $K$  to minimize

$$\frac{\sum_i^N \left[ Y_i - X'_{1i} \hat{\beta} - \hat{\phi}(X_{2i}) \right]^2}{\left( 1 - \left( \frac{K}{n} \right) \right)^2}$$

Finally, the leave-one-out cross-validation consists of selecting  $K$  to minimize

$$CV_K = \sum_i^N \left[ Y_i - X'_{1i} \hat{\beta}_{-i} - \hat{\phi}_{-i}(X_{2i}) \right]^2$$

where  $\hat{\beta}_{-i}$  and  $\hat{\phi}_{-i}$  are estimated removing  $(Y_i, X'_{1i}, X_{2i})$ .

We apply the procedures above to our data sample for both Specification I and II fixing the maximum of  $K$  to 4. As showed in Table 1, the majority of the criteria point to a first order polynomial as the best fitting option, so we stick to it.

Next we compare our OLS estimates to the 2SLS ones by means of a Hausman test. As in Barro (1995) we instrument inflation by its first lag. Using this instrument entails assuming that it should not enter the econometric model for growth. This is consistent with our theoretical model, where there is not any delay in the impact of money growth on real growth<sup>13</sup>. Table 2 shows our results. Regressing present inflation on its first lag (and on all the controls of our model) returns a p-value of 0.00 with a t-statistic of 4.91 and an F-statistic 24.06 for Specification I and a t-statistic of 4.65 and an F-statistic of 21.61 for Specification II. So the instrumented variable is highly significantly correlated with its instrument and our results should not be biased by a weak instrument problem. The Hausman test would not reject the null that the OLS and 2SLS estimators are equal and, indeed, for both the specifications the inflation coefficients are very close. Inflation is therefore confirmed to have a negative linear impact on growth.

In order to check the robustness of our results, we first split our sample

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<sup>13</sup>More in general, this modelling choice is consistent with two plausible thoughts. First, past inflation has an impact on current growth only because it is correlated with current inflation and, second, current real growth might have a countereffect on inflation by overheating the economy, but not on past inflation. Given that we are using multi-year averages it is implausible to think, for instance, that current inflation is high *in the first half of a decade* because agents expect high growth *in the second half of the decade* to exacerbate inflationary bottlenecks.

between developed and developing countries (Table 3). For the latter ones, all the criteria point to a linear model as the best fitting one. For the former ones, instead, there is a discrepancy between Specifications I and II. In the first case a fourth order polynomial would be selected by all the criteria, but the Schwarz one, which would select a second order polynomial. Once adding to the model the growth rate of the terms of trade and their 5-years standard deviation, the majority of the criteria would select a linear model. Such a discrepancy could be caused by the omission of a relevant variable, given that the growth rate of the terms of trade is highly significantly correlated with economic growth<sup>14</sup> and that non-linear terms can be used to detect the omission of a relevant variable (see for instance Ramsey, 1969).

Finally, Vaona and Schiavo (2007) found that a mild positive effect of inflation on growth appears at low inflation rates in developed countries for Specification I when considering 8 years means instead of 5 years ones. We run again our model selection criteria for this case, but we do not find any non-linearity in the data as showed in Table 4. A linear model appears to be the best fitting one.

Our final robustness check is to compute a semiparametric local linear kernel estimator, whose main advantage, compared to the Nadaraya-Watson one, is not to be so sensitive to boundary observations. In our context, this advantage is very important given that the threshold level of inflation has

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<sup>14</sup>Once restricting the analysis to developed countries it has a coefficient equal to 0.12 with a p-value of 0.006.

often been found to be rather low. When using a semiparametric kernel estimator, one can obtain an estimate of  $\beta$  as follows:

$$\hat{\beta} = \left[ \sum_{i=1}^N (X_{1i} - \hat{m}_{12i})(X_{1i} - \hat{m}_{12i})' \right]^{-1} \left[ \sum_{i=1}^N (X_{1i} - \hat{m}_{12i})(Y_i - \hat{m}_{2i})' \right]$$

where  $\hat{m}_{12i}$  and  $\hat{m}_{2i}$  are the non-parametric estimators of  $m_{12i} = E(X_{1i}|X_{2i})$  and  $m_{2i} = E(Y_i|X_{2i})$ , where  $E$  is the expectation operator. Using a local linear estimator, one has:

$$\hat{m}_{2i} = e_1' \left( n^{-2} + \sum_{i=1}^n a_i K \left( \frac{X_{2i} - X_2}{h(X_{2i})} \right) a_i' \right)^{-1} a_i K \left( \frac{X_{2i} - X_2}{h(X_{2i})} \right) Y_i$$

where  $a_i' = [1, (X_{2i} - X_2)]$ ,  $e_1$  is a column vector of the same dimension as  $a_i$ , with unity as first element and zero elsewhere, and  $K(\cdot)$  is the kernel function (Pagan and Ullah, 1999). Here we use a Gaussian kernel. After Fan and Gijbels (1992), we work with a variable bandwidth  $h(X_{2i}) = hf^{-\frac{1}{5}}(X_{2i})$ , where  $h$  was chosen to be equal to Silverman's optimal smoothing bandwidth and  $f(X_{2i})$  is a kernel estimate of the density of  $X_{2i}$ .  $\hat{m}_{12i}$  assumes a similar form to  $\hat{m}_{2i}$ . Recall that

$$\phi(X_{2i}) = E \left[ \left( Y_i - X_{1i}' \hat{\beta} \right) | X_{2i} \right]$$

Given our results above, we mainly focus on Specification II as Specifica-

tion I omits a relevant variable, the growth rate of terms of trade. Parameter estimates for the control variables are set out in Table 5. Figure 9 shows the estimates of the partial effect of inflation on growth for the whole sample, for developed countries and for developing ones. Once again no threshold level of inflation can be detected, the effect of inflation on growth appears to be just negative and linearity is a good approximation. Shifting to 8 years averages for developed countries and to Specification I would not change our results.

Our empirical results can shed light on the theoretical results above. Though we cannot prove that the labour market channel highlighted by our theory is the dominant one compared to other channels already illustrated by the literature, we show that, once assuming a non-null intertemporal elasticity of substitution of working time, our model does not produce unrealistic results. So our empirical results help us to select the most suitable parametrization for our model.

## 8 Conclusions

This paper extends the results of the New-Keynesian literature with wage-staggering from the relation between inflation and the level of output to the inflation-growth nexus. In this way, it is possible to explore how inflation affects growth passing through the labour market instead of capital, credit or product markets. Once allowing for an intertemporal elasticity of substi-

tution of working time greater than zero, inflation has sizeable real effects: it harms economic growth and reduces economic agents' welfare. Assuming an endogenous length of the labour contract does not in general lead to money superneutrality. In presence of a fixed cost to changing wages, agents prefer wage staggering to wage flexibility unless inflation is high - how high depending on the magnitude of the fixed cost.

Our empirical results confirm that inflation has a negative impact on growth and no-threshold level of inflation is found. We argue that this is consistent with our theoretical model with generalized preferences.

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## 9 Appendices

### A The Solution of The Model

Recall that the macroeconomic model consists of the equations: (1), (2), (4), (5), (6), (9), (12), (19), (22), (23), (24), and (25).

Let us first consider the price index (2) in steady state. Given the absence of price staggering and due to symmetry all the firms will charge the same price  $p_{ft}$ . So (2) will become:

$$\frac{p_f}{p} = 1$$

By substituting the steady state price index into (9) it is possible to obtain that in steady state the marginal cost is the same across all the firms and that in its turn it can be substituted, together with (8), into the first order conditions for capital and the composite labour input to obtain

$$\frac{w}{p} = \frac{\theta_p - 1}{\theta_p} \alpha \frac{y}{\hat{n}} \quad (39)$$

$$r = \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{y}{k} \quad (40)$$

Having in mind the equation for capital remuneration, it is possible to obtain the solution for the steady state growth rate of the economy, by using (22) and (23):

$$\gamma = \beta \left[ 1 - \delta + \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{y}{k} \right] \quad (41)$$

Consider the production function in steady state, the output-capital ratio is given by:

$$\frac{y}{k} = (\hat{n})^\alpha \quad (42)$$

where the firm subscript was dropped because all the firms are symmetric.

Substituting (42) into (41), it is possible to find that in steady state the real growth rate of the economy depends on the composite labour input:

$$\gamma = \beta \left[ 1 - \delta + \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \hat{n}^\alpha \right] \quad (43)$$

Therefore it is necessary to find the steady state level of the composite labour input in order to solve the model. As a first step let us focus on (24). As showed in Graham and Snower (2004), the labour supply of each cohort can be reduced to the labour supply of cohort zero times a term accounting for money growth:

$$\frac{n_0}{n_j} = \mu^{-\theta_w j}$$

Therefore, considering that  $\tilde{\rho}_{t,t+j} = \beta^j$  the left hand side of (24), becomes in steady state

$$\sum_{j=0}^{N-1} n_{j,t+j} \beta^j = n_0 \frac{1 - \beta^N \mu^{N\theta_w}}{1 - \beta \mu^{\theta_w}} \quad (44)$$

As far as the right hand side of (24) is concerned, divide both the numerator and the denominator by  $y_{t+j}$ :

$$\frac{\theta_w - 1}{\theta_w \varsigma} \sum_{j=0}^{N-1} \frac{\beta^j w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j c_{t+j}} = \frac{\theta_w - 1}{\theta_w \varsigma} \sum_{j=0}^{N-1} \frac{\frac{\beta^j w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j} \frac{n_{j,t+j}}{y_{t+j}}}{\frac{c_{t+j}}{y_{t+j}}} \quad (45)$$

Focusing on the numerator, in steady state, one has:

$$\sum_{j=1}^{N-1} \beta^j \left( \frac{w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j} \frac{n_{j,t+j}}{y_{t+j}} \right) = \frac{w^* n_0}{p y} \sum_{j=1}^{N-1} \beta^j \mu^{(\theta_w - 1)j} \quad (46)$$

where  $w^*$  is the steady state nominal wage rate of cohort zero.

Consider that, due to (11) and to (39) one has:

$$\frac{w \hat{n}}{p y} = \frac{\sum_{j=0}^{N-1} \frac{w_{j,t} \hat{n}_{j,t}}{\mu^j} \frac{1}{N}}{p y} = \frac{w^* n_0}{p y N} \sum_{j=0}^{N-1} \mu^{(\theta_w - 1)j} = \frac{w^* n_0}{p y N} \frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{(\theta_w - 1)}} = \frac{\theta_p - 1}{\theta_p} \alpha$$

hence

$$\frac{w^* n_0}{p y} = N \frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha$$

So it is possible to rewrite (46) as follows

$$\sum_{j=1}^{N-1} \beta^j \left( \frac{w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j} \frac{n_{j,t+j}}{y} \right) = N \frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \quad (47)$$

Considering the denominator of the right hand side of (45), both con-

sumption and output are growing at the same rate so their ratio is a constant in steady state and one can extract it from the sum. Furthermore, given the aggregate budget constraint

$$\frac{c}{y} = 1 - \frac{i}{y} \quad (48)$$

Consider that

$$\frac{i}{y} = \frac{i}{k} \frac{k}{y} \quad (49)$$

The capital-output ratio can be recovered from the first order conditions for cost minimization ( $r = \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{y}{k}$ ), whereas the investment-capital ratio from the law of motion of capital:

$$\frac{i}{k} = \gamma - 1 + \delta$$

So by substituting the capital-output ratio and the investment-capital ratio into (49) and taking into consideration (40) and (41), it is possible to get:

$$\frac{i}{y} = \frac{i}{k} \frac{k}{y} = [(1 - \delta) \beta + \beta r + \delta - 1] \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{1}{r}$$

Simplifying and substituting for  $r$  by taking into consideration (40) and

(42), it is possible to perform two further steps

$$\begin{aligned}
\frac{i}{y} &= (1 - \delta) (\beta - 1) \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{1}{r} + \beta \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \\
&= (1 - \delta) (\beta - 1) (\hat{n})^{-\alpha} + \beta \frac{\theta_p - 1}{\theta_p} (1 - \alpha)
\end{aligned} \tag{50}$$

Therefore the investment-output ratio depends on labor in efficiency units.

It is time to take stock. By considering (44), (45), (47),(48) and (50), it is possible to rewrite (24) as:

$$n_0 = \frac{1 - \beta \mu^{\theta_w}}{1 - \beta^N \mu^{N\theta_w}} \frac{\left( N \frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right)}{\left[ 1 - (1 - \delta) (\beta - 1) (\hat{n})^{-\alpha} - \beta \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right]} \frac{\theta_w - 1}{\theta_w \varsigma} \tag{51}$$

It is worth noting that the labour supply of cohort zero depends on the composite labour input via the investment share of output.

In steady state, the demand for the labour input of cohort zero is equal to:

$$n_0 = \left( \frac{w^*}{w} \right)^{-\theta_w} \hat{n} \tag{52}$$

where, considering (12),

$$\frac{w^*}{w} = \left( \frac{1}{N} \frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{1}{\theta_w - 1}}$$

By substituting (51) into (52), it is possible to obtain:

$$\hat{n} = \frac{\left( N \frac{1-\mu^{(\theta_w-1)}}{1-\mu^{N(\theta_w-1)}} \frac{\theta_p-1}{\theta_p} \alpha \frac{1-\beta^N \mu^{N(\theta_w-1)}}{1-\beta \mu^{(\theta_w-1)}} \right)}{\left[ 1 - (1-\delta)(\beta-1)(\hat{n})^{-\alpha} - \beta \frac{\theta_p-1}{\theta_p} (1-\alpha) \right]} \frac{1-\beta \mu^{\theta_w}}{1-(\beta \mu^{\theta_w})^N} \frac{\theta_w-1}{\theta_w \varsigma} \left( \frac{1}{N} \frac{1-\mu^{N(\theta_w-1)}}{1-\mu^{\theta_w-1}} \right)^{\frac{\theta_w}{\theta_w-1}} \quad (53)$$

Simplifying

$$\hat{n} \left[ 1 - \beta \frac{\theta_p-1}{\theta_p} (1-\alpha) \right] - (1-\delta)(\beta-1)\hat{n}^{1-\alpha} - \quad (54)$$

$$\left( N \frac{1-\mu^{(\theta_w-1)}}{1-\mu^{N(\theta_w-1)}} \frac{\theta_p-1}{\theta_p} \alpha \frac{1-\beta^N \mu^{N(\theta_w-1)}}{1-\beta \mu^{(\theta_w-1)}} \right) \frac{1-\beta \mu^{\theta_w}}{1-(\beta \mu^{\theta_w})^N} \frac{\theta_w-1}{\theta_w \varsigma} \left( \frac{1}{N} \frac{1-\mu^{N(\theta_w-1)}}{1-\mu^{\theta_w-1}} \right)^{\frac{\theta_w}{\theta_w-1}} = 0 \quad (55)$$

Suppose, as customary, that  $\alpha = \frac{2}{3}$  :

$$\hat{n} \left[ 1 - \beta \frac{\theta_p-1}{\theta_p} (1-\alpha) \right] - (1-\delta)(\beta-1)\hat{n}^{\frac{1}{3}} - \quad (56)$$

$$\left( N \frac{1-\mu^{(\theta_w-1)}}{1-\mu^{N(\theta_w-1)}} \frac{\theta_p-1}{\theta_p} \alpha \frac{1-\beta^N \mu^{N(\theta_w-1)}}{1-\beta \mu^{(\theta_w-1)}} \right) \frac{1-\beta \mu^{\theta_w}}{1-(\beta \mu^{\theta_w})^N} \frac{\theta_w-1}{\theta_w \varsigma} \left( \frac{1}{N} \frac{1-\mu^{N(\theta_w-1)}}{1-\mu^{\theta_w-1}} \right)^{\frac{\theta_w}{\theta_w-1}} = 0 \quad (57)$$

Define:

$$\begin{aligned}
A & : = \left[ 1 - \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right] \\
C & : = (1 - \delta) (\beta - 1) \\
B & : = \left( N \frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right) \frac{1 - \beta \mu^{\theta_w}}{1 - (\beta \mu^{\theta_w})^N} \frac{\theta_w - 1}{\theta_w \varsigma} \left( \frac{1}{N} \frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{\theta_w}{\theta_w - 1}}
\end{aligned}$$

As a consequence the solution for labour in efficiency units is given by the roots of the following polynomial <sup>15</sup>

$$A^3 \hat{n}^3 - 3A^2 B \hat{n}^2 + (3AB^2 - C^3) \hat{n} - B^3 = 0$$

With  $\hat{n}$  in mind, one can easily compute  $\gamma$  from (43).

## B Optimal Monetary Policy

As stated above welfare is given by:

$$W = \sum_{j=0}^{\infty} \beta^j (\log C_{t+j} - \varsigma n_{jt+j})$$

Keeping in mind (31) and that  $C_{t+j} = c_t \gamma^j$ , it is possible to write for the

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<sup>15</sup>To see this, consider that

$$\begin{aligned}
\hat{n}A - B & = C \hat{n}^{\frac{1}{3}} \\
(\hat{n}A - B)^3 & = C^3 \hat{n}
\end{aligned}$$

After expanding the polynomial to the right of the equation and subtracting from both terms  $C^3 \hat{n}$ , it is possible to obtain (27).

steady state:

$$W = \sum_{j=0}^{\infty} \beta^j [\log \gamma^j - \varsigma n_0 \mu^{\theta_w(j)} + \log c.] \quad (58)$$

Given the recursive structure of labour contracts, following Graham and Snower (2004, p. 23), (58) can be written, considering the steady state, as

$$W = \sum_{j=0}^{\infty} \sum_{l=0}^{N-1} \beta^l \log \gamma^l - \sum_{j=0}^{\infty} \sum_{l=0}^{N-1} \beta^l \varsigma n_0 \mu^{\theta_w(l)} + \sum_{j=0}^{\infty} \sum_{l=0}^{N-1} \beta^l \log c.$$

Making explicit the relation of the initial level of consumption with the initial level of the capital stock one has

$$c = y - i$$

$$c = y - (\gamma - 1 + \delta) k$$

$$c = (\hat{n}^\alpha - \gamma + 1 - \delta) k$$

Being the value of  $k$  given by the initial condition, one obtains without loss of generality:

$$W' = \frac{1}{1 - \beta^N} \log \gamma \sum_{l=0}^{N-1} \beta^l l - \varsigma n_0 \frac{1}{1 - \beta^N} \frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}} + \frac{1}{1 - \beta^N} \log (\hat{n}^\alpha - \gamma + 1 - \delta) + const.$$

Consider that

$$(1 - \beta) \sum_{l=0}^{N-1} \beta^l l = \beta + \beta^2 + \dots + \beta^{N-1} + \beta^N - \beta^N N$$

consequently

$$\sum_{j=0}^{N-1} \beta^j l = \beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta}$$

and therefore

$$W' = \frac{1}{1 - \beta^N} \left[ \beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta} \right] \log \gamma - \varsigma n_0 \frac{1}{1 - \beta^N} \frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}} + \frac{1}{1 - \beta^N} \log (\hat{n}^\alpha - \gamma + 1 - \delta)$$

Figure 1 – The effect of  $\theta_p$  on real growth

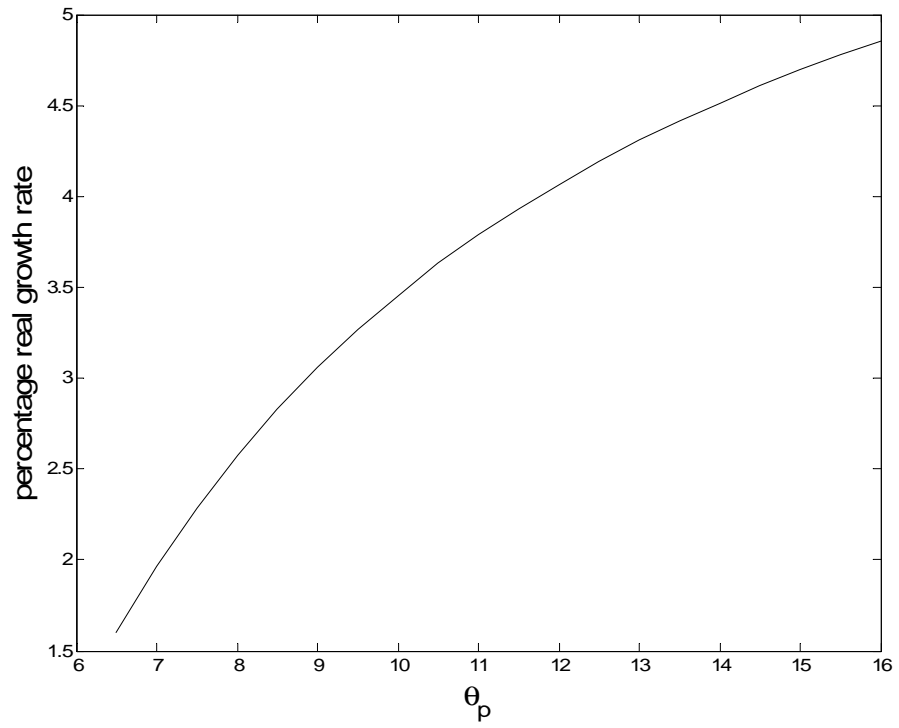


Figure 2 – The effect of  $\theta_w$  on real growth

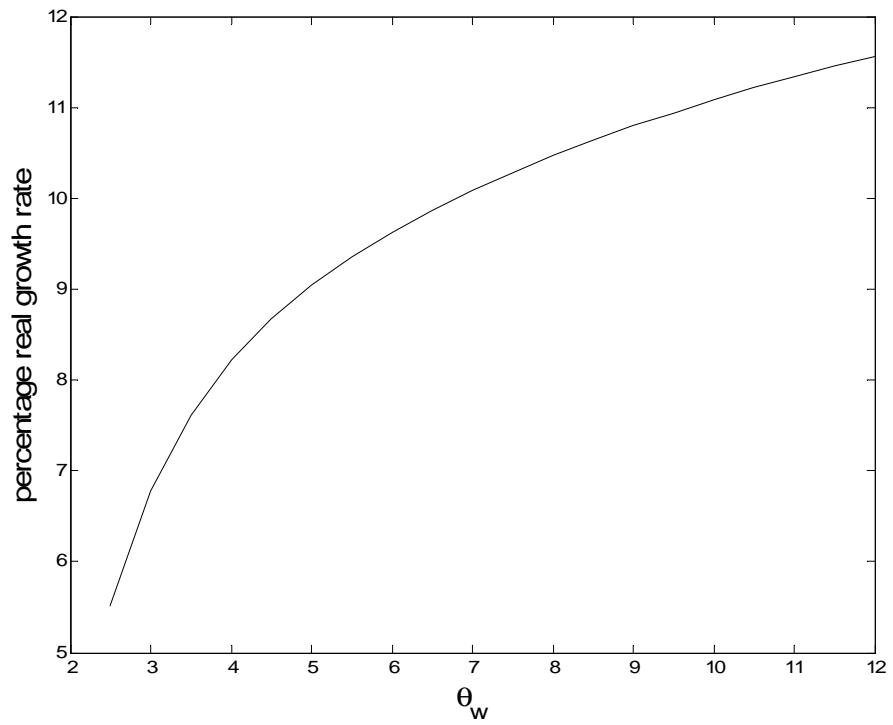


Figure 3 – The effect of money growth on real growth

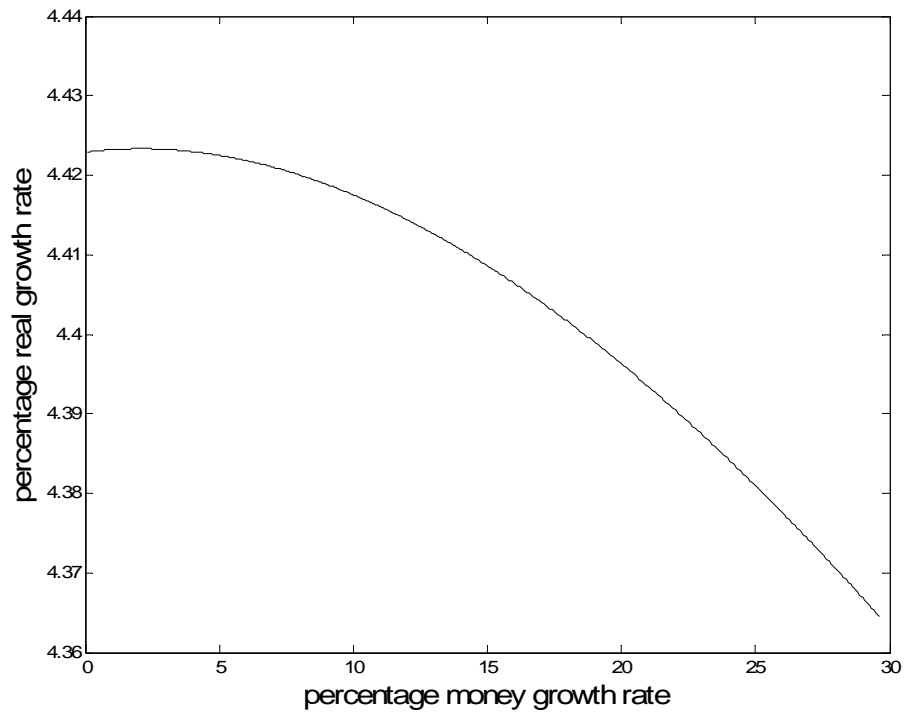


Figure 4 – The effect of money growth on real growth for different values of  $\theta_p$

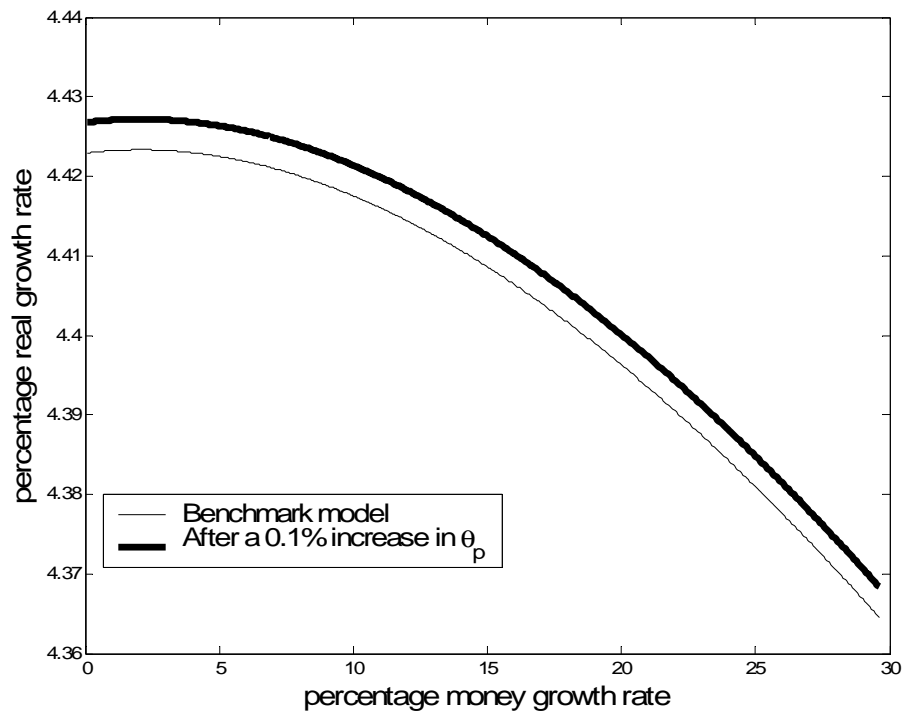


Figure 5 – The effect of money growth on real growth for different values of  $\theta_w$

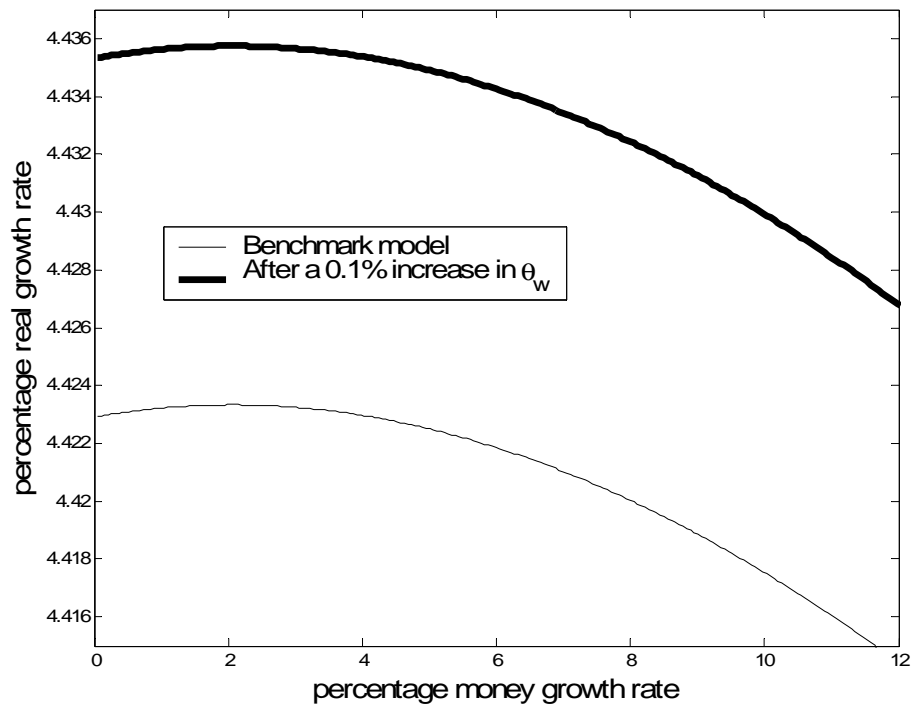


Figure 6 – The effect of money growth on welfare

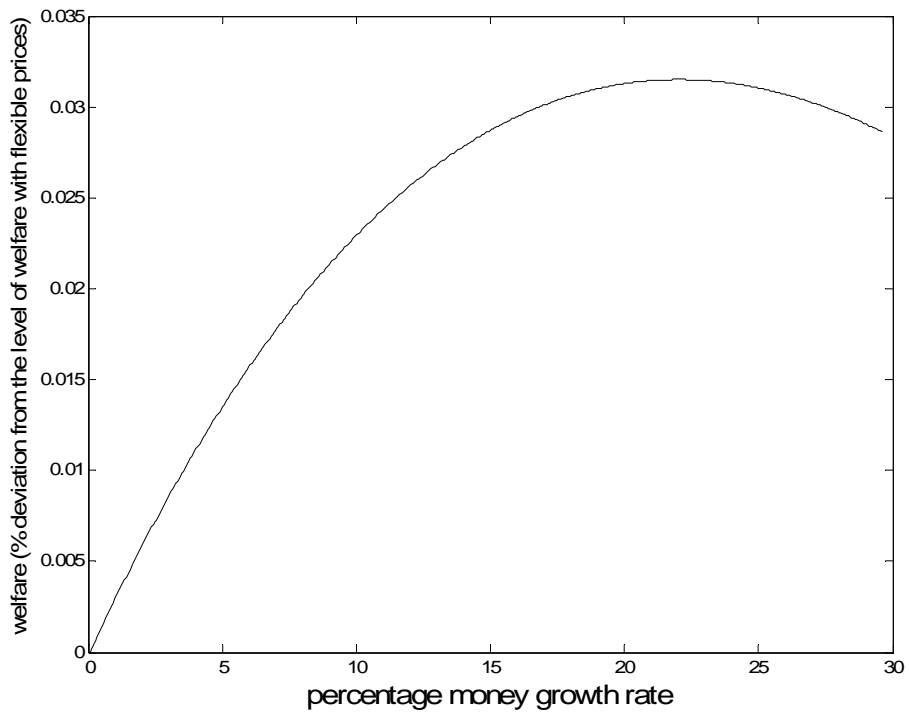
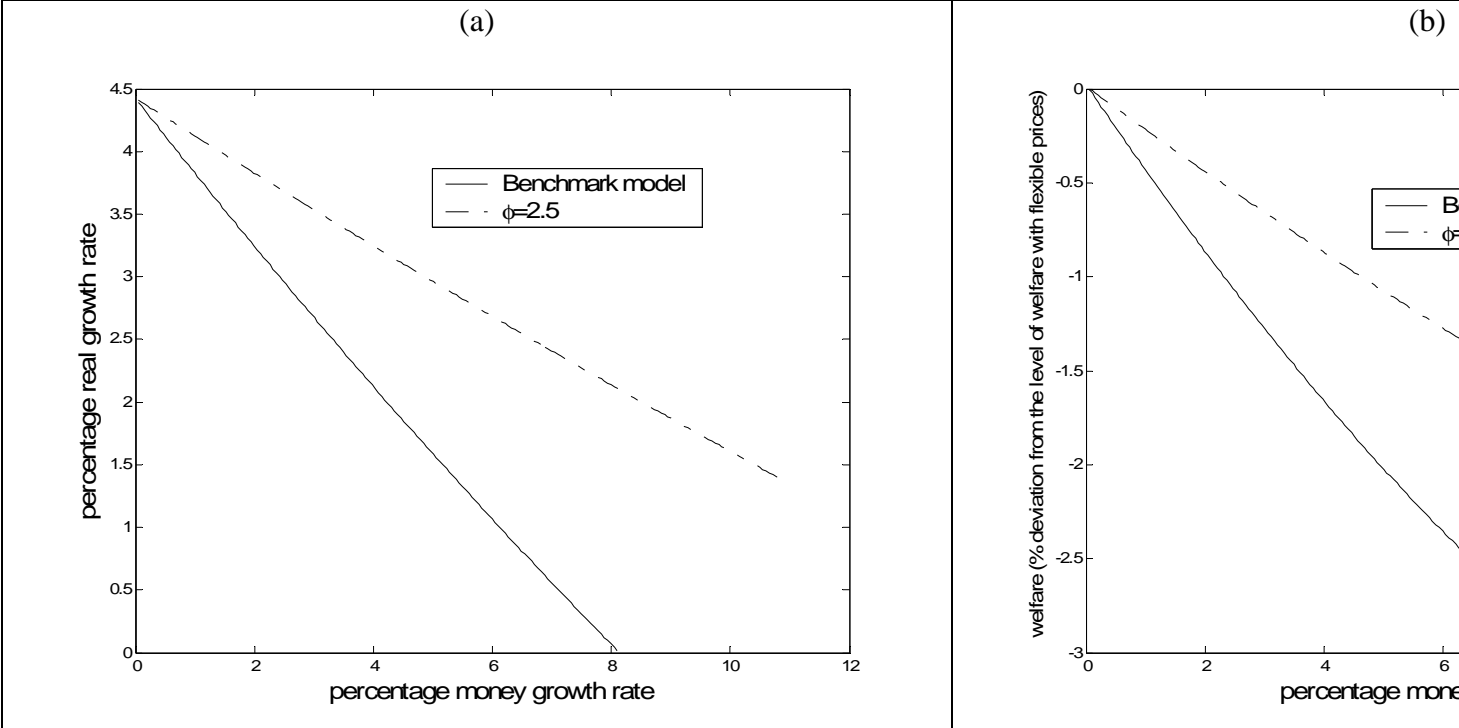
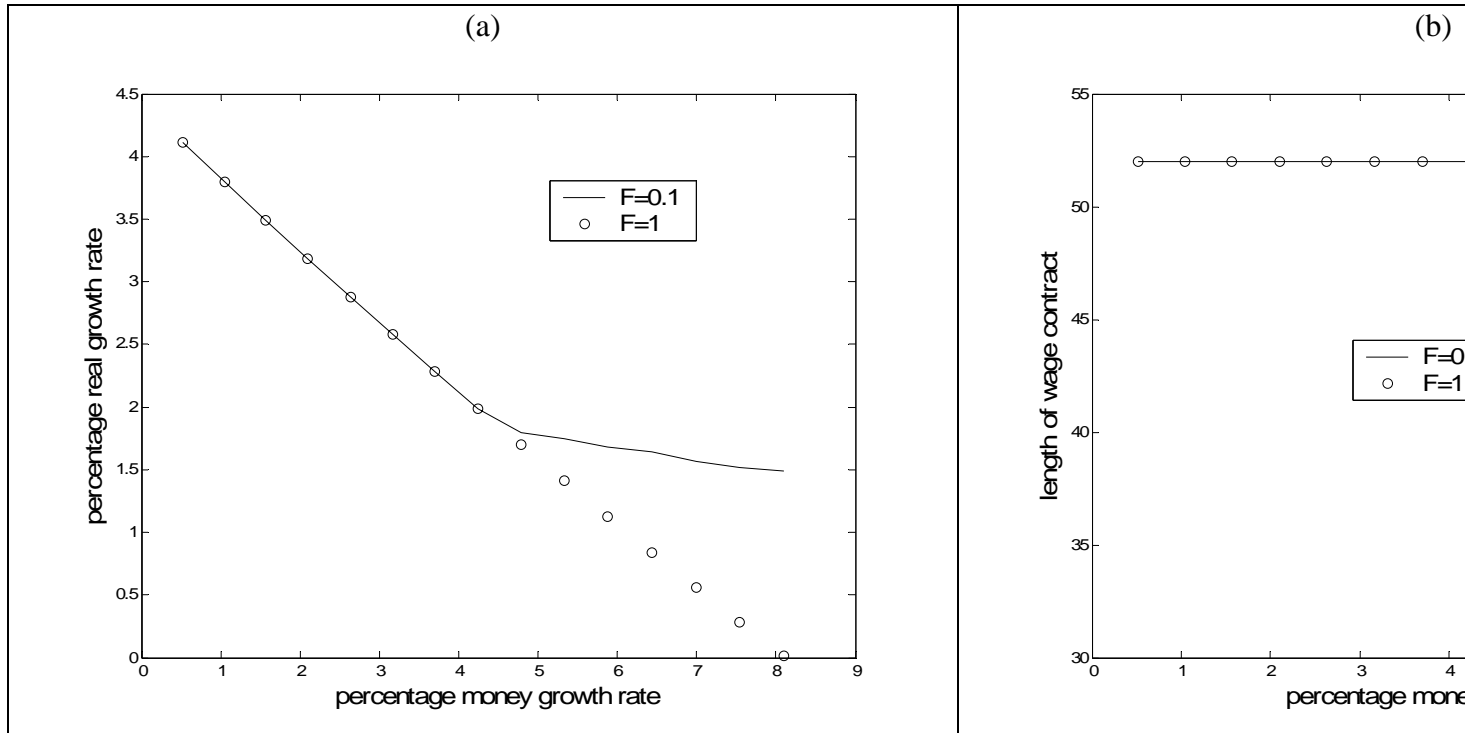


Figure 7 – The effect of money growth on real growth and welfare for generalized preferences



The intertemporal elasticity of substitution for working time is set equal to 5.

Figure 8 – The effect of money growth on real growth and on wage contract length with endogenous labour contract



The intertemporal elasticity of substitution for working time is set equal to 5.

Table 1 – Model selection criteria

Criteria	Whole sample: specification I				Whole sample: specification II			
	Polynomial order				Polynomial order			
	1	2	3	4	1	2	3	4
Mallow's $C_L$	4.0004	3.9954	3.9902	3.9936	2.7151	2.7023	2.7130	2.7265
Generalized cross-validation	4.5821	4.5882	4.5942	4.6103	3.2551	3.2589	3.2914	3.3280
Leave-one-out cross-validation	1.2893	1.2911	1.2889	1.2901	428.8411	429.0025	431.2143	434.8971
Akaike Information Criterion	1987.257	1987.294	1987.308	1988.337	929.1067	928.6779	930.2978	932.1711
Schwartz Information Criterion	2413.22	2417.393	2421.543	2426.707	1140.4	1143.435	1148.519	1153.856

Table 2 - Coefficient estimates and t-statistics of the control variables

	Specification I		Specification II	
	OLS	2SLS	OLS	2SLS
Inflation	-0.09*	-0.09	-0.16*	-0.23*
t-statistics	-4.76	-1.23	-4.51	-2.15
GDP per capita	-4.74*	-4.76*	-6.46*	-6.57*
t-statistics	-8.25	-6.50	-7.01	-6.95
Population Growth	1.50*	1.50*	0.64	0.70*
t-statistics	9.12	8.93	1.92	2.02
Investment/GDP	0.20*	0.20*	0.30*	0.31*
t-statistics	7.73	7.17	8.15	8.06
Government consumption/GDP	-0.03	-0.03	-0.02	0.01
t-statistics	-1.30	-1.00	-0.70	0.02
Education	0.19	0.19	0.11	0.08
t-statistics	0.96	0.85	0.46	0.33
Terms of trade growth	-	-	0.11*	0.11*
t-statistics	-	-	3.07	3.11
Terms of trade standard deviation	-	-	0.89*	0.86*
t-statistics	-	-	2.22	2.14
First-stage F-statistic (p-value)	-	0.00	-	0.00
Hausman test (p-value)	-	1.00	-	1.00
Adjusted R <sup>2</sup>	0.56	0.56	0.60	0.60
Observations	462	462	236	236

\*: significant at the 5% level. The models also include a set of country and time-period dummies.

Table 3 – Robustness checks for model selection criteria

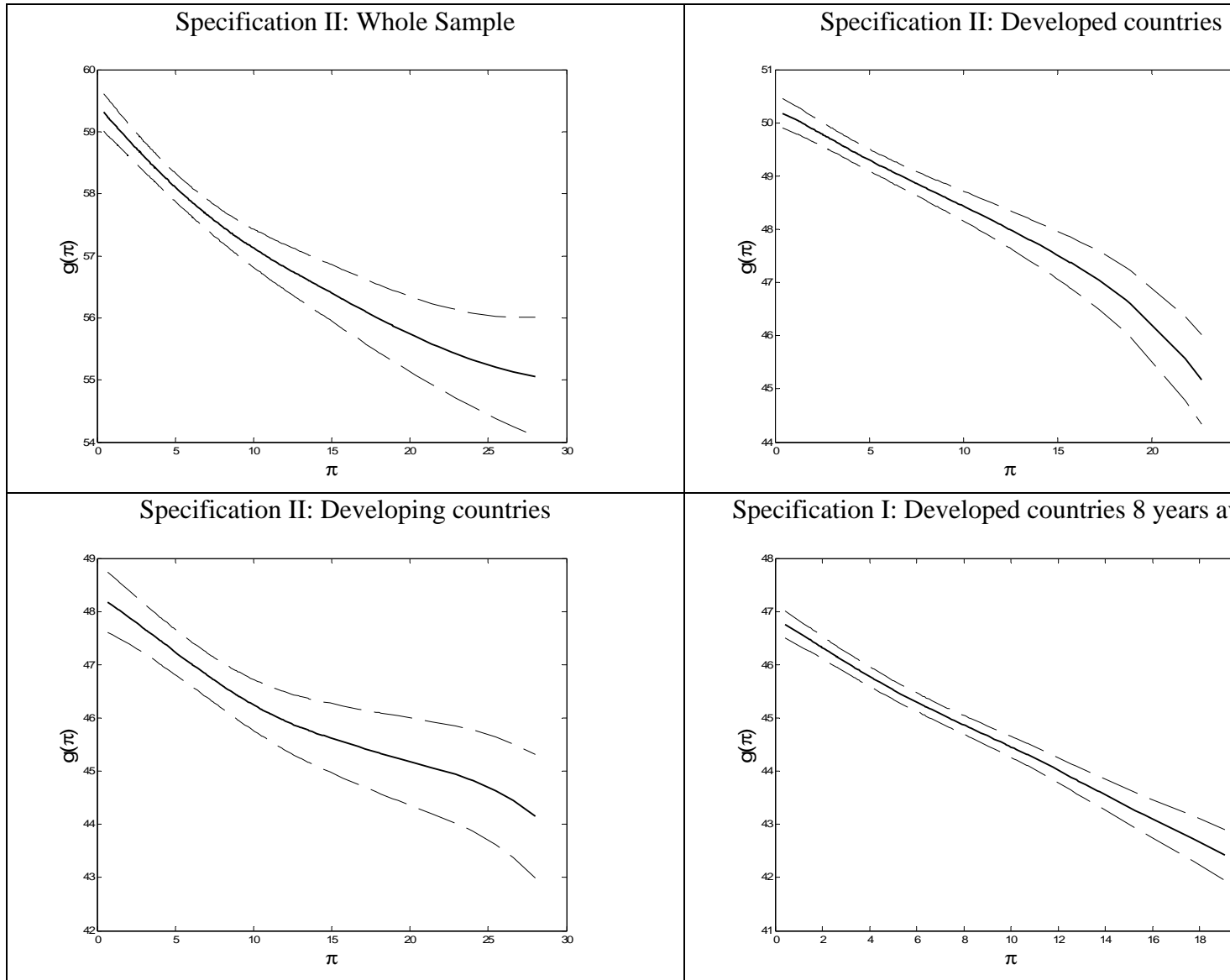
	<b>Developed countries: specification I</b>				<b>Developed countries: specification II</b>			
	<b>Polynomial order</b>				<b>Polynomial order</b>			
<b>Criteria</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>Mallow's <math>C_L</math></b>	1.4604	1.4246	1.3982	1.3856	1.3996	1.4060	1.4038	1.3957
<b>Generalized cross-validation</b>	1.7016	1.6739	1.6571	1.6568	1.7071	1.7339	1.7509	1.7611
<b>Leave-one-out cross-validation</b>	156.7271	153.7707	150.4885	149.8273	124.3379	126.8406	126.6606	127.6177
<b>Akaike Information Criterion</b>	501.9681	498.8436	496.6573	495.9556	436.5977	437.9164	438.428	438.3876
<b>Schwartz Information Criterion</b>	610.8278	610.7271	611.5647	613.8869	540.6503	544.8593	548.2613	551.1112

	<b>Developing countries: specification I</b>				<b>Developing countries: specification II</b>			
	<b>Polynomial order</b>				<b>Polynomial order</b>			
<b>Criteria</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>Mallow's <math>C_L</math></b>	5.1022	5.1224	5.1116	5.1242	4.2152	4.2274	4.2635	4.2776
<b>Generalized cross-validation</b>	6.0865	6.1377	6.1522	6.1953	6.0263	6.1554	6.3259	6.4710
<b>Leave-one-out cross-validation</b>	1060.6	1067.3	1063.4	1067.0	277.4053	281.1286	284.6060	287.3215
<b>Akaike Information Criterion</b>	1415.244	1417.148	1417.180	1418.631	490.3006	491.4675	493.2615	932.1711
<b>Schwartz Information Criterion</b>	1710.434	1716.074	1719.842	1725.030	598.3198	602.1872	606.6817	610.6177

Table 4 – Model selection criteria using 8 years averages

	<b>Developed countries: specification I</b>			
	<b>Polynomial order</b>			
<b>Criteria</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>Mallow's <math>C_L</math></b>	0.6674	0.6745	0.6748	0.6813
<b>Generalized cross-validation</b>	0.9851	1.0202	1.0466	1.0849
<b>Leave-one-out cross-validation</b>	34.6133	35.7789	36.2510	37.1464
<b>Akaike Information Criterion</b>	228.6061	230.3979	231.3121	1418.631
<b>Schwartz Information Criterion</b>	309.9811	314.2387	317.6189	321.827

Figure 9 – Semiparametric estimation of the effect of inflation on economic growth



Note:  $\pi$  is the inflation rate,  $g(\pi)$  is the partial effect of inflation on real economic growth. The other regressors in the model are shown in Table 5. Dotted lines mark 95% confidence intervals.

Table 5 – Coefficient estimates and t-statistics of the control variables of the semi-parametric estimation of the inflation-growth nexus

	Specification II			Specification I
	All	Developed	Developing	Developed / 8 years averages
GDP per cap	-6.44*	-5.54*	-6.02*	-4.96*
<i>t</i> -statistics	-7.07	-4.27	-4.34	-3.92
Population Growth	0.65	1.09*	0.71	0.94
<i>t</i> -statistics	1.97	2.44	1.30	1.93
Investment/GDP	0.31*	0.19*	0.34*	0.17*
<i>t</i> -statistics	8.33	2.90	5.20	2.58
Government consumption/GDP	-0.01	-0.10*	0.05	-0.08*
<i>t</i> -statistics	-0.35	-2.61	0.69	-2.54
Education	0.10	0.45*	-0.32	0.36
<i>t</i> -statistics	0.45	2.07	-0.64	1.85
Terms of trade growth	0.10*	0.12*	0.10	-
<i>t</i> -statistics	3.04	2.81	1.73	-
Terms of trade standard deviation	0.86*	-5.22	0.83	-
<i>t</i> -statistics	2.15	-1.41	1.37	-

\*: significant at the 5% level. The models also include a set of country and time-period dummies.